Modal Logic 1

Preliminaries and basic terminology

- **Modal Logic:**

  - Axiomatic at the beginning of the last century
    (C.I. Lewis)

  - ‘50 – ‘60, possible worlds semantics
    (Kripke, Marcus, Hintikka)

- Propositional logic → Predicate logic → Modal logic

All men are mortal

Socrates is man.

Therefore, Socrates is mortal.

→ Not valid in propositional logic. Predicate logic is needed.

→ But for other good arguments, also predicate logic is not enough, and modal logic is needed...

Example:

- If a new course is to be offered next year, then submissions **must** be made to the Faculty Board before April.

- If submissions are to be made to the Faculty Board before April, then a Departmental meeting **must** be called.

- A week's notice **must** be given, if a Departmental meeting is

to be called.

- Since it is **not possible** to give such notice,

It follows that

- It is **not possible** to offer a new course next year.

Possibility and necessity

- It is **possible** that p =

  *there is **at least one** possible world in which p is true*

- It is **necessary** that p =

  ***in all** possible worlds p is true*

- We consider statements of the form:

It is possible that p ---> ◊p (also Mp)

It is necessary that p ---> □ p (also Lp)

Modal Equivalences (MN)

1. -◊p <=> □ -p
2. -□ p <=> ◊ -p

3. -◊- p <=> □ p
4. -□- p <=> ◊p

- Check that these are correct using your intuitions and possible worlds.

- They are similar to quantifiers.

Other modal expressions

*p is **Contingent*** = (◊p & ◊-p) (also ∇p)

*p is **Analytic*** = (□ p V □ -p)

→ Tautologies (analytic truths) should be modally valid.

*p is **Contradictory*** = □ -p

*p and q are **Compatible*** = ◊ (p & q) (also p o q)

*p and q are **Incompatible*** = - ◊ (p & q)

Translations

- *It is necessary that we study*.

  we study = P

  It is necessary that = □

  → Translation: □ P

- *It is not possible to eat and it is necessary to drink*.

  Translation:

  -◊Q & □R

More difficult:

*“**If** the course is important, **then** you **must** attend it.”*

- “The course is important” = N
- “you attend it” = S
- “must” = □

There are two possible translations:

- 1. $N \rightarrow \Box S$
- 2. $\Box (N \rightarrow S)$

- Different logical forms. Different results.

- Often the second reading is the intended one.

- In natural language it is not always clear which one is needed.

→ Some practice is required to learn how to translate.

Strict implication

- C.I. Lewis proposed an implication that captures the notion of “**necessarily** follows”

Strict implication:

$\Box (A \rightarrow B)$

Kind of modalities/logics

- **Alethic (truth) / Objective modality → Modal logic**

- **Other modalities:**

  Implication → Conditional logic

  Knowledge → Epistemic logic

  Belief → Doxastic logic

  Time → Temporal logic

  Obligation → Deontic logic

  Change → Dynamic logic

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