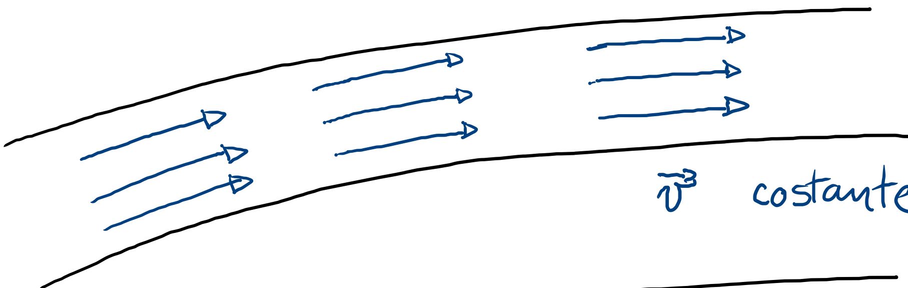
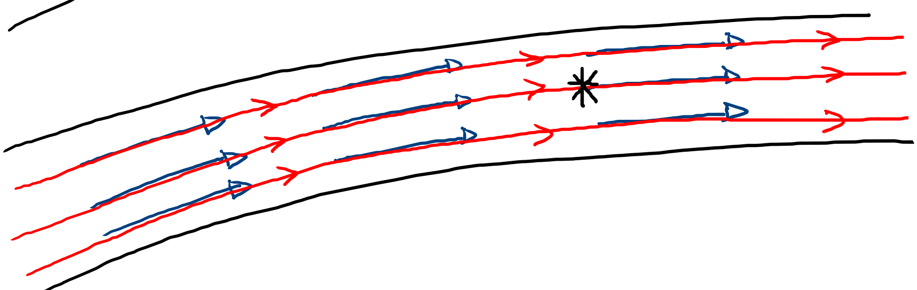
FLUIDODINAMICA

- LIQUIDO IDEALE $\begin{cases} \rho = \text{cost} & \text{(incomprimibile)} \\ \eta = 0 & \text{(senza attrito)} \end{cases}$

- FLUSSO $\begin{cases} \text{STAZIONARIO} & (\vec{v} \text{ in ogni punto è costante}) \\ \text{IRROTAZIONALE} & \vec{\nabla} \cdot \vec{v} = 0 \quad \text{(divergenza nulla NO VORTICI!)} \end{cases}$

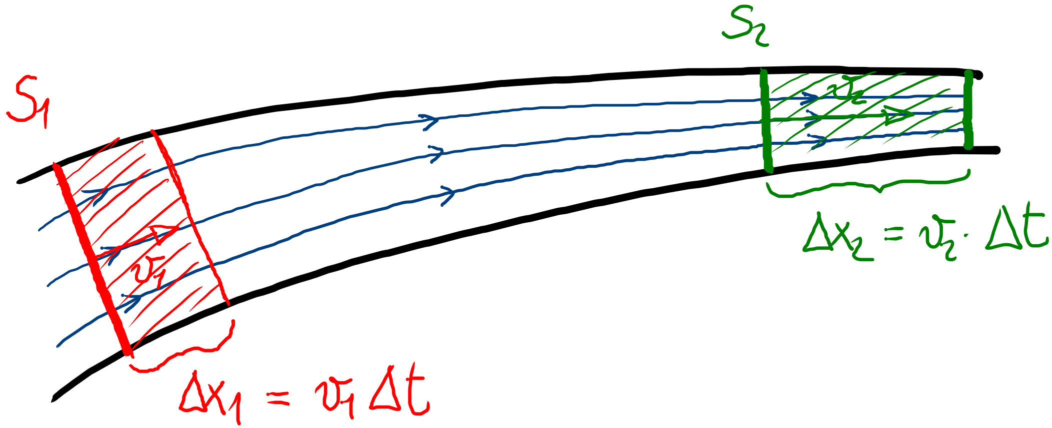


$\vec{v}$ costante nel tempo



linee di flusso

EQUAZIONE DI CONTINUITÀ (TEOREMA DI LEONARDO)



Δt

In Δt, Nel volume tra S_1 ed S_2 è entrato: $S_1 \cdot \Delta x_1$

è uscito: $S_2 \cdot \Delta x_2$

$$\Rightarrow \quad S_1 \cdot \Delta x_1 = S_2 \cdot \Delta x_2$$

$$S_1 v_1 \cancel{\Delta t} = S_2 \cdot v_2 \cdot \cancel{\Delta t}$$

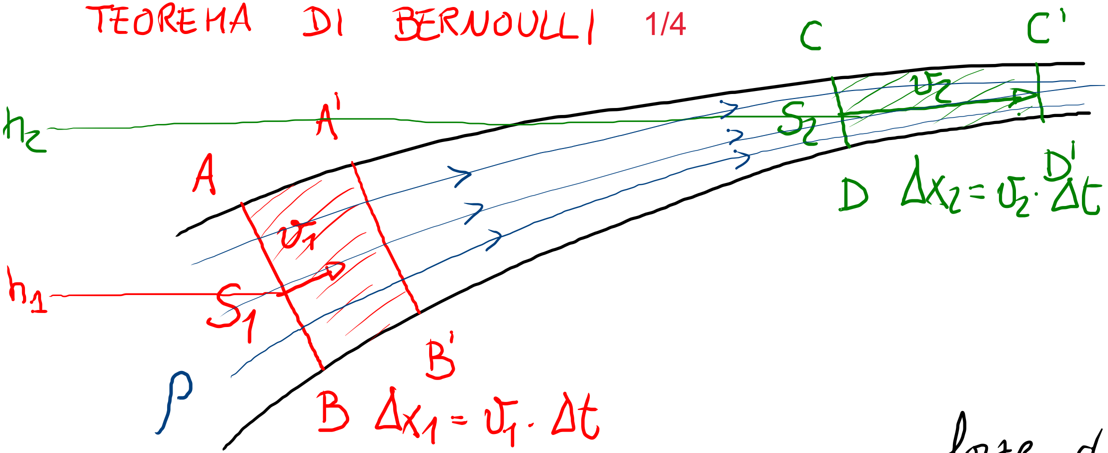
$$\boxed{S_1 v_1 = S_2 v_2}$$

$$Q = S \cdot v \qquad \text{portata in volume}$$

$$[Q] = m^2 \cdot \frac{m}{s} = \frac{m^3}{s}$$

$$\boxed{Q_1 = Q_2}$$

TEOREMA DI BERNOULLI 1/4



Δt

$\downarrow \bar{g}$

forze di pressione

ABCD : p_1 agisce su S_1 $\Rightarrow$ $F_1 = p_1 \cdot S_1$ favorisce

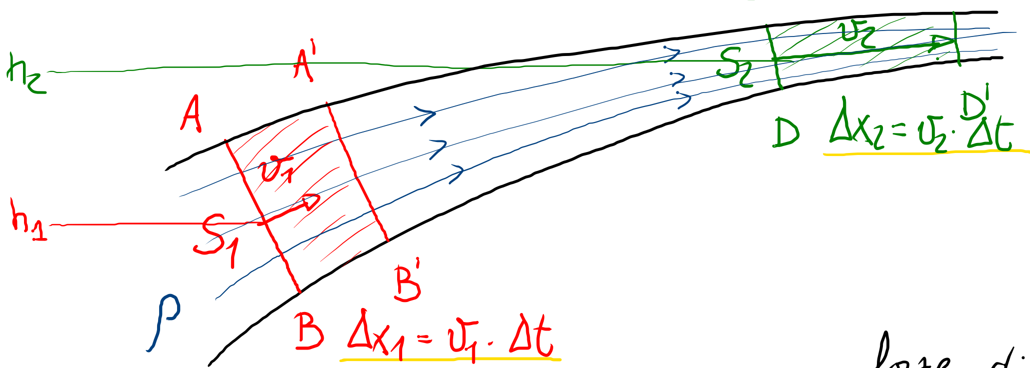
p_2 agisce su S_2 $\Rightarrow$ $F_2 = p_2 S_2$ si oppone

$\mathcal{L} = \Delta K$ teorema lavoro-energia

$\mathcal{L}_p + \mathcal{L}_g = \Delta K$ $\qquad$ $\mathcal{L}_g = -\Delta U_g$

$$\boxed{\overset{\mathrm{I}}{\mathcal{L}_p} = \overset{\mathrm{II}}{\Delta U_g} + \overset{\mathrm{III}}{\Delta K}}$$

TEOREMA DI BERNOULLI 2/4



Δt

$\bar{g}$

forze di pressione

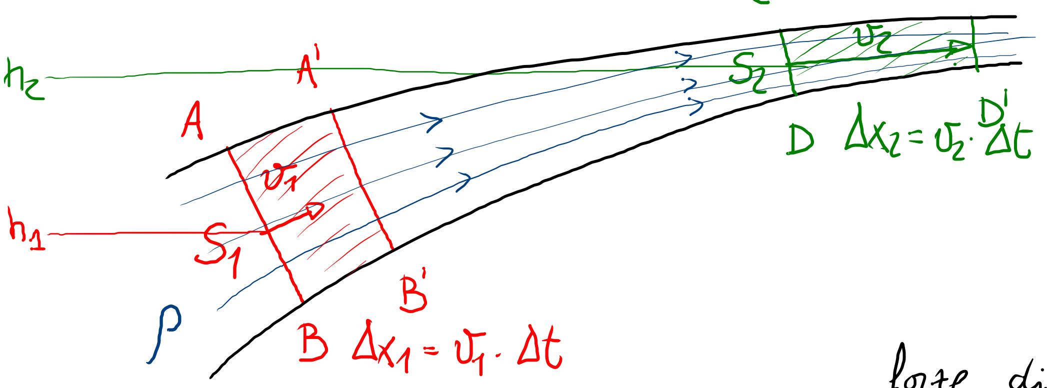
ABCD : p_1 agisce su S_1 $\Rightarrow$ $F_1 = p_1 \cdot S_1$ favorisce $\oplus$

p_2 agisce su S_2 $\Rightarrow$ $F_2 = p_2 S_2$ si oppone $\ominus$

$$\text{I} \quad \mathcal{L}_p = F_1 \cdot \Delta x_1 - F_2 \Delta x_2 = p_1 S_1 v_1 \Delta t - p_2 S_2 v_2 \Delta t = p_1 V - p_2 V = (p_1 - p_2) \cdot V$$

$$S_1 v_1 = S_2 v_2$$

$$S_1 v_1 \Delta t = S_2 v_2 \Delta t \equiv V$$



Δt

$\downarrow \bar{g}$

forze di pressione

$$V \equiv V_1 = S_1 \cdot \Delta x_1 = S_1 v_1 \Delta t = S_2 v_2 \Delta t = S_2 \cdot \Delta x_2 = V_2$$

II
$$\Delta U_g = U_g^{A'B'C'D'} - U_g^{ABCD} = U_g^{CDD'C'} - U_g^{ABB'A'} = \rho V g h_2 - \rho V g h_1$$

III
$$\Delta K = K^{A'B'C'D'} - K^{ABCD} = K^{CDD'C'} - K^{ABB'A'} = \frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2$$

Teorema di Bernoulli 4/4

$$\underset{\mathrm{I}}{\mathcal{L}_p} = \underset{\mathrm{II}}{\Delta U_g} + \underset{\mathrm{III}}{\Delta K}$$

$$(p_1 - p_2)\cancel{V} = \rho \cancel{V} g (h_2 - h_1) + \frac{1}{2}\rho \cancel{V}(v_2^2 - v_1^2)$$

$$\boxed{p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2}$$

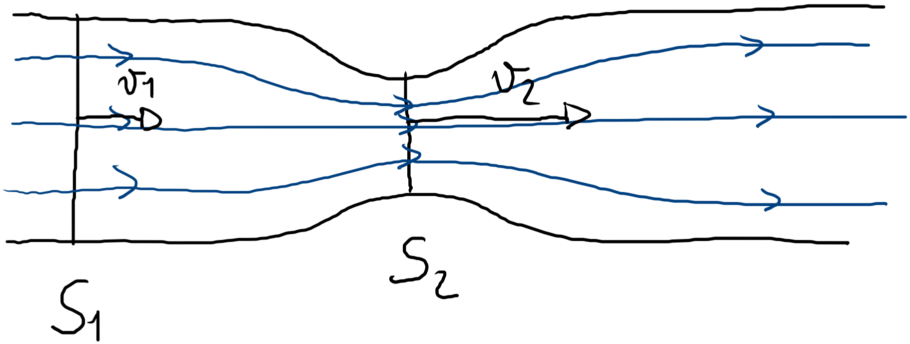
$$\boxed{p + \frac{1}{2}\rho v^2 + \rho g h \quad \text{è la stessa in ogni punto}}$$

$$p \to \frac{N}{m^2}\cdot\frac{m}{m} \to \frac{J}{m^3} \qquad \underbrace{\frac{1}{2}\rho v^2 \to \frac{J}{m^3} \qquad \rho g h \to \frac{J}{m^3}}_{}$$

densità di energia = $\dfrac{\text{energia}}{\text{volume}}$

→ anche la p è densità di energia

TUBO DI VENTURI



$S_1 v_1 = S_2 v_2$

$v_2 = \underbrace{\frac{S_1}{S_2}}_{>1} \cdot v_1$

$v_2 > v_1$

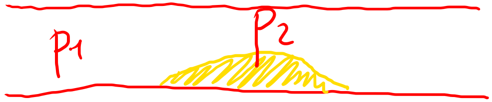
$$p_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g h_1} = p_2 + \frac{1}{2}\rho v_2^2 + \cancel{\rho g h_2}$$

$h_1 = h_2 = h$

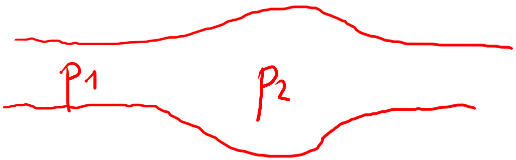
$$p_1 - p_2 = \underbrace{\frac{1}{2}\rho\left(v_2^2 - v_1^2\right)}_{>0}$$

perché $v_2 > v_1$

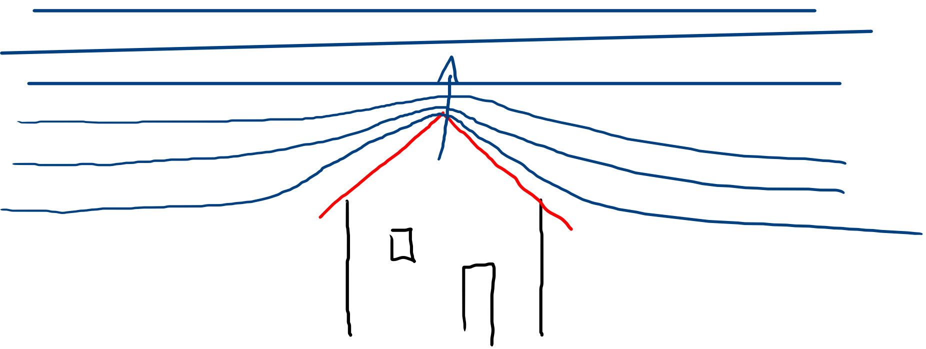
$p_1 > p_2$

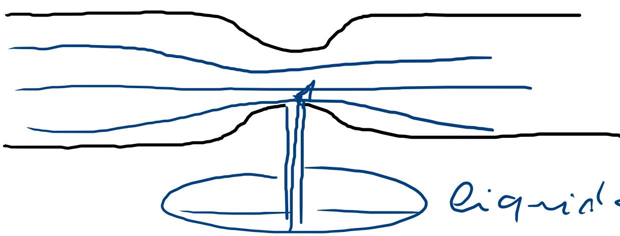


$p_2 < p_1$ stenosi



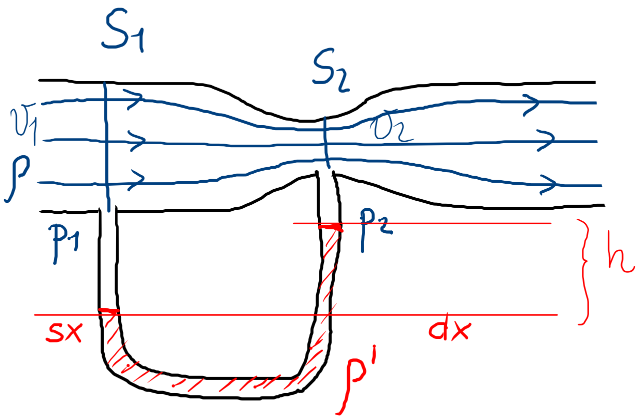
$p_2 > p_1$ aneurisma





liquido

VENTURIMETRO
(misura la velocità del flusso di un liquido)



Leo. $S_1 v_1 = S_2 v_2 \qquad v_2 = \frac{S_1}{S_2} v_1$

Bern. $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

$$p_1 - p_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left[\left(\frac{S_1}{S_2}v_1\right)^2 - v_1^2\right]$$

$$p_1 - p_2 = \frac{1}{2}\rho\left[\frac{S_1^2}{S_2^2} - 1\right]v_1^2$$

Stev. $\underset{sx}{p_1} = \underset{dx}{p_2 + \rho' g h}$

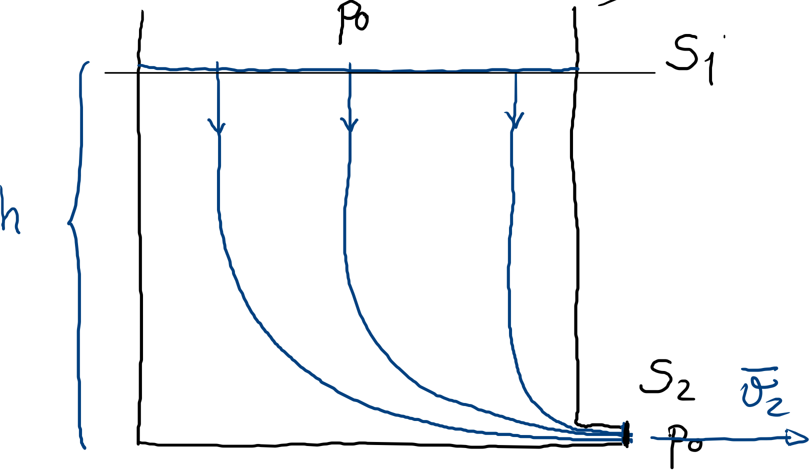
$p_1 - p_2 = \rho' g h$

$$\rho' g h = \frac{1}{2}\rho\left[\frac{S_1^2}{S_2^2} - 1\right]v_1^2$$

$$h = \underbrace{\frac{1}{2}\frac{\rho}{\rho'}\left(\frac{S_1^2}{S_2^2} - 1\right)\frac{1}{g}}_{C} v_1^2 \qquad h = C v_1^2 \qquad v_1^2 = \frac{1}{C} h$$

TEOREMA DI TORRICELLI

cisterna aperta!



dev: $S_1 v_1 = S_2 v_2$

$$v_1 = \frac{S_2}{S_1} v_2 \qquad \frac{S_2}{S_1} \ll 1$$

$$v_1 \ll v_2$$

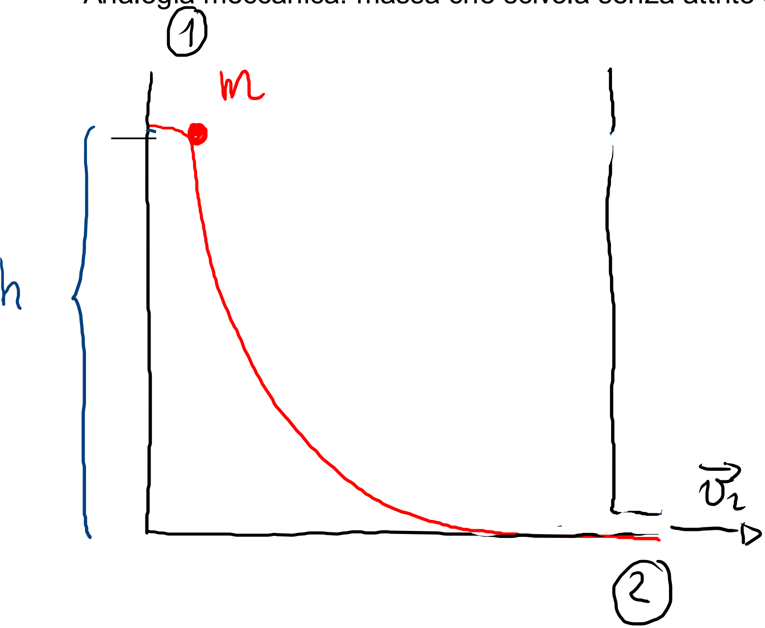
Bern.: $\cancel{p_1} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = \cancel{p_2} + \frac{1}{2}\rho v_2^2 + \rho g h_2$ $\qquad p_1 = p_2 = p_0$

$$\frac{1}{2}\rho \underbrace{\left(v_2^2 - v_1^2\right)}_{\sim v_2^2} = \rho g \underbrace{\left(h_1 - h_2\right)}_{h}$$

$$\frac{1}{2}\cancel{\rho} v_2^2 = \cancel{\rho} g h$$

$$\boxed{v_2 = \sqrt{2gh}}$$

Analogia meccanica: massa che scivola senza attrito su uno scivolo posto all'interno della cisterna vuota



1 $\quad v_1 = 0$

$E_{mecc} = U_g = mgh$

2 $\quad v_2 \neq 0$

$U_g = 0$

$E_{mecc} = K = \frac{1}{2} m v_2^2$

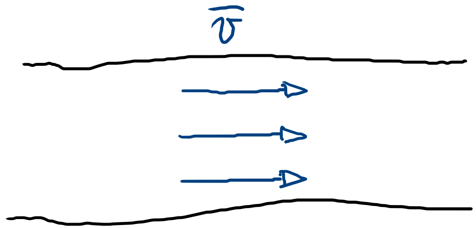
$$E_{mecc_1} = E_{mecc_2}$$

$$\cancel{m} g h = \frac{1}{2} \cancel{m} v_2^2$$

$$v_2 = \sqrt{2gh}$$

FLUIDI "REALI"

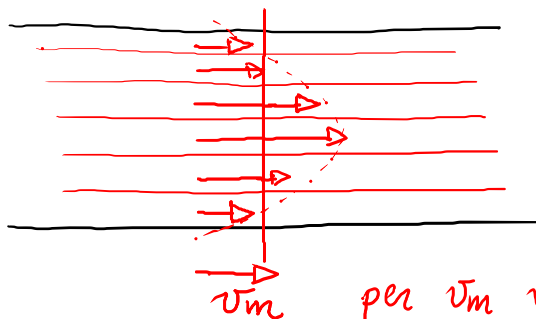
ideali



$\eta = 0$

$\bar{v}$ è la stessa in tutti i punti della sezione

reali



$\eta \neq 0$

moto laminare

$\bar{v}$ ha profilo parabolico

per v_m vale: $S_1 v_{m_1} = S_2 v_{m_2}$

(TEO. DI LEONARDO)

moto stationario

moto laminare (se v_m è piccola)

$v_m < v_c = N_R \dfrac{\eta}{\rho r}$

di Reynolds
~ 1000
in questo caso

NO BERNOULLI

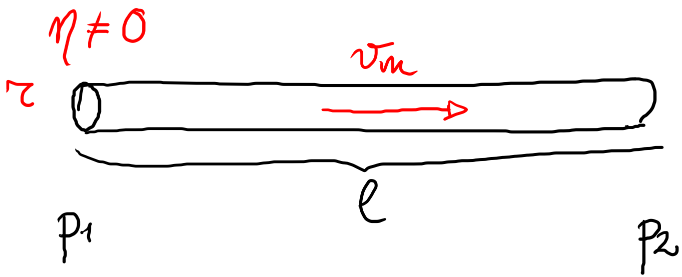
altrimenti c'è moto turbolento

EQUAZIONE DI POISEUILLE
(liquidi newtoniani)

$$\boxed{Q = \frac{\pi}{8}\,\frac{r^4}{\eta}\,\frac{(p_1-p_2)}{l}}$$

$$\frac{(p_1-p_2)}{l} \sim -\frac{\Delta p}{\Delta l} \sim -\frac{\partial p}{\partial l}$$

gradiente di pressione



condutture orizzontale
(per Bernoulli sarebbe $p_1 = p_2$)
MA

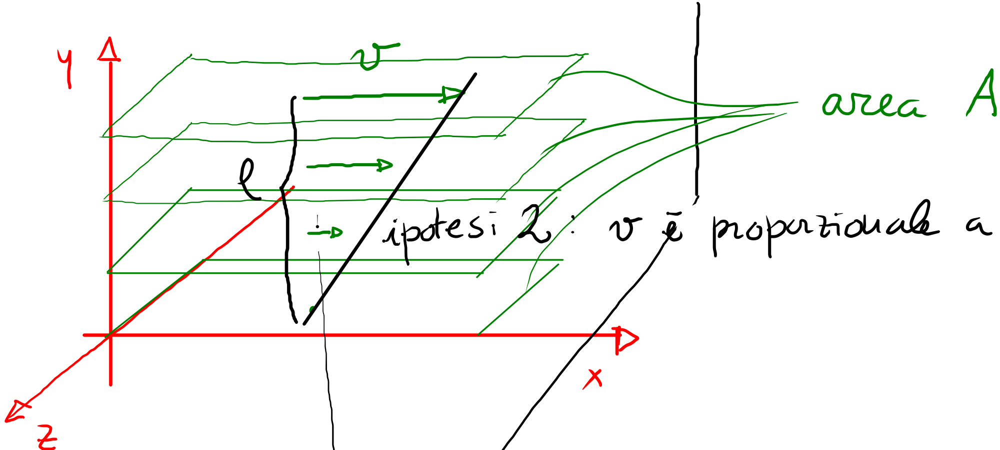
$p_1 > p_2$

$p_1 > p_2$
- per contrastare le forze d'attrito e garantire un moto stazionario
- a causa delle forze d'attrito (dissipative)

$\longrightarrow$ la densità di energia in 2 (p_2) è minore di quella in 1

NOTA:
le slide che seguono costituiscono un approfondimento e non sono materia per la prova SCRITTA
possono pero' essere oggetto di una eventuale prova ORALE

APPROFONDIMENTO (non serve x lo scritto)



ipotesi 2: v è proporzionale a y : $\frac{dv}{dy}$ è cost.

$$\frac{dv}{dy} = \frac{v}{\ell}$$

ipotesi 1: moto laminare

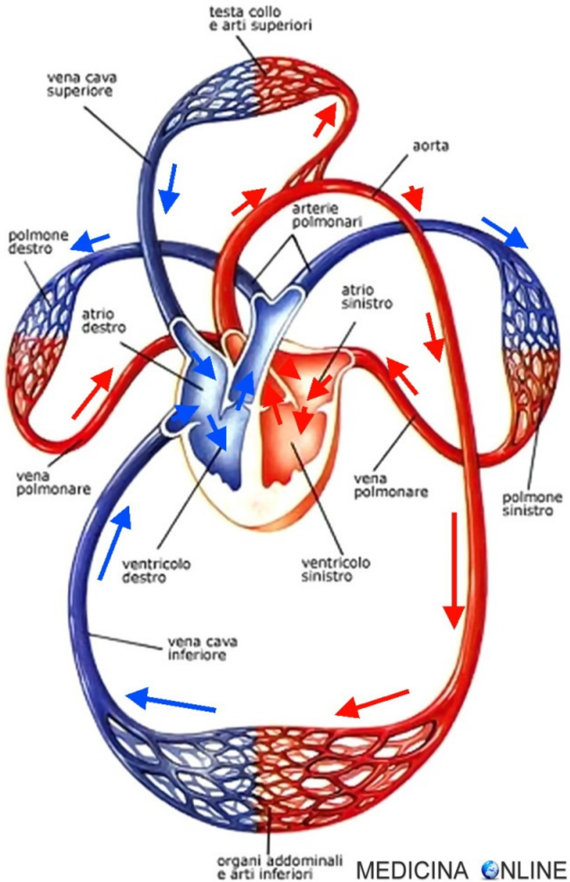
ipotesi 3: $F \propto \left(\frac{dv}{dy}\right) \cdot A \implies \boxed{F = \eta \left(\frac{dv}{dy}\right) \cdot A}$

forza d'attrito

(def. formale di viscosità)

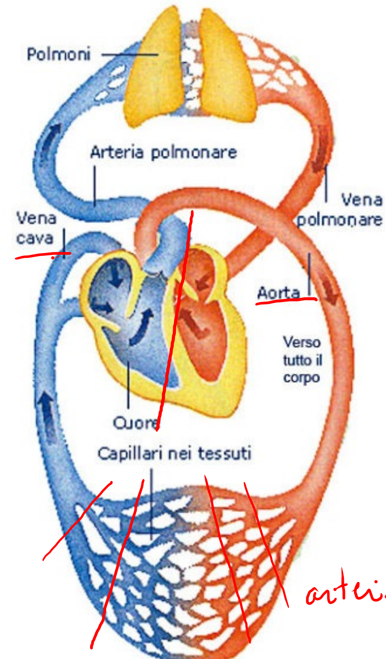
<u>carico di scorrimento</u> $\quad \frac{F}{A} = \eta \cdot \left(\frac{dv}{dy}\right)$

CIRCOLAZIONE SANGUIGNA



(polmone)

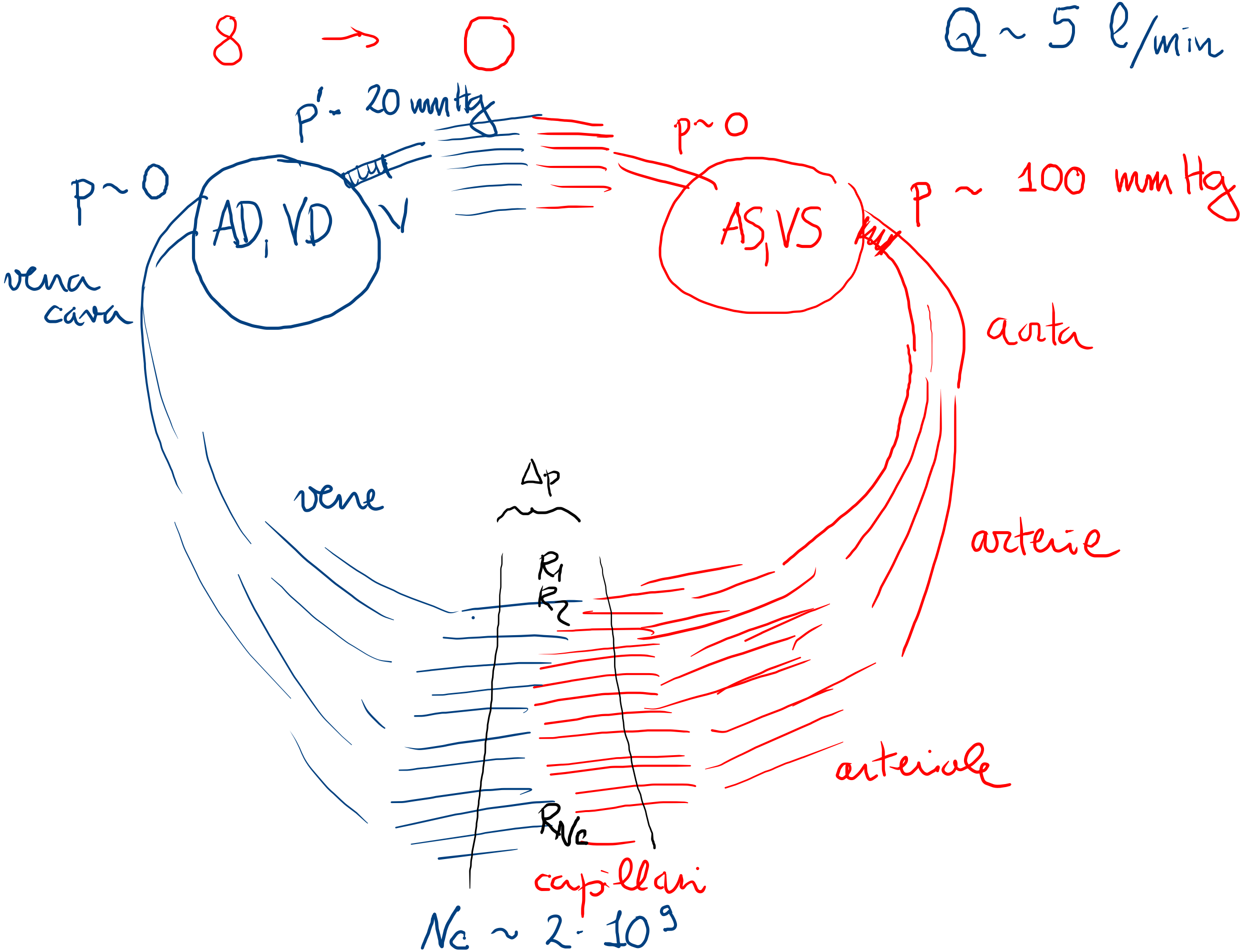
piccola circolazione



arteriole

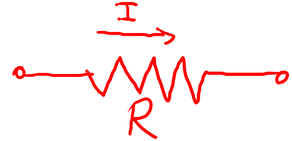
grande circolazione

(sistemica)



8 → 0
Q ~ 5 l/min
p' ~ 20 mmHg
p ~ 0
p ~ 0
AD, VD
V
AS, VS
p ~ 100 mmHg
vena cava
aorta
vene
Δp
R1
R2
arterie
arteriole
RNc
capillari
Nc ~ 2 · 10^9

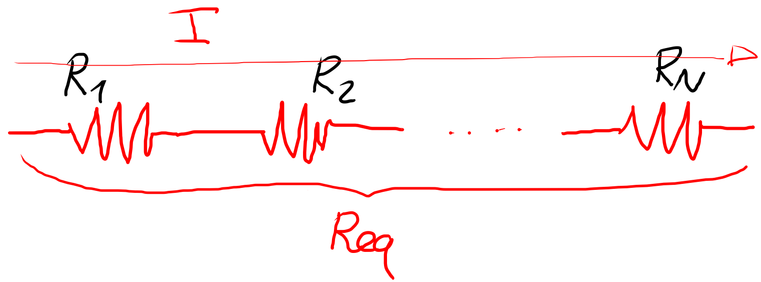
$$Q = \frac{\pi}{8} \frac{r^4}{\eta} \frac{\Delta p}{\ell}$$



I legge di Ohm

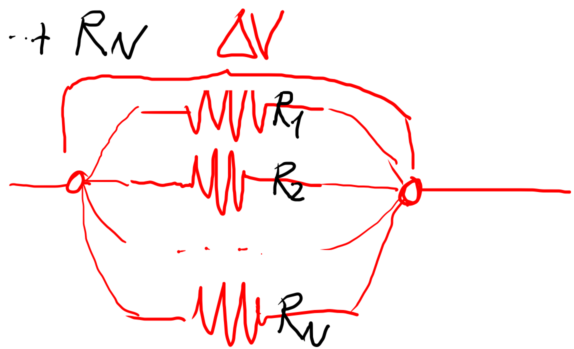
$$\Delta p = \underbrace{\frac{8 \eta \ell}{\pi r^4}}_{\sim R} \cdot Q$$

$$\Delta V = R I$$



- Resistente in serie

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$



- Resistente in parallelo

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

Resistenza eq. del letto capillare

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_{N_C}}$$

N_C = numero capillari $\sim 2 \cdot 10^9$

assumiamo che i capillari siano tutti uguali:

$$R_1 = R_2 = \dots = R_{N_C} \equiv R_C$$

ed abbiano tutti resistenza R_C

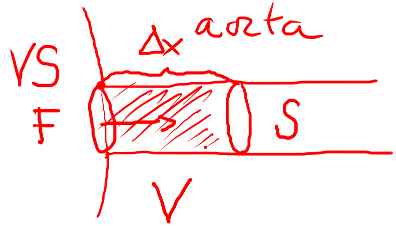
$$\frac{1}{R_{eq}} = \frac{1}{R_C} + \frac{1}{R_C} + \dots + \frac{1}{R_C} = N_C \cdot \frac{1}{R_C} = \frac{N_C}{R_C}$$

$$R_{eq} = \frac{R_C}{N_C}$$

resistenza di un capillare

$2 \cdot 10^9$

LAVORO CARDIACO



$V \sim 60\ cm^3$

$p = \frac{F}{S}$

$$\mathcal{L}_{VS} = F \cdot \Delta x = \overbrace{pS \cdot \Delta x}^{V} = pV$$

$$\mathcal{L}_{VD} = \cdots\cdots = p'V$$

$p \sim 100\ mmHg$

$p' \sim 20\ mmHg$

$$\mathcal{L} = \mathcal{L}_{VS} + \mathcal{L}_{VD} = pV + p'V = (p+p')V$$

$$= (120\ \cancel{mmHg})\left(\frac{10^5\ Pa}{760\ \cancel{mmHg}}\right) \cdot 60 \cdot (10^{-2} m)^3$$

$$\simeq 10 \cdot 10^5\ Pa \cdot 10^{-6} m^3 = 1\ J \qquad P = \frac{1J}{1s} = 1\ W$$

modello in 4 fasi: $P \rightarrow \times 3$

lavoro interno

rendimento 10 %

200 $\frac{Kcal}{giorno}$

VISCOSITÀ DEL SANGUE (non newtoniano)

Non è costante ma dipende da τ $\quad \eta(\tau)$

$$\eta(\tau) = \frac{\eta_\infty}{\left(1 + \frac{d}{\tau}\right)^2}$$

η_∞: la viscosità che corrisponde ad $\tau \to \infty$

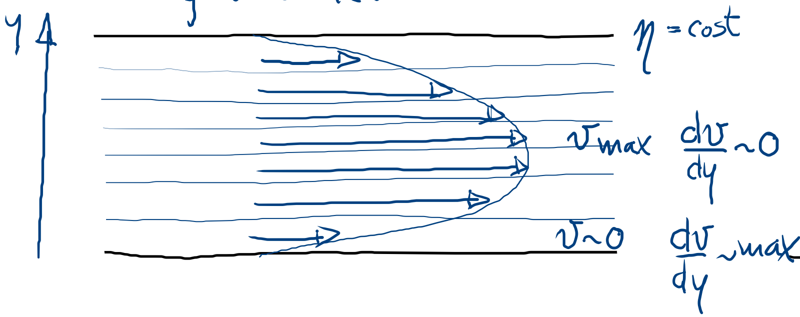
d: costante diametro eritrociti 6-9 µm

$\tau \gg d \qquad \frac{d}{\tau} \to 0 \qquad \eta(\tau) \to \eta_\infty$

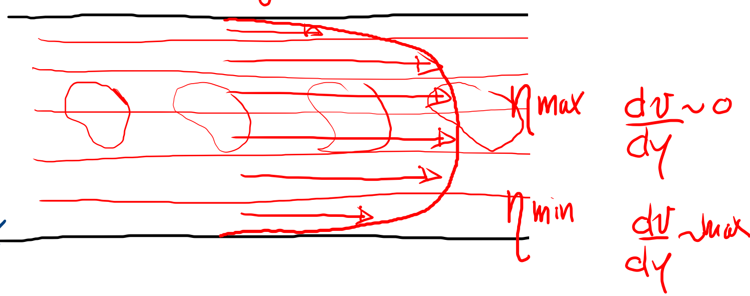
$\tau \sim d \qquad \qquad \eta < \eta_\infty$

carico scorrimento tende a valori piccoli $\eta \cdot \frac{dv}{dy}$

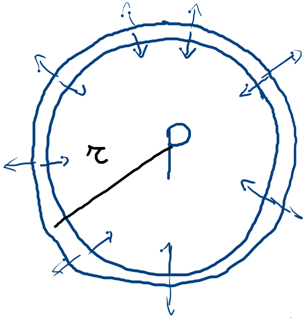
fluido newtoniano



sangue



LEGGE DI LAPLACE



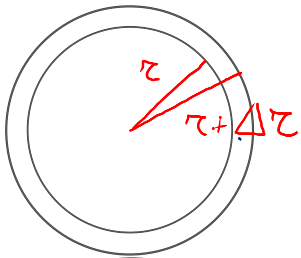
p = sovrappressione rispetto alla p esterna

$$p = \frac{4\tau}{r}$$

4τ ← tensione superficiale $\tau = \frac{\mathcal{L}}{\Delta S}$

r ← raggio della bolla

Δr è piccolo



$$\mathcal{L} = \tau \cdot \Delta S \cdot 2$$

perché ci sono 2 superfici (int ed est.)

$$\stackrel{!}{=} \tau \left[4\pi (r+\Delta r)^2 - 4\pi r^2 \right] \cdot 2$$

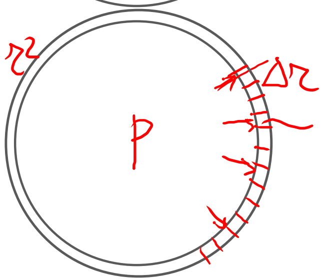
$$\stackrel{!}{=} \tau \left[4\pi (r^2 + 2r\Delta r + \Delta r^2) - 4\pi r^2 \right] \cdot 2$$

$$\stackrel{!}{=} \tau \left[\cancel{4\pi r^2} + 8\pi r \Delta r + \cancel{4\pi \Delta r^2} - \cancel{4\pi r^2} \right] \cdot 2$$

trascurabile

$$\stackrel{!}{=} 16\pi \, \tau \cdot r \cdot \Delta r$$

$\Sigma_i \Delta S_i = 4\pi r^2$



ΔS_i

$F_i = p \Delta S_i$

$$\mathcal{L}_i = F_i \cdot \Delta r = p \Delta S_i \cdot \Delta r$$

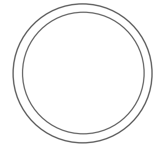
$$\mathcal{L} = \Sigma_i \mathcal{L}_i = \Sigma_i \, p \Delta S_i \cdot \Delta r = p \Delta r \Sigma_i \Delta S_i$$

$$\stackrel{!}{=} p \Delta r \, 4\pi r^2$$

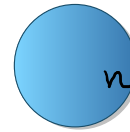
4

$$\cancel{16}\cancel{\pi}\,\tau \cdot \cancel{r}\,\cancel{\Delta r} = p\,\cancel{\Delta r}\,\cancel{4}\cancel{\pi}\,r^{\cancel{2}}$$

$$\frac{4\tau}{r} = p \qquad \text{bolla}$$

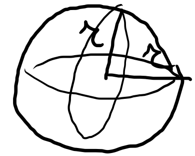


$$\frac{2\tau}{r} = p \qquad \text{sfera}$$

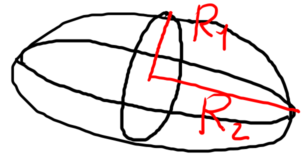


nella sfera $R_1 = R_2 = r$

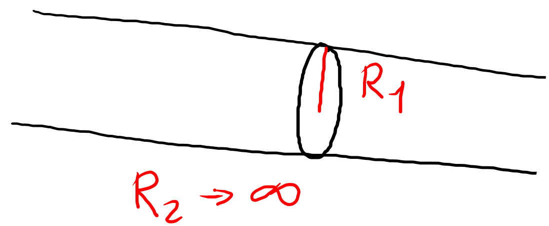
$$\frac{\tau}{r} + \frac{\tau}{r} = p \qquad \text{sfera}$$



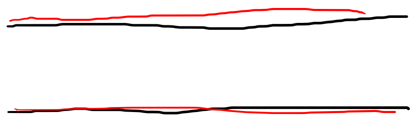
$$\frac{\tau}{R_1} + \frac{\tau}{R_2} = p \qquad \text{elissoide}$$



$$\frac{\tau}{R_1} = p \qquad \text{cilindro}$$



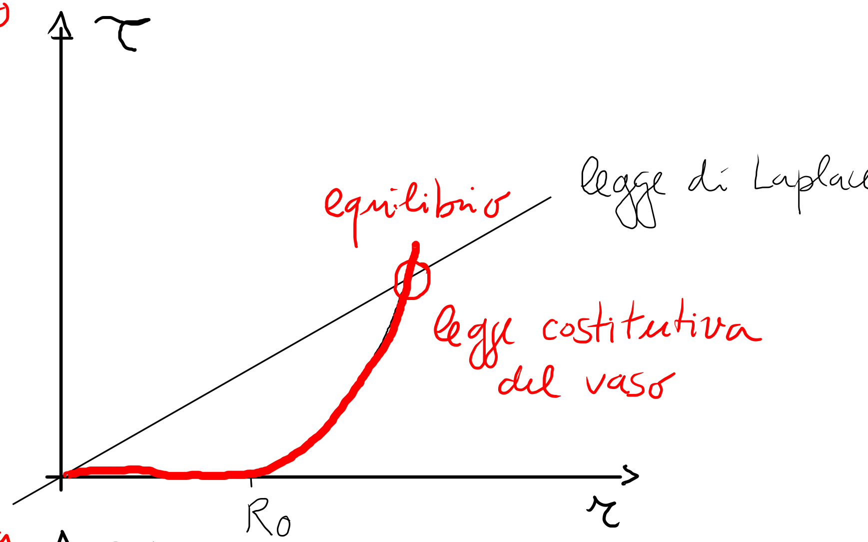
TENSIONE DI UN VASO SANGUIGNO



$$p = \frac{\tau}{r}$$

$$\tau = r\,p \quad \text{legge laplace}$$

$$\tau = \tau(r) \quad \text{legge costitutiva}$$

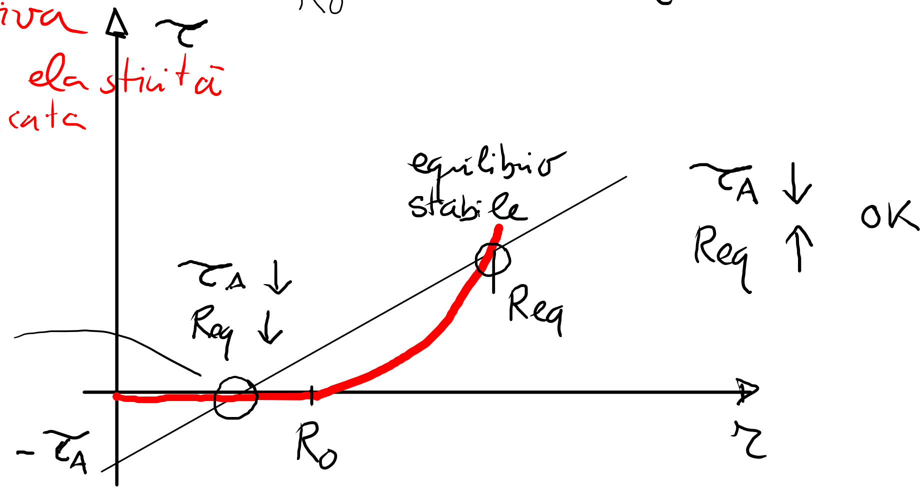


se il vaso perde di elasticità
legge laplace modificata

$$\tau + \tau_A = r\,p$$

$$\tau = r\,p - \tau_A$$

equilibrio instabile



Fine Approfondimento

I GAS

μscopico → teoria cinetica dei gas ✗

macroscopico → equilibrio ρ, V, T, p ✓

gas perfetto → molecole non interagiscono

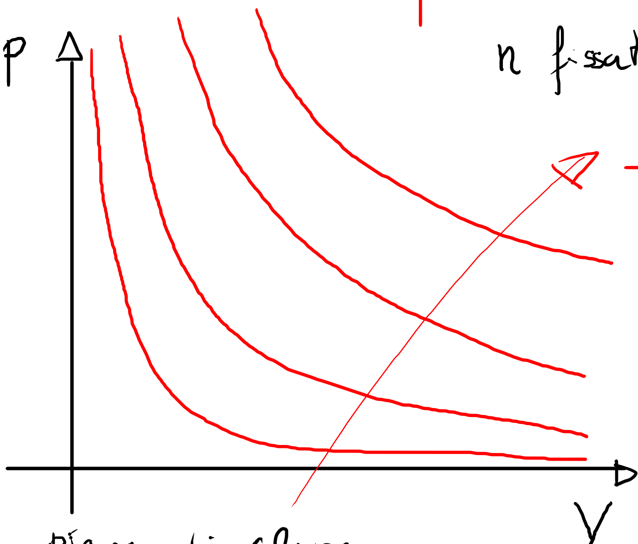
→ puntiformi

$$pV = nRT$$

n fissato

$$R = 0{,}082 \ \frac{l \, atm}{mol \, K}$$

$$R = 8{,}314 \ \frac{J}{mol \, K}$$

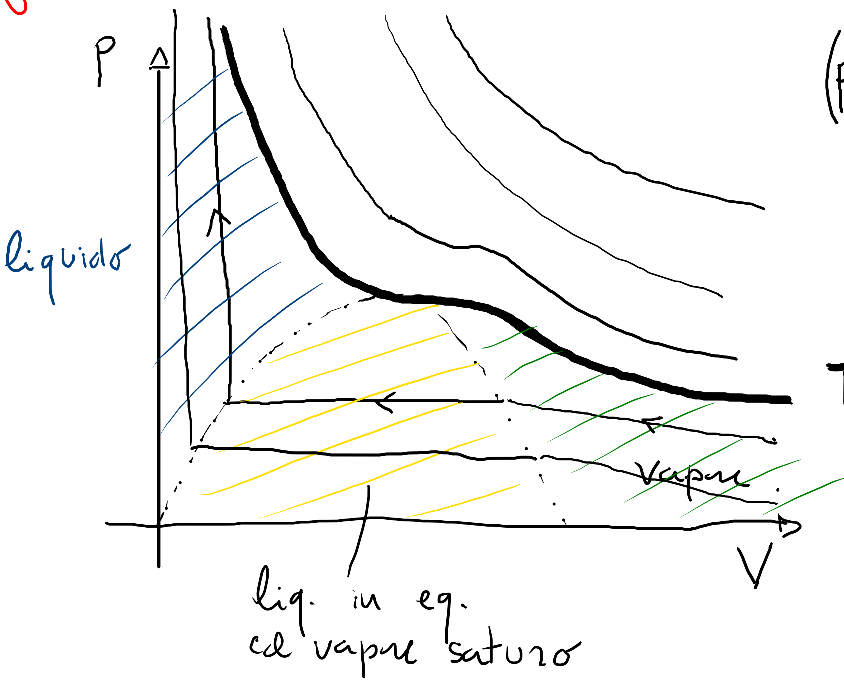


T crescente

curve isoterme (T definito)

piano di Clapeyron

gas "reali" (approfondimento)



v vol. specifico per mole

$$\left(p + a\frac{n^2}{V^2}\right)\left(\frac{V}{n} - b\cdot\cancel{n}\right) = \cancel{n}RT$$

$$\left(p + \frac{a}{v^2}\right)\left(v - b\right) = RT$$

eq. di Van der Waals

Tc temperatura critica

Fine Approfondimento