PHYS 3650: Observational Astronomy

Properties of Light and Optical Observations from the Earth

Learning Objectives

- \triangleleft Telescope (angular) resolution:
	- diffraction limit
	- diffraction and Airy pattern
	- Rayleigh's criterion
	- diffraction patterns produced by telescopes
- Effects of Earth's atmosphere on telescope observations:
	- refraction by Earth's atmosphere
	- atmospheric seeing and scintillation
	- speckle imaging
	- adaptive optics
	- dome seeing
	- dispersion
	- absorption and scattering (extinction)
	- air mass
	- dependence on altitude

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 \blacklozenge Why is there a limit to the (angular) resolution of the human eye?

Lower Resolution (Human Eye)

Higher Resolution (Telescope)

(Telescope)

 Ω

 \blacklozenge Why is there a limit to the (angular) resolution of telescopes?

- Quantum mechanics teaches us that everything in the Universe have both particlelike and wave-like properties, although wave-like properties are appreciable only for things having very low mass.
- \blacklozenge Light (electromagnetic radiation, which has no mass) exhibits:
	- particle-like properties as it interacts with matter
	- wave-like properties as it propagates through space

Diffraction through Aperture

- \blacklozenge When light passes through an aperture, light is diffracted (i.e., waves spread out after passing through opening).
- \bullet It does not matter if the aperture comprises the entrance to the telescope tube, a lens, or a mirror.

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- \bullet It does not matter if the aperture comprises the entrance to the telescope tube, a lens, or a mirror.

- \blacklozenge Notice that the pattern that forms on the screen is a series of bright spots parallel to the slit width.
- \bullet This pattern is produced by the interference of light waves diffracted by the slit, and is called a diffraction pattern.

- \blacklozenge To compute the pattern produced by the interference of light waves, we use:
	- Huygen's principle that every point on a wavefront can be thought of as a source of secondary waves traveling in the forward direction; and
	- Fresnel's principle of interference

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Constructive Interference Destructive Interference

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- So, imagine that every point at the slit opening generates a secondary wave, and consider how these waves interfere with each other at some distance behind the slit.

 \bullet For simplicity, consider the interference pattern produced at a screen placed at a distance, *D*, that is far larger than the slit width, *a*.

 \blacklozenge Interference pattern produced by plane-parallel waves passing through slit.

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 \blacklozenge Intensity of diffraction pattern decreases away from the central maximum, a condition we will not derive in this course.

 \blacklozenge The diffraction pattern through a circular aperture with a diameter *D*, such as a telescope, was first derived by the English mathematician and astronomer George Airy in the 19th century. The derivation is more complex than for a 1-dimensional opening, and only the solution is presented in this course. By analogy, the diffraction pattern comprises a series of bright (maxima) and dark (minima) rings with maxima/minima at

$$
\sin \theta = m \frac{\lambda}{D} \frac{\sin \theta}{2}
$$

diameter of cular aperture

but for non-integer values of *m* as given in the table below.

- \blacklozenge The diffraction pattern produced by a circular aperture is known as the Airy pattern, which comprises a bright central disk known as the Airy disk surrounded by much dimmer diffraction rings.
- \blacklozenge Note that the intensity drops off rapidly beyond the Airy disk, which encompasses 83.8% of the energy in an Airy pattern. The first ring is only 1.75% as bright, and the second ring only 0.42% as bright, as the center of the Airy disk.

- \triangle The image of a star seen through a circular aperture human eye or telescope is therefore an Airy pattern rather than a single bright point.
- \blacklozenge Why do we not perceive an Airy pattern when looking at stars in the sky with the naked eye?

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- \blacklozenge Why do we not perceive an Airy pattern when looking at stars in the sky with the naked eye? Not sufficiently sensitive to see diffraction rings

• Consider two closely-separated stars. In which of the three examples below would you consider the two stars to be separated?

• Consider two closely-separated stars. In which of the three examples below would you consider the two stars to be separated? This was a question debated in the 19th century, and the answer is entirely subjective.

Rayleigh's Criterion

 \triangle Lord Rayleigh introduced an empirical criterion – now known as the Rayleigh criterion – for separating two point sources: the central maximum of one diffraction pattern should lie at or beyond the first minimum of the other. That is, two point sources are considered separated if their minimum angular separation in the sky satisfies

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\theta_{\min} = 1.22 \frac{\lambda}{D}
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(measured in radians) where λ is the received wavelength of light and *D* is the entrance diameter of the telescope.

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Angular Resolution

Consider two closely-separated stars. In which of the three examples below is the binary system resolved according to the Rayleigh criterion?

• Consider two closely-separated stars. In which of the three examples below is the binary system resolved according to the Rayleigh criterion?

 \triangle The angular resolution of a telescope as defined according to Rayleigh's criterion is therefore

$$
\theta_{\min} = 1.22 \frac{\lambda}{D}
$$

Notice that the Rayleigh criterion corresponds closely to the diameter of the Airy disk as measured at full-width half-maximum (FWHM), which provides a more convenient measure of the angular resolution.

Angular Resolution

- Angular resolution at optical wavelengths (0.5 um) of human eye at night is about 20". For comparison, α Centauri A and B have an angular separation ranging from 2" to 22" over an orbital period of ~ 80 years.
- Angular resolution at optical wavelengths (0.5 um) of telescopes with the following apertures:
	- 25 cm, $\theta_{\min} = 0.50''$
	- -50 cm, $\theta_{\min} = 0.25$ "
	- 1 m, $\theta_{\min} = 0.125''$
	- 2.4 m, $\theta_{\min} = 0.05$ " (Hubble Space Telescope)
	- -4 m, θ_{\min} = 0.03" (Canadian-France-Hawaii Telescope)
	- 8.2 m, $\theta_{\text{min}} = 0.015$ " (Subaru Telescope)

 \blacklozenge If the light path in a telescope is partially blocked (e.g. by a secondary mirror, support for the secondary mirror, other components), the diffraction pattern is more complicated than just an Airy pattern.

 \blacklozenge Diffraction pattern produced by telescope with secondary mirror of different sizes.

 \blacklozenge Diffraction pattern produced by telescopes with different secondary mirror supports, along with the addition of mirror clamps, heating element for secondary mirror, and eyepiece tube.

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mirror.
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Refraction by Earth's Atmosphere

- Light from astronomical objects are refracted by the Earth's atmosphere before reaching telescopes on the ground.
- \triangle Refraction by the Earth's atmosphere produces a shift in the observed (apparent) positions from the true positions of astronomical objects.

Refraction by Earth's Atmosphere

- \triangle Refractive index of air depends on:
	- composition
		- (e.g., water vapor content)
	- density
	- temperature
- \blacklozenge All these parameters change with height in the Earth's atmosphere.

- Because of turbulence, the density, temperature, and water vapor content and hence refractive index at a given height in the Earth's atmosphere can change rapidly with time.
- \triangle This causes the apparent position of an astronomical object to change rapidly with time, an effect known as seeing.

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- As imaged by the human eye or telescope, the position of the image on the focal plane changes with time.
- Thus, rather than seeing an Airy pattern for a star, we see a "blob." More frequently, the star is deflected by small angles from the center of the blob. Less frequently, the star is deflected by large angles from the center of the blob. At any given time, the total deflection depends on the sum of individual deflections at many different heights.
- Because individual deflections at different heights vary independently with time, the total deflection caused by atmospheric seeing has a normal or Gaussian distribution.

- A cross-section through a seeing-dominated stellar image therefore resembles a Gaussian function.
- The 2-dimensional Gaussian image of a star produced by seeing is commonly referred to as the seeing disk (by analogy with the Airy disk).
- \blacklozenge The size of the seeing disk is measured at the full-width half-maximum (FWHM), and is typically \gtrsim 1" at most locations on the Earth.

- \blacklozenge The intensity profile of the Airy disk is close to that of a Gaussian function.
- Recall that the diffracted-limited angular resolution is conveniently defined at the FWHM of the Airy disk. Similarly, the size of the seeing disk is defined at its FWHM. Thus, whether diffraction or seeing limited, the size of a stellar image as measured at FWHM defines the angular resolution of that image.

Speckle Imaging

- Speckle imaging employs:
	- extremely short exposures (typically over 100 frames a second)
	- shifting the image in each exposure so that the image is aligned across all exposures
	- adding all the exposures together to create the final image
- \cdot Speckle images (stitched into a slow-motion movie) of a single star:

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- Speckle imaging possible only on bright and compact objects, and works better on smaller telescopes. Why?

Seeing-limited image With Speckle Imaging

Speckle Imaging

- Speckle imaging employs:
	- extremely short exposures (typically over 100 frames a second)
	- shifting the image in each exposure so that the image is aligned across all exposures
	- adding all the exposures together to create the final image
- exposure, need compact objects to align separate images. • Speckle imaging possible only on bright and compact objects, and works better on smaller telescopes. Why? Need bright objects so can be detected in a short

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 \blacklozenge This is generally not the case (depending on the wavelength of light and the sizescale of atmospheric turbulence) and therefore the incoming wavefront is distorted.

Plane waves from distant point source

• Why does speckle imaging therefore work better on smaller telescopes?

 \blacklozenge Why does speckle imaging therefore work better on smaller telescopes? The wavefront only appears to be planar, or close to planar, if we restrict ourselves to a sufficiently small portion of the wavefront; i.e., small telescope apertures, up to diameters of \sim 20 cm at the best sites, and \sim 5 cm at sea level.

Scintillation

 \blacklozenge For a point source (star), turbulence in the Earth's atmosphere results in apparent image motion (seeing) as well as scintillation. At a given time, if more light rays are directed into an aperture, the object will appear brighter. Conversely, if fewer light rays are directed into an aperture, the object will appear dimmer. This effect is known as scintillation.Plane waves from distant point source

• For an extended source, turbulence in the Earth's atmosphere results in different amounts of image motion (seeing) for different positions in the sky and therefore image distortion.

- One of the best sites in the world for astronomical observations is the summit of Mauna (elevation of 4200 m) in Hawaii. The atmospheric seeing at this site is typically 0.6″.
- \triangle Figure below shows number of nights over a year where a star image has a certain size as measured at FWHM at Mauna Kea.

 \blacklozenge Winds from east travels over a vast area of the pacific ocean before arriving at Hawaii. Air flow over Mauna Kea is closely laminar (i.e., little turbulence).

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Atmospheric Seeing Limited

- One of the best sites in the world for astronomical observations is the summit of Mauna (elevation of 4200 m) in Hawaii. The atmospheric seeing at this site is typically 0.6″.
- Angular resolution at optical wavelengths (0.5 um) of telescopes with the following apertures:
	- 25 cm, $\theta_{\min} = 0.50''$
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	- -1 m, $\theta_{\min} = 0.125''$
	- 2.4 m, $\theta_{\min} = 0.05$ " (Hubble Space Telescope)
	- -4 m, θ_{\min} = 0.03" (Canadian-France-Hawaii Telescope)
	- 8.2 m, $\theta_{\text{min}} = 0.015$ " (Subaru Telescope)
- \bullet So, even a 25-cm telescope has an angular resolution that is limited by atmospheric seeing rather than diffraction! Without correcting for atmospheric seeing, the angular resolution of all the telescopes on Mauna Kea is no better than that of a approximately 25-cm telescope.

Atmospheric Seeing Limited

Angular resolution of HST is 0.05". Angular resolution of Subaru is 0.015", but without correction for atmospheric seeing is typically 0.6".

Adaptive optics employs a deformable mirror to correct for the distortion of the incoming wavefront. At the present time, adaptive optics works sufficiently well for ground-based telescopes to attain (close to) their diffraction limit at near-IR wavelengths (2 um) but not at optical wavelengths.

The deformable mirror can be the secondary mirror (not common), or another mirror in the light path following the secondary mirror (as is usually the case on large telescopes).

 \blacklozenge A deformable secondary mirror.

Deformable Secondary Mirror on the Multi-Mirror Telescope (6.5 m)

 \blacklozenge Small deformable mirrors of the type placed in the light path following the secondary mirror.

Examples of Deformable Mirrors

- Adaptive optics require a bright compact object preferably a point source to derive corrections from the incoming distorted wavefront to apply to the deformable mirror. Usually bright stars next to the target object (natural guide stars) or artificially-created laser guide stars are used.
- \blacktriangleright A wavefront sensor is used to measure distortions in the incoming wavefront, and the correction factors derived applied to the deformable mirror in a feedback loop.

Basic concept of a Shack-Hartmann wavefront sensor. An image of the exit pupil is projected onto a lenslet array - a collection of small identical lenses. Each lens takes a small part of the aperture, called sub-pupil, and forms an image of the source. All images are formed on the same detector, typically a CCD.

An image of the trapezium region in the Orion Nebula with (left; resolution 0.06) and without (right; resolution 0.6″) adaptive optics in the near-IR on the Subaru telescope.

^u Natural guide stars need not always be stars. Sometime, a planetary moon can be used as AO natural guide star for the planet.

- \blacklozenge What if target object does not have a bright star conveniently located next to it? In such cases, artificially-created laser guide stars have to be used.
- \triangle Laser guide stars are created by shining a laser next to the target object, exciting atoms (typically sodium) in the upper layer of the Earth's atmosphere. (In the case of sodium, at an altitude of around 90 km.)

Absorption by Earth's Atmosphere

A schematic of the lower portion of the Earth's atmosphere.

Absorption by Earth's Atmosphere

 \blacklozenge The ionosphere comprises a part of the mesosphere, the entire thermosphere, and a part of the exosphere, spanning altitudes of about 85-600 km. Notice that the ionosphere is highly ionized **EXOSPHERE** (hence its 600 km· name).**IONOSPHERE THERMOSPHERE** 300 km E 85 km **MESOSPHERE** 45 km **STRATOSPHERE** $12 km$ TROPOSPHERE

^u Animation of the laser guide star and adaptive optics system at the Gemini North telescope on Mauna Kea.

Dome Seeing

- \blacktriangleright Air turbulence above telescope also produced by telescope (i.e., convective motion of air because telescope is warmer than the surrounding air).
- \triangle Telescopes built with open structures, and dome vented before observing, to equalize temperature with surrounding air and hence reduce dome seeing.

8.1 m Gemini Telescope 8.3 m SubaruTelescope

- \bullet So far we have ignored that the refraction of light by the Earth's atmosphere depends on wavelength (higher refractive index at shorter wavelengths).
- \triangle The different amounts of refraction at different wavelengths is known as dispersion.
- Dispersion causes image to be increasingly elongated with decreasing elevation.

60°

 $1.04"$

 $\begin{array}{c} 45^\circ \\ 0.60" \end{array}$

 \mathbf{n}°

 $0.00"$

 30°

 $0.35"$

 \circ \circ \circ

 \blacklozenge Green flash from the Sun produced by atmospheric dispersion.

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- Extinction is the dimming of light by absorption and scattering.
- At optical λ , there is relatively little absorption of light in the Earth's atmosphere.
- At optical wavelengths, the primary cause of extinction are:
	- scattering by gas molecules (Rayleigh scattering)
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- At optical λ , there is relatively little absorption of light in the Earth's atmosphere.
- \blacklozenge At optical wavelengths, the primary cause of extinction are:
	- scattering by gas molecules (Rayleigh scattering)
	- scattering by dust and aerosol particles
- \blacklozenge Rayleigh scattering occurs when the particle size (molecules have sizes ~10⁻¹⁰ m or 1 Angstrom) \ll wavelength (10⁻⁷-10⁻⁶ m for optical wavelengths). Scattered intensity propto $1/\lambda^4$, explaining why the sky is blue.
- In the opposite regime (particle size larger than wavelength; e.g., dust, smoke, mist, fog), scattered intensity independent of wavelength.

• Daytime picture of Hong Kong. Why does the view across the harbor look "whitish?"

• Daytime picture of Hong Kong. Why does the view across the harbor look "whitish?" Scattering of sunlight by small particles in the air.

• Nighttime picture of Hong Kong. Even when the daytime view looks "whitish," the nighttime view looks clearer. Why?

 \blacktriangleright Nighttime picture of Hong Kong. Even when the daytime view looks "whitish," the nighttime view looks clearer. Why? No sunlight, so therefore no scattering of sunlight by small particles in the air.

Extinction

Zenithal extinction measured at Flagstaff, Arizona, USA. (Recall that 1 mag corresponds to a factor of 2.512.)

Extinction

 \triangle At lower elevations, the extinction increases as we are looking through more of the Earth's atmosphere.

Plane-Parallel Atmosphere

- Except at low elevations, the Earth's atmosphere can be modeled as a stack of plane-parallel layers whose properties only depend on their height above the ground.
- \blacklozenge The curvature of the Earth's atmosphere compared to its depth drawn to scale. The heavy line at the bottom represents the surface of the Earth. The light lines are the levels at which the pressure is (from the top down) $10\%, 20\%, \ldots, 90\%$ of the pressure at the surface. All these lines are arcs of circles, concentric with the Earth's center (which is far below the bottom of your screen). The top (10% pressure) level is about 16 km above the surface.

Ground

Air mass is defined according to the relationship:

observed magnitude

intrinsic magnitude zenith extinction (in magnitude)

 $m_{\lambda} = m_0 + \Delta m_0 M(\zeta)$

- What is the air mass at zenith? 1
- How would you expect the air mass to change with elevation? Sec *ζ* for sufficiently high elevations where the Earth's atmosphere can be modeled as plane parallel.

Air mass is defined according to the relationship: $m_{\lambda} = m_0 + \Delta m_0 M(\zeta)$ Air mass (function of ζ) observed magnitude zenith extinction intrinsic magnitude (in magnitude) $\blacklozenge M(\zeta) \approx 1/\cos \zeta = \sec \zeta$ for $\zeta \le 60^\circ$ (plane **Source** Zenith parallel). $\blacklozenge M(\zeta) \approx \sec \zeta - 0.0018167$ (sec ζ -1) dz sec ζ \mathcal{F}_{2} 0.002875(sec *ζ*-1)2-0.0008083 (sec *ζ*-1)3 for ζ < 85[°] (Hardie 1962). dz F_{λ} + d F_{λ} \blacktriangleright For ζ > 85°, no single one-size-fits-all relationship.**Absorption** coefficient z $k(z, \lambda)$ 'Zenith distance'

Ground

• Dependence of air mass on zenith angle ζ at $\zeta > 80^\circ$.

Air mass is defined according to the relationship: $m_{\lambda} = m_0 + \Delta m_0 M(\zeta)$ Air mass (function of ζ) observed magnitude zenith extinction intrinsic magnitude (in magnitude) ^u How can we measure the zenith extinction Δm_0 and hence intrinsic magnitude m_0 ? $m₀$ - observe the object at different elevations Δm_0 m Observed points Measurements must be done over a time = 0°: Zenith frame shorter than any changes in atmospheric conditions (sometimes 2 3 0 repeatedly over a single night).sec č

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- ^u How can we measure the zenith extinction Δm_0 and hence intrinsic magnitude m_0 ?
	- observe the object at different elevations
	- plot the observed magnitude vs. sec *ζ*

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	- observe the object at different elevations
	- plot the observed magnitude vs. sec *ζ*
	- extrapolate measurements to sec $\zeta = 0$ to derive the intrinsic magnitude m_0
		- difference between intrinsic and observed magnitude at zenith is the zenith extinction
- \bullet Measurements must be done over a time frame shorter than any changes in atmospheric conditions (sometimes repeatedly over a single night).

 $|m_0|$ Δm_0 m Observed points $= 0^\circ$: Zenith $\overline{2}$ 3 0 sec č