Entropy injection

2111.11444, 1109.2829

$$\begin{cases} \dot\rho_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi \\ \dot\rho_r + 4H\rho_r = +\Gamma_\phi \rho_\phi \end{cases} \Rightarrow \quad \rho_\phi(t) = \rho_\phi(t_i)\left(\frac{a_i}{a}\right)^3 e^{-\Gamma t} \quad (1)$$

from the continuity eq

$$\dot\rho_\phi = \rho_\phi(t_i)\left[-3\frac{a_i^3}{a_i^4}\dot a_i - \left(\frac{a_i}{a}\right)^3 \frac{1}{\tau}\right] e^{-t\Gamma_\phi}$$

$$\dot\rho_\phi + 3H\rho_\phi = -\rho_\phi \Gamma_\phi$$

I'm assuming instantaneous thermalization $(\Gamma \gg H)$

I also assume all radiation species are in eq: $g_* = g_{*s}$.

Otherwise, I would have to keep track of what species does X decay into.

I can rewrite

$$s = \frac{2\pi^2}{45} g_{*s} T^3 \Rightarrow T = \left(\frac{45\, s}{2\pi^2 g_{*s}}\right)^{1/3} \Rightarrow$$

$$\Rightarrow \rho_r = \frac{\pi^2}{30} g_* \left(\frac{45\, s}{2\pi^2 g_{*s}}\right)^{4/3} = \frac{\pi^2}{30} g_* \frac{45}{2\pi^2} \frac{1}{g_{*s}} \left(\frac{45\, s^4}{2\pi^2 g_{*s}}\right)^{1/3}$$

$$\Rightarrow \rho_r = \frac{3}{4}\left(\frac{45\, S^4}{2\pi^2 g_{*s}}\right)^{1/3} R^{-4} \quad (2)$$

The Hubble rate is $\quad H^2 = \dfrac{\rho_\phi + \rho_r}{3M_p^2} \quad (3)$

$$\text{Entropy: } dS = \frac{\overbrace{dQ}^{dU+PdV}}{T} = -\frac{d(a^3\rho_\phi)}{T} = \frac{a^3}{T}\Gamma_\phi \rho_\phi\, dt$$

$$\Rightarrow \dot S = \frac{a^3}{T}\Gamma_\phi\rho_\phi = a^3 \Gamma_\phi \rho_\phi \left(\frac{2\pi^2 g_{*s} a^3}{45\, S}\right)^{1/3} \Rightarrow \dot S = \left(\frac{2\pi^2 g_{*s}}{45\, S}\right)^{1/3} a^4 \Gamma_\phi \rho_\phi$$

$$\dot S S^{1/3} = \left(\frac{2\pi^2}{45}\right)^{1/3} g_{*s}^{1/3} a^4 \Gamma_\phi \rho_\phi \quad (4)$$

(1) – (4) are enough to close the system

④ : $\int S^{1/3}\, dS = \left(\frac{2\pi^2}{45}\right)^{1/3} \int dt\; a^4 \rho_\phi \Gamma_\phi\, g_{*s}^{1/3}$

$$\Rightarrow S_f^{4/3} = S_i^{4/3} + \frac{4}{3}\left(\frac{2\pi^2}{45}\right)^{1/3} \int_{t_i}^{t_f} dt\; a^4 \rho_\phi \Gamma_\phi\, g_{*s}^{1/3}$$

- exponential decay: Γ_ϕ = constant $\Rightarrow$ need numerical solution
- sudden decay approximation: $\Gamma_\phi = \delta(t - \tau_d)$

$\rightsquigarrow$ allows analytic treatment

$$S_f^{4/3} = S_i^{4/3} + \frac{4}{3}\left(\frac{2\pi}{45}\right)^{1/3} a_d^4\, \rho_\phi^d \left(g_{*s}^d\right)^{1/3}$$

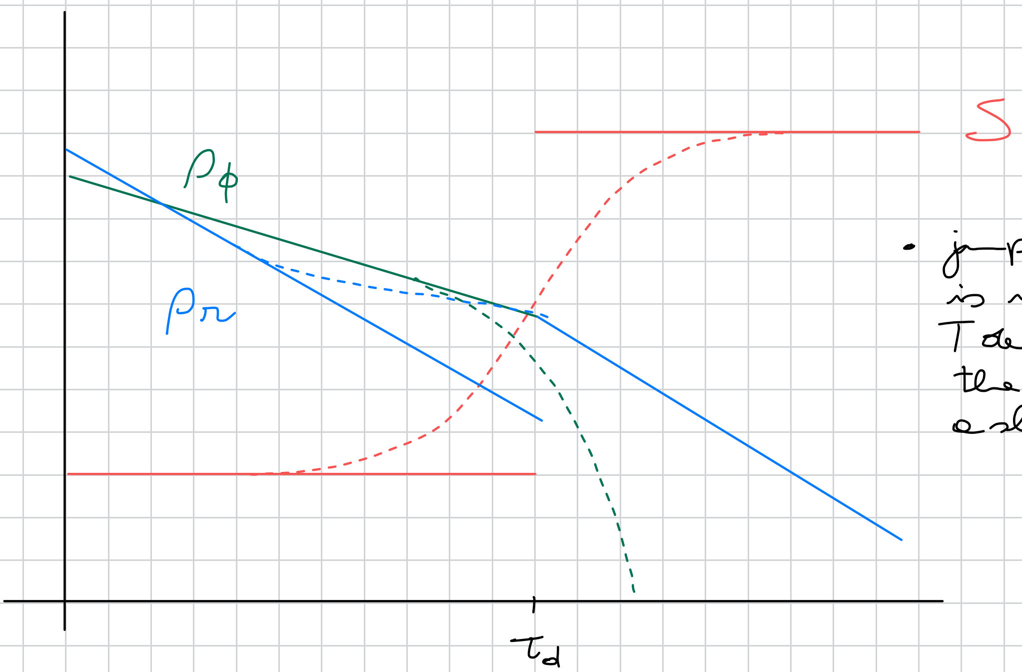
Assume $\rho_\phi \gg \rho_r$ at $t = \tau_d$

$$\Rightarrow S_f^{4/3} \approx \frac{4}{3}\left(\frac{2\pi}{45}\right)^{1/3} a_d^4\, \rho_\phi^d \left(g_{*s}^d\right)^{1/3}$$

$$\Delta \equiv \frac{S_f}{S_i} \gg 1$$

$$\Rightarrow \rho_r|_{\tau_d^+} = \left(\frac{3}{4}\left(\frac{45}{2\pi^2 g_{*s}}\right)^{1/3} a^{-4}\, \frac{4}{3}\left(\frac{2\pi}{45}\right)^{1/3} a_d^4\, \rho_\phi^d \left(g_{*s}^d\right)^{1/3}\right)_{\tau_d^+} = \rho_\phi|_{\tau_d^-}$$

$$\rho_\phi|_{\tau_d^+} = 0$$



• jump in ρ_r ($\Rightarrow T$) is not real: T decreases during the decay, just at a slower rate

Consequence for a decoupled specie

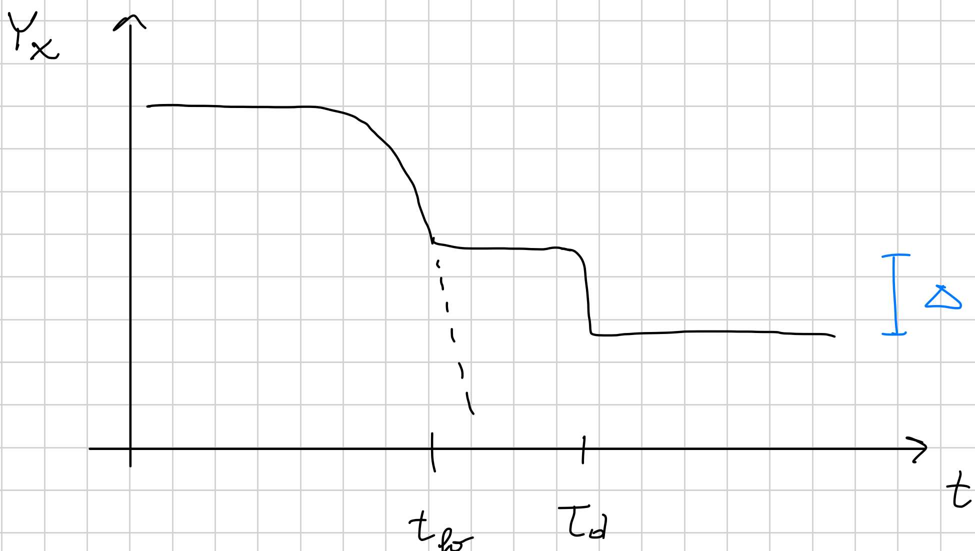
Suppose we have a decoupled specie X (DM? SM ν? Extra ν? GW?)

No number changing processes : $n_X a^3 = \text{const}$

$Y_X = \frac{n_X}{s}$ constant before and after decay, changes at decay

$$Y_0 = Y_{\tau_d^+} = \frac{n_X(\tau_d)}{s(\tau_d^+)} = \frac{n_X(\tau_d)}{\Delta\, s(\tau_d^-)} = \frac{1}{\Delta} Y_{\tau_d^-}$$

$\Rightarrow$ The density of any decoupled particles gets diluted by the decay of some heavy particles into thermal radiation

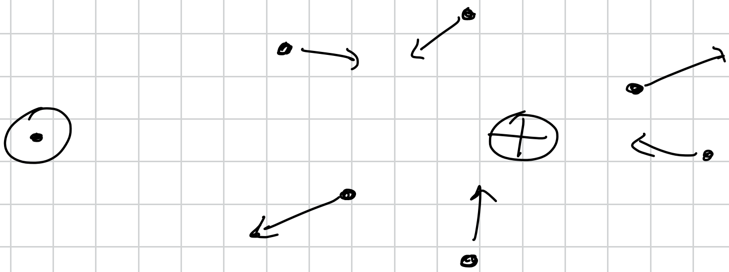


WIMP dark matter searches: Direct detection

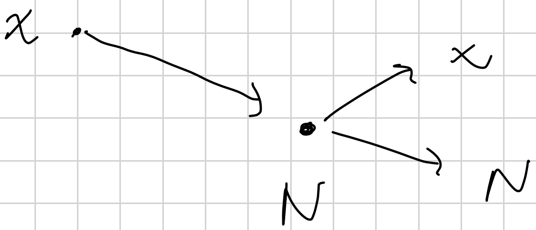
DM produced via freeze-out: it has to be in thermal eq with the SM ⇒ there must be sizeable interactions. Can we test them?

Direct detection

Idea: Earth is constantly hitted by DM particles



Some of them (depending on the cross section) will scatter against some particle (nuclei) of the stuff Earth is made of. If we take a "target" and we equip it to see any recoil of the nuclei it is made of, we can hope to detect these scattering processes and measure the cross section.

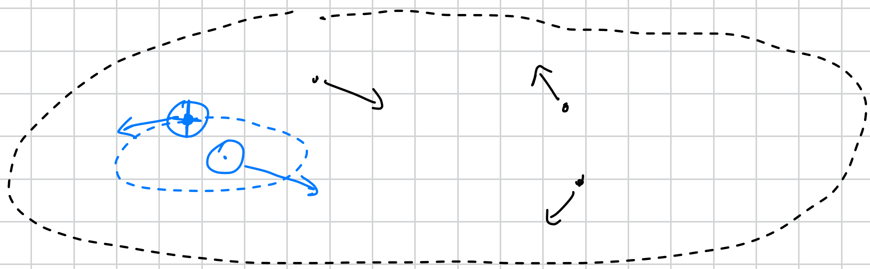


elastic scattering.

Typical numbers: $\rho_{DM} \sim 0.3 \text{ GeV cm}^{-3}$ $v \sim 220 \text{ km s}^{-1}$

$v/c \sim 10^{-3}$

Earth is moving $\left(v_{\oplus} \simeq 30 \text{ km s}^{-1} \sim \frac{v_\chi}{10} \right)$



DM halo

The Sun moves in the galaxy, Earth moves around the Sun

$\Rightarrow$ Expect annual modulation of the incoming flux (and hence of the signal)

Estimate the flux

$$n_\chi = \frac{\rho_\chi^{local}}{m_\chi}$$

ρ_χ^{local} fixed $\Rightarrow$ n_χ decreases for larger m_χ

heavier χ harder to detect (there are fewer around)

Typical flux: $\phi_\chi = n_\chi v_\chi = \frac{\rho_\chi^{local}}{m_\chi} v_\chi$

for $m_\chi = 100$ GeV:

$$\phi_\chi = \frac{0.3 \text{ GeV cm}^{-3} \quad 220 \text{ km s}^{-1}}{100 \text{ GeV}} \approx 10^5 \text{ cm}^{-2} \text{ s}^{-1}$$

For comparison: the flux of solar neutrinos is $\sim 10^{11} \text{ cm}^{-2} \text{ s}^{-1}$

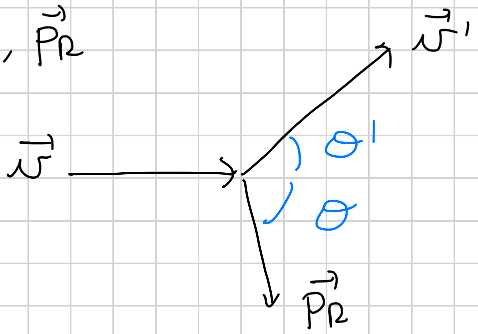
Background: - cosmic rays (and their secondary products)
- natural radioactivity

Setup: specify the target (mass and spin of the nucleus) and the detection method (electrons emitted from ionization, heat and phonons from vibrations (→ temperature increase), light emitted by scintillating crystals ...)

Kinematics

$$\chi \, N \to \chi \, N$$

In the plane of $\vec{v}\,'$, $\vec{p}_R$



$$\begin{cases} \frac{1}{2} m_\chi v^2 = \frac{1}{2} m_\chi v'^2 + \frac{1}{2} m_N v_R^2 \\ m_\chi \vec{v} = m_\chi \vec{v}\,' + m_N \vec{v}_R \end{cases} \Rightarrow \begin{cases} v'^2 = v^2 - \frac{m_N}{m_\chi} v_R^2 & (1) \\ \vec{v}\,' = \vec{v} - \frac{m_N}{m_\chi} \vec{v}_R & (2) \end{cases}$$

$$(2) \Rightarrow \quad v'^2 = v^2 + \left(\frac{m_N}{m_R}\right)^2 v_R^2 - 2 v v_R \frac{m_N}{m_\chi} \cos\theta \qquad (3)$$

$$(1),(3) \Rightarrow \quad \cancel{v^2} - \cancel{\frac{m_N}{m_\chi}} v_R^{\cancel{2}} = \cancel{v^2} + \left(\frac{m_N}{m_\chi}\right)^{\cancel{2}} v_R^{\cancel{2}} - 2 v \cancel{v_R} \cancel{\frac{m_N}{m_\chi}} \cos\theta$$

$$v_R \left(\frac{m_N}{m_\chi} + 1\right) = 2 v \cos\theta$$

$$\Rightarrow \quad v_R = 2 \frac{m_\chi}{m_N + m_\chi} v \cos\theta$$

$$\Rightarrow \quad E_R = \frac{1}{2} m_N v_R^2 = 2 \frac{\mu^2}{m_N} v^2 \cos\theta$$

where $\mu = \dfrac{m_\chi m_N}{m_\chi + m_N}$

Energy thresholds

Maximal velocity: $E_{max} = 2\frac{\mu^2}{m_N} v^2$

This needs to impose some detector thresholds:

$$E_{max} > E_{th} \Rightarrow v > v_{th} = \sqrt{\frac{m_N E_{th}}{2\mu^2}}$$

$$\mu \approx \begin{cases} m_N & m_\chi \gg m_N \\ m_\chi & m_\chi \ll m_N \end{cases}$$

$$\Rightarrow v_{th} = \begin{cases} \frac{E_{th}}{2m_N} & m_\chi \gg m_N \\ \frac{m_N E_{th}}{2m_\chi^2} & m_\chi \ll m_N \end{cases}$$

$\Rightarrow$ detection becomes problematic for $m_\chi \ll m_N$

Typical recoil energy: $m_\chi \sim 100\,\mathrm{GeV}$, $m_N \sim 100\,\mathrm{GeV}$ (Xenon atoms) ^{54}Xe

$v \sim 10^{-3}$

$$E_{max} \sim \frac{2\,(100\,\mathrm{GeV})^2\,10^{-6}}{100\,\mathrm{GeV}} \approx 200\,\mathrm{keV}$$

Minimal recoil energy in an experiment: $\sim \mathcal{O}(\mathrm{keV})$

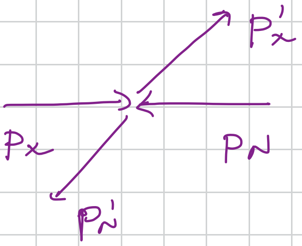
$\rightsquigarrow$ corresponds to mass $\quad (m_\chi \ll m_N \Rightarrow \mu \approx m_\chi)$

$$m_\chi \approx \left(\frac{m_N E_{th}}{v^2}\right)^{1/2} \approx \frac{(100\,\mathrm{GeV}\;1\,\mathrm{keV})^{1/2}}{10^{-3}} \approx 10\,\mathrm{GeV}$$

Center of mass kinematics

You can work out the kinematics and get $E_R = \frac{\mu^2 v_{rel}^2}{m_N}(1-\cos\theta_\chi)$

Solution:



$$p'_\chi = p'_N \qquad p_N = p_\chi$$

$$\frac{p_N^2}{2m_N} + \frac{p_\chi^2}{2m_\chi} = \frac{p_N'^2}{2m_N} + \frac{p_\chi'^2}{2m_\chi} \qquad \Rightarrow p_\chi = p_N = p'_\chi = p'_N$$

$$p_N = m_N v_N = m_\chi v_\chi \quad \Rightarrow \quad v_{rel} = v_\chi + v_N = p_N\left(\frac{1}{m_N} + \frac{1}{m_\chi}\right) = \frac{p_N}{\mu}$$

$$\Rightarrow p = \mu v_{rel}$$

define $\vec{q} = \vec{p}' - \vec{p} \Rightarrow q^2 = 2\mu^2 v_{rel}^2 (1+\cos\theta_\chi)$

q^2 is a Lorentz invariant! $Q = P'_\chi - P_\chi \Rightarrow Q^2 = -q^2$

$$P_\chi = \begin{pmatrix} E_p \\ \vec{p} \end{pmatrix} \qquad P'_\chi = \begin{pmatrix} E'_p \\ \vec{p}' \end{pmatrix} = \begin{pmatrix} E_p \\ \vec{p}' \end{pmatrix} \qquad Q = \begin{pmatrix} 0 \\ \vec{p}' - \vec{p} \end{pmatrix}$$

In the lab frame instead:

$$P'_\chi - P_\chi = P_N - P'_N = \begin{pmatrix} m_N \\ \vec{0} \end{pmatrix} - \begin{pmatrix} \sqrt{m_N^2 + p_R^2} \\ \vec{p}_R \end{pmatrix}$$

$$\Rightarrow Q^2 = \left(\sqrt{m_N^2 + p_R^2} - m_N\right)^2 - p_R^2 = m_N^2 + p_R^2 - p_R^2 + m_N^2 - 2m_N\sqrt{m_N^2 + p_R^2}$$

$$= 2m_N^2 - 2m_N^2\left(1 + \frac{1}{2}\frac{p_R^2}{m_N^2}\right) = -p_R^2$$

$$\Rightarrow |\vec{p}_R| = |\vec{q}| \quad \text{and} \quad E_R = \frac{q^2}{2m_N} = \frac{\mu^2 v_{rel}^2}{m_N}(1-\cos\theta_*)$$

$$\Rightarrow \frac{d}{dE_R} = \frac{m_N}{\mu^2 v_{rel}^2} \frac{d}{d\cos\theta_*}$$

Scattering rate

Event rate: $\Gamma \equiv \frac{dN_T}{dV\,dt} \equiv n_\chi n_N v \sigma$ → in general, v_{rel}, but we can write it as v_{rel} because the two are equal in frames in which $v_\chi \parallel v_N$

Γ is Lorentz invariant

Differential rate:

$$\frac{d^2\Gamma}{dE_R\,d\varphi} = \frac{m_N}{\mu^2 v_{rel}^2}\frac{d^2\Gamma}{d\varphi\,d\cos\theta_*} = \frac{m_N}{\mu^2 v_{rel}^2} n_N n_\chi v_{rel} \underbrace{\frac{d^2\sigma}{d\varphi\,d\cos\theta_*}}_{\frac{d\sigma}{d\Omega_*} \text{ or } \sigma(\varphi,\theta_*)}$$

For NR relativistic scattering

$$\frac{d\sigma}{d\Omega_*} = \frac{1}{(m_\chi+m_N)^2}\frac{|\mathcal{M}|^2}{64\pi^2}$$

$\mathcal{M}$ depends on how does DM interact with the nucleus.

NB: here we fixed a velocity, but DM actually has a distribution. On Earth, this is not the simple maxwellian distribution, because 1) I have to boost it to the Earth frame ($\oplus$ is moving around $\odot$ which moves around the galaxy) and 2) there is an escape velocity.

$$\int_0^{v_{esc}} f_\chi(v_{rel})\,dv_{rel} = 1$$

and

$$\frac{d\Gamma}{dE_R\,d\varphi} = \frac{m_N n_T n_\chi}{\mu^2 v_{rel}}\frac{d\sigma}{d\Omega_*\,dv}\,dv$$

$n_\chi \to \frac{\rho_\chi}{m_\chi}$ and $\rho_\chi = 0.3\ \text{GeV cm}^{-3}$ is purely astro

Non-relativistic operators

How does DM interact w/ nuclei?

Two observations:

1) $q \sim \mu v_{rel} \sim 10^{-3} \frac{m_N m_\chi}{m_N + m_\chi} \sim 10^{-3}\, 100\,\text{GeV} \sim 100\,\text{MeV}$

$\Delta x \sim 10^{-2}\,\text{MeV}^{-1}$ where $\hbar = 1 = 197.3\,\text{MeV fm}$

$\sim 2\,\text{fm} \gtrsim$ proton size

$\Rightarrow$ χ doesn't see quark and gluons, it sees protons and neutrons!

2) $v \ll 1$ then the scattering is non relativistic $\Rightarrow$ use the tools of non relativistic QM rather than QFT.

Procedure:

1) consider an operator mediating the χ - quarks (gluon) interaction, eg

$$\mathcal{O}_5^q = \bar{\chi}\gamma^\mu \chi\; \bar{q}\gamma_\mu q$$
$$\mathcal{O}_8^q = \bar{\chi}\gamma^\mu\gamma_5 \chi\; \bar{q}\gamma_\mu\gamma_5 \chi$$

2) Derive what operators does this induce at the nucleon level

$$\mathcal{O}_5^q \longrightarrow \mathcal{O}_5^N = \bar{\chi}\gamma_\mu \chi\, \bar{N}\gamma^\mu N$$

(with an appropriate coefficients)

$$\mathcal{O}_8^q \longrightarrow \mathcal{O}_8^N = \bar{\chi}\gamma^\mu\gamma_5 \chi\, \bar{N}\gamma_\mu\gamma_5 N$$

3) Consider the non-relativistic limit. Limits of fermion bilinears are of the form

$$u^s(p) = \begin{pmatrix} \sqrt{p^\mu \sigma_\mu}\ \xi^s \\ \sqrt{p^\mu \bar\sigma_\mu}\ \xi^s \end{pmatrix} \approx \frac{1}{\sqrt{4m}} \begin{pmatrix} (2m - \vec{p}\cdot\vec{\sigma})\,\xi^s \\ (2m + \vec{p}\cdot\vec{\sigma})\,\xi^s \end{pmatrix}$$

$$\bar{u}(p')\gamma^\mu u(p) \approx \begin{pmatrix} 2m \\ \vec{P} + 2i\,\vec{q}\times\vec{s} \end{pmatrix}$$

$$\bar{u}(p')\gamma^\mu\gamma_5 u(p) \approx \begin{pmatrix} 2\vec{P}\cdot\vec{s} \\ 4m\vec{s} \end{pmatrix}$$

$$\vec{P} = \vec{p} + \vec{p}\,' \qquad \vec{q} = \vec{p} - \vec{p}\,' \qquad \vec{s} = \xi'\,\frac{\vec{\sigma}}{2}\,\xi$$

We obtain

$$\mathcal{O}_s^N \longrightarrow \mathcal{O}_1^{NR} = \mathbb{1} \qquad \text{"SI: spin-independent"}$$

$$\mathcal{O}_8^N \longrightarrow \mathcal{O}_4^{NR} = \vec{s}_\chi \cdot \vec{s}_N \qquad \text{"SD: spin-dependent"}$$

Where the non rel operators are "used" with non-rel fields containing only creation or annihilation ops

$$\mathcal{L}_{int} = \sum_{N=n,p} \sum_i c_i^N\, \mathcal{O}_i\, \chi^+ \chi^- N^+ N^-$$

$$N^-(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2m_N}}\, e^{-ik\cdot y}\, a_k^+ \qquad N^+(x) = \left(N^-(x)\right)^+$$

4) Compute the χ-nucleus cross section. SI scattering wins a factor $A^2 \sim \mathcal{O}(10^4)$ because the matrix elements with individual nuclei add opp coherently.

Example: SI

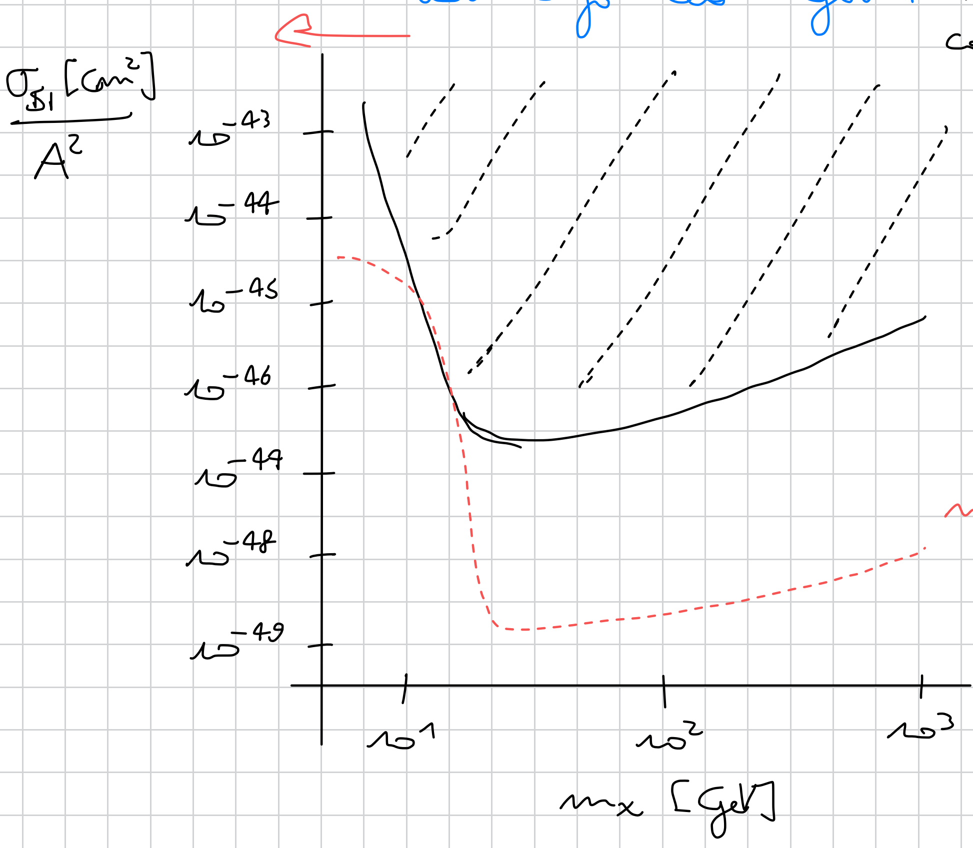
$$\mathcal{M}^{SI}_{\chi N} = F(E_R)\left[Z f_p + (A-Z) f_n\right]$$

"nucleon form factor" obscures from matching the quark operator onto the nucleus

$$\frac{d\sigma}{d\Omega_\chi} = \frac{\sigma}{4\pi} = \frac{\sigma(E_R=0)}{4\pi} F(E_R)^2$$

$$\frac{d\Gamma}{dE_R} = \underbrace{(N_N m_N)}_{\text{target}} \underbrace{\left(\frac{\sigma^{SI}_{\chi N}(E_R=0)}{\mu^2 m_\chi}\right)}_{\text{microscopic DM model}} \underbrace{F(E_R)^2}_{\text{nuclear physics: from } q, g \text{ to nucleus}} \underbrace{\left(\rho_\chi \int_{v_{min}(E_R)}^{v_{esc}} \frac{f_\chi(v_{rel})}{2 v_{rel}} dv_{rel}\right)}_{\text{astrophysics}}$$

how to go below GeV? scattering off electrons, collective scatterings.



Xenon 1T
(1t of Xenon × 1 yr)

neutrino floor: at this sensitivity the experiment starts to see νs from the sun (very rare at these energies)

→ a nightmare background

need directional detection?

Low mass limit : - below ~ 10 GeV ⇒ scattering w/ electrons

- below ~ MeV : $q \sim$ keV ⇒ $\Delta\lambda \sim$ Å

⇒ DM interacts with many nuclei at the same time , Excite collective modes of the target

Indirect detection

General idea

Thermal DM production $\Rightarrow$ DM particles can annihilate into pairs of SM ones.

Thermal freeze-out benchmark: $\langle \sigma v_{rel} \rangle \simeq 3 \times 10^{-26}\, cm^3 s^{-1}$

Is this a big number? Number of annihilations in our solar system, since when the galaxy exists:

$$N^{local}_{ann} \sim (n_\chi^{loc})^2 \langle \sigma v \rangle V \tau \sim \left(\frac{\rho_\chi^{loc}}{m_\chi}\right)^2 \langle \sigma v \rangle R^3_{sol\,sys} \tau$$

$$\sim \left(\frac{0.3\, GeV\, cm^{-3}}{100\, GeV}\right)^2 3 \times 10^{-26}\, cm^3 s^{-1} \, (10^5 AU)^3 \, (5 \times 10^9\, yr)$$

$$\sim 10^{41}$$

We need to point our telescope at regions with high DM density

- galactic centre: large ρ, but large / complicated bkg from stellar processes
- dwarf-galaxies: most precise signal because DM-dominated

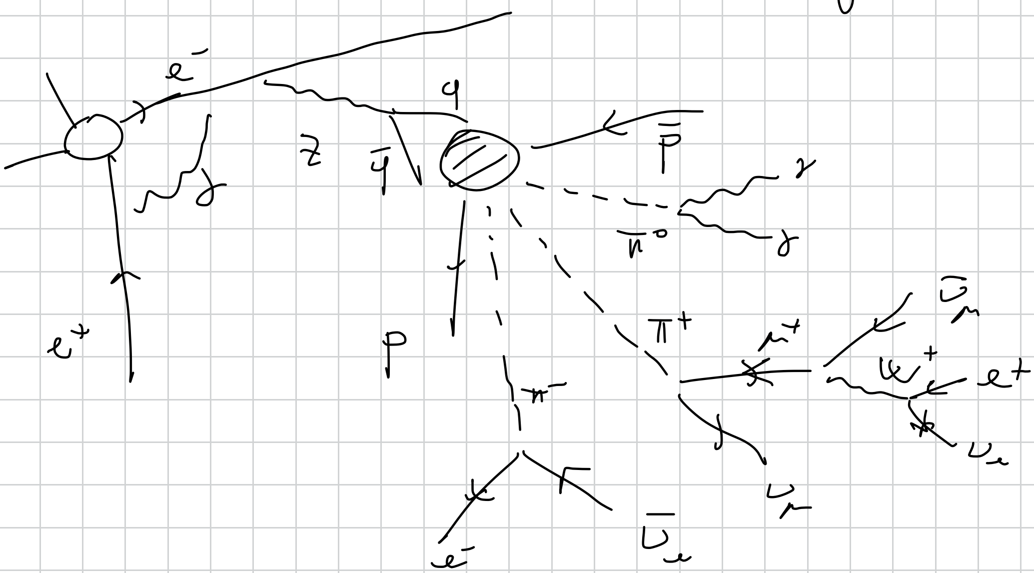
Final states

main channel : pair annihilation

$$\chi\chi \to q\bar{q},\ e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-,\ \nu\nu,\ \gamma\gamma,$$
$$W^+W^-,\ ZZ,\ Zh,\ hh$$

- quarks hadronize $\to p, n, \pi$'s, ...
- unstable particles decay
- EW corrections are important: emission of a soft/collinear W boson is enhanced by $\log \frac{m_\chi}{m_W}$, $\log\log \frac{m_\chi}{m_W}$ and can alter the spectrum.

$\Rightarrow$ the emitted W/Z induces production of other particles



Propagation

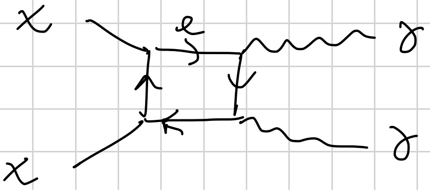
neutral stuff (γ and ν) : travel undisturbed on a straight line

charged stuff ($e^\pm$, $p^\pm$) : scattered and bended by galactic fields in the galaxy : need some propagation model (with uncertainties)

ID w/ photons

Two channels :

$\chi\chi \to \gamma\gamma$ eg via a box diagram

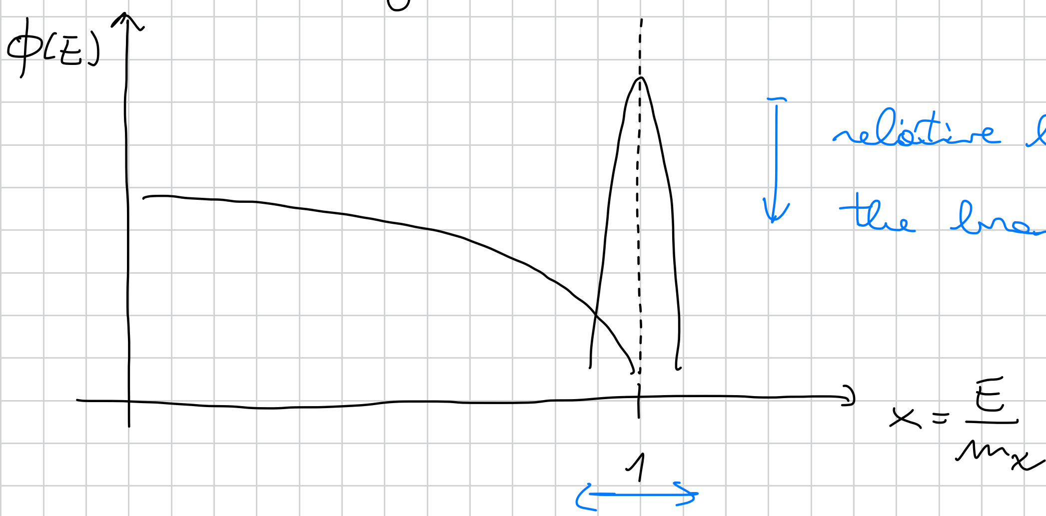


$$s = (p_1 + p_2)^2 \approx 4 m_\chi^2 \qquad (v \ll 1)$$

$\Rightarrow E_\gamma = m_\chi \quad \Rightarrow$ X-ray line

Final state radiation / decay : continuous spectrum

We expect something like



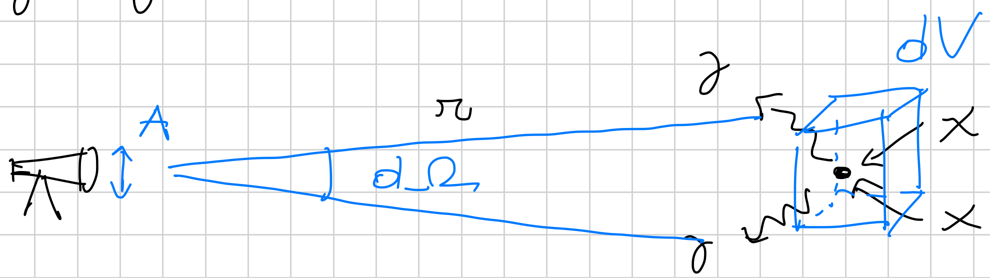
relative height depends on the branching fraction

detector resolution

$\chi\chi \to W^+W^-$ is the same as $e^+e^- \to W^+W^-$, as long as FSR is considered. $\Rightarrow$ we can use all the machinery developed for collider experiments (hadronization, radiative corrections, etc)

Photon flux

Consider photons produced at one particular location in the galaxy



$$\frac{dN_\gamma}{dE\, dV} = \frac{1}{4\pi r^2} \frac{dn_\gamma(m_\chi)}{dE_\gamma} \frac{1}{2} n_\chi^2 \langle \sigma v \rangle \quad A\, \Delta t$$

spectrum of produced photons

avoid double counting in the number of distinct DM pairs

Integrate over the volume w/ $dV = r^2\, dr\, d\Omega$

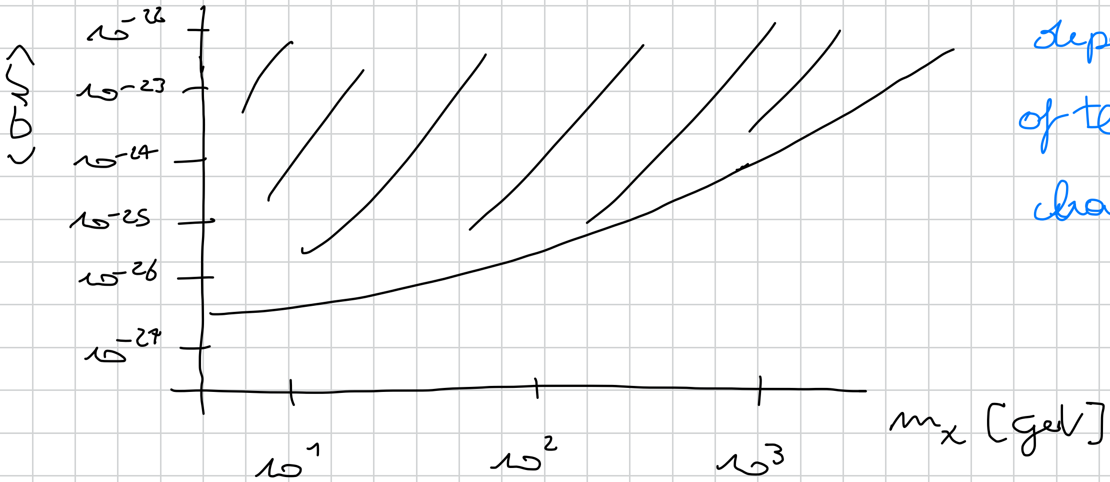
$$\frac{dN_\gamma}{dE_\gamma\, d\Omega} = \underbrace{(A\,\Delta t)}_{\text{detector}} \underbrace{\left(\frac{\langle \sigma v_{rel} \rangle}{m_\chi^2}\right)}_{\text{particle physics}} \underbrace{\left(\frac{dn_\gamma(m_\chi)}{dE_\gamma}\right)}_{\text{SM}} \underbrace{\left(\frac{1}{8\pi}\int \rho_\chi^2\, dr\right)}_{\text{astrophysics}}$$

$\equiv J$ "J-factor"

Main uncertainty : J - factor (up to 2 orders of magnitude)
We do not know whether the distribution is peaked or cored

Results

Typical bound looks like



depends on the assumption of the dominant annihilation channel

Decaying DM

$\chi \to$ SM stuff w/ $\tau_\chi > \tau_U$

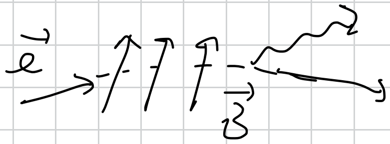
Same as above, w/ $m_\chi^2 \langle \sigma v \rangle$ replaced by $m_\chi \Gamma_\chi$ w/ $\Gamma_\chi = \tau_\chi^{-1}$

$\Rightarrow \quad J_{decay} = \frac{1}{8\pi} \int dl \, \rho_\chi \quad \Rightarrow$ less uncertainty

Other photon sources (DM related)

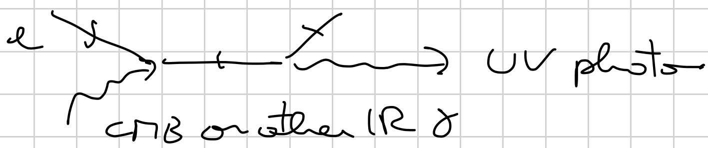
- synchrotron emission in the galactic magnetic field $\vec{B}$

$\chi\chi \to e^+e^- \quad +$



- Inverse Compton scattering

$\chi\chi \to e^+e^- \quad +$



Again, for $v \to 0$, Sommerfeld enhancement & bound state formation can be very important

Claimed excesses [2109.02696]

Background estimation + statistics is a nasty business

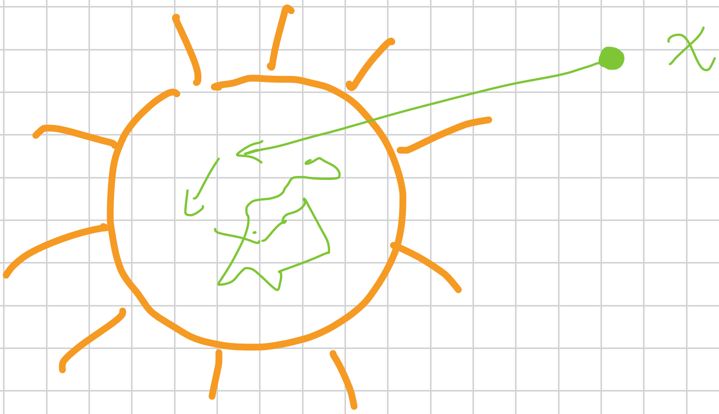
If any signal is seen, 2 questions:

1) is it really there? Or is it just some statistical fluctuation? Or maybe there is just no excess?

2) if an excess is found, is it due to DM? Or do I need to remodel my bkg?

- cosmic rays positrons, antiprotons, at high energy
- anti-^{4}He
- 3.5 keV line in galaxy clusters (decay of 7 keV sterile neutrinos)
- 21 cm absorption
- 1-3 GeV γ-ray excess around the galactic centre,

Neutrinos from the Sun



Scatterings slow down χ that is trapped in the Sun

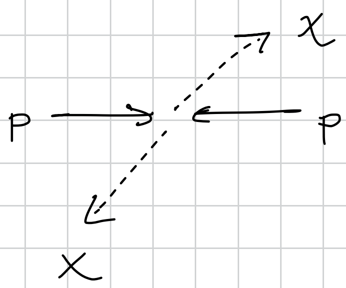
$$\Gamma_{ann} = \Gamma_{capture} \propto \langle \sigma v \rangle \big|_{scattering}$$

$\chi\chi \to \nu\bar{\nu}$

put bounds on $\sigma v|_{scatt}$ (strong also for SD)

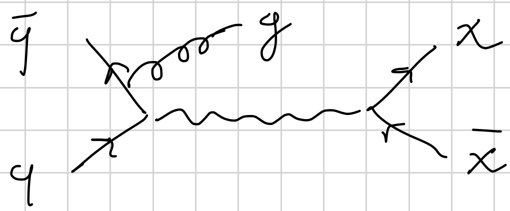
Collider searches

Produce DM at colliders. χ leave the detector without being detected



Such a event is useless because I cannot detect it. Need to have some recoiling SM particle.

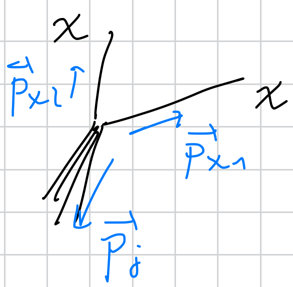
Eg $\quad q\bar{q} \to \chi\chi g \rightsquigarrow$ mono-jet + missing energy



in general "mono-X + MET" searches

$(X = \text{jet}, \gamma, Z, h, \dots)$

MET = missing $E_T = -\sum \vec{p}_T$ $\quad$ momentum in the transverse plane is imbalanced

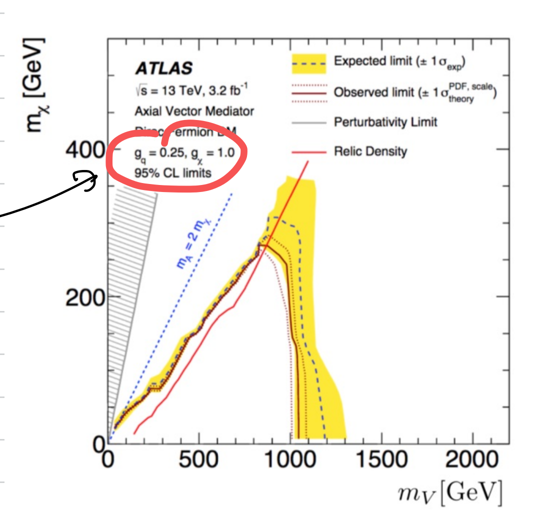


$\vec{p}_j + \vec{p}_{\chi_1} + \vec{p}_{\chi_2} = 0$ but I don't see $\vec{p}_{\chi_{1,2}}$

At a proton collider the longitudinal momentum is not 0

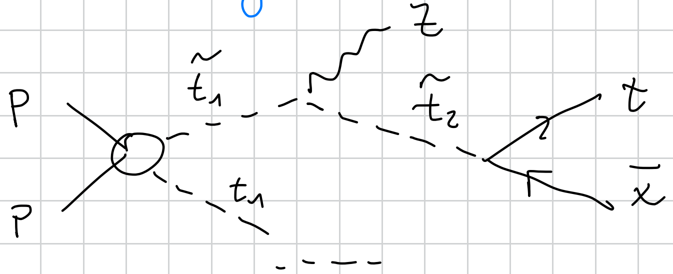
Typical result:

couplings are fixed



Other search channels:

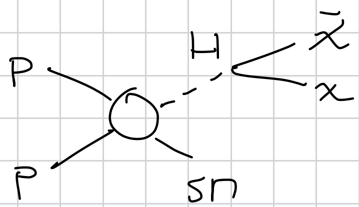
- cascade decays



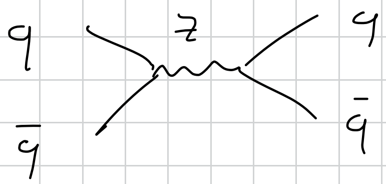
t quark is the least constrained

top partners such as $\tilde t$ are well motivated (SUSY) and less constrained

- $H \to$ invisible



- Mediator searches (this is what LHC does best)



dijet channel

$$m_{Z'} \gtrsim 2 - 2.5 \text{ TeV} \qquad \text{for } g \simeq 0.1$$

Early Universe bounds

DM annihilation can be dangerous if they deposit too much energy into the plasma around the time of nucleosynthesis or CMB.

Energy injected in the plasma after freeze-out:

$$\frac{dE}{dV\,dt} = m_\chi\, n_\chi^2 \langle \sigma v \rangle \qquad n_\chi = \left(\frac{a_f}{a}\right)^3 n_\chi^f = \left(\frac{T}{T_f}\right)^3 n_\chi^f$$

$$= m_\chi\, n_\chi \left(\frac{T}{T_f}\right)^3 n_\chi^f \langle \sigma v \rangle$$

$$= m_\chi\, n_\chi \left(\frac{T}{T_f}\right)^3 H_f$$

$$= \rho_\chi \left(\frac{T}{T_f}\right)^3 H_f$$

In one Hubble time, for each baryon I inject an energy

$$\frac{E}{N_b} = \left(\rho_\chi \left(\frac{T}{T_f}\right)^3 H_f\right) \frac{a^3}{n_b a^3} H^{-1} \qquad H \propto T^{-2}$$

$$= \frac{\rho_\chi}{\rho_b / 1\,\text{GeV}} \left(\frac{T}{T_f}\right) \qquad m_\chi \approx 100\,\text{GeV} \quad x_f \approx 25 \quad T_f \approx 4\,\text{GeV}$$

$$\approx 5\,\text{GeV}\,\frac{T}{4\,\text{GeV}} \approx T$$

BBN: $T \sim \text{MeV} \Rightarrow \frac{E}{N_b} \approx \text{MeV} \approx$ binding energy of nuclei

$\Rightarrow$ dangerous!

CMB: $T \sim \text{eV} \Rightarrow \frac{E}{N_b} \approx \text{eV} \lesssim$ binding energy of H atom

$\Rightarrow$ dangerous

For the actual bound I have to go back to the expression for the energy deposit and evolve it in time over the duration of BBN or CMB:

$$\frac{dE}{dV\,dt} = m_\chi\, n_\chi^2 \langle \sigma v \rangle = \frac{\rho_\chi^2}{m_\chi} \langle \sigma v \rangle = (1+z)^6\, \rho_{\chi,0}^2\, \frac{\langle \sigma v \rangle}{m_\chi}$$

Strongest bound from CMB: $\langle \sigma v \rangle < 3 \times 10^{-27} \frac{cm^3}{s} \left(\frac{m_\chi}{1\,GeV} \right)$

$\Rightarrow$ $m_\chi \gtrsim 10\,GeV$ for thermal fo production
(unless the dominant channel is into neutrinos, that decouple around BBN (way before CMB)

Axions

CP violation in QCD

CP violation & the quark mass

Consider QCD around 1GeV : only 3 quarks, but decoupled

$$\mathcal{L}_{QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}\, i \not{D}\, q - \left(\bar{q}\, m_q\, q + h.c. \right)$$

Notation : $q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ (all Dirac fermions)

Strong interactions do not mix flavours $\Rightarrow$ m_q is diagonal

but it can be complex

Some thoughts on fermion mass terms:

If I use a Dirac mass, I'm automatically writing a CP invariant lagrangian

USE DIRAC VECTORS HERE

$$\bar{u}\, m_q\, u = u_R^\dagger\, m_u\, u_L + u_L^\dagger\, m_u\, u_R$$

$$(\bar{u}\, m_q\, u)^\dagger = u_L^\dagger\, m_u^*\, u_R + u_R^\dagger\, m_u^*\, u_L$$

$$\bar{u}\, m_q\, u + h.c. = (2 \mathrm{Re}\, m_u)(u_R^\dagger u_L + u_L^\dagger u_R)$$ CP invariant

Instead, I can write the mass term as $\bar{u}_R\, m_u\, u_L + h.c.$

and allow m_u to have a phase at this level:

$$\left(u_R^\dagger\, m_u e^{i\theta_u}\, u_L \right)^\dagger = u_L^\dagger\, m_u e^{-i\theta_u}\, u_R$$

$\Rightarrow$ mass term:

$$m_u\, u_R^\dagger u_L (\cos\theta + i \sin\theta) + m_u\, u_L^\dagger u_R (\cos\theta - i \sin\theta)$$

$$= m_u \cos\theta\, (u_R^\dagger u_L + u_L^\dagger u_R) + \underbrace{i\, m_u \sin\theta\, (u_R^\dagger u_L - u_L^\dagger u_R)}_{\text{CP - odd}}$$

CP violation in the QCD gluonic Lagrangian

I can add a term $\quad \mathcal{L}_{QCD} \to \mathcal{L}_{QCD} + \Delta\mathcal{L}_{QCD}$

$$\Delta\mathcal{L}_{QCD} = \Theta \frac{g_s^2}{32\pi} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

w/ $\tilde{G}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} G^{a,\rho\sigma}$

$$G\tilde{G} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \left(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \right)$$

$$\left(f^{abc} = -\frac{1}{4} i \, \mathrm{tr}\left(\lambda_a [\lambda_b, \lambda_c] \right) \right)$$

CP $\quad G\tilde{G} \to -G\tilde{G}$

$G\tilde{G} \sim \vec{E} \cdot \vec{B}$

$\vec{E}$ is a vector

$\vec{B}$ is an axial vector (or pseudo-vector)

$$\left(\begin{array}{ll} \text{CP}: & A^a_0 \to -A^a_0 (t, \vec{x}) \\ & \vec{A}^a (t, \vec{x}) \to \vec{A}^a (t, -\vec{x}) \\ & \partial_0 \to \partial_0 , \quad \vec{\nabla} \to -\vec{\nabla} \end{array} \right)$$

Anomalous rotations

Interestingly, the two are connected

In order to get rid of the phase of the quark masses, I can perform a chiral rotation

$$\psi_L \to e^{i\alpha_L \theta_L}\psi_L \qquad \psi_R \to e^{i\alpha_R \theta_R}\psi_R$$

or, in a simple notation, $\psi \to e^{i\theta_V} e^{i\theta_A \gamma_5}\psi$

w/ $\theta_V = \frac{\theta_L + \theta_R}{2}$ $\quad \theta_A = \frac{\theta_R - \theta_L}{2}$ $\qquad (\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3)$

Back to the Lagrangian

$$\mathcal{L} = \bar{q}\, i\not{D}\, q - \sum_j \left(m_j e^{i\theta_j}\, \bar{q}_{jR}\, q_{jL} + h.c.\right) =$$

$$= \bar{q}\, i\not{D}\, q - \sum_j \left(m_j \cos\theta_j\, \bar{q}_j q_j - m_j \sin\theta_j\, \bar{q}_j\, i\gamma_5\, q_j\right)$$

Transformations under chiral rotations:

$$V: \quad \bar{\chi}\chi \to \bar{\chi} e^{-i\alpha_V} e^{i\alpha_V}\chi = \bar{\chi}\chi$$

$$\bar{\chi}\, i\gamma^5 \chi \to \bar{\chi} e^{-i\alpha_V}\, i\gamma_5\, e^{i\alpha_V}\chi = \bar{\chi}\, i\gamma_5\, \chi$$

$$A: \quad \bar{\chi}\chi \to \bar{\chi} e^{i\alpha_A\gamma_5} e^{i\alpha_A\gamma_5}\chi = \cos 2\alpha_A\, \bar{\chi}\chi + \sin 2\alpha_A\, \bar{\chi}\, i\gamma_5\, \chi$$

$$\bar{\chi}\, i\gamma_5\, \chi \to \bar{\chi} e^{i\alpha_A\gamma_5}\, i\gamma_5\, e^{i\alpha_A\gamma_5}\chi = \cos 2\alpha_A\, \bar{\chi}\, i\gamma^5 \chi - \sin 2\alpha_A\, \bar{\chi}\chi$$

$$\left(e^{2i\alpha_A\gamma_5} = \cos 2\alpha_A + i\gamma^5 \sin 2\alpha_A\right)$$

$\Rightarrow$ eliminate the imaginary piece upon choosing

$$\tan 2\alpha_A = \tan \theta_\chi$$

$$\rightsquigarrow \mathcal{L} = \bar{q}' i \not{D} q' - \bar{q}_R' m_q q_L' =$$

$$= \bar{q}' i \not{D} q' - \bar{q}' m_q q$$

Anomalous rotation:

- vector rotation: it's a symmetry of the theory also at the quantum level, for $m = 0$ or $m \neq 0$

- axial rotation: at the classical level it's a symmetry if $m_q = 0$. classical level $\equiv$ apply the transformation to the Lagrangian only

Noether theorem:

$$\langle \partial_\mu j_V^\mu \rangle = 0 \qquad w/ \quad j_V^\mu = \bar{\chi} \gamma^\mu \chi$$

$$\langle \partial_\mu j_A^\mu \rangle = \sum_j 2 m_j \bar{q}_j i \gamma^5 q_j \qquad w/ \quad j_A^\mu = \bar{q}_j \gamma^\mu \gamma^5 q_j$$

At the quantum level: the symmetry is anomalous if q are charged under $SU(3)$

An anomaly means that the measure in the path integral is not invariant

Recap: for any gauge invariant operator

$$\langle \mathcal{O}(x_1 \dots x_n) \rangle = \frac{1}{Z[0]} \int \mathcal{D}\bar{\psi} \, \mathcal{D}\psi \, \exp\left[i \int d^4x \; \bar{\psi} i \not{D} \psi \right] \mathcal{O}(x_1, \dots)$$

Topological term

$G\tilde{G}$ is a total derivative

$$G\tilde{G} = \partial_\mu K^\mu \qquad K^\mu = \varepsilon^{\mu\nu\rho\sigma} A^a_\nu \left(F^a_{\rho\sigma} - \frac{g}{3} f^{abc} A^b_\rho A^c_\sigma \right)$$

I cannot integrate it out because $\exists$ solutions with finite energy such that A_μ does not go to zero at infinity

Transform the fields:

$$\psi(x) \to \Delta(x)\,\psi(x), \quad \bar\psi(x) \to \Delta^{+}(x)\,\bar\psi(x)$$

$$\Rightarrow \mathcal{D}\bar\psi\,\mathcal{D}\psi \to (\det\Delta)^{-2}\,\mathcal{D}\bar\psi\,\mathcal{D}\psi$$

If $\det\Delta \neq 1$, the measure is not invariant. This is equivalent to a shift in the Lagrangian:

$$\mathcal{L} \to \mathcal{L} + \Delta\mathcal{L}$$

In our case ($U(1)_A$ rotation in QCD)

$$\Delta\mathcal{L} = \theta_A \frac{\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

This has the same form of the "theta-term".

Thus, I can always rewrite the Lagrangian in a form in which quark masses are real and

$$\Delta\mathcal{L}_{QCD} = \bar\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

with $\bar\theta = \theta + \arg\det m_q = \theta - \theta_u - \theta_d$

Massless quark

Important observation: if $m_u = 0$ the CP violating terms can be set to $0 \Rightarrow \bar\theta$ non physical

I can make a rotation such that $\theta_u = \theta - \theta_d \Rightarrow \bar\theta = 0$ but now $\bar\theta$ does not reappear in a mass term!

Why is it important?

Electric dipole moment of a neutron

$d_n \propto \bar{\theta}$

Use a classical analogy

$\vec{d} = \sum q \vec{r}$

$r_n \sim 1/m_\pi$

$\Rightarrow |d_n| \sim 10^{-13} \sqrt{1-\cos\theta}\ e\,cm$

- expect $|d_n| \sim 10^{-13}$
- experimentally $|d_n| < 10^{-26}\ e\,cm$

$\Rightarrow \theta < 10^{-13}$ why?

$d - u - d$

Why does $\vec{d}$ violate CP?

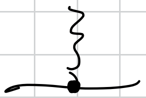
Start with magnetic dipole moments of elementary particles

$$H = -\mu\, \vec{S} \cdot \vec{B}$$

$\mu \equiv$ magnetic dipole moment

spin precession: $\omega = 2\mu B \sin\theta$

μ can be computed in QFT



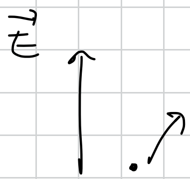
$$\mu = \frac{eg}{2m} \qquad g = 2 \times (1.00118 \pm \dots)$$

Electric dipole moment

For an elementary particle the only vector is $\vec{S}$

$\Rightarrow \quad \vec{d} \propto \vec{E}$

In an external electric field it must be



$$H = -d\, \vec{S} \cdot \vec{B}$$

I need to use time reversal because CP is not properly defined w/o antiparticles

Time reversal (equivalent to CP):

$\vec{S} \to -\vec{S}$ (spin is like angular momentum $\vec{L} \sim \vec{r} \times \vec{p}$)

$\vec{B} \to -\vec{B}$ (all currents that generate $\vec{B}$ change sign)

$\vec{E} \to \vec{E}$

Thus

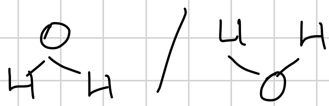
$$H \xrightarrow{CP} -\mu\, \vec{S}\cdot\vec{B} + d\, \vec{S}\cdot\vec{E}$$

H is not invariant under CP $\Rightarrow$ in a CP invariant theory $d = 0$

Same thing happens with parity: $\vec{S} \to \vec{S}$, $\vec{B} \to \vec{B}$, $\vec{E} \to -\vec{E}$

If an <u>elementary particle</u> has $d \neq 0$ this violates CP

In a P, T symmetric world, composite objects like water molecules do not have permanent edm.

Eg water: two degenerate states 

Parity eigenstates: $|+\rangle$ has lower energy than $|-\rangle$ ($\varepsilon_+ < \varepsilon_-$)

Place it in an external electric field. The potential is zE.
The new hamiltonian is

$$H = \begin{pmatrix} \varepsilon_+ & 0 \\ 0 & \varepsilon_- \end{pmatrix} + \begin{pmatrix} 0 & \Delta \\ \Delta^* & 0 \end{pmatrix} \qquad \text{w/} \begin{cases} \langle \pm | zE | \pm \rangle = 0 \\ \langle \pm | zE | \mp \rangle = \Delta \end{cases}$$

Compute the new eigenvalues

$$\varepsilon_{1,2} = \frac{1}{2}(\varepsilon_+ + \varepsilon_-) \pm \sqrt{\frac{1}{4}(\varepsilon_+ - \varepsilon_-)^2 + |\Delta|^2}$$

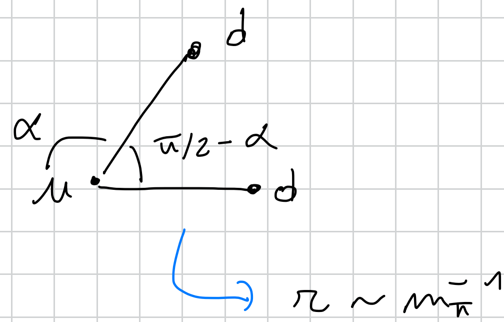
$$\simeq \frac{1}{2}(\varepsilon_+ + \varepsilon_-) \pm \frac{1}{2}(\varepsilon_+ - \varepsilon_-)\left(1 + \frac{1}{2}\frac{|\Delta|^2}{\frac{1}{4}(\varepsilon_+ - \varepsilon_-)^2}\right)$$

$$= \varepsilon_\pm \pm \frac{|\Delta|^2}{\varepsilon_+ - \varepsilon_-}$$

$\Rightarrow$ energy shift $\propto E^2$. This is different for the case of <u>permanent</u> EDM.

Suppose now P, T are violated. Nothing would prevent a permanent edm.

Classical analogy for the neutron



$$\vec{d} = \sum q \vec{r} = \frac{2}{3} e \, \vec{0} - \frac{1}{3} \frac{e}{m_\pi} \begin{pmatrix} 1 - \cos\alpha \\ \sin\alpha \end{pmatrix}$$

$$|\vec{d}_n| \approx \frac{e}{3 m_\pi} \left[1 + \cos^2\alpha - 2\cos\alpha + \sin^2\alpha \right]^{1/2} = \frac{2e}{3 m_\pi} \sqrt{1 - \cos\alpha}$$

$$\approx 10^{-13} \sqrt{1 - \cos\alpha} \; e\,\mathrm{cm}$$

if α is somewhat smaller than 1

$$|d_n| \approx 10^{-13} \alpha$$

Compute d_n in QFT: $d_n \approx 3 \times 10^{-16} \, \bar{\theta} \; e\,\mathrm{cm}$

$$\Rightarrow \bar{\theta} \lesssim 10^{-10}$$

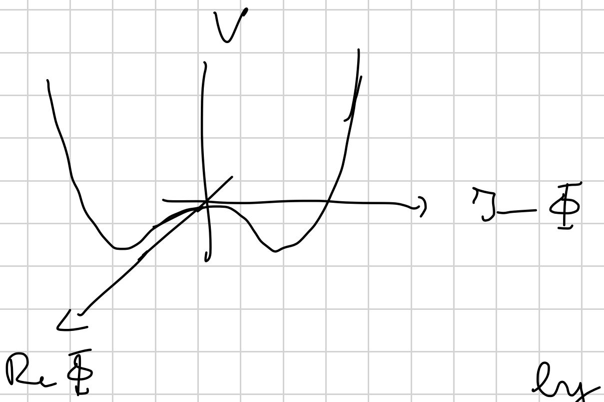
The QCD axion

The solution to this involves the introduction of a new pseudoscalar field: the axion

Take a complex scalar Φ, with a $U(1)$ sym $\Phi \to e^{i\alpha}\Phi$
and a pair of new coloured Dirac fermions

$$V = -m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 + y \Phi \bar{\psi} \psi + h.c.$$

The negative mass term causes symmetry breaking



$$\Phi = (f_a + r)\, e^{i a / f_a}$$

Get rid of the phase in the Yukawa term by means of an axial transformation

$$\psi \to \exp\left(i \frac{a}{f_a} \gamma_5\right) \psi$$

$\rightsquigarrow$ through the anomaly, a coupling is induced:

$$\left(\frac{a}{f_a} + \bar{\theta}\right) \frac{g_s^2}{32\pi^2} G \tilde{G}$$

At low temperature / energy QCD confines $(q, g \rightsquigarrow \pi, \eta, \rho \dots)$
Induced potential:

$$V(\phi) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\left(\frac{a}{2f} + \frac{\bar{\theta}}{2}\right)}$$

minimum: $\frac{a}{f_a} = -\bar{\theta} \Rightarrow \left(\frac{a}{f_a} + \bar{\theta}\right) G\tilde{G} = 0$

strong CP problem is solved

axion mass: $V \approx -m_\pi^2 f_\pi^2 \left[1 - \frac{1}{2} \frac{4 m_u m_d}{(m_u+m_d)^2} \left(\frac{a}{2f_a} - \frac{\bar{\theta}}{2} \right)^2 \right]$

define $\frac{a'}{f_a} = \frac{a}{f_a} - \bar{\theta}$

$$V \supset m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u+m_d)^2} \frac{1}{2} \left(\frac{a'}{f_a} \right)^2$$

$$\Rightarrow m_a = \left(\frac{m_u m_d}{(m_u+m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} \right)^{1/2} \approx 5.7\,\mu eV \frac{10^{12}\,GeV}{f_a}$$

simple form

$$V = -m_a^2 f_a^2 \cos \frac{a}{f_a}$$

temperature dependence

Dilute Instanton Gas: $m_a^2 = \frac{m_u m_d m_s}{f_a^2} \frac{\Lambda_{QCD}^9}{T^8}$

Interacting Instanton Liquid $m_a^2 = m_a^2(0) \left(\frac{\Lambda_{QCD}}{T} \right)^n$ $n \approx 6.8 - 7$

$\Lambda_{QCD} \approx 200\,MeV$

Another form (1511.02867 Villadoro +)

$$m_a^2(T) = m_a^2(0) \frac{\chi(1\,GeV)}{\chi(0)} \left(\frac{GeV}{T} \right)^\alpha$$

good convergence only above $T \gtrsim 10^{5-6}\,GeV$

expected: instantons: $\chi(1)/\chi(0) \approx 10^{-7}$, $\alpha = 8$

lattice: $\chi(1)/\chi(0) \approx 10^{-2}$, $\alpha \approx 2$

axion couplings

At energies GeV $\lesssim E \lesssim$ 100 GeV

$$\mathcal{L} = \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G\tilde{G} + \frac{g_{a\gamma\gamma}}{4} a F\tilde{F} + \frac{\partial_\mu a}{2 f_a} \sum_\psi c_\psi \bar{\psi} \gamma^\mu \gamma^5 \psi$$

where $g_{a\gamma\gamma} \sim 1/f_a^2$, and c_ψ are expected $\sim \mathcal{O}(1)$. The actual values are model dependent, but can be set to 0.

Below QCD confinement $\exists$ couplings w/ nucleons and mesons.

value of f

f_a can be anything. First axion model by Weinberg et al had $f_a \simeq$ New.

Soon excluded because it would mediate a fast decay $K^+ \to \pi^+ a$.

stability: axions can be DM

Exercise: compute $\Gamma_{a\to\gamma\gamma}$

$$\rightsquigarrow \quad \Gamma_{a\to\gamma\gamma} = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi}$$

$$\tau = \Gamma^{-1} \simeq 1.3 \times 10^{43} \left(\frac{f}{10^{12}\,\mathrm{GeV}}\right)^5 \text{yrs}$$

$$\sim 10^{50}\ \mathrm{s}$$

Axion quality problem and other criticisms

Some extra care is needed: we wrote a $U(1)$ symmetric Lagrangian, we treat a as a Goldstone boson (i.e. a non-linear realization of the symmetry) but the symmetry was explicitly broken from the start by the anomaly.

All the discussion about the θ-term requires special care in handling the $V \to \infty$ limit. Some recent claims that this was not done properly.

ALPs : axion-like particles

Can we be more general? What are the key ingredients?

Similar particles can emerge from different physics models

- UV QFT constructions, as Goldstone bosons of some spontaneous symmetry breaking

- in string theory as the result of compactification of some extra dimension

$\Rightarrow$ axion-like particles defined by:

- potential

$$V(\phi) = -m^2 f^2 \cos\frac{\phi}{f} \qquad \left(\text{or } \Lambda^4 \cos\frac{\phi}{f}, \quad \Lambda^4 = m^2 f^2\right)$$

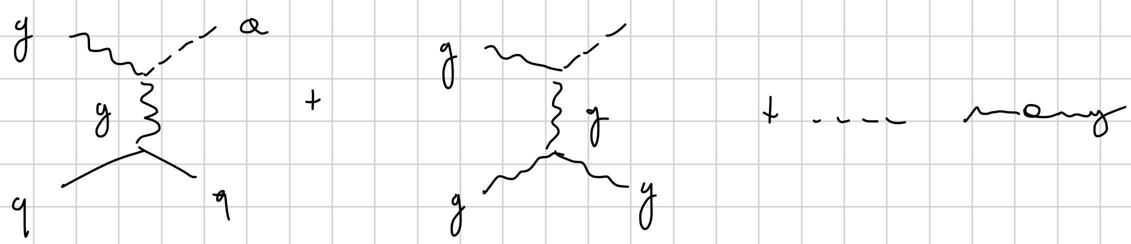
- no temperature dependence for simplicity

- couplings to SM same as for QCD axion, but w/ generic couplings

Axion Dark Matter (thermal axions)

Axions are neutral and stable. How can I produce them as DM?

scattering w/ the plasma



Even though it's freeze-in (not freeze-out) the distribution is almost thermal $\Rightarrow$ require $m_a \gtrsim \mathcal{O}(10\,\text{keV})$

For QCD axion

$$10\,\text{keV} \approx \mu\text{eV}\,\frac{10^{13}\,\text{GeV}}{f_a} \Rightarrow f_a \lesssim 10^3\,\text{GeV} \quad \text{excluded}$$

but it works for ALPs

Lifetime:

$$\tau = \frac{64\pi}{g_{a\gamma\gamma}^2\, m_a^3} \approx f_a^2\,\frac{64\pi}{m_a^3} \Rightarrow f_a \approx \frac{\tau\, m_a^3}{64\pi}$$

$$\tau \gtrsim 10^{28}\,\text{s} \qquad (\text{eg NuSTAR } 2207.04572)$$

$$\Rightarrow f_a \gtrsim 2.7\times10^{17}\,\text{GeV}\left(\frac{m_a}{10\,\text{keV}}\right)^{3/2}\left(\frac{\tau_{\text{limit}}}{10^{28}\,\text{s}}\right)^{1/2}$$

very much borderline

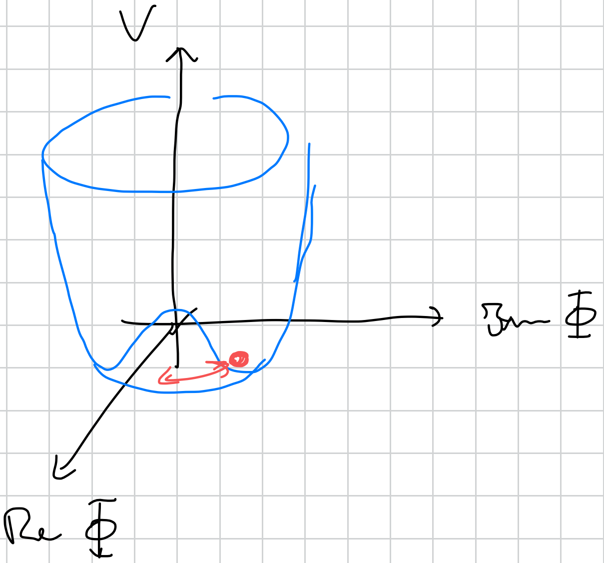
Decay of topological objects (cosmic strings)

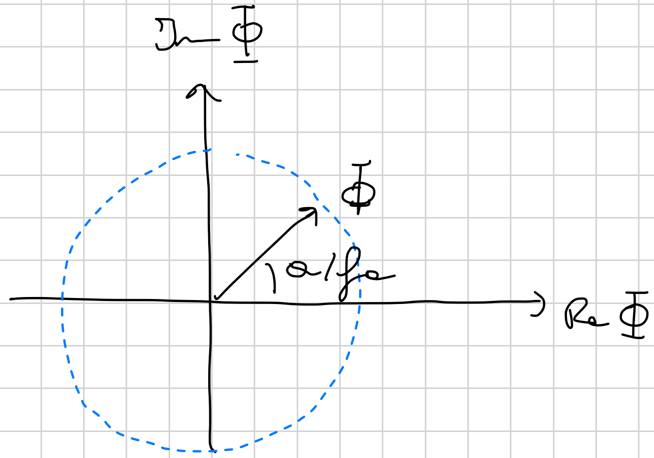
Treat the axion as a classical field (more about that later)

Suppose it can take many values within our visible horizon.

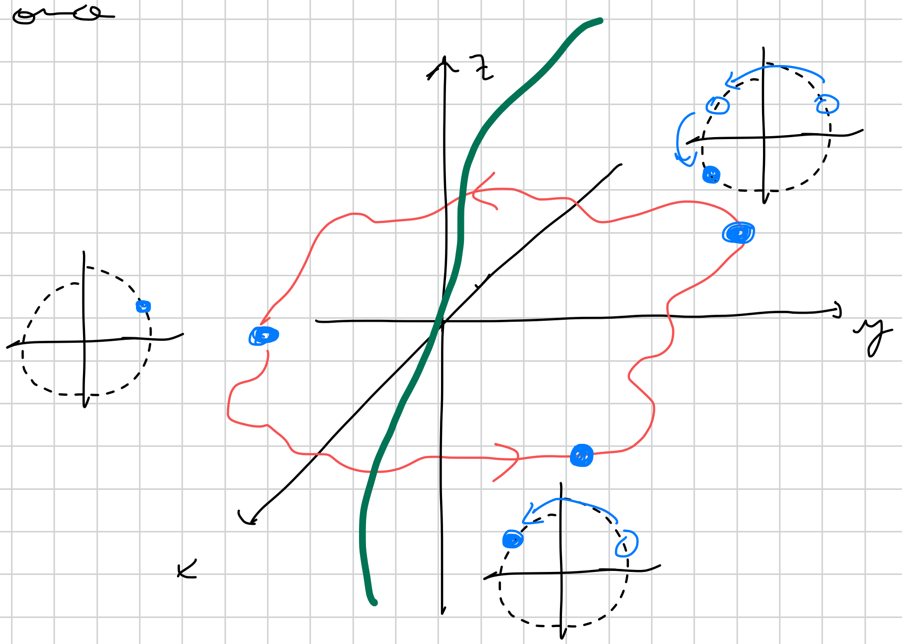
$$\mathcal{L} = (\partial_\mu \Phi)^\dagger \partial^\mu \Phi - \frac{\lambda}{4}\left(|\Phi|^2 - \eta^2\right)^2$$

$$\Phi \rightarrow (f_a + r)\, e^{i a/f_a}$$





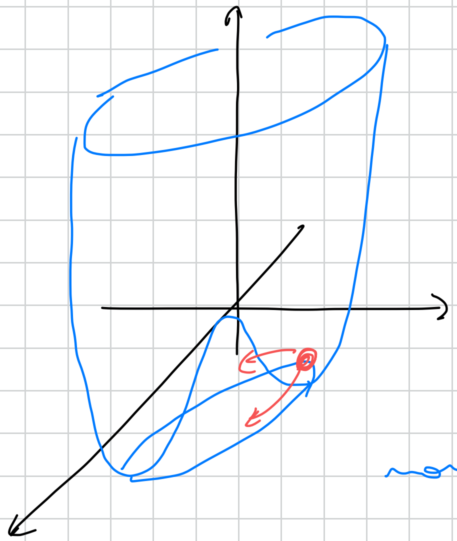
$\exists$ special configurations in which the field winds up at least once



There exists a manifold (a line) along which

$$\Phi = 0$$

"Cosmic string"



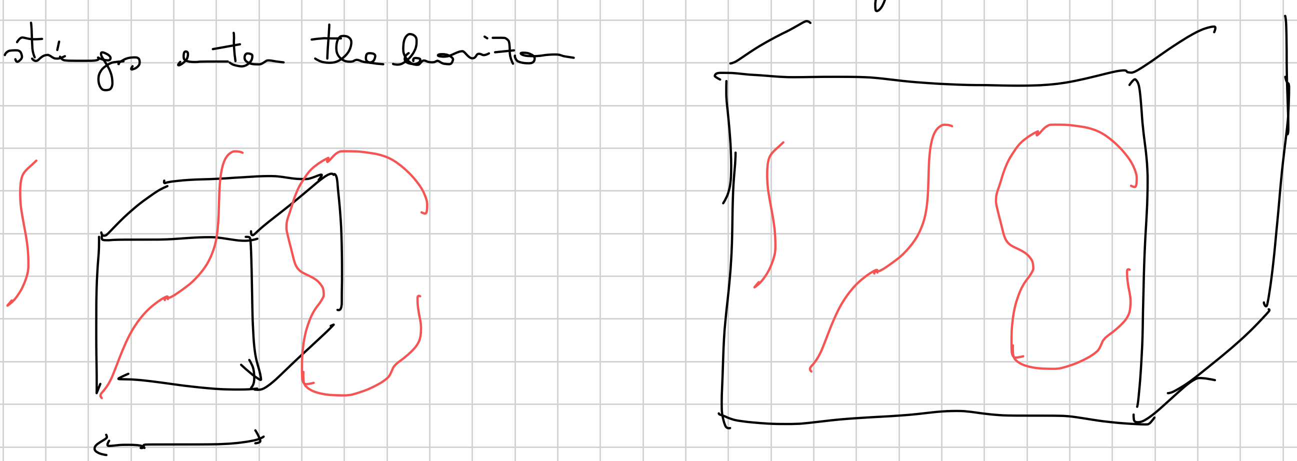
When the axion gets a potential the strings become unstable and decay into axion particles

These particles are very cold: it's a non-thermal, cold DM candidate

Abundance: depends on the abundance of strings in the Universe

Vague idea: string can interact, cut one another, close on closed paths which shrink and decay...

At the same time, as the horizon grows, more and more strings enter the horizon

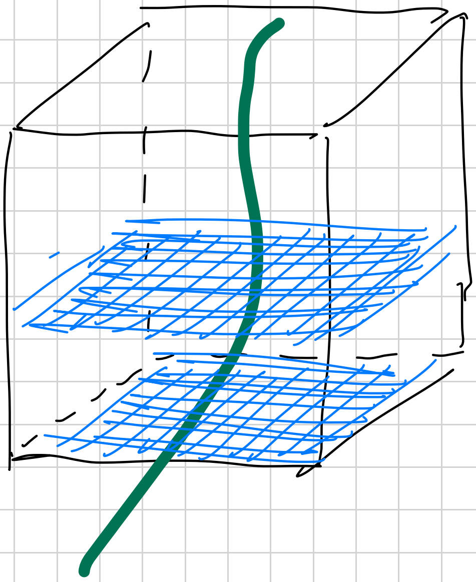


H^{-1}

$\Rightarrow$ attractor solution: ~ 1 string of length H^{-1} per Hubble volume at any time

Requires very difficult simulations involving two well separated scales $H \sim m_a$ and f_a (typically some 15 orders of magnitude apart)

Simulations = solve the classical eom on a discretized space time ("lattice")



The grid should ideally be $\ll$ string width

Huge computational problem

- can I work with a small hierarchy and extrapolate?

- let params vary to capture the axion dynamics at all times (fat string)?

Indications in the literature of a deviation from scaling solution. String axions are the dominant source of DM?

2007.04990

The existence of defects depends on whether the initial conditions are set before or after inflation

- before : no strings
- after : strings

Axion misalignment

Axions can be treated as a classical field when the state is significantly displaced from the vacuum

[Think of this fact as in electromagnetism: when $\vec{E}, \vec{B} \approx 0$ we talk of photons (quanta of quantum fields), when $\vec{E}, \vec{B}$ are large they behave classically following Maxwell's equations]

Start from the action of a minimally coupled scalar field in the expanding universe

change notation: ϕ axion, a scale factor

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right)$$

EoM are found by varying $\phi \to \phi + \delta\phi$ and imposing $\delta S = 0$

$$S \to \int d^4x \sqrt{-g} \left[\frac{1}{2} \partial_\mu (\phi + \delta\phi) \, \partial^\mu (\phi + \delta\phi) - V(\phi + \delta\phi) \right]$$

$$\delta S = \int d^4x \sqrt{-g} \left[\underbrace{\frac{1}{2} \partial_\mu \delta\phi \, \partial^\mu \phi \times 2}_{\text{by parts}} - V'(\phi) \, \delta\phi \right]$$

$$= - \int d^4x \, \delta\phi \left[\partial_\mu \left(\sqrt{-g} \, \partial^\mu \phi \right) + \sqrt{-g} \, V'(\phi) \right]$$

$$= - \int d^4x \sqrt{-g} \, \delta\phi \left[\frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \, g^{\mu\nu} \partial_\nu \phi \right) + V'(\phi) \right]$$

$$\equiv - \int d^4x \sqrt{-g} \, \delta\phi \left(\Box \phi + V'(\phi) \right)$$

(NB: $\Box = \nabla_\mu g^{\mu\nu} \nabla_\nu$)

D'Alembertian

Eq of motion: $\quad \Box\phi + V'(\phi) = 0$

In a FRLW universe

$$g_{\mu\nu} = \left(1,\ -a(t)^2,\ -a(t)^2,\ -a(t)^2\right)$$

$$\sqrt{-g} = a(t)^3$$

$$\Box\phi = \frac{1}{a^3}\left[\partial_0\left(a^3\,\partial_0\phi\right) + \partial_i\left(a^3\left(-\frac{1}{a^2}\right)\partial_i\phi\right)\right]$$

$$= \frac{1}{a^3}\left[a^3\,3H\dot\phi + a^3\ddot\phi - a\,\nabla^2\phi\right]$$

$$= \ddot\phi - \frac{1}{a^2}\nabla^2\phi + 3H\dot\phi$$

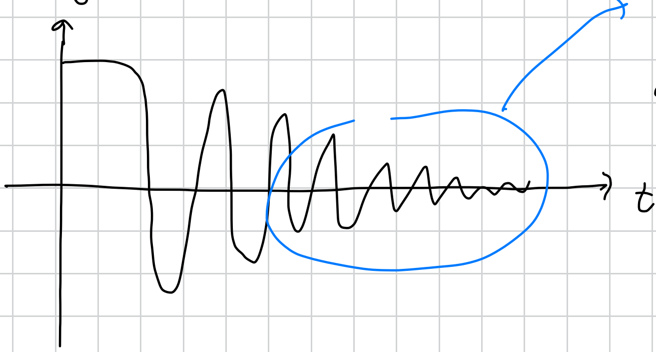
$$\Rightarrow \ddot\phi - \frac{1}{a^2}\nabla^2\phi + 3H\dot\phi + V'(\phi) = 0$$

Gradient: $\frac{1}{a^2}\nabla^2\phi$ decays w/ expansion $\Rightarrow$ can neglect
(perturbations are treated separately)

$$\ddot\phi + 3H\dot\phi + V'(\phi) = 0$$

Hubble friction: dissipate energy. Oscillations are damped.

Expect something like:



We quickly oscillate around the minimum $V(\phi) \approx \frac{1}{2}m_a^2\phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2 \phi = 0$$

H decreases with time, m_a is constant or increases.

ALP case $m_a = \text{const}$

During matter / radiation $a \propto t^p \rightarrow H = \frac{p}{t}$

exact solution $\phi = a^{-3/2} \left(\frac{t}{t_i}\right)^{1/2} \left(C_1 J_n(m_a t) + C_2 Y_n(m_a t) \right)$

(if ϕ is subdominant) $(n = (3p-1)/2)$

WKB solution: at first neglect $H\dot{\phi}$

$$\ddot{\phi} + m_a^2 \phi = 0 \quad \Rightarrow \phi = A \cos(m_a t)$$

$$\dot{\phi} = \dot{A} \cos(m_a t) - m_a A \sin(m_a t)$$
$$\ddot{\phi} = \ddot{A} \cos(m_a t) - 2 m_a \dot{A} \sin(m_a t) - m_a^2 A \cos(m_a t)$$

$$\ddot{A} c - 2 m_a \dot{A} s - \cancel{m_a^2 A c} + 3H\dot{A} c - 3H m_a A s + \cancel{m_a^2 A c} = 0$$

divide by $A m_a^2$ $\frac{\ddot{A}}{A m_a^2} c - 2 \frac{\dot{A}}{A m_a} s + 3 \frac{H\dot{A}}{m_a^2 A} c - 3 \frac{H}{m_a} s \simeq 0$

assumption: $\frac{\dot{A}}{A m_a} \sim \frac{H}{m_a} \sim \varepsilon$, $\frac{\ddot{A}}{A m_a^2} \sim \frac{H\dot{A}}{m_a^2 A} \sim \varepsilon^2$

at order ε: $\frac{\dot{A}}{A} = -\frac{3}{2} H = -\frac{3}{2} \frac{\dot{a}}{a} \Rightarrow A \propto a^{-3/2}$

$$\Rightarrow \phi(t) \approx \phi_{osc} \left(\frac{a_{osc}}{a}\right)^{3/2} \cos(m_a t)$$

Check: $\frac{H\dot{A}}{m_a^2 A} \sim \frac{H^2}{m_a^2} \sim \varepsilon^2$ $\frac{\ddot{A}}{A m_a^2} \sim$

For $m_a \ll H$ $\rightarrow$ $\ddot{\phi} + 3H\dot{\phi} = 0$ $\Rightarrow$ $\dot{\phi} \approx 0$ overdamped

Transition: around $3H(T_{osc}) = m_a(T_{osc})$

Energy density $\rho_\phi = \rho_K + \rho_V = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m_\phi^2\phi^2$

averaging over many oscillations $\langle \rho_K \rangle = \langle \rho_V \rangle = \frac{\langle \rho_\phi \rangle}{2}$

$\Rightarrow$ at late times $\langle \rho_\phi \rangle \propto A^2 \propto a^{-3}$ $\Rightarrow$ scales as dark matter

$$\frac{d\rho_\phi}{dt} = \dot{\phi}\ddot{\phi} + m_\phi^2\dot{\phi}\phi = \dot{\phi}\left(\ddot{\phi} + m_\phi^2\phi\right) = -3H\dot{\phi}^2 = -3H\rho_\phi$$

(continuity eq of matter)

Pressure $\langle P_\phi \rangle = \frac{1}{2}\langle \dot{\phi}^2 \rangle - \frac{1}{2}m^2\langle \phi^2 \rangle \approx 0$

Oscillating axions are good DM candidates!

QCD axions

Now we let the mass vary.

Oscillations start when

$$3H(T_{osc}) = m(T_{osc})$$

In rad dominance

$$g_*^{1/2}\frac{T_{osc}^2}{M_p} = m(T_{osc}) = m_0\left(\frac{\chi(1\,GeV)}{\chi(0)}\right)^{1/2}\left(\frac{GeV}{T}\right)^{\frac{\alpha}{2}}$$

$$\Rightarrow T_{osc} = 10^{\frac{6.35}{4+\alpha}}\,GeV\left(\frac{10^{12}\,GeV}{f_0}\right)^{\frac{2}{4+\alpha}}\left(\frac{g_*(5\,GeV)}{g_*(T_{osc})}\right)^{\frac{1}{4+\alpha}}\left(\frac{\chi(1\,GeV)}{\chi(0)}\right)^{\frac{1}{4+\alpha}}$$

$$\simeq 5.3\,GeV$$

$$m(T_{osc}) \approx 0.1\,\mu eV$$

Now solve the equation of motion

Define $\theta = \frac{\phi}{f}$: $\quad \ddot{\theta} + 3H\dot{\theta} + m^2\theta = 0 \qquad -\pi < \theta \leq \pi$

Assume Hubble friction is not efficient during a single oscillation

Given the energy

$$\rho = f^2\left(\dot{\theta}^2 + m^2\theta^2\right)/2$$

I have $\rho_\phi = f^2\langle\dot{\theta}^2\rangle = f^2 m^2\langle\theta^2\rangle$

I multiply the eom by $f^2\dot{\theta}$ and take a time average over a single oscillation

$$f^2\left\langle\left(\dot{\theta}\ddot{\theta} + 3H\dot{\theta}^2 + m^2\theta\dot{\theta}\right)\right\rangle =$$

$$= f^2\left\langle\frac{d}{dt}\left(\frac{1}{2}\dot{\theta}^2 + \frac{m^2}{2}\theta^2\right) - \frac{1}{2}m\dot{m}\theta^2 + 3H\dot{\theta}^2\right\rangle =$$

$$= f \left\langle \frac{d}{dt}\left(\frac{1}{2}\dot\theta^2 + \frac{m^2}{2}\theta^2\right) - \frac{1}{2} m\dot m \theta^2 + 3H\dot\theta^2 \right\rangle =$$

$$= \langle \dot\rho \rangle - \frac{1}{2} f^2 m^2 \frac{\dot m}{m} \langle \theta^2 \rangle + 3H \langle \dot\theta^2 \rangle =$$

$$= \dot\rho - \frac{\dot m_a}{m_a}\rho + 3H\rho = 0$$

$$\Rightarrow \dot\rho = -\left(3H - \frac{\dot m_a}{m_a}\right)\rho$$

$$\Rightarrow \rho_\phi = \text{const}\ \frac{m_\phi(T)}{a^3}$$

Define the "number of axions"

$$n_\phi = \frac{\rho_\phi}{m_\phi} = \frac{\text{const}}{a^3} \Rightarrow n_\phi a^3 = \text{const}$$ even when the mass vary!

$\Rightarrow$ with no entropy production the axion yield is constant

$$Y_\phi = \frac{n_\phi}{s} = \text{const}$$

$$= \frac{\rho_\phi}{m_\phi s} = \frac{\frac{1}{2} m_\phi^2 f_\phi^2 \theta_i^2}{m_\phi s} = \frac{m_\phi(T) f_\phi^2 \theta_i^2}{2s}$$

Abundance today

$$\rho_{\phi,0} = m_{\phi,0}\, n_{\phi,0} = m_{\phi,0} Y_{\phi,osc}\, s_0 = \frac{m_{\phi,0}^2 f_\phi^2}{2}\left(\frac{m_{\phi,osc}}{m_{\phi,0}}\right)\frac{s_0}{s_{osc}}\theta_i^2$$

$$= \frac{m_{\phi,0}^2 f_\phi^2}{2}\frac{3H(T_{osc})}{m_{\phi,0}}\frac{g_{*S,0}\,T_0^3}{g_{*S,osc}\,T_{osc}^3}\theta_i^2$$

$$\Rightarrow \Omega_\phi h^2 \approx 0.12 \quad \text{for} \quad f_\phi \approx 10^{16}\ \text{GeV} \quad \text{for}\ \alpha = 2$$

Alternative way (more powerful technique):

use an adiabatic invariant from Lagrangian mechanics

Define $\theta = \frac{\phi}{f}$: $\ddot{\theta} + 3H\dot{\theta} + m^2\theta = 0$ $\quad -\pi < \theta \leq \pi$

The eom can be derived from the Lagrangian

$$L = a^3\left(\frac{1}{2}\dot{\theta}^2 - \frac{1}{2}m^2\theta^2\right) \longrightarrow \frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = 0$$

Assume $H \ll m$ $\longrightarrow$ obvious, since oscillations are frozen before

$$\frac{\dot{m}}{m} \ll m \; : \; \frac{\dot{m}}{m} = \frac{1}{m}\frac{dm}{dT}\frac{dT}{dt} = -n\frac{1}{T}\frac{dT}{da}\frac{da}{dt} = nH$$

Define $p_\theta \equiv \frac{\partial L}{\partial\dot{\theta}} = a^3\dot{\theta}$

$$\Rightarrow I = \frac{1}{2\pi}\oint p_\theta \, d\theta$$

I is invariant in time as we go on with the damped oscillations

Let's compute I:

Consider a single oscillation. The energy, within one oscillation, is almost conserved and a is constant.

$$E = a^3\left(\frac{1}{2}\dot{\theta}^2 + \frac{1}{2}m^2\theta^2\right) = \frac{p_\theta^2}{2a^3} + \frac{1}{2}m^2\theta^2 a^3 = \frac{1}{2}m^2\theta_c^2 a^3$$

$$p_\theta = m a^3\left(\theta_c^2 - \theta^2\right)^{1/2}$$

$$\Rightarrow I = \frac{m a^3}{2\pi} \, 2 \times \int_{-\theta_c}^{\theta_c}\left(\theta_c^2 - \theta^2\right)^{1/2} d\theta$$

$$= \frac{m a^3}{\pi}\,\frac{1}{2}\pi\theta_c^2 = \frac{a^3 m \theta_c^2}{2}$$

$$\Rightarrow a_0^3 m_0 \theta_0^2 = a_{osc}^3 m_{osc} \theta_i^2$$

Initial conditions

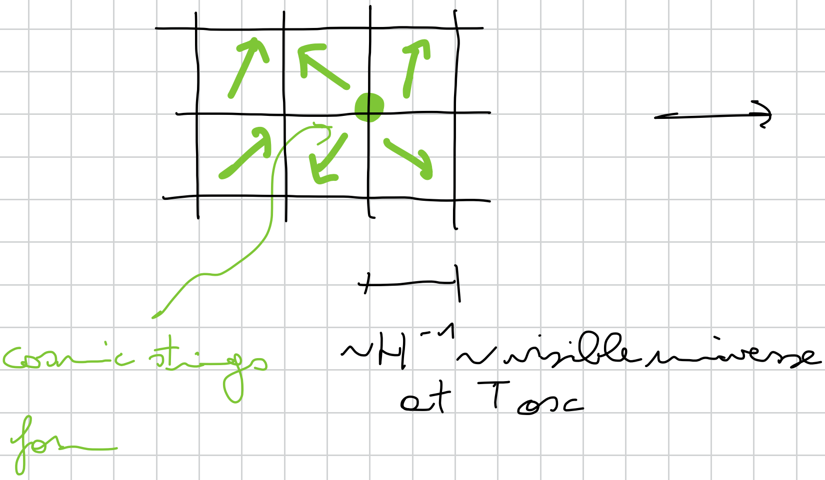
It all depends on when the PQ symmetry is spontaneously broken (ie $\Phi \to (f+r)\, e^{i\phi/f}$)

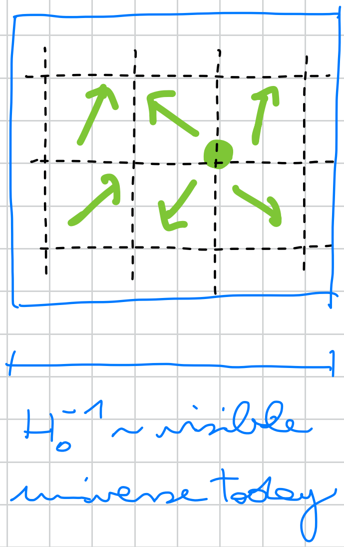
Remember that: Hubble patches exist as causally separated worlds, because light takes $\Delta t \sim H^{-1}$ to travel $\Delta x \sim H^{-1}$.

- post-inflationary breaking:

$$H_I > f \quad \text{and} \quad T_{rh} > f$$

In each patch SSB proceeds independently





$\frac{\phi_i}{f}$ assume all values in the range $(0, 2\pi)$. Thus, I take an average of θ_i^2:

$$\theta_i^2 \to \langle \theta^2 \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} \theta^2 d\theta = \frac{1}{2\pi} \frac{\pi^3}{3} 2 = \frac{\pi^2}{3}$$

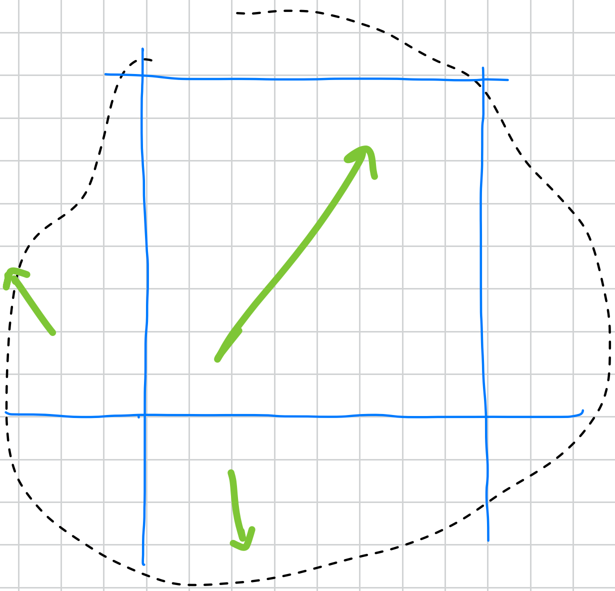
NB: we have approximated $|\theta| \ll \pi$, but we average over the full range. One can be more precise and get a $\mathcal{O}(1)$ correction.

- breaking before / during inflation

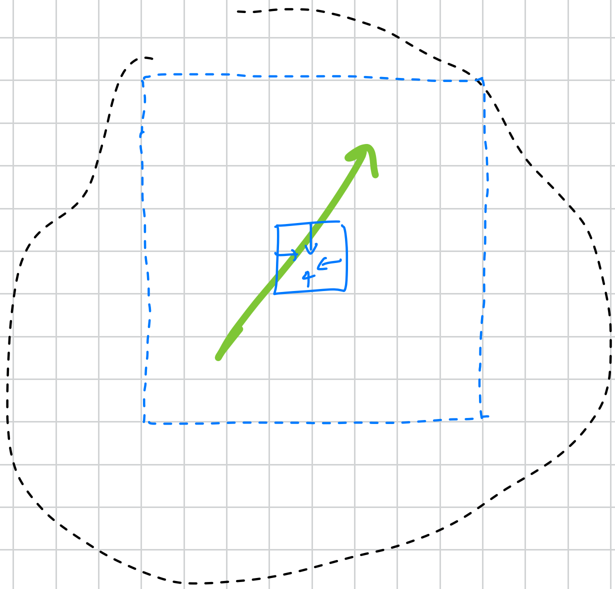
As before, each Hubble patch has a different value of ϕ.

This time, patches are inflated outside the Hubble horizon (remember: during inflation the physical horizon $(aH)^{-1}$ shrinks)

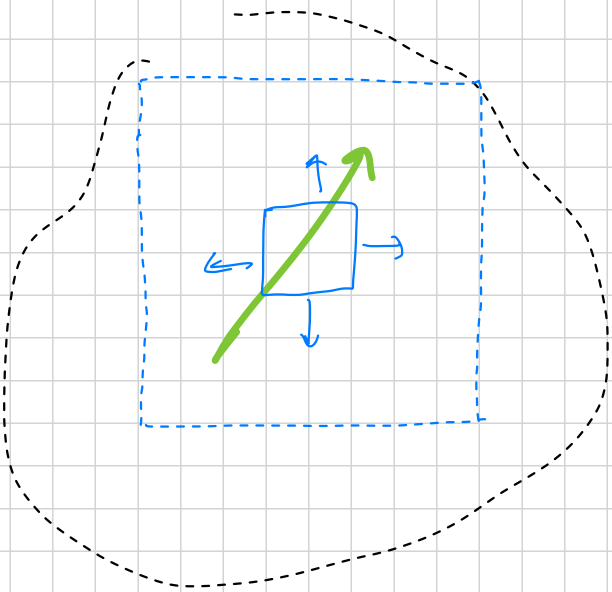
When the horizon grows again after inflation, ϕ is the same everywhere



before inflation



during inflation: horizon shrinks



after inflation: horizon grows again, but slowly

$\frac{\phi_i}{f}$ is constant everywhere

$\Rightarrow$ randomly distributed $(-\pi, \pi)$

- if $\theta_i \ll 1$ I can tune the abundance to be small
- if $\theta_i \approx \pm\pi$ I can delay the start of oscillation for some time (increase the final abundance!)

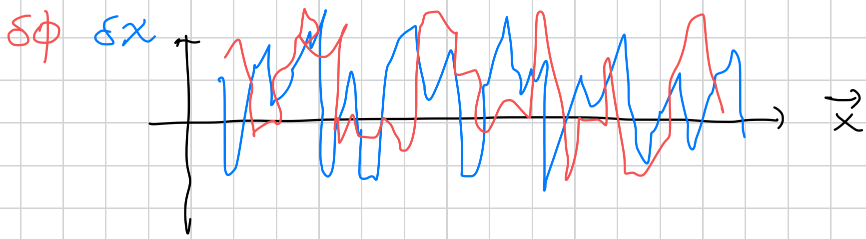
One quick word on isocurvature perturbations:

If PQ is broken during inflation, the axion exists as a light field and obtains some random fluctuations during inflation

Statistically speaking, each Hubble time one gets a "kick" with st. dev.

$$\sqrt{\langle \delta\phi^2 \rangle} = \frac{H_I}{2\pi}$$

The same happens to the inflaton χ, independently. At the end of inflation, ϕ and χ have different distributions



The correct quantity that measures this difference is

$$\frac{\delta\phi}{\dot\phi} \neq \frac{\delta\chi}{\dot\chi}$$

χ decays to the thermal bath $\Rightarrow$ $\delta\chi \leftrightarrow \delta T$. All particles in the thermal bath share the same temperature thus

$$\frac{\delta\rho_i}{(1+w_i)\,\bar\rho_i} = \frac{\delta\rho_j}{(1+w_j)\,\bar\rho_j} \neq \frac{\delta\rho_\phi}{(1+w_\phi)\,\rho_\phi}$$

There exist strong limits on these differences (isocurvature perturbations)

Axion searches (main reference: 1812.02669)

Two broad categories:

DM independent: the axion is just another particle in the spectrum, and I look for it through its couplings to SM

Axion DM searches: I exploit the fact that I have a large classical background axion field and look for its influence on other stuff in the lab, typically a large EM field.

DM independent searches

1.) rare meson decays

axion - meson mixing + $a\gamma\gamma$ coupling

relevant for $m_a \lesssim 100\,\text{MeV}$ (light compared to the meson)

$f_a \lesssim 10^4\,\text{GeV}$

2.) stellar cooling

$f_a \lesssim 10^7 - 10^8\,\text{GeV}$ and $m_a \lesssim 100\,\text{keV}$ ($\sim T_{core}$)

the axion can be produced in the star and escape, draining some of the energy.

eg $e + N \to e + N + a$, $e + \gamma \to e + a$

3) Supernovae:

$$10^6\,\text{GeV} \leq f_a \lesssim 10^8\,\text{GeV} \qquad m_a \lesssim 100\,\text{MeV} \quad (\sim T_{core})$$

impose that the energy carried away by axions is $\lesssim$ than that into neutrinos

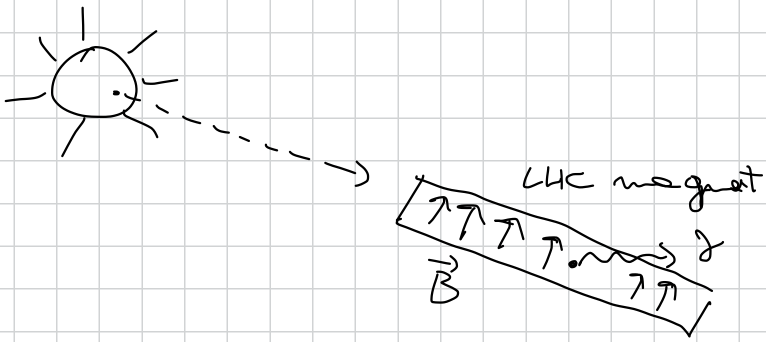
For small f_a axions do not escape

4) Axion helioscopes (CAST, (baby-) IAXO)

$$g_{a\gamma\gamma}\, a F\tilde{F} \sim g_{a\gamma\gamma}\, a\, \vec{E}\cdot\vec{B}$$

in a very large external $\vec{B}$ field, this is a mixing term for axions and photons.

⇒ look for photons generated inside the experiment from converting some axion produced in the Sun.



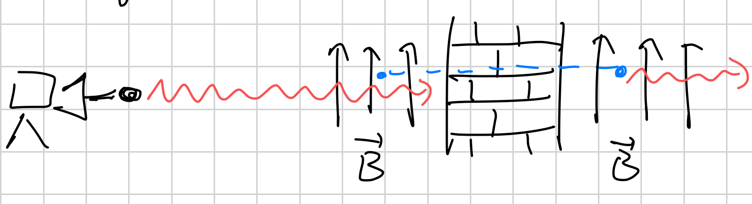
$g_{a\gamma\gamma} \lesssim 10^{-10}\,\text{GeV}^{-1}$

for $m_a \lesssim \text{eV}$

5) Similar idea: light shining through walls

$$g_{a\gamma\gamma} \lesssim 10^{-7}\,\text{GeV}^{-1} \qquad \text{for } m_a \lesssim 10^{-3}\,\text{eV}$$

Shoot a laser at a wall w/ a strong magnetic field. Some γs converted to axions and traverse the wall. Convert back to γs after the wall



6) polarization: axions mix only with the photon's mode perpendicular to the $\vec{B}$ field. A laser travelling through a region w/ a $\vec{B}$ field can have its polarization messed up

7) BH superradiance

Spinning BHs have a region (ergosphere) in which, if an object falls in, the object is accellerated and thrown away.

If axions exist, they can use this to subtract energy from the BH. The existence of fast spinning BHs constrains the existence and properties of axions.

8) Fifth force: axions mediate the following forces

* if $\bar{\theta} \neq 0$, $\quad \bar{\theta}\, \psi \bar{\psi}\, a \frac{m_\psi}{f_a} \Rightarrow$ new Yukawa force $\sim \frac{\bar{\theta}^2}{r^2}$
* $\frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma_5 \psi \Rightarrow \frac{1}{f^2 r^4}$ spin-dependent force
* cross-term $\Rightarrow \frac{\bar{\theta}}{f_a r^3}$

9) $\bar{\theta} = \pi$ inside NS

For light axions, the potential flips at finite density
$\Rightarrow \bar{\theta} = \pi$ inside a NS, 0 outside

Affect NS mergers measured by LIGO.

10) $\bar{\theta} = \pi$ inside the Sun. Also excluded

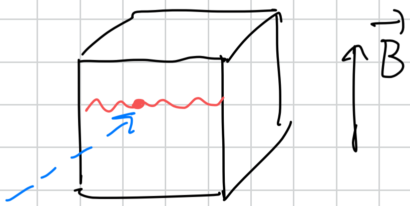
Axion DM searches

1) Astrophysical probes : DM - γ conversion in astrophysical magnetic fields

2) Haloscopes (ADMX)

DM converting into photons in a resonating cavity with magnetic fields.

Cavity modes' frequencies have to match m_a



w/ cavities (ADMX)

or dielectrics (MADMAX)

3) Similarly, axions can act as a current

$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J} + \frac{\dot{a}}{f} \vec{B}$$

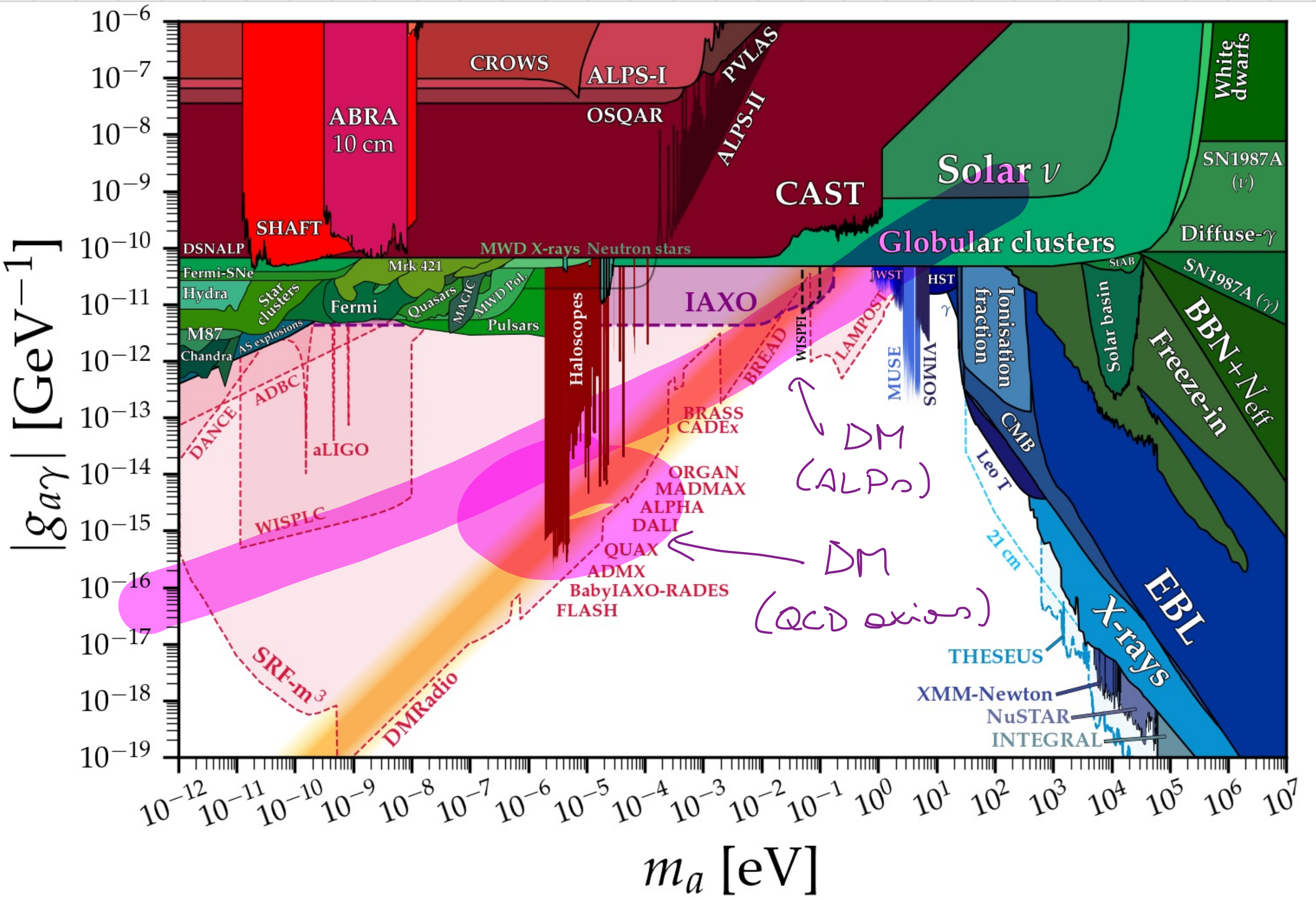
$\Rightarrow$ generates $\vec{B} \neq 0$ outside solenoids

4) Nuclear magnetic resonance

oscillating $a \sim \bar{\theta} \neq 0 \sim$ electric dipole $\neq 0$

$\Rightarrow$ nuclei spins oscillate in external $\vec{E} \neq 0$ (precess)

Limits on axion-photon couplings



https://cajohare.github.io/AxionLimits/

(visited: 05/2024)