Conditional Uniform Graph (CUG) tests

- Is a certain network feature *more prevalent* than expected *by chance*?
  - Operationalization of a network feature (“more prevalent”)
  - Definition of hypotheses (H_0: less prevalent or equally present, H_1: more prevalent)
  - Definition of a reference/null network model (“expected by chance”)

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- Steps:
  1. Calculate the number of local configurations on the observed network
     (statistic := number of local configurations)
  2. Generate networks from the reference model and
     compute the value of the statistic for each generated network
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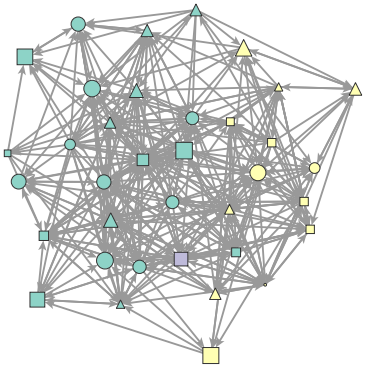
- The simplest reference distribution is the $G(n, m)$ model

CUG test: example

Evidence for/against social mechanisms in an advice network? Lazega et al. (2001)



https://www.stats.ox.ac.uk/~snijders/siena/

- 36 partners in a Northeastern US corporate law firm
- Advice relation:
  "Think back over the past year, consider all the lawyers in your Firm. To whom did you go for basic professional advice?"
- Friendship relation (not shown in the picture):
  "Would you go through this list, and check the names of those you socialize with outside work. You know their family, they know yours, for instance. I do not mean all the people you are simply on a friendly level with, or people you happen to meet at Firm functions."
- Vertex attributes:
  office (green=Boston; yellow=Hartford; violet=Providence)
  school (circle=Harvard, Yale; Triangle: Ucon; Square: other)
  years with the firm (node area)

CUG test: example

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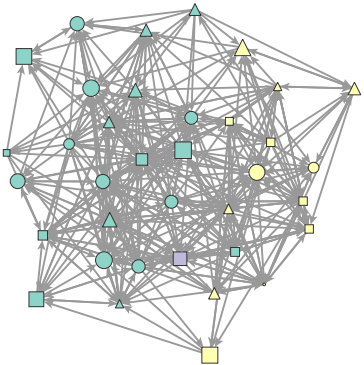


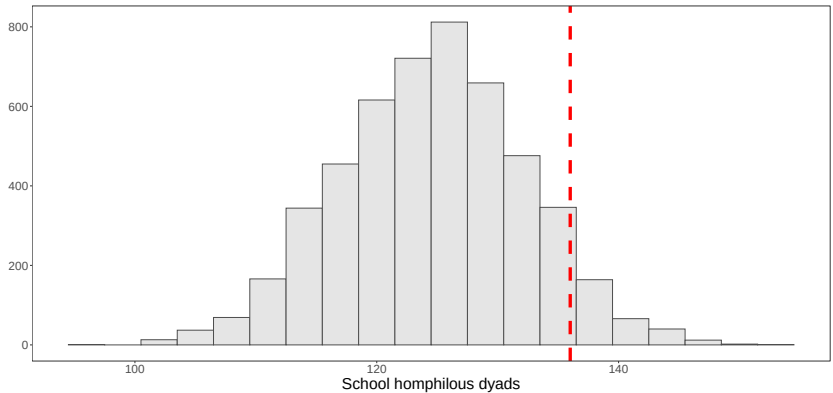
- Is there evidence for reciprocity?
- Is there evidence for transitivity?
- Is there evidence for school homophily?

Reference/Null model: $G(n, m)$, $n = 36$, $m = 395$

Full description and data download:
https://www.stats.ox.ac.uk/~snijders/siena/

CUG test: example

Is there evidence for school homophily? No!



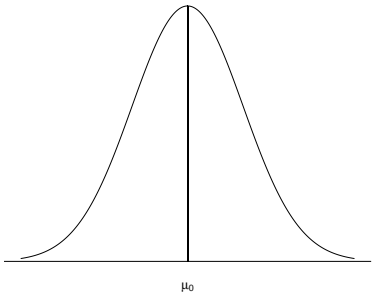
- 7.8% of the random networks has equal or more school homophilous dyads (p-value = 0.078)
- Under a significance level of $\alpha = 0.05$ we consider this result not significant

Statistical and CUG tests

State the hypotheses

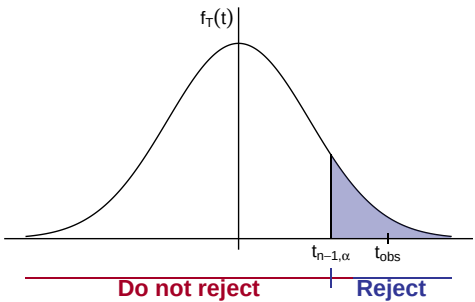
$H_0 : \mu = \mu_0$
$H_1 : \mu > \mu_0$

Distribution of the test statistic (under H_0)



Statistical and CUG tests

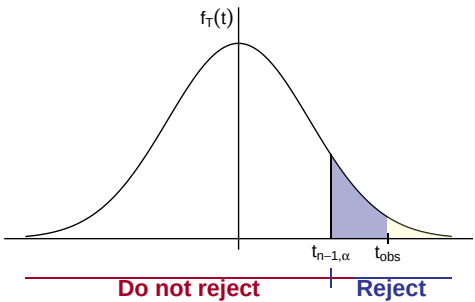
Decision rule



Using thresholds

Statistical and CUG tests

Decision rule



Using p-value

Statistical and CUG tests

State the hypotheses

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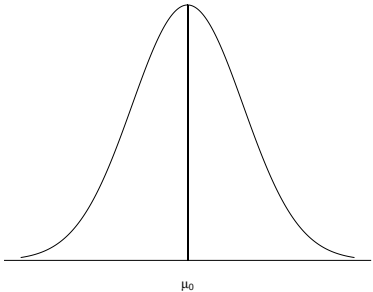
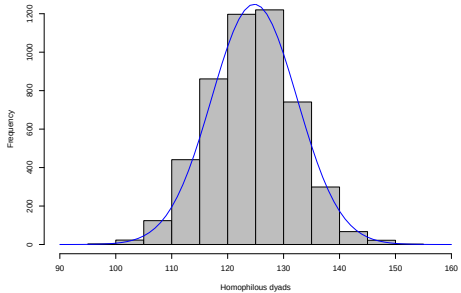
H_0:same number of configurations as by chance
H_1: more configurations than by chance

Distribution of the test statistic (under H_0)





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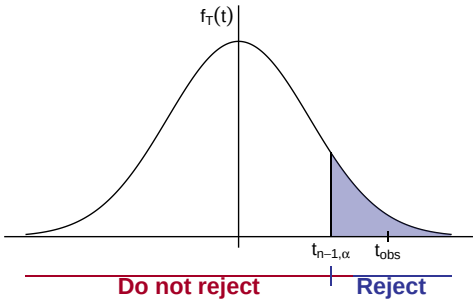

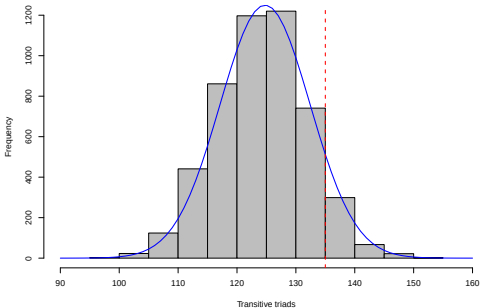

Using thresholds

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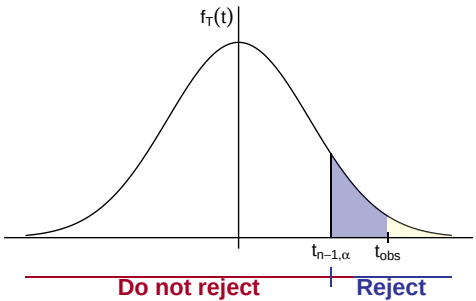

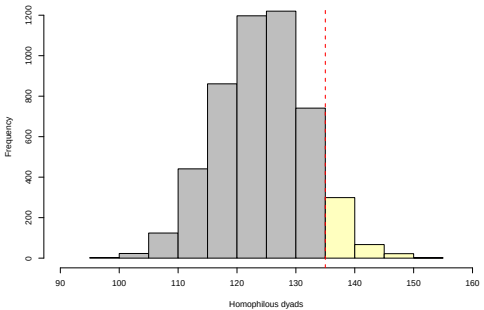

Using p-value

References

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