# Multi state modeling of life course trajectories

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## Outline

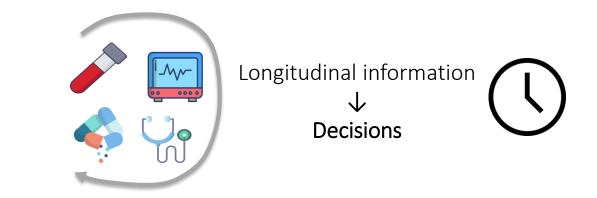
#### 1. Motivation

- Longitudinal data & multiple-events
- Objectives and Applications in chronic illnesses
- 2. Competing-risk analysis
  - Definition
  - Estimation

#### 3. Multi-state models

- Definitions
- Formulation
- Time scales
- Estimation
- Microsimulation
- 4. Case-studies
  - Optimal timing for a transplant
  - Cost-effectiveness analysis
- 5. Conclusions

Life courses of individuals can be defined by multiple events that happen to them over time. Such events determine different life trajectories depending on the type of events and their timing.



Depending on the **phenomenon under study**, and the **population of interest**, different events can be recorded together with additional subjects' information recorded at specific times.

Multi state models extend classical survival analysis, allowing to describe from a mathematical point of view multiple events over time. They represent natural statistical methods to model subjects' life trajectories.

Time dimension Data sources Data sources Time Time Time

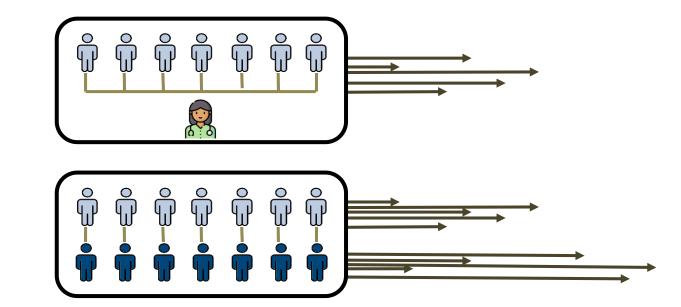
In clinical and epidemiological research of chronic illnesses, subjects:

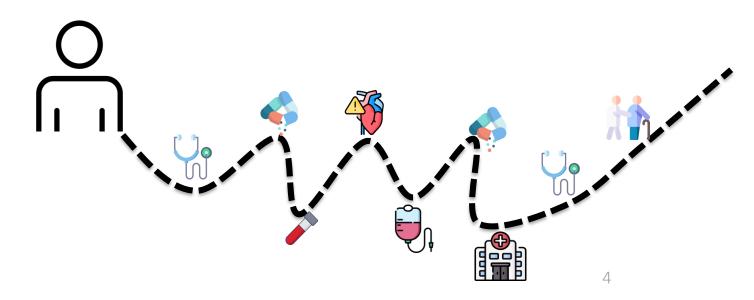
- are at risk of different type of events
- can experience multiple events of the same time
- can go through different «phases» in life

In many cases, focusing only on a single outcome is not enough ...

Some objectives of time-to-events analyses:

- Estimation the **effect** of a treatment/exposure
- Cost-effectiveness of a treatment
- Risk Stratification and prediction

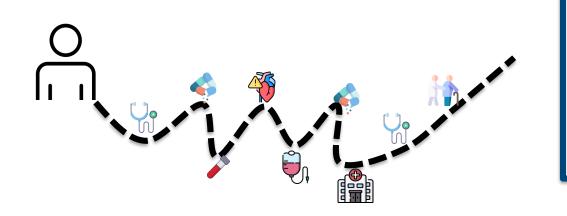


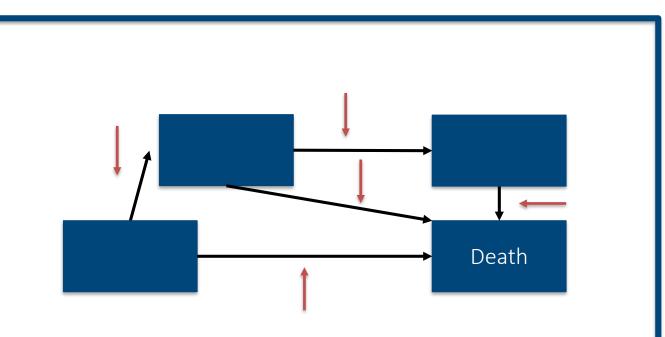


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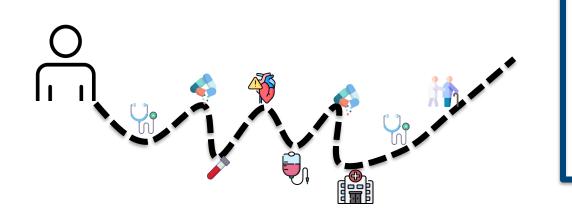


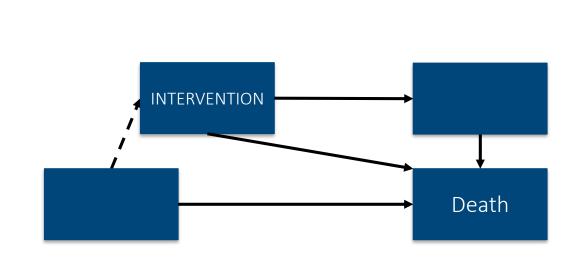


- Exposure may impact differently each transition
- Long-term effect on survival needs to take into account of intermediate events

Some objectives of time-to-events analyses:

- Estimation the effect of a treatment/exposure
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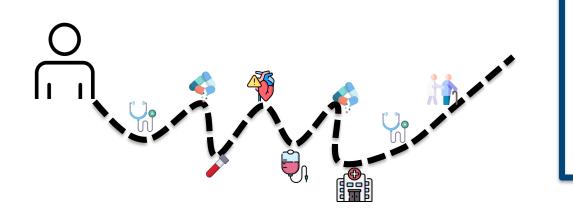


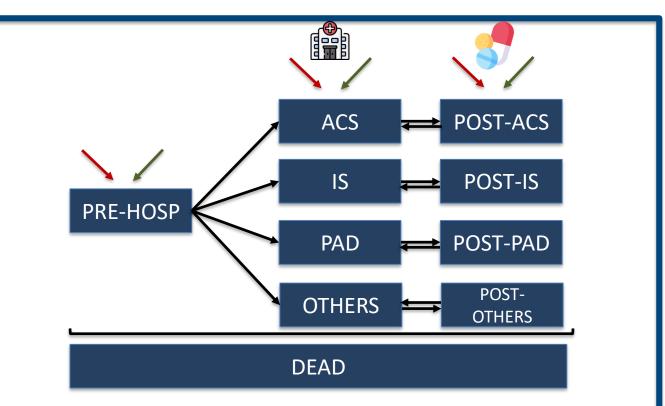


- Exposure/treatment can itself be something that can be assigned over time
- It can be of interest to study the optimal timing for an intervention

Some objectives of time-to-events analyses:

- Estimation the effect of a treatment/exposure
- <u>Cost-effectiveness of a treatment</u>
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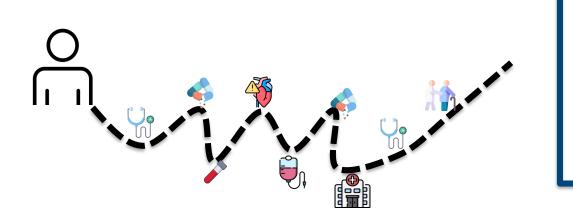


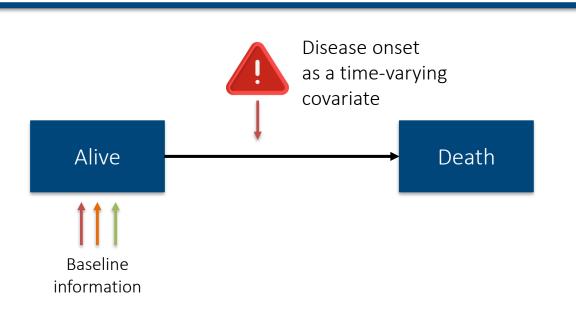


• Cost-effectiveness needs to take into account the costs and utilities associated with each "life-phase"

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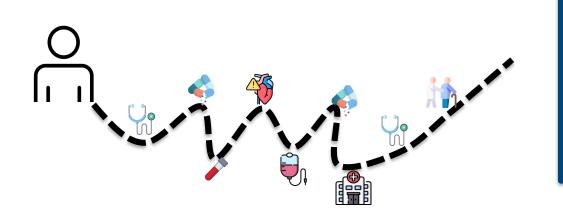


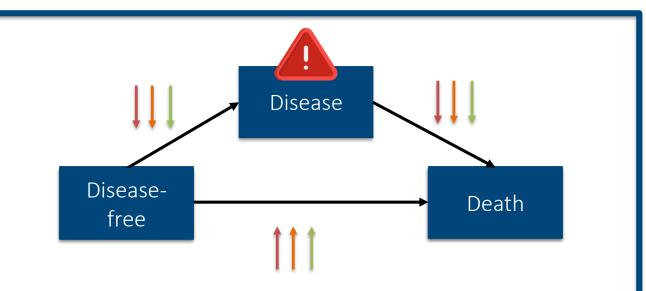


• When predicting survival, it is also important to model the process which underlies the onset of the disease

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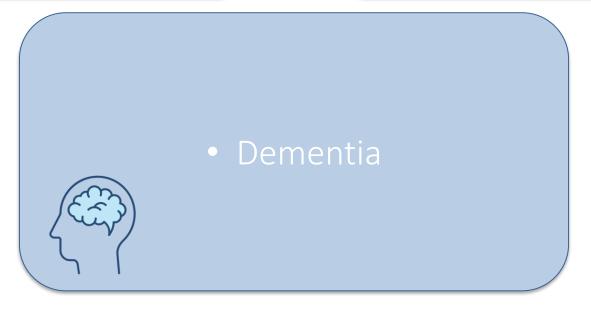


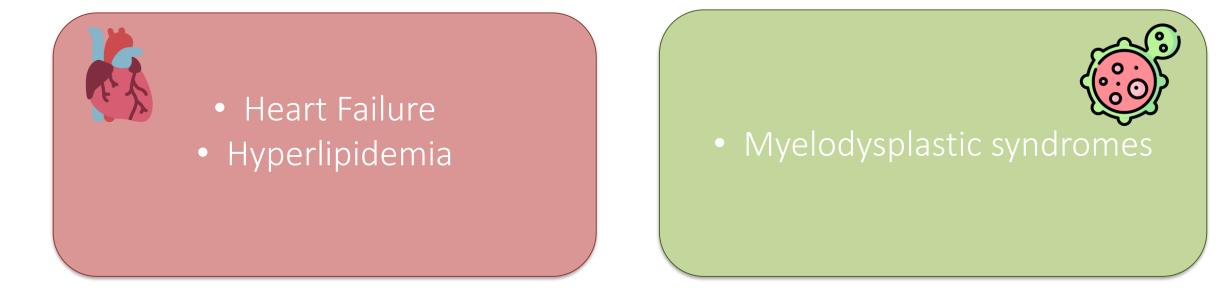


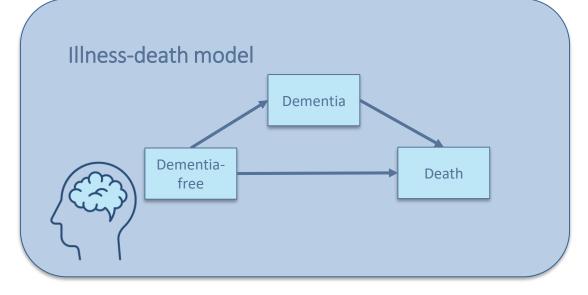
- When predicting survival, it is also important to model the process which underlies the onset of the disease
- The 3-states models allows to consider the process which underlies the onset of the disease



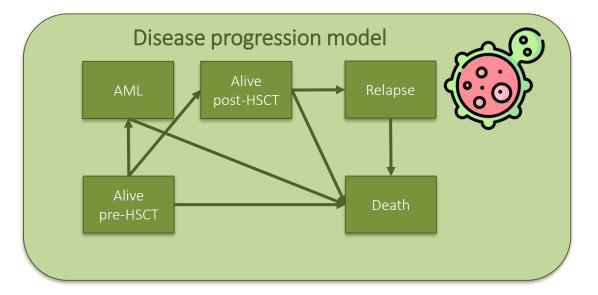




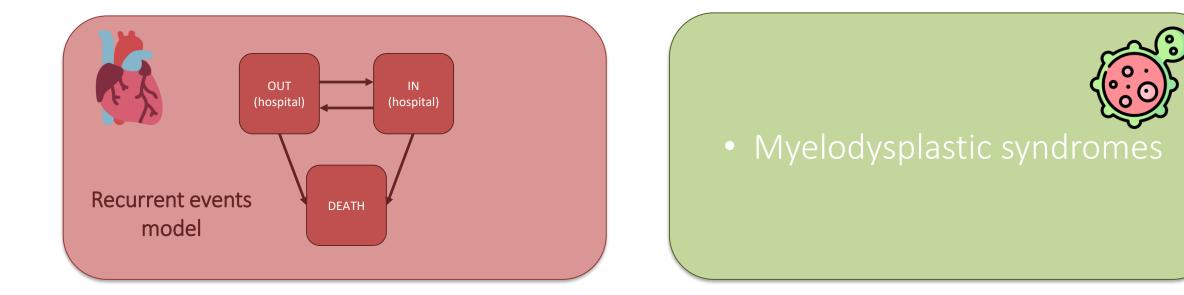




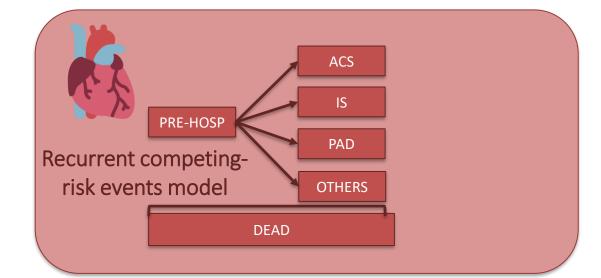
Heart FailureCardiovascular disease







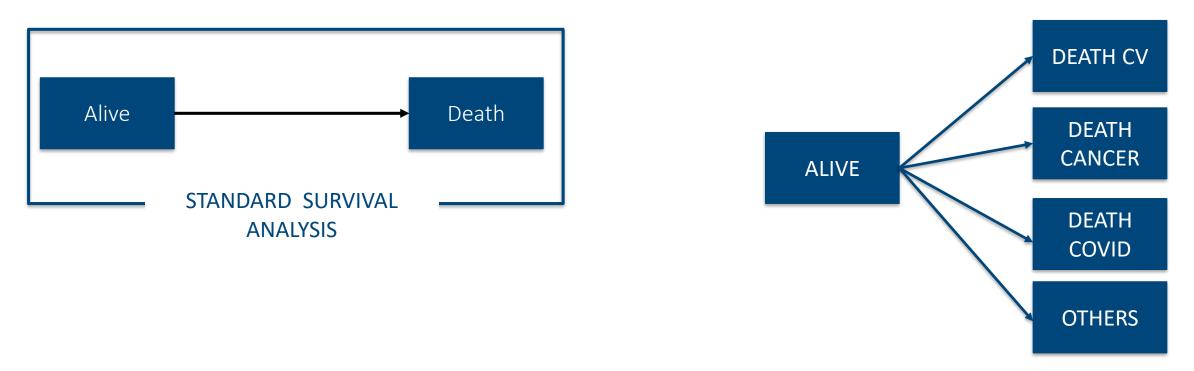
### • Cognitive decline and dementia



# • Myelodysplastic syndromes

### Cognitive decline and demential

## Competing risks



By definition, a **competing event** is an event that prevents one or more events from happening. Possible choices to deal with it:

- Marginal analysis Outside of the scope of this presentation!
- <u>Competing risk analysis</u>

## Competing risks

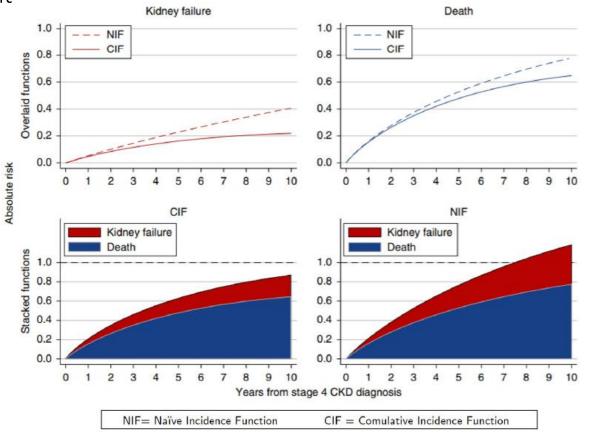
Whenever we have a setting where there are **competing events**, we need to modify the definitions of the quantities of interest for our time-to-event analysis. For each event k = 1, ..., K we now define:

CUMULATIVE  
INCIDENCE 
$$Pr(T \le t, E = k) = \int_{0}^{t} S(h)\lambda_{k}(h)dh$$

where T is the event time, E is the event type.

It depends on two other quantities:

- Overall survival:  $S(t) = \prod_{e=1}^{K} \exp\{-\int_{0}^{t} \lambda_{e}(h) dh\}$
- Cause-specific hazard:  $\lambda_k = \lim_{\Delta t \downarrow 0} \frac{\Pr(T \le t < t + \Delta t, E = k | T \ge t)}{\Delta t}$



## Competing risks

Whenever we have a setting where there are competing events, we need to modify the definitions of the quantities of interest for our time-to-event analysis. For each event k = 1, ..., K we now define:

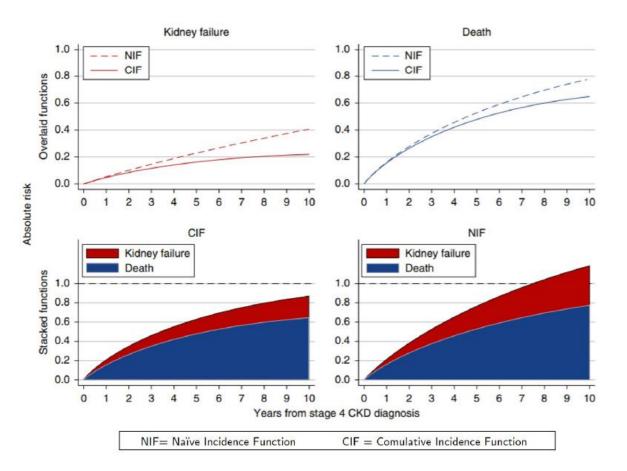
$$\Pr(T \le t, E = k) = \int_{0}^{t} S(h)\lambda_{k}(h)dh$$

This quantity is equivalent to the 1-survival probability ir the standard survival setting.

It can be estimated as:

$$\int_{0}^{t} \hat{S}(h)\hat{\lambda}_{k}(h)dh$$

Only if K=1, we retrieve the Kaplan-Meier estimator as a special case.



A multi state model can be described by a set of **discrete states**, *S={1,...,s}*, which individuals can visit over time and by corresponding **possible transitions** between them.

Formally, an individual in the model can be denoted as a random process,  $\{X(t); t > 0\}$ , with X(t) taking the values between  $S = \{1, ..., s\}$ ,

Each possible transition between a state g and a state h can be defined through the **intensity** function,  $\lambda_{gh}(t)$ , which corresponds to **instantaneous probability** at each time t of moving from one state to the other, taking into account the **past state occupation history** up to time t,  $\mathcal{H}_t = \{X(u); 0 \le u \le t\}$ :

$$\lambda_{gh}(t) = \lim_{\Delta t \downarrow 0} \frac{\Pr(X(t + \Delta t) = h | X(t) = g, \mathcal{H}_t)}{\Delta t}$$

The transition hazard generalizes the cause-specific hazard to the multi-state settings.

### Definitions

On the cumulative scales, other quantities of interest in a multi-state model are:

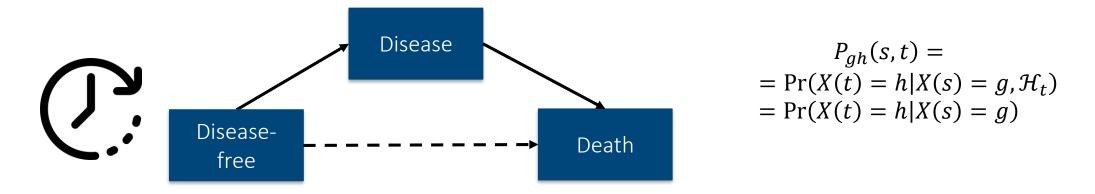
- $P_{gh}(s,t) = \Pr(X(t) = h | X(s) = g, \mathcal{H}_t)$  TRANSITION PROBABILITY
- P(X(t) = g) STATE OCCUPANCY PROBABILITY

In a competing-risks model the two quantities are the same.

Others could be:

- S(t) = 1 Pr(X(t) = a) OVERALL SURVIVAL FUNCTION where a *is* the absorbing state
- $\int_{0}^{w} S(h) dh$  RESTRICTED (over horizon w) MEAN SURVIVAL TIME

A Markov multi-state model assumes that all information about the future is contained in the present state. In these models time is measured since «time 0» e.g. age at study entry. *Clock forward* 

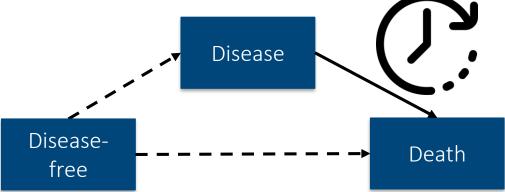


A Semi-Markov model assumes that the risk of transitioning into the next state depends on time spent in the present state. *Clock reset* 

Most of the phenomenon of interest in chronic illnesses are not markovian/semi-markovian in a strict sense but ...

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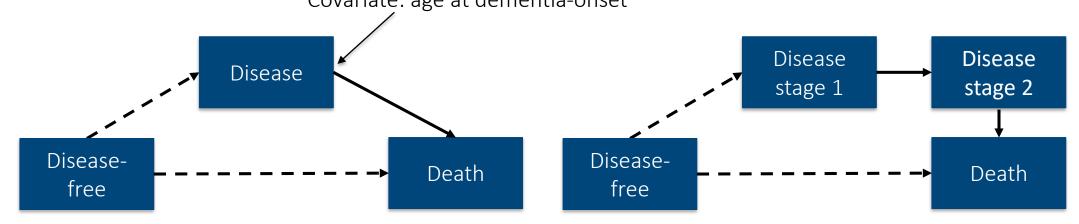


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Most of the phenomenon of interest in chronic illnesses are not markovian/semi-markovian in a strict sense but... Covariate: age at dementia-onset



## Estimation of the hazards

We can estimate hazards from a generic state *g* to *h* using for example a generalized parametric survival model:

$$\begin{split} \lambda_{gh}(\mathbf{t}|\boldsymbol{x}_{i}) &= \lambda_{gh}^{0}(t) \exp\{\boldsymbol{\beta}^{T} \boldsymbol{x}_{i}\} \\ \lambda_{gh}(\mathbf{u}|\boldsymbol{x}_{i}) &= \lambda_{gh}^{0}(u) \exp\{\boldsymbol{\beta}^{T} \boldsymbol{x}_{i}\} \\ \end{split}$$
 clock-reset approach

→The baseline hazard is estimated together with the coefficients
→More flexibility in the shape of the baseline hazards, only on the number of knots
→It can be easily extended to include time-varying effects for the covariates

Specifically, the logarithm of the baseline cumulative hazard function is modelled as a natural cubic spline function of log time:

$$logH_{gh}(t|\mathbf{x}_i) = logH_{gh}^0(t) + \boldsymbol{\beta}^T \mathbf{x}_i = b_{gh}^m(log(t); \boldsymbol{\gamma}) + \boldsymbol{\beta}^T \mathbf{x}_i$$

where  $b_{gh}^m(\log(t); \mathbf{\gamma})$  is a natural cubic spline with m knots. The parameters in the model can be estimated using Maximum Likelihood (ML) and their uncertainly can be evaluated using standard ML asymptotic theory.

## Estimation of the hazards

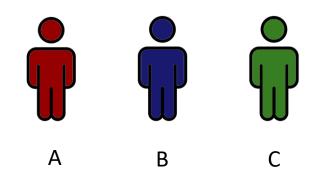
We can estimate hazards from a generic state g to h using for example a generalized parametric survival model:

$$\begin{split} \lambda_{gh}(\mathbf{t}|\boldsymbol{x_i}) &= \lambda_{gh}^0(t) \exp\{\boldsymbol{\beta}^T \boldsymbol{x_i}\} \quad \text{clock-forward approach} \\ \lambda_{gh}(\mathbf{u}|\boldsymbol{x_i}) &= \lambda_{gh}^0(u) \exp\{\boldsymbol{\beta}^T \boldsymbol{x_i}\} \quad \text{clock-reset approach} \end{split}$$

Regardless of whether the models are parametrically or semi-parametrically specified, the estimation can be obtain using standard software assuming that the format of the dataset is correct.

The model changes according to the structure of the dataset. Basic rules for constructing the dataset in the correct format are (also called **counting-process formulation**):

- Each transition will have its own sub-dataset
- Each time a subject **start being "at risk"** for a transition a new row needs to be created considering:
  - time for which subject **started being at risk** for that transition
  - time for which subject **stopped being at risk** for that transition
  - at the end of the "at-risk" interval **did it experience the transition**?
- A subject can be at risk of multiple events at the same time but only one can be experience in the same interval.
- Every time subject experience an event (transition to a different state) he enters a "new interval" so possibly multiple new rows needs to be created (unless the state entered is an absorbing state).



A, B and C enter the study when they are diagnosed with Heart Failure at age 60, 73 and 80 respectively.

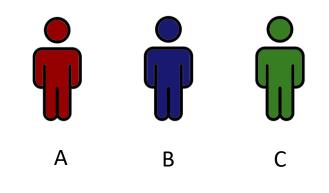
It is of interest to study the **time to death** and by the administrative end of our study, 11 years later, we know that A is still alive, B and C died aged 80 and 90 respectively.

STANDARD SURVIVAL	ANALYSIS
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ID	Tstart	Tstop	time	status	trans
A	0	11	11	0	DEATH
В	0	7	7	1	DEATH
С	0	10	10	1	DEATH

- One possible transition
- All subjects enter at time 0 so time since entering is equal to the time spent in the state.
- Each subject has only one row in the dataset

It this the only possible scale to measure time?



A, B and C enter the study when they are diagnosed with Heart Failure at age 60, 73 and 80 respectively.

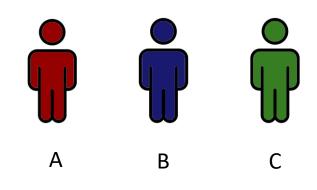
It is of interest to study the **time to death for heart failure** and by the administrative end of our study, 11 years later, we know that A is still alive, B and C died aged 80 and 90 respectively. We also know that B died of cancer while C died because of its underlying heart failure.

COMPETING RISK ANALYSIS

ID	Tstart	Tstop	time	status	trans	
А	0	11	11	0	HF DEATH	
В	0	7	7	0	HF DEATH	
С	0	10	10	1	HF DEATH	
ID	Tstart	Tstop	time	status	trans	
ID A	Tstart 0	Tstop 11	time 11	status O	<b>trans</b> O DEATH	
		-				

- Two possible transitions  $\rightarrow$  two sub-datasets
- All subjects enter at time 0 so time since entering is equal to the time spent in the state.
- Each subject has one row in each sub-dataset

Models can be either estimated separately or together depending on what we think they have in common...



A, B and C enter the study when they are diagnosed with Heart Failure at age 60, 73 and 80 respectively.

It is of interest to study the **time to death taking into account the onset of chronic kidney disease (ckd)** and by the administrative end of our study, 11 years later, we know that A is still alive, B and C died aged 80 and 90 respectively. We also know that B died of cancer while C died because of its underlying heart failure. Subjects A and B were diagnosed with ckd age 70 and 79 respectively.

#### ILLNESS-DEATH MODEL

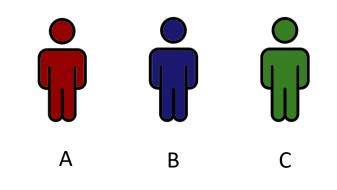
ID	Tstart	Tstop	time	status	Trans	
А	0	10	10	1	CKD	
В	0	6	6 6 1		CKD	
С	0	10	10	0	CKD	
ID	Tstart	Tstop	time	Status	trans	
ID A	Tstart 0	Tstop 10	time 10	Status 0	<b>trans</b> DEATH PRE	
	Tstart 0 0	-			DEATH	

#### • Three possible transitions $\rightarrow$ three sub-datasets

- For death after ckd, we need to choose a time scale for our model.
- Each subject has either 1 or 0 row per sub-dataset.

ID	Tstart	Tstop	time	Status	trans
А	10	11	1	0	DEATH POST
В	6	7	1	1	DEATH POST

27



**REPEATED EVENTS AND DEATH** 

A, B and C enter the study when they are diagnosed with Heart Failure at age 60, 73 and 80 respectively.

It is of interest to study the **time to death taking into hospitalizations** and by the administrative end of our study, 11 years later, we know that A is still alive, B and C died aged 80 and 90 respectively. We know that A was hospitalized at age 61.3 and it stayed in hospital 20 days while subject C was hospitalized at age 83.4 (10 days) and 89.8 (died in hospital).

ID	Tstart	Tstop	time	status	Trans	ID	Tstart	Tstop	time	status	Trans
А	0	1.3	1.3	1	IN	А	0	1.3	1.3	0	OUT-D
А	1.35	11	9.65	0	IN	А	1.35	11	9.65	0	OUT-D
В	0	7	7	0	IN	В	0	7	7	1	OUT-D
С	0	3.4	3.4	1	IN	С	0	3.4	3.4	0	OUT-D
С	3.7	9.8	6.1	1	IN	С	3.7	9.8	6.1	0	OUT-D
ID	Tstart	Tstop	time	status	trans	ID	Tstart	Tstop	time	status	trans
А	1.3	1.35	0.05	1	OUT	А	1.3	1.35	0.05	0	IN-D
С	3.4	3.7	0.03	1	OUT	С	3.4	3.7	0.03	0	IN-D
С	9.8	10	0.02	0	OUT	<b>C</b> <sup>28</sup>	9.8	10	0.02	1	IN-D

## Estimation of the hazards

- Creating the dataset is intuitive, yet **time consuming** with a large sample size. ٠
- Depending on the structure of the dataset there are **different functions in R** which can help, even though for ۲ general structures/repeated events hard coding is sometimes still necessary.
- As the number of transitions increase, of course more **choices regarding the modelling needs to be made**: •
  - Time-scale
  - Baseline hazards
  - "Similarities" between transitions
- Even the **structure of the multi state model** can depend on assumptions and modelling choices • E.g. the first hospitalization is the same as the second one? COODRESS OF F
- How to decide when my estimated model is good enough? Standard methods are valid: •
  - ✓ AIC and BIC
  - Comparison with non-parametric estimates
  - Residuals

Estimation of the cumulative quantities are a matter algebra under the Markov assumption, once the transition-hazards have been estimated.

$$\hat{P}_{gh}(s,t) = \prod_{s \le u \le t} \{ I + d\widehat{\Lambda(u)} \}$$
 AALEN-JOHANSEN ESTIMATOR

Where  $d\widehat{\Lambda(u)}$  is the KxK matrix that has transition hazards on off-diagonal elements while on the diagonal it has minus the sum of all the off-diagonal elements from the same row.

For state-occupancy probabilities, we can use the same estimator since they correspond to transition probabilities from the initial state.

The formula for the cumulative incidence function in the competing risks setting is also special case of this formula.

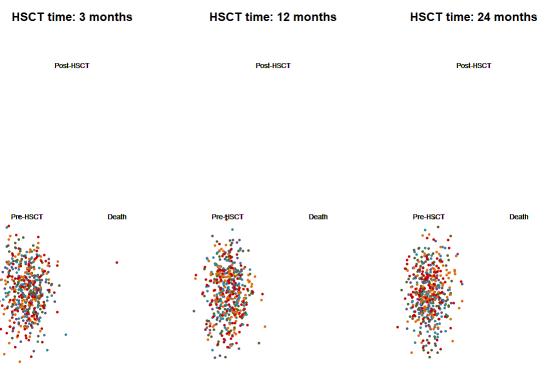
#### What is (dynamic) microsimulation?

They are **individual based** simulations from an **estimated multi-state model**.

They simulate **individual trajectories** in continuous time between **health states using random number generation**.

Why to simulate?

- Cumulative quantities can be analytically obtained only under markov assumption and the computation can be tedious as the model move away from the simple illness-death model
- Perform scenario analyses



#### What is microsimulation?

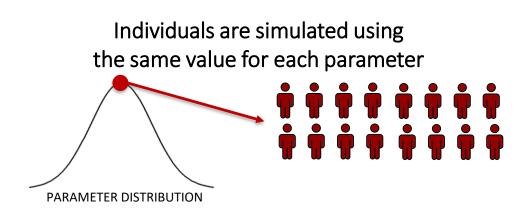
The term microsimulation was introduced by **Orcutt** in **1957** in the context of economic applications. Since then, they have been used also extensively in **health applications**.

From a statistical perspective, they belong to the class of **Monte Carlo simulation methods**.

HSCT time: 3 months HSCT time: 12 months HSCT time: 24 months Why to simulate? Post-HSC Post-HSC Post-HSCT Cumulative quantities can be analytically obtained only under markov assumption and the computation can be tedious as the model move away from the simple illness-death model Perform scenario analyses

## More on microsimulations: Probability Sensibility Analysis

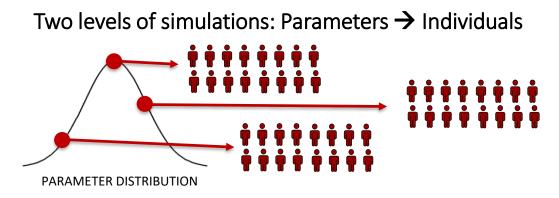
- It is used take into account uncertainty in the estimations of the parameters of the disease model into the microsimulation
- It is inherently a Bayesian idea even tough it can be used also under a frequentist framework



STANDARD MICROSIMULATION

• Disease model → MLE point estimates

#### PSA MICROSIMULATION

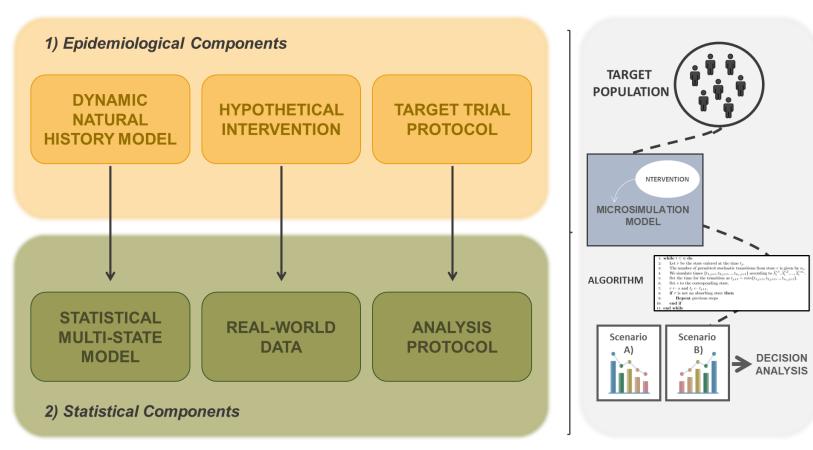


FREQUENTIST FRAMEWORK: we sample the parameters from the asymptotic normal distribution of the MLE (Parametric Bootstrap) or suitable parametric family as appropriate

BAYESIAN FRAMEWORK: we sample the parameters from their posterior probability density

## Dynamic scenario analysis

Dynamic scenario analyses can be useful to answer clinical or public health research question which requires following the life course trajectories of a individuals from a population across different health states.



Why scenario analyses?

- Time requirements
- Target trial emulation
- Analytical difficulties
- Combine evidence from multiple sources of information

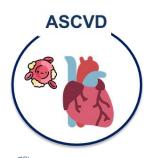


# Personalized optimal timing for a treatment

Gregorio, C., Spreafico, M.,..., Della Porta, M.G., Ieva, F. **Personalized timing for allogeneic stem cell** transplantation in haematological neoplasms: a target trial emulation approach using multi-state modelling and microsimulation. JCO CCI. doi: <u>10.1200/CCI.23.0020</u>

Tentori\*, C.A., Gregorio, C.\*,,..., R., Ieva\*\*, F., Della Porta\*\*.

**Clinical and genomic-based Decision Support System to define the optimal timing of allogeneic hematopoietic stem cell transplantation in patients with myelodysplastic syndromes**.\* co-first; \*\*co-last. Journal of Clinical Oncology. doi:10.1200/JCO.23.02



### **Cost-effectiveness of a treatment**

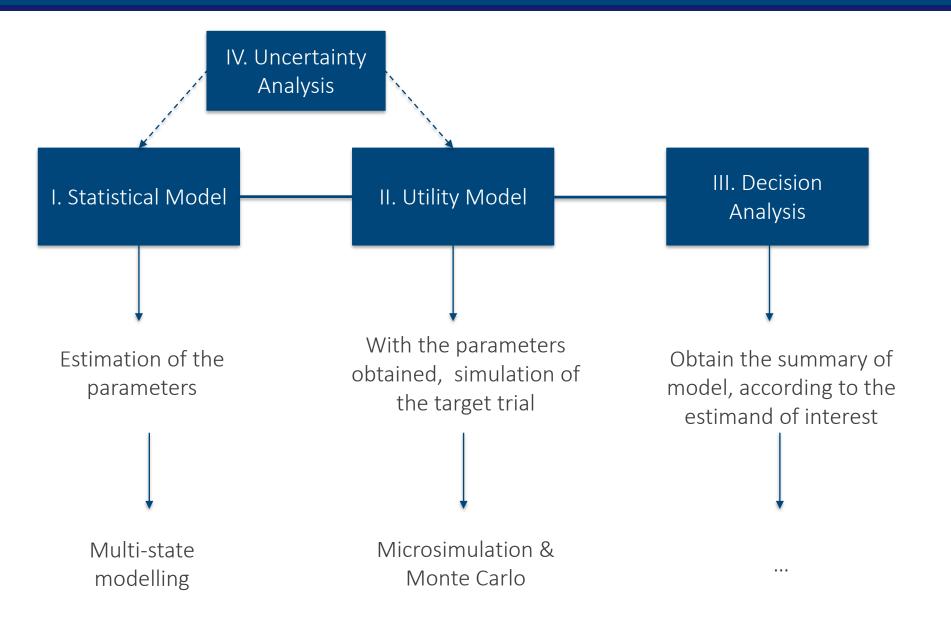
Gregorio, C., Rea, F., Ieva, F., Scagnetto, A., Indennidate, C., Cappelletto, C., Di Lenarda, A., Barbati, G. **Flexible approaches based on multistate models and microsimulation to perform real-world cost effectiveness analyses: an application to PCSK9-Inhibitors.** Value in Health. doi: <u>10.1016/j.jval.2024.03.008</u>



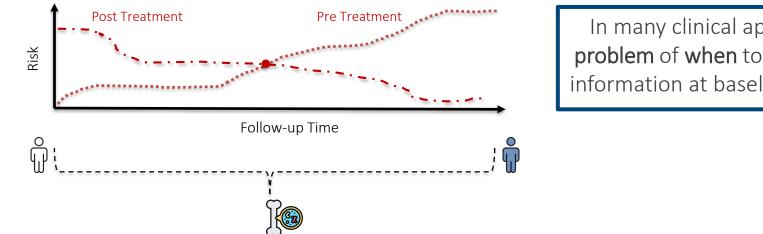
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TRIESTE

### Overview of the statistical steps



## Case study: identification of a personalized treatment timing



In many clinical applications, it is of interest to study the **decision problem** of **when** to perform a **treatment** based on available patient information at baseline to **minimize** the **overall risk** of **adverse events**.

Ideally, we would like to design a randomized clinical trial but it is often impossible for practical or ethical reason. → The aim of this work is to propose statistical methods to tackle this problem using available observational data



Personalized treatment decision

Myelodysplastic syndromes (MDS) are a group of myeloid **neoplasms disorders** caused by poorly formed blood cells or ones that do not work properly.

- MDS condition range from near normal life expectancy to acute myeloid leukaemia with highly variable clinical courses
- Only curative treatment: allogeneic hematopoietic stem cell transplantation (HSCT)
  - Transplantation failure due to toxicity or disease relapse

#### Patient's characteristics

- Age at diagnosis
- Gender



#### Hematological marker

- Hemoglobin [g/l]
- Neutrophils [x10<sup>9</sup>/L]
- Platelets [x10<sup>9</sup>/L]
- % bone marrow blasts





- Relapse after HSCT
- Outcome at last follow-up ${f C}$

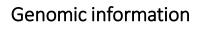


## TreatmentHSCT status

Timing

Around 7000 patients with MDS diagnosis coming from a international multi-center clinical register





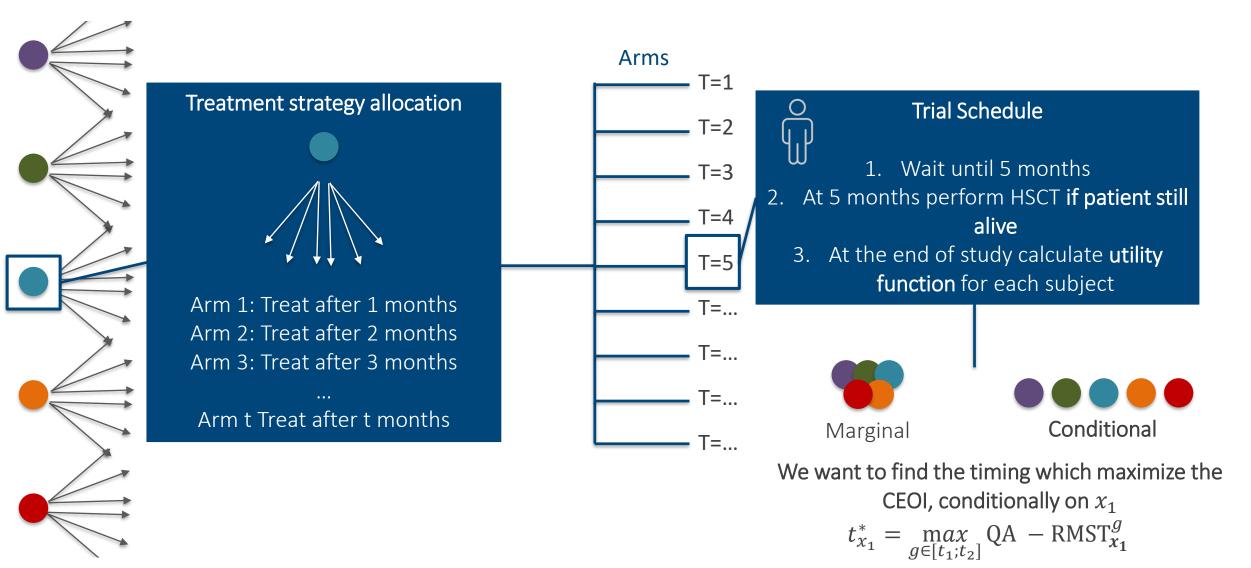
MDS score proposed by Bersanelli et al. 2021

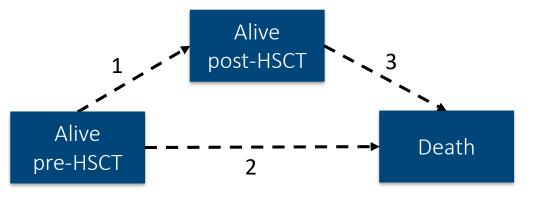
#### Risk stratification

IPSSR score

## **Target Trial**

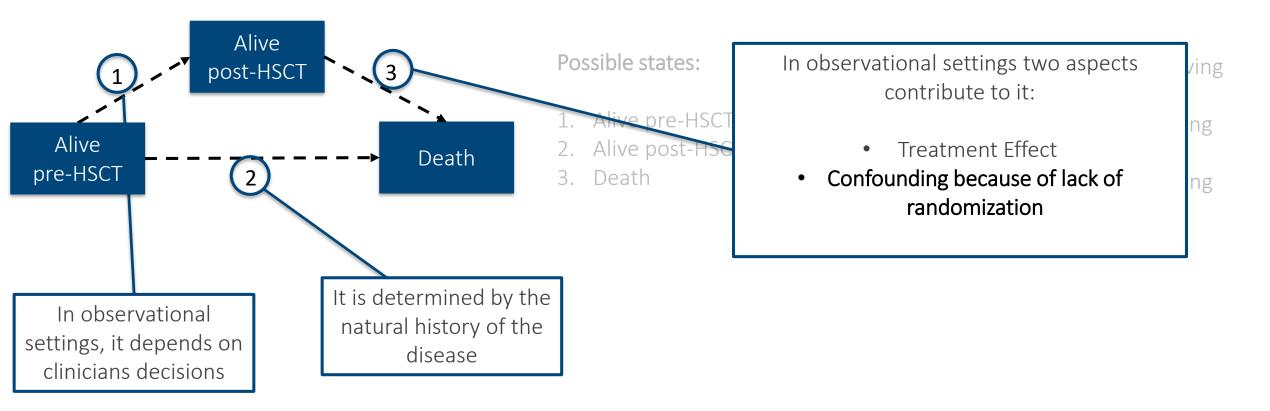
According to Hernan and Robins (2006), we imagine how a clinical randomized trial would look like in this setting

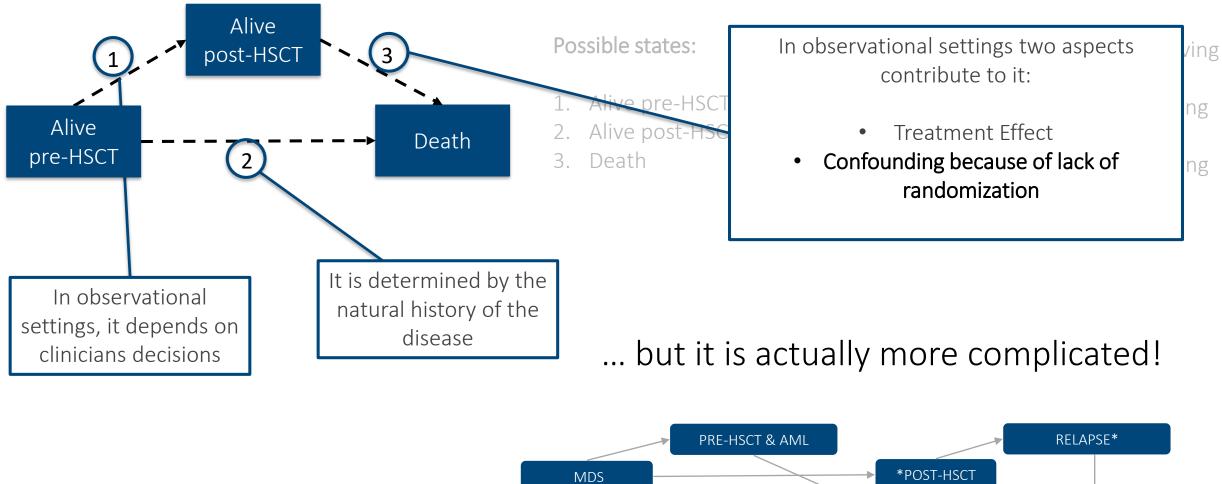


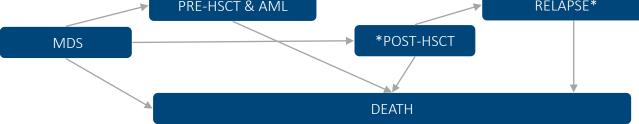


- Possible states:
- 1. Alive pre-HSCT
- 2. Alive post-HSCT
- 3. Death

- Transition 1: model for the hazard of having had HSCT over time
- **Transition 2:** model for the hazard of dying before receiving HSCT over time
- **Transition 3**: model for the hazard of dying after receiving HSCT over time



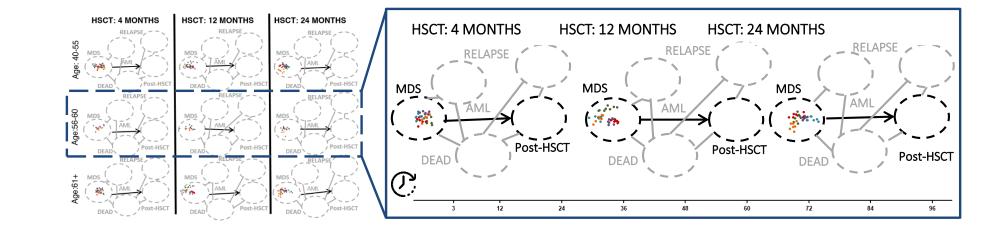




## Microsimulation and decision analyses step

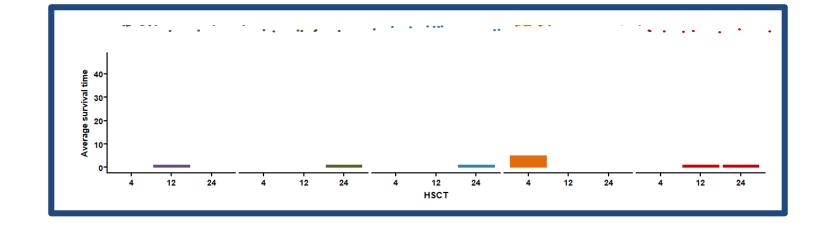
Scenario analysis

Decision Model Through Microsimulation



#### **Decision Analysis**

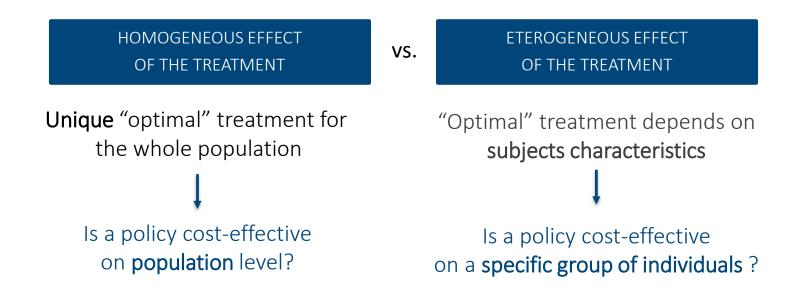
Optimal timing for HSCT



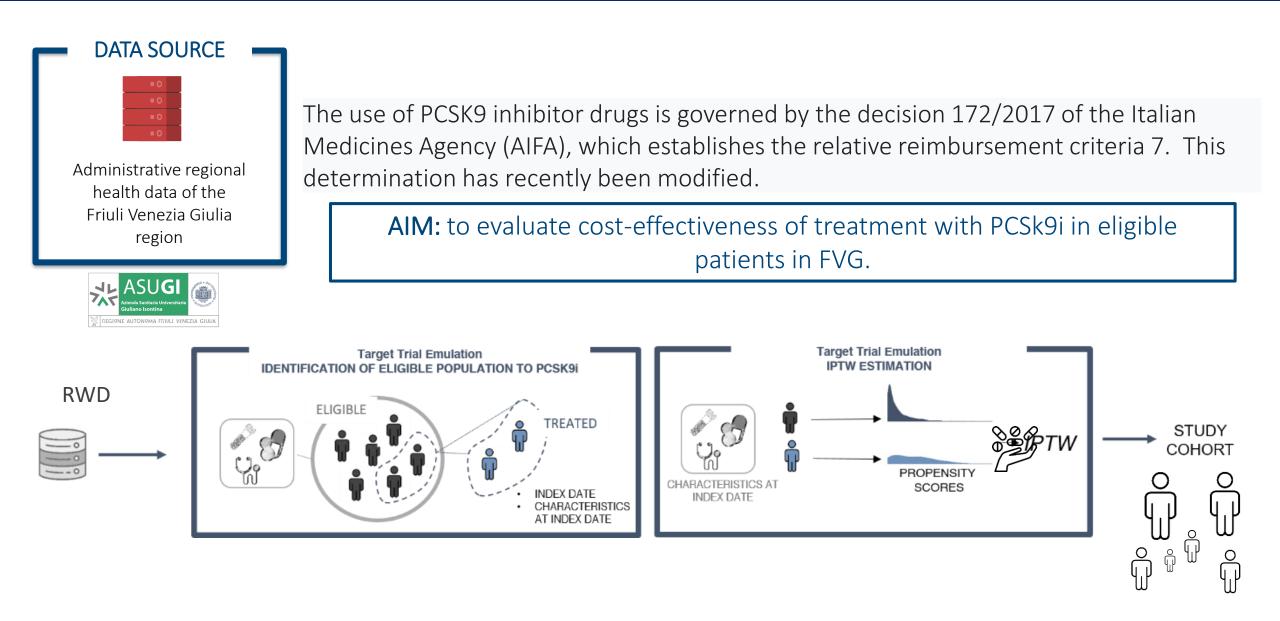
In many clinical applications, it is of interest to study the **impact** of a given intervention taking into its **costs & benefits** in order to allocate resources optimally.

Randomized clinical trials have many limitations

→ Cost-effectiveness analyses propose statistical methods to tackle this problem using available observational data



#### Data & clinical study



## Statistical models in event-history settings for cost-effectiveness analysis

The statistical model are defined by three components:

- **DISEASE MODEL:** multi-state model containing all states describing events/disease stages affecting costs and benefit over time;
- COST MODEL: it defines the costs associated with each state, it can be either deterministic of stochastic.
- HEALTH OUTCOME MODEL : it defines the benefit associated with each state.

In this case:

- **DISEASE MODEL:** estimated from the data;
- **COST MODEL:** it is defined by regional price list for FVG.

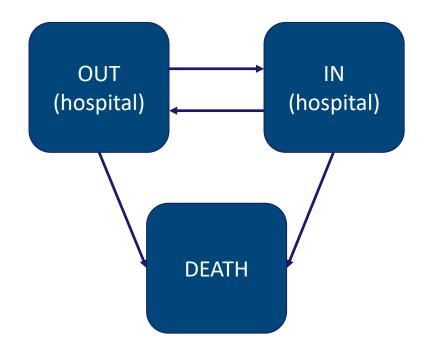


We know the daily cost of being treated with statins and PCSKi9

We know the cost of hospitalizations depending of type and length  $\amalg$ 



- HEALTH OUTCOME MODEL :
  - Weight fixed to 1 for all states.
  - Defined with QALY after hospitalizations



We can model the transition rates using a **proportional hazard** model (leva et al.,2017):

$$\lambda_i^{rs}(u|\mathbf{x}_i) = \lambda_0^{rs}(u) \exp\{\boldsymbol{\beta}_{rs}^T \mathbf{x}_i\}$$

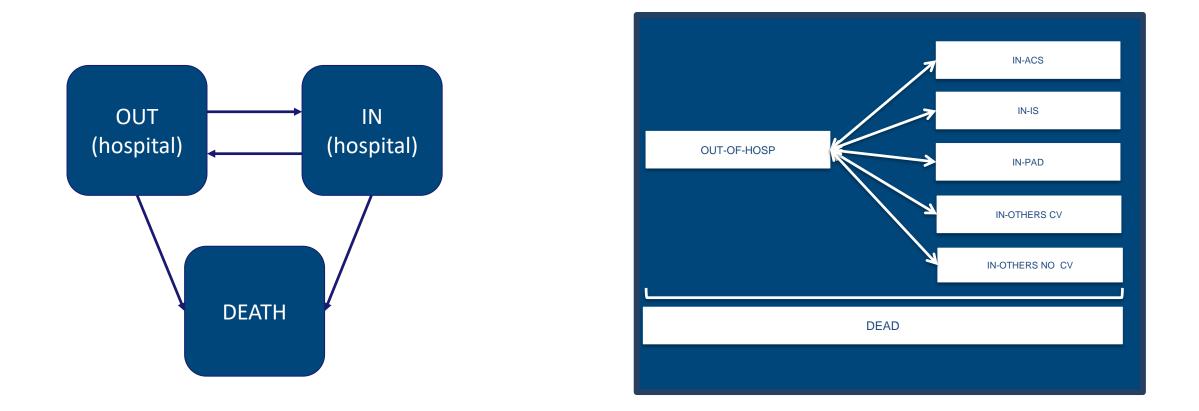
Treatment and optional baseline covariates

To study the:

- Re-admission rate
- Discharge rate
- IN-hospital death
- OUT- of-hospital death

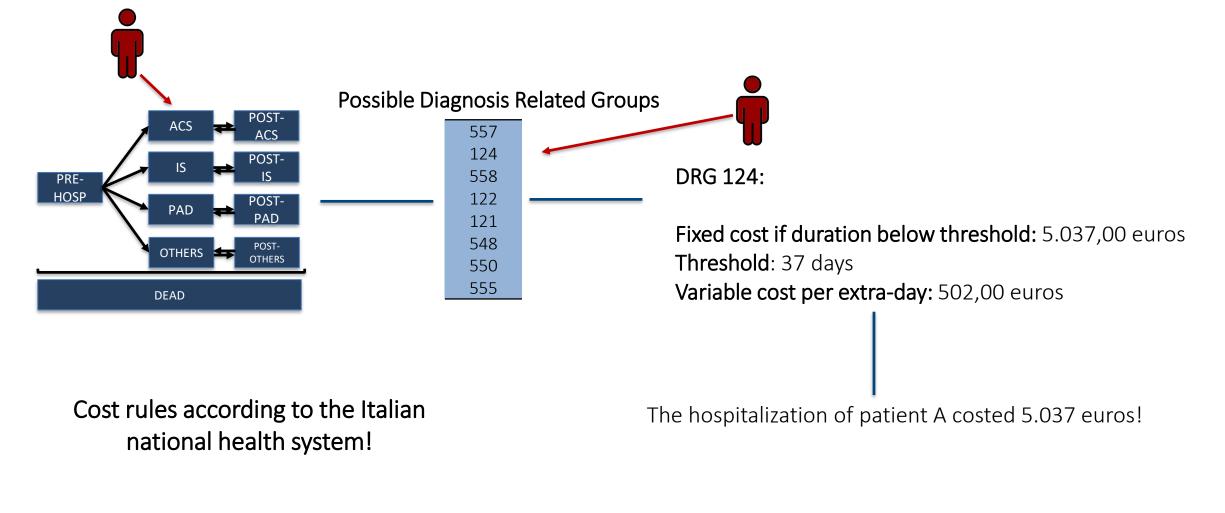
→ Clock reset time scale (Semi-Markov model)

... it is also important to take into account for **previous history**: e.g. number of past hospitalizations

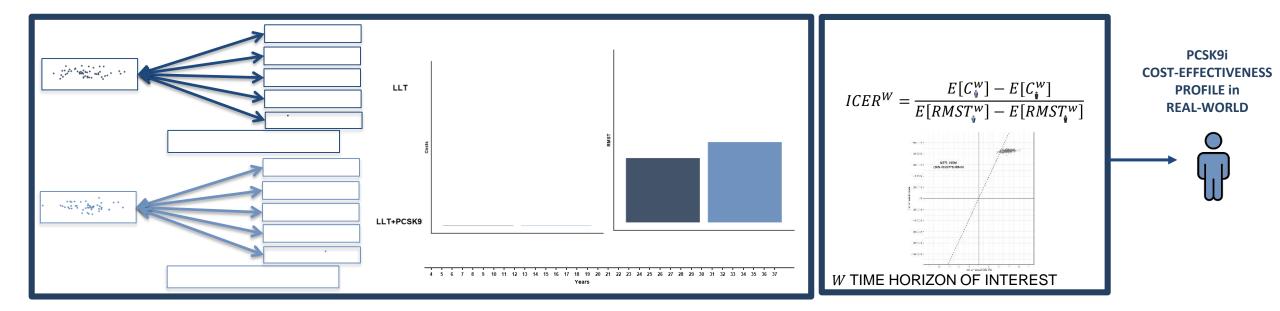


... it is also important to take into account for **previous history**: e.g. number of past hospitalizations

What is the cost attributed to a subjects spending 10 days in hospital for Acute Coronary Syndromes?



#### Microsimulation and health-economic evaluation



Given the large amount of **observational longitudinal data available**, this modeling framework provides a powerful and flexible way to try to answer a variety of questions that relates to developing and progression of **chronic diseases** over the **lifespan of individuals**.

... using models that taking into account the complexities of the phenomena under study

- ... from an individualized prospective
- ... possibly integrating causal inference methods

... allowing for simulating under "alternative scenarios" and assess the impact of possible interventions

With respect to few years ago, what makes a lot of this possible is the advance in computational power!

# Thank you for the attention!

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