Classical predicate logic

A brief reminder

- Propositional logic is not enough to account for all valid arguments in natural language.

- For example:

  All humans are mortal,

  Socrates is human,

  therefore, Socrates is mortal.

  Cannot be accounted for in propositional logic (Try!)

- To cover these arguments a richer language is needed.

- We need a language able to analyse the internal structure of sentences (beyond just P or Q) and capturing quantifiers such as “all” and “some”.

- This is done by predicate logic.

  In what follows a quick review of predicate logic is given.

The formal language of classical predicate logic

- The classical predicate language L1 is given by extending the classical propositional language L.

- We extend: the vocabulary, the syntax, the semantics of the propositional language L.

Vocabulary
(extension)

- We have two kinds of new expressions:

- **Individual constants**: p,q,r,s,… (like names of individuals)
- **Individual variables**: x,y,z, … (like names of unspecified individuals)

- **N-place predicates**: P(…), Q(…), … (like expressions for properties and relations)

- **Quantifiers:**

∀ (The universal quantifier: *For all*)

∃ (The existential quantifier: *For some/There exists at least one*)

Syntax of L1

(Extension)

- The syntax of L1 is obtained by adding the following clauses to the propositional syntax.

1.

If A( ) is a predicate symbols with n-places, then writing n individual constants or variables, separated by commas, in A( ), what is obtained a WFF.

2.

If A is a WFF with no occurrences of $\forall v$ or $\exists v$ for the variable v,

Then $\forall vA$ and $\exists vA$ are WFF.

- The syntactic notions are defined as in L.

- An additional notion is that of **free variable** and **bound variable**.

- A variable v is free in wff if it is not in the range of a quantifier.

  It is bounded otherwise.

- A wff is **closed** if it no free variables occur in it.

  Otherwise the formula is **open**.

  (Note: these definition are intended to be intuitive reminder, but they are not fully rigorous. For full definition I refer to standard textbooks introducing predicate logic).

- Closed wffs are also called **sentences**.

- We only consider sentences (closed formulas) from now on.

Semantics

- The semantics for the predicate language is much more complex than the semantics for the propositional language.

- In particular, truth tables do not extend to the new expressions (predicates, individual constants, variables, quantifiers).

- To fix these a more complex semantics, based on the notions like that of domain, interpretation, assignment, satisfaction, model, … must be introduced.

- Here, however, we just give an intuitive semantic reading for the new expressions, and then turn to semantic trees.

- Notice that, unlike truth tables, refutation trees can be easily extended to the language of predicate logic.

→ And also to modal predicate logic.

We thus follow that route.

- Intuitively, the domain is a set of objects. The set of objects we intende to speak about.

- Interpretations give meaning relatively to that set of objects.

- **Individual constants** are given a meaning by associating to every constant an object of the domain.

- **Individual variables** are assigned a “temporary” and variable objects by various assignments.

→ A complete treatment would be needed to give rigorous semantic clauses for quantifiers.

- A **predicate symbol** with n individual constants is read as meaning that a certain relation hold among those n individuals.

  For example P(t,r) means that t and r are in the relation P.

  (e.g. if P stands for “loves”, t loves r).

- For simplicity, we limit our treatment to unary predicate symbols ascribing a property to certain objects.

- For example, Q(r) means that r is Q.

- $\forall x Q(x)$ means that all objects x are Q

- $\exists y R(y)$ means that at least one object is R

- In complete treatment this should be phrased in terms of assignments of values to individual variables.

- The details are not obvious and we skip them here.

- Note that:

  All humans are mortal can be translated as:

  $\forall y(H(y) \rightarrow M(y))$ ∀ ∃

  Namely: for all y, if y us human, then y is mortal.

  With a **conditional**!

Some humans are mortal can be translated as:

$\exists y(H(y) \& M(y))$

Namely: there is at least one y, such that y is human and y is mortal.

With a **conjunction**!

- For other notable translations, such as *no human is mortal*, see an introduction to predicate logic.

Semantics

Appendix

Note:

What follows is an additional appendix not required for the course on modal logic, and just put as material for the interested reader.

- A **little** more rigorously, the semantics for the predicate language could be given as follows.

- For predicative logic, we need a more complex structure (also called a model) consisting of:

1. A domain D and
2. An interpretation I.

- D is a set of element.
- It is intended to provide the things we talk about.
- I is the semantic interpretation that gives meaning to the expression of L1.

  I interprets L1 in D.

- In particular, for the basic vocabulary:

  I gives an element to each individual constant.

  I gives a set of elements to unary predicates (the set of things that have the intended property)

- Then we can define **truth in the model M** = (D,I) as follows. The definition is inductive:

  Base of induction:

  An atomic sentence, like P(a), is true in M if the element given to a by I is in the set given to P( ) by I.

Inductive steps:

- Propositional combinations of formulas follows truth tables.

  (so A&B is true in M iff A is true in M and B is true in M)

- For the quantifiers, we have that

  $\forall vA$ is true in M, if everything in M has the property A.

  $\exists vA$ is true in M, if something in M has the property A.

- With the notion of truth in M, we can define logical validity as usual.

- An argument is logically validity when:

  if the premises are true in every model, the conclusion is true in every model.

- Notice that the above is not the fully rigorous treatment, which require the notion of satisfaction and assignment of values to variables.

- For such a treatment, refer, again, to a textbook in predicate logic.

Semantics for modal predicate logic

- The semantics works like in predicate logic, interpreting and defining truth in a model (M = D,I).

- Additionally however, M specifies a set W, the set of “possible worlds” of M,

  one of which is designated its “actual world”,

  and each world w in W is assigned its own domain of quantification, **d(w) included in D (**intuitively,the set of individuals that exist in w).

- For each basic expression (constant and predicate) we have an **extension**, relative to each world,

  and an **intension**. (the function that assigns the extension).

- The definition of truth is the same of predciate logic but relativized to possible worlds.

  So we have **Truth in M, relatively to a world w**.

- For example, an atomic sentence like P(a) is true in M, in world w iff

  the extension of a in w belongs to the extension of P( ) in w.

- The clauses for connective remains the same.

- For quantifiers, we need to specify that quantifiers are limited to a single world, and they do not run on the entire domain (given by the union of all elements in all possible worlds).

$\forall vA$ is true in M,w, if everything in w has the property A.

$\exists vA$ is true in M,w if something in w has the property A.

- The clauses of modal operators also remains the same, just relativized to possible worlds.

□P is true in M,w if and only if p is true in all possible worlds.

◇P is true in M,w if and only if p is true in some possible worlds.

- We can then define logical validity, as usual just relativized to possible worlds.

De dicto and *de re*

- An application of the possible world analysis concerns the venerable distinction between *de re* and *de dicto* modality.

- Among the strongest modal intuitions is that the possession of a property has a modal character — that things exemplify some properties necessarily, or essentially, and others only accidentally.

- We can say that an individual a has a property P essentially if a has P in every world in which it exists.

Algol is a dog essentially: $\Box G(a)$

Algol is a pet accidentally: $Ta \,\&\, \Diamond\neg T(a)$

Sentences like these, in which properties are ascribed to a specific individual in a modal context, signaled formally by the occurrence **of a name or the free occurrence of a variable** in the scope of a modal operator, are de re.

- Often, *de re* statements are identified, more strictly, with those in which there is quantification in the scope of modal operator.

  So, there is a free variable in the scope.

- These are easily obtained by existential generalizations, even if individual constants are involved.

  So, this sense includes the former.

- Necessarily, all dogs are mammals:

  $\Box\,\forall x(D(x) \rightarrow M(x)$

  $\Box\,(PvQ)$

  Where in the scope of the modal operators only bound variables, at most, occur are *de dicto*.

- Note that propositional modal logic only expresses *de dicto* modalities.

- Sometimes, de re statement can be derived from *de dicto*, by logical steps, in modal predicate logic.

- This is a problem for those, like Quine, who rejects de re statements.

  Indeed, Quine only accepts modal propositional logic at most.

- Modality *de re* involves in addition a commitment to the meaningfulness of ***transworld identity***.

  The thesis that, necessarily, individuals exist and exemplify (often very different) properties in many different possible worlds.

- More specifically, basic possible world semantics:

  (i) permits world domains to overlap

  (ii) assigns intensions to predicates, thereby, in effect, relativizing predicate extensions to worlds

- Both features are absent in Lewis concretism.