CARICHE ELETTRICHE E CORRENTI

$$e = 1{,}602 \cdot 10^{-19} \; C$$

C: unità SI carica elettrica

e: carica elementare

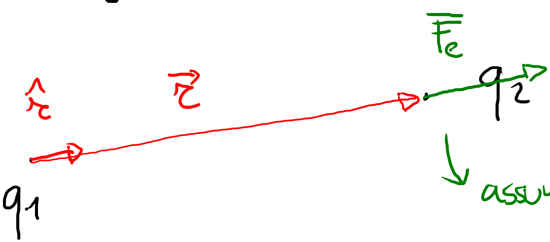
$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0 \varepsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$$

K

ε_r: relativa: numero puro ≥ 1

$= 1$ nel vuoto

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r} \quad \text{con } r = |\vec{r}|$$



assumendo q_1 e q_2 con lo stesso segno

$$\varepsilon_0 = 8{,}854 \cdot 10^{-12} \; \frac{C^2}{m^2 N}$$

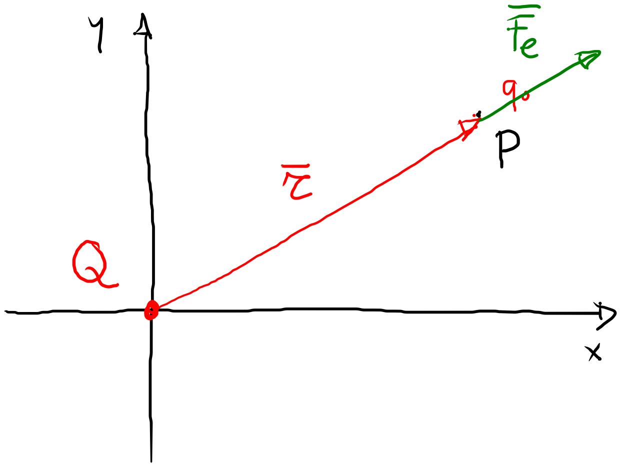
$$K = \frac{1}{4\pi\varepsilon_0} = 9 \cdot 10^9 \; \frac{m^2 N}{C^2}$$

CAMPO ELETTRICO

q_0 carica di prova

$\overline{F_e}$ forza elettrica a cui è soggetta q_0

$$\vec{E} = \frac{\vec{F_e}}{q} \qquad \left(\frac{N}{C}\right)$$

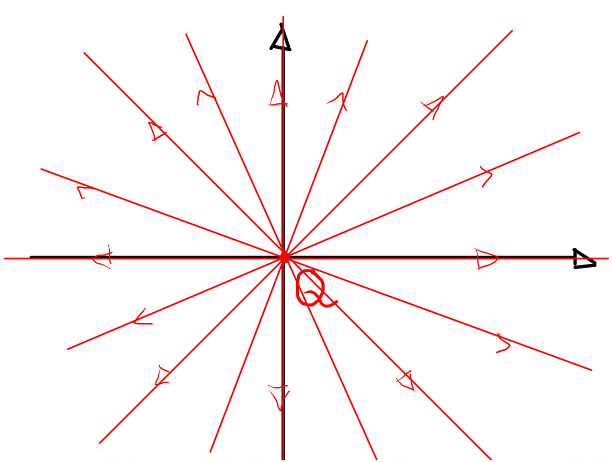


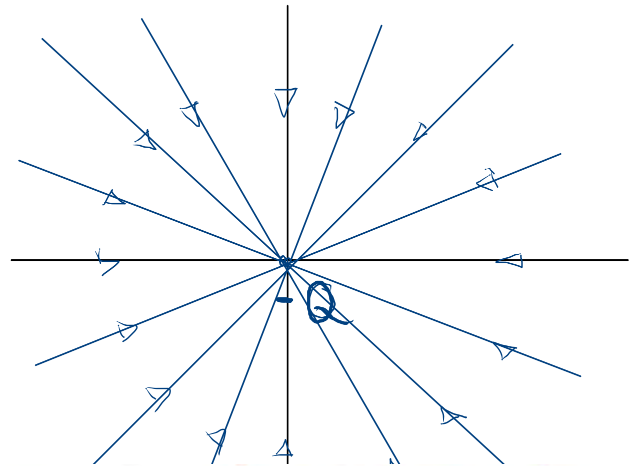
$$\overline{F_e} = \frac{1}{4\pi\varepsilon_0}\frac{Q q_0}{r^2}\hat{r}$$

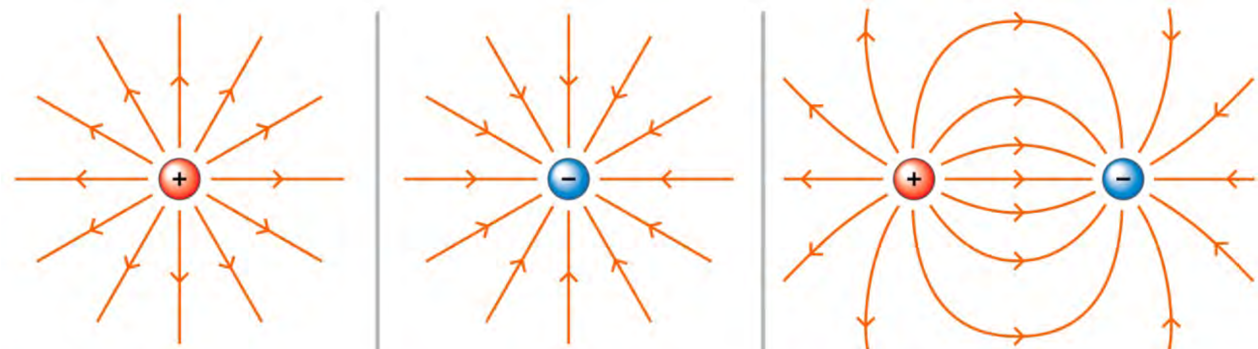
$$\vec{E}(\bar{r}) = \frac{\overline{F_e}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r} \qquad Q > 0$$







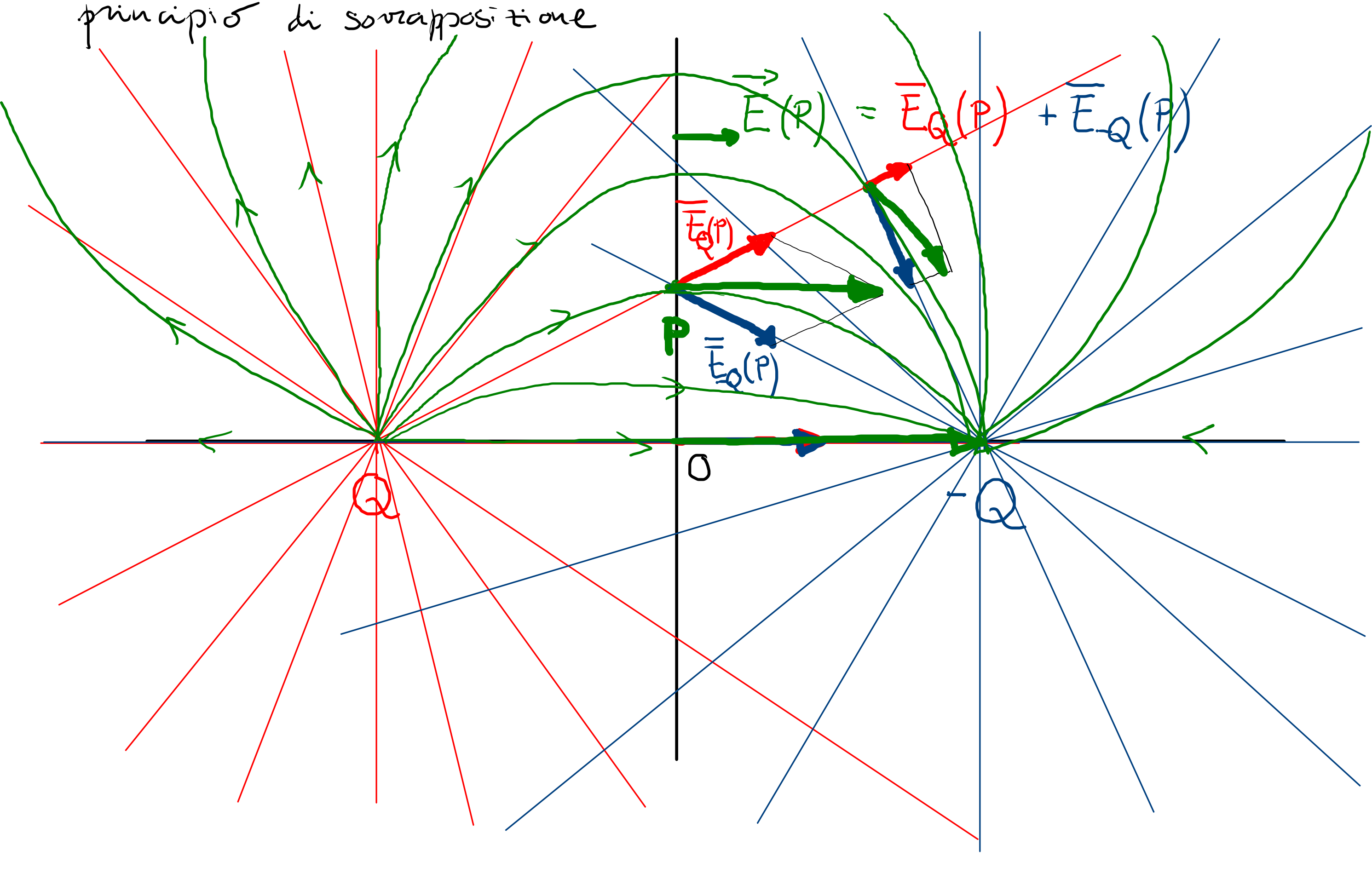
a Linee di forza del campo elettrico creato da una carica positiva.

b Linee di forza del campo creato da una carica negativa.

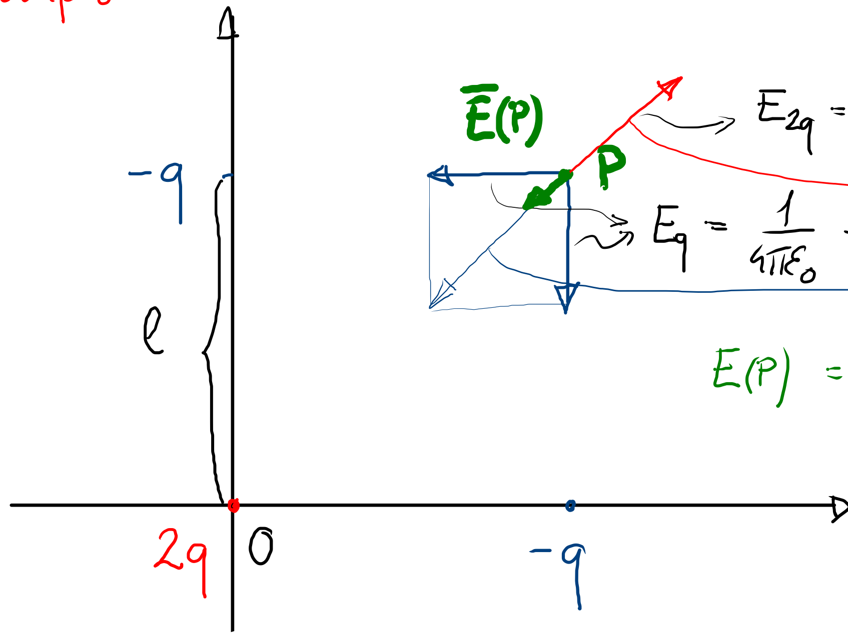
c Linee di forza del campo creato da due cariche uguali e opposte (dipolo elettrico).

CAMPO DI DIPOLO (= +Q e −Q a d)

principio di sovrapposizione



Esempio

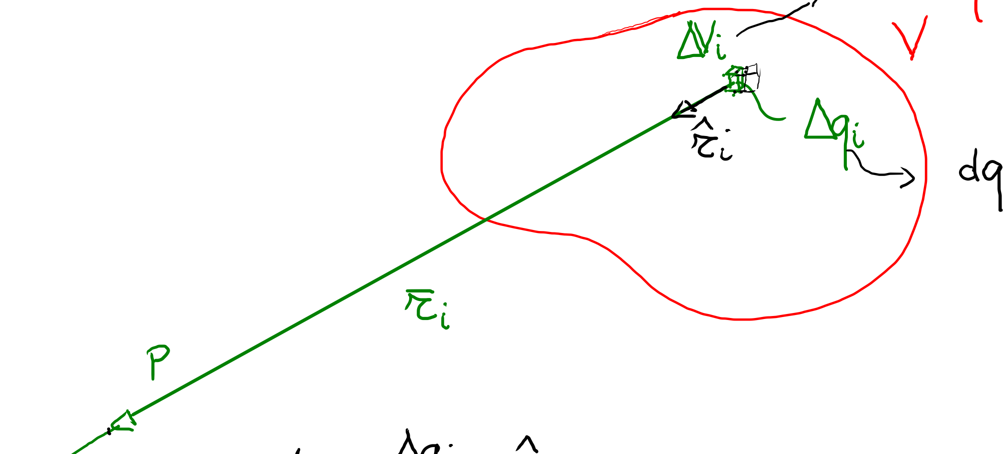


$$E_{2q} = \frac{1}{4\pi\varepsilon_0}\,\frac{\cancel{2}q}{(\ell\sqrt{2})^{\cancel{2}}} = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\ell^2}$$

$$E_q = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\ell^2}$$

$$E(P) = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{\ell^2}\left(\sqrt{2}-1\right)$$

CAMPO $\overline{E}$ in P da una distribuzione qualsiasi di carica



$$\Delta\vec{E}_i = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \sum_i \Delta\vec{E}_i = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{\Delta q_i}{r_i^2}\hat{r}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{\Delta q_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{r}$$

ESEMPI

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_V \frac{dq}{r^2} \hat{r}$$

→ Q è distribuita uniformemente in V ⇒ $\rho_e \equiv \frac{Q}{V}$

ρ_e densità di carica elettrica

$$dq = \rho_e \, dV$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \rho_e \int_V \frac{dV}{r^2} \hat{r}$$

→ Q è distribuita uniformemente su S ⇒ $\sigma = \frac{Q}{S}$

σ densità superficiale di carica

$$dq = \sigma \, dS$$

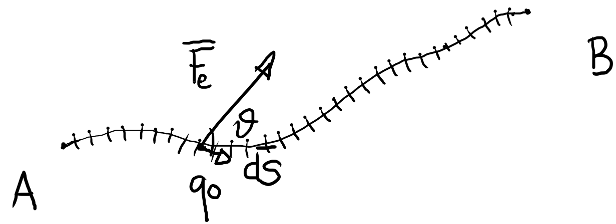
→ Q è distribuita uniformemente su ℓ ⇒ $\lambda = \frac{Q}{\ell}$

λ densità lineare di carica

$$dq = \lambda \, d\ell$$

ENERGIA POTENZIALE ELETTRICA

$* \quad \mathcal{L} = -\Delta U = U_A - U_B$



$** \quad \mathcal{L} = \int_A^B \vec{F}_e \cdot d\vec{s} = \int_A^B q_0 \vec{E} \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$

$* + ** \qquad \Delta U = -\mathcal{L} = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$

ΔU dipende da q_0, quindi si introduce ....

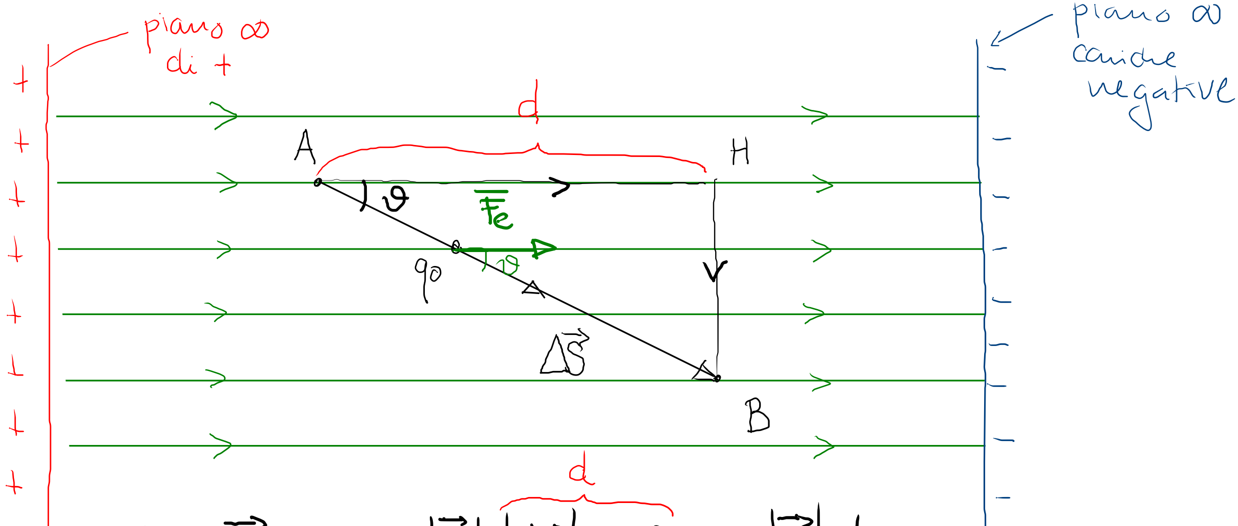
POTENZIALE ELETTRICO

$$V = \frac{U}{q_0} \qquad [V] = \frac{1\,J}{1\,C} = 1\,\text{Volt} = 1\,V$$

$$\Delta V = V_B - V_A = \frac{\Delta U}{q_0} = -\int_A^B \bar{E} \cdot d\bar{s}$$

$$[E] = \frac{N}{C} = \frac{N \cdot m}{C \cdot m} = \frac{J}{C \cdot m} = \frac{V}{m}$$

ESEMPIO



$$\mathcal{L} = \vec{F_e}\cdot\Delta\vec{S} = |\vec{F_e}|\cdot\overbrace{|\Delta\vec{S}|\cos\vartheta}^{d} = q_0|\vec{E}|\,d = q_0 E d$$

$$\mathcal{L} = -\Delta U$$

$$\Delta U = -q_0 E d$$

$$\Delta V = \frac{\Delta U}{q_0} = -Ed$$

$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s}$$

POTENZIALE ELETTRICO IN UN PUNTO P

$$\Delta V = -\int_A^B \vec{E}\cdot d\vec{s} = V_B - V_A$$

$$\downarrow$$

$B \to P$

$A \to \infty \qquad V_A = 0$

$$\downarrow$$

$$-\int_\infty^P \vec{E}\cdot d\vec{s} = V_P$$

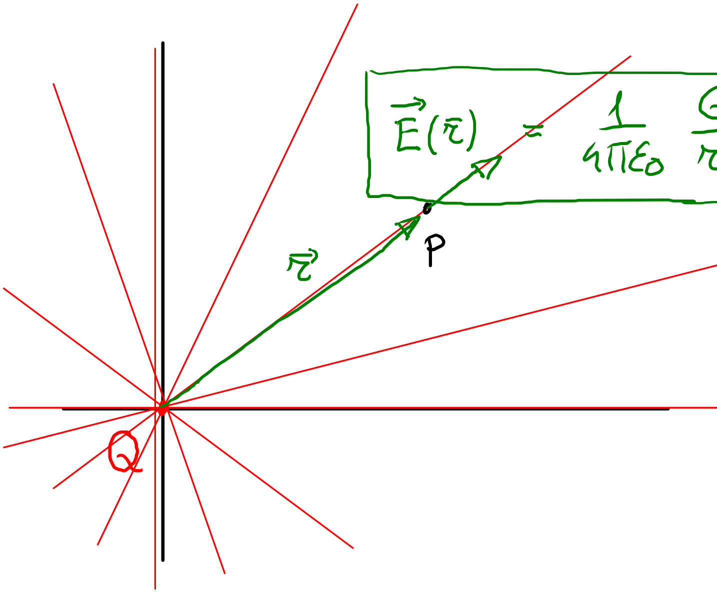
$$\boxed{V_P = -\int_\infty^P \vec{E}\cdot d\vec{s}}$$

V_P rappresenta il lavoro per unità di carica necessario per portare q_0 dall'∞ a P contro le forze elettriche

Dal punto di vista matematico:

$$V_P = \int_P^\infty \vec{E}\cdot d\vec{s}$$

POTENZIALE ELETTRICO di Q posta in O



$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}}$$

dimostrazione negli appunti.

$$\boxed{V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}}$$

$V(\vec{r})$ è uno scalare con segno.

$$dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$V(\vec{r}) = \int dV(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

POTENZIALE E CAMPO ELETTRICO (approfondimento)

$$\Delta V = \int_A^B dV = -\int_A^B \overline{E} \cdot d\overline{s}$$

$$\boxed{\vec{E} = -\overline{\nabla} V}$$

$$dV = -\overline{E} \cdot d\overline{s}$$

$$\downarrow$$

1D (asse x)

$$\vec{E} \longrightarrow E_x \hat{i}$$

$$d\vec{s} \longrightarrow dx \hat{i}$$

$$\downarrow$$

$$dV = -E_x \, dx$$

$$E_x = -\frac{dV}{dx} \qquad E_y = -\frac{dV}{dy} \qquad E_z = -\frac{dV}{dz}$$

$$\vec{E} = -\frac{dV}{dx}\hat{i} - \frac{dV}{dy}\hat{j} - \frac{dV}{dz}\hat{k} = -\overline{\nabla} V$$

CONDUTTORI (metallici)

Le cariche sono libere di muoversi $\Rightarrow$ è allo stesso V

$Q = 0$ neutro $\Rightarrow$ $V = 0$

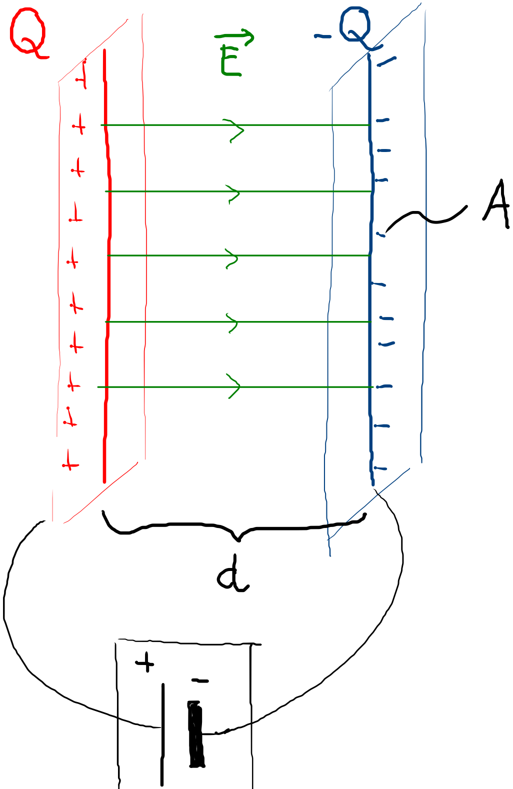
$Q \neq 0$ carico $V \neq 0$

cost. di prop. CAPACITÀ DEL CONDUTTORE

$$\boxed{Q = C\,V}$$

$$C = \frac{Q}{V} \qquad [C] = \frac{[Q]}{[V]} = \frac{C}{V} = F \qquad 1 \text{ Farad}$$

CAPACITÀ DI UN CONDENSATORE a facce piane e parallele



generatore di tensione
ΔV

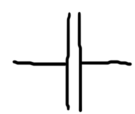
$$|\vec{E}| = \frac{1}{\varepsilon_0}\frac{Q}{A} \qquad Q = \varepsilon_0 A E$$

$$\Delta V = E \cdot d$$

$$C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A \cancel{E}}{\cancel{E} d} = \varepsilon_0 \frac{A}{d}$$

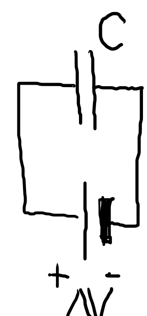
$$\boxed{C = \varepsilon_0 \frac{A}{d}}$$

\+

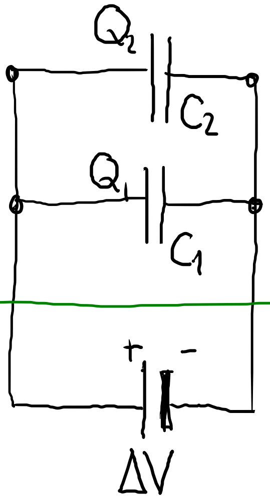


condensatore

=



CONDENSATORI IN PARALLELO



$$C_2 = \frac{Q_2}{\Delta V_2}$$

$$C_1 = \frac{Q_1}{\Delta V_1}$$

$$C_{eq} = \frac{Q_{eq}}{\Delta V}$$

1) $\Delta V = \Delta V_1 = \Delta V_2$

2) $Q_{eq} = Q_1 + Q_2 = Q$

da 2 $C_{eq}\,\Delta V = C_1\,\Delta V_1 + C_2\,\Delta V_2$

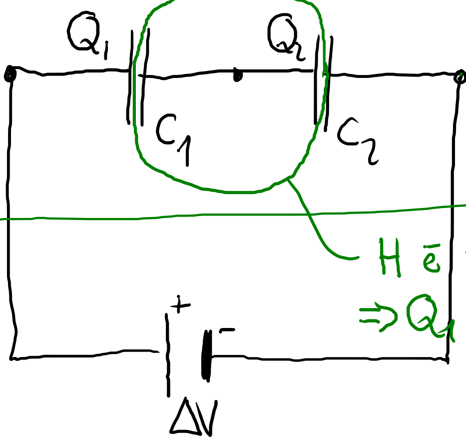
da 1 $C_{eq}\,\cancel{\Delta V} = C_1\,\cancel{\Delta V} + C_2\,\cancel{\Delta V}$

$$\boxed{C_{eq} = C_1 + C_2}$$

per n condensatori in parallelo

$$C_{eq} = C_1 + C_2 + \cdots\cdots + C_n$$

CONDENSATORI IN SERIE

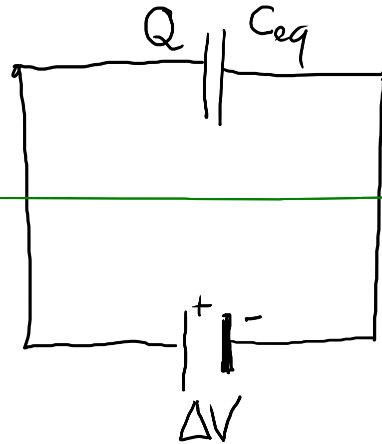


$$C_2 = \frac{Q_2}{\Delta V_2}$$

$$C_1 = \frac{Q_1}{\Delta V_1}$$

H è neutra

$\Rightarrow Q_1 = Q_2$



$$C_{eq} = \frac{Q}{\Delta V}$$

1) $\Delta V_1 + \Delta V_2 = \Delta V$

2) $Q_1 = Q_2 = Q$

da 1) $\dfrac{Q_1}{C_1} + \dfrac{Q_2}{C_2} = \dfrac{Q}{C_{eq}}$

da 2) $\dfrac{\cancel{Q}}{C_1} + \dfrac{\cancel{Q}}{C_2} = \dfrac{\cancel{Q}}{C_{eq}}$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

per n condensatori in serie

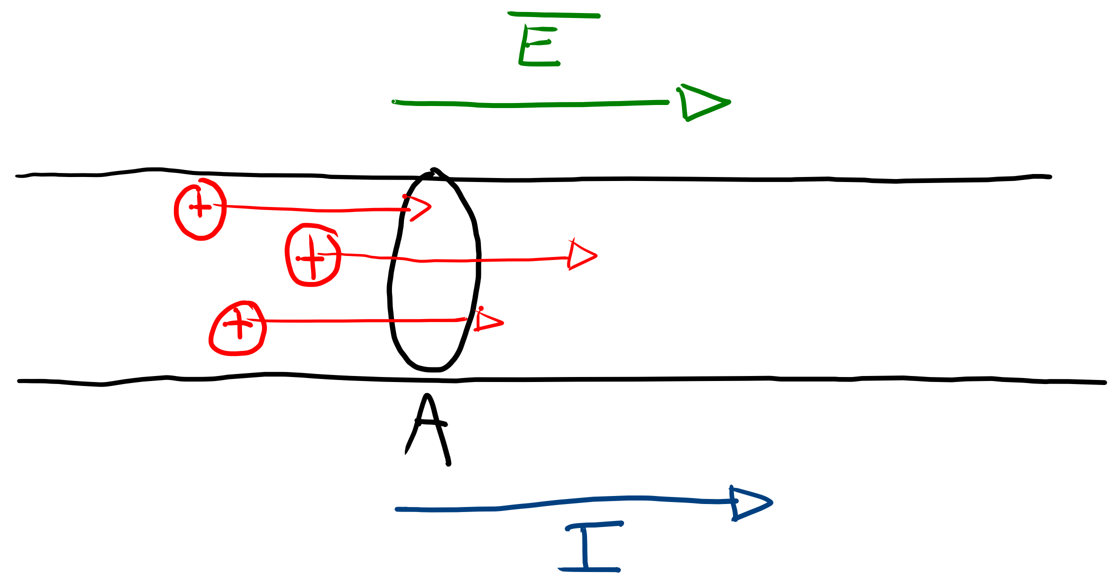
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

ENERGIA DI UN CONDENSATORE CARICO

$$\boxed{U = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2 = \frac{1}{2} \frac{Q^2}{C}}$$

$$C = \frac{Q}{\Delta V}$$

CORRENTE ELETTRICA



ΔQ la carica che attraversa A in Δt

$$I_m = \frac{\Delta Q}{\Delta t} \qquad [I_m] = \frac{C}{s} = \text{Ampere}$$

fondamentale in SI

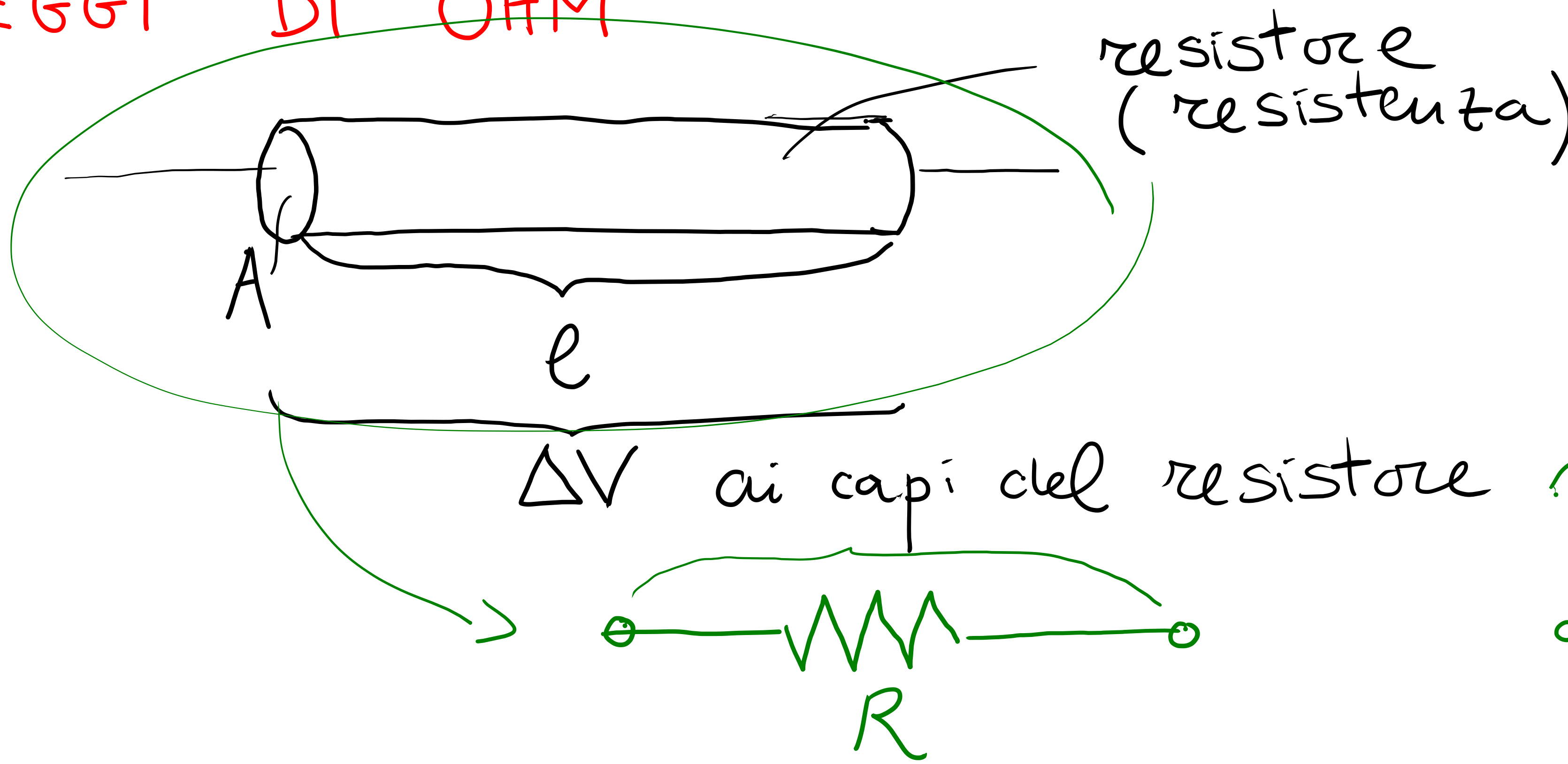
$$1\,A = \frac{1\,C}{1\,s}$$

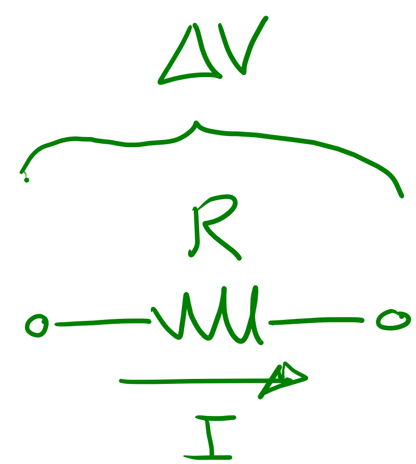
$$1\,C = 1 \cdot A \cdot s$$ derivata in SI

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

il segno di I ne determina il verso

LEGGI DI OHM





1) $\boxed{\Delta V = R I}$

resistenza

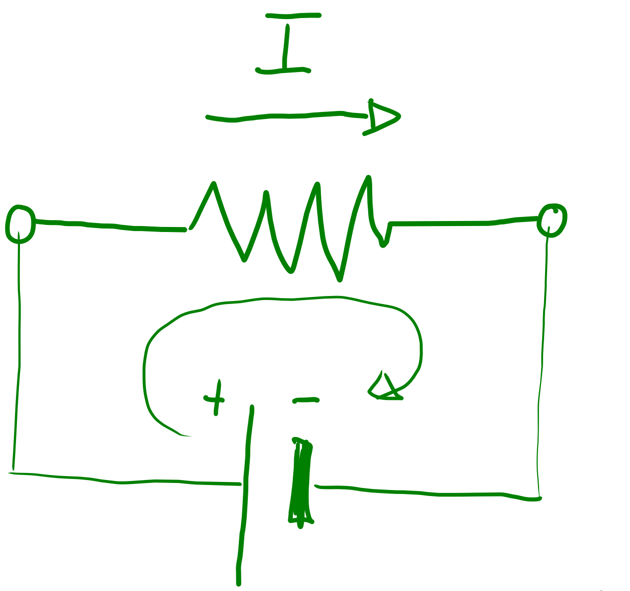
$$[R] = \frac{[\Delta V]}{[I]} = \frac{V}{A} = \Omega \quad (\text{Ohm})$$

2) $R = \rho \dfrac{\ell}{A}$

resistività del materiale
(aumenta con T)

$$[\rho] = \frac{[R][A]}{[\ell]} = \frac{\Omega\, m^{\cancel{2}}}{\cancel{m}} = \Omega \cdot m$$

POTENZA TRASFERITA AL RESISTORE



$$\Delta V = \frac{\Delta U}{q}$$

$$\mathcal{P} = \frac{\Delta U}{\Delta t} = \frac{q\,\Delta V}{\Delta t} = \underbrace{\frac{q}{\Delta t}}_{I} \cdot \Delta V$$

$$I \cdot \Delta V$$

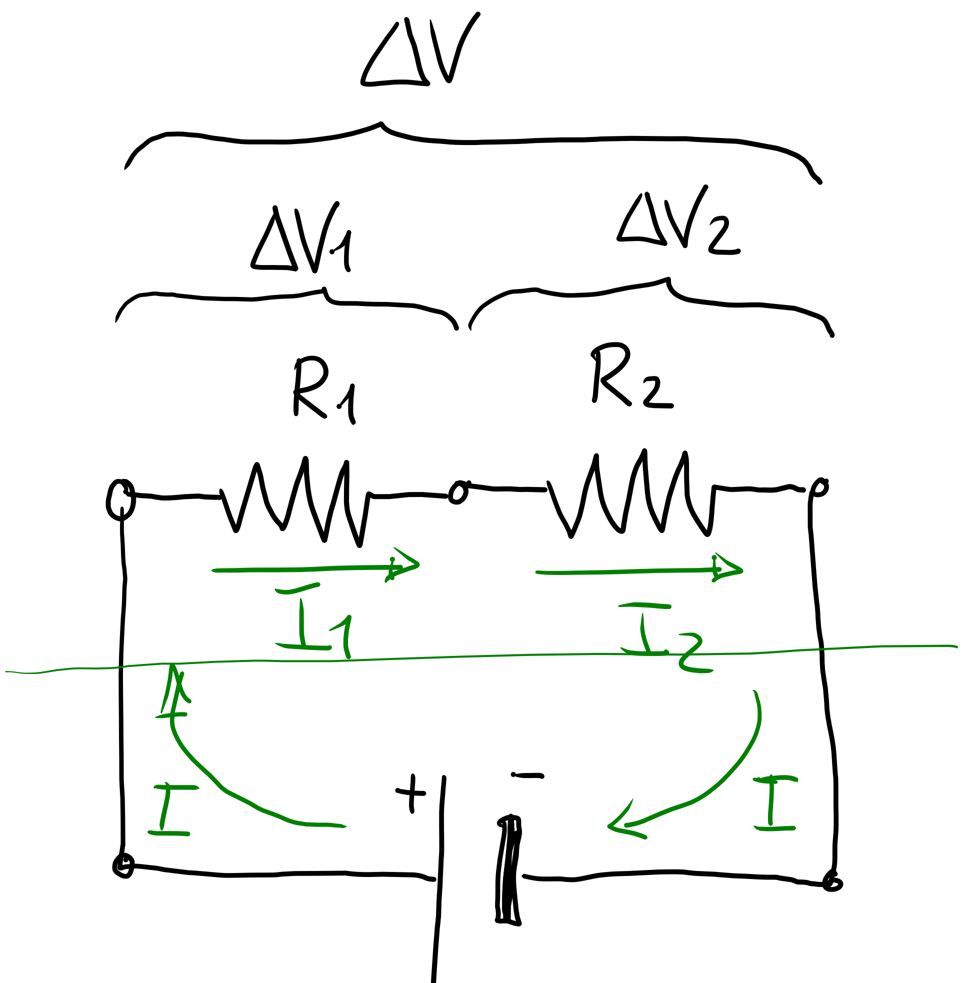
$$\mathcal{P} = I \cdot \Delta V$$

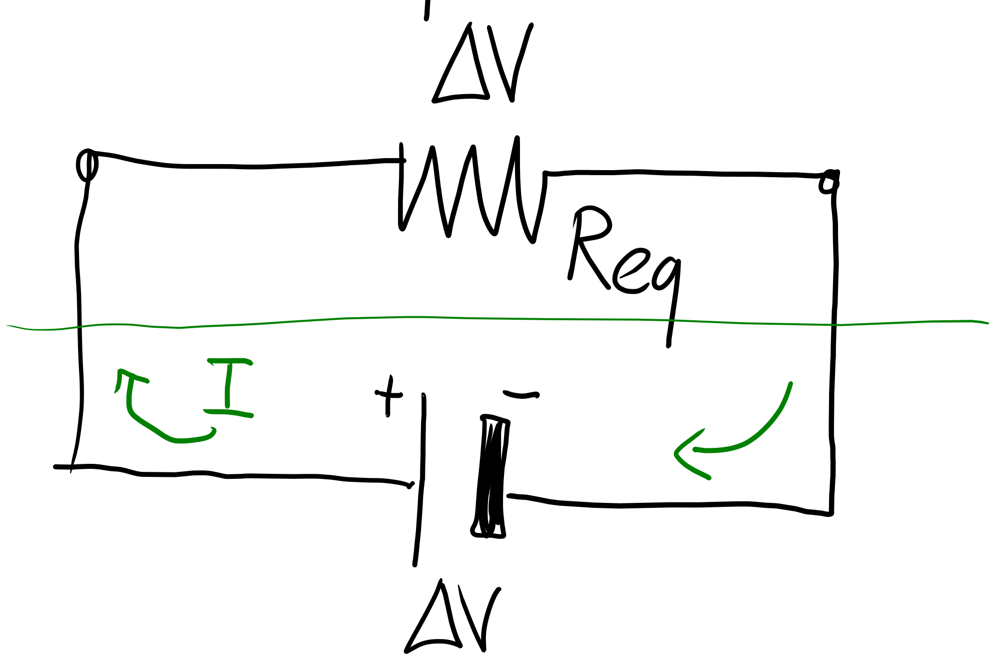
$$\Delta V = RI$$

$$\boxed{\mathcal{P} = I\,\Delta V = RI^2 = \frac{\Delta V^2}{R}}$$

RESISTENZE IN SERIE

$(I_1 = I_2 = I)$





$$\Delta V_1 = R_1 I_1 = R_1 I$$

$$\Delta V_2 = R_2 I_2 = R_2 I$$

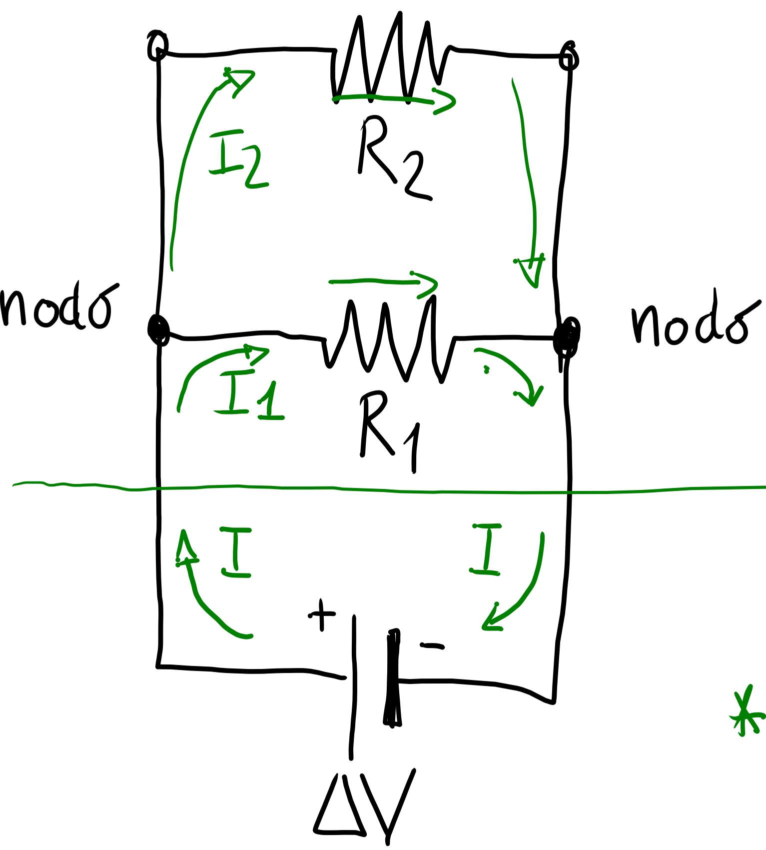
$$\Delta V_1 + \Delta V_2 = \Delta V$$

$$\left.\begin{array}{l} R_1 I + R_2 I = \Delta V \\ \\ \Delta V = R_{eq} \cdot I \end{array}\right\} *$$

$$* \quad \boxed{R_1 \cancel{I} + R_2 \cancel{I} = R_{eq}} \cancel{I}$$

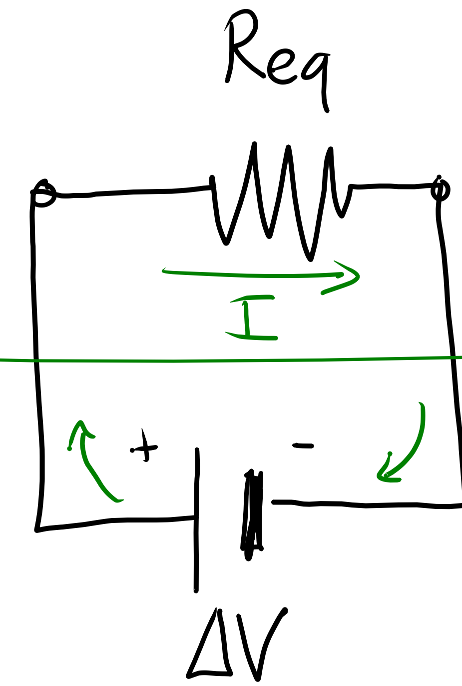
$$R_{eq} = R_1 + R_2 + \dots + R_n$$

RESISTENZE IN PARALLELO $(\Delta V_1 = \Delta V_2 = \Delta V)$ **



* $I_1 + I_2 = I$

$$\Delta V_1 = R_1 I_1$$
$$\Delta V_2 = R_2 I_2$$



* $\Delta V = R_{eq} I$

* $$\frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\Delta V}{R_{eq}}$$

** $$\frac{\cancel{\Delta V}}{R_1} + \frac{\cancel{\Delta V}}{R_2} = \frac{\cancel{\Delta V}}{R_{eq}}$$

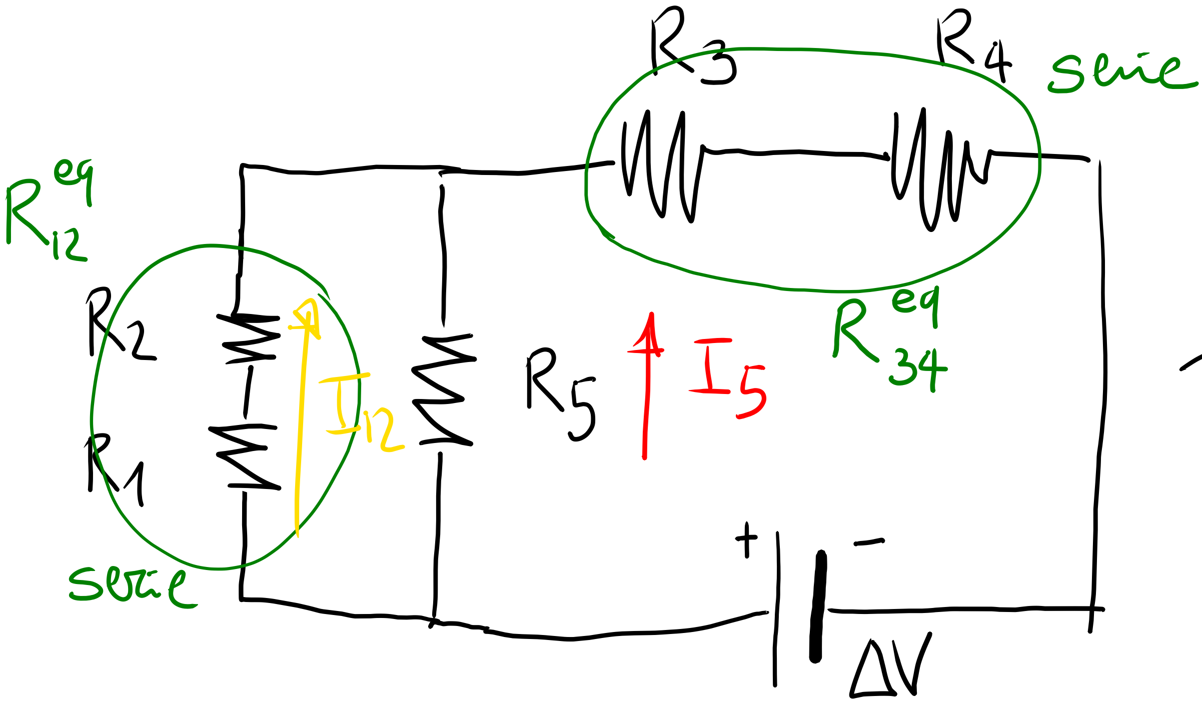
$$\boxed{\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}}}$$

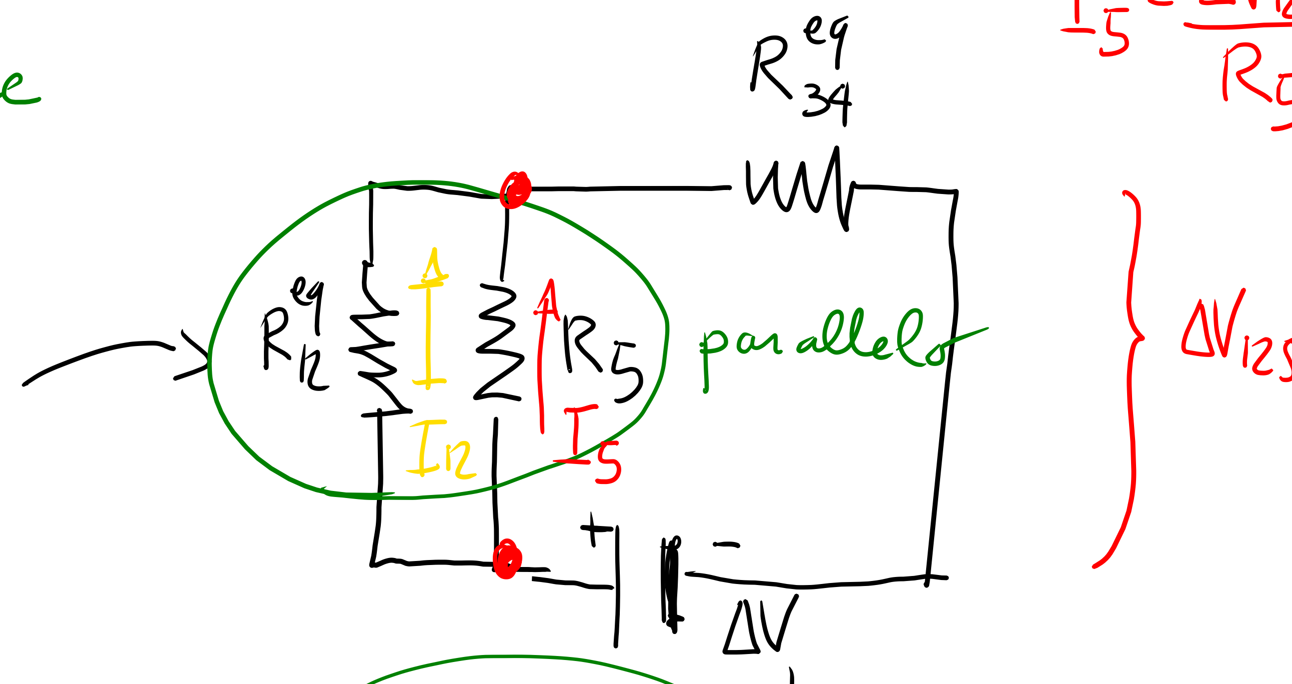
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$

CIRCUITI IN CORRENTE CONTINUA

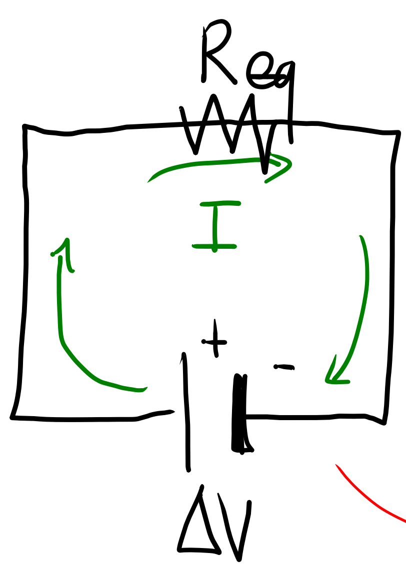
Quanto vale I_5 ?

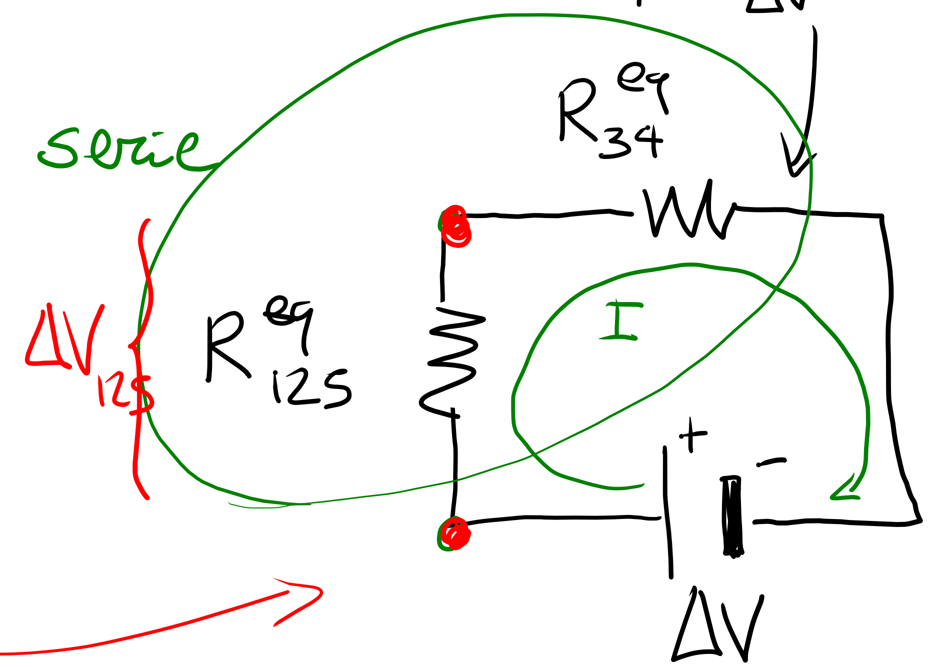
$$I_5 = \frac{\Delta V_{125}}{R_5}$$





$$I = \frac{\Delta V}{R_{eq}}$$

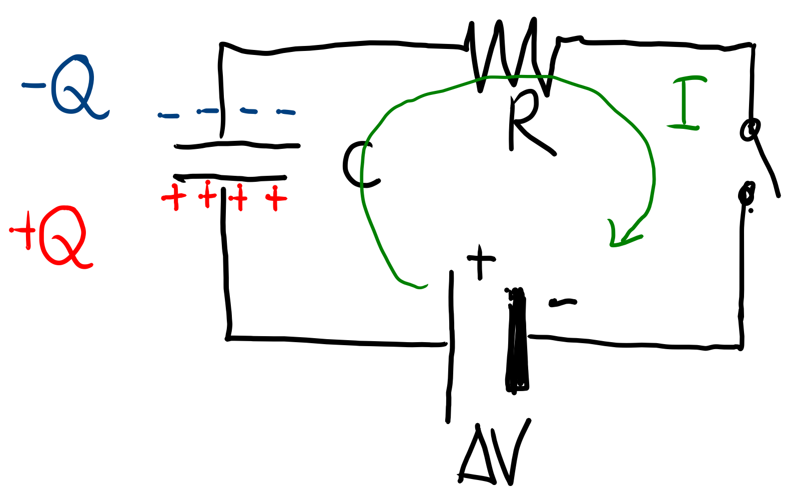




$$\Delta V_{125} = R^{eq}_{125} \cdot I$$

I è la stessa

CIRCUITO RC (carica)



$t = 0$

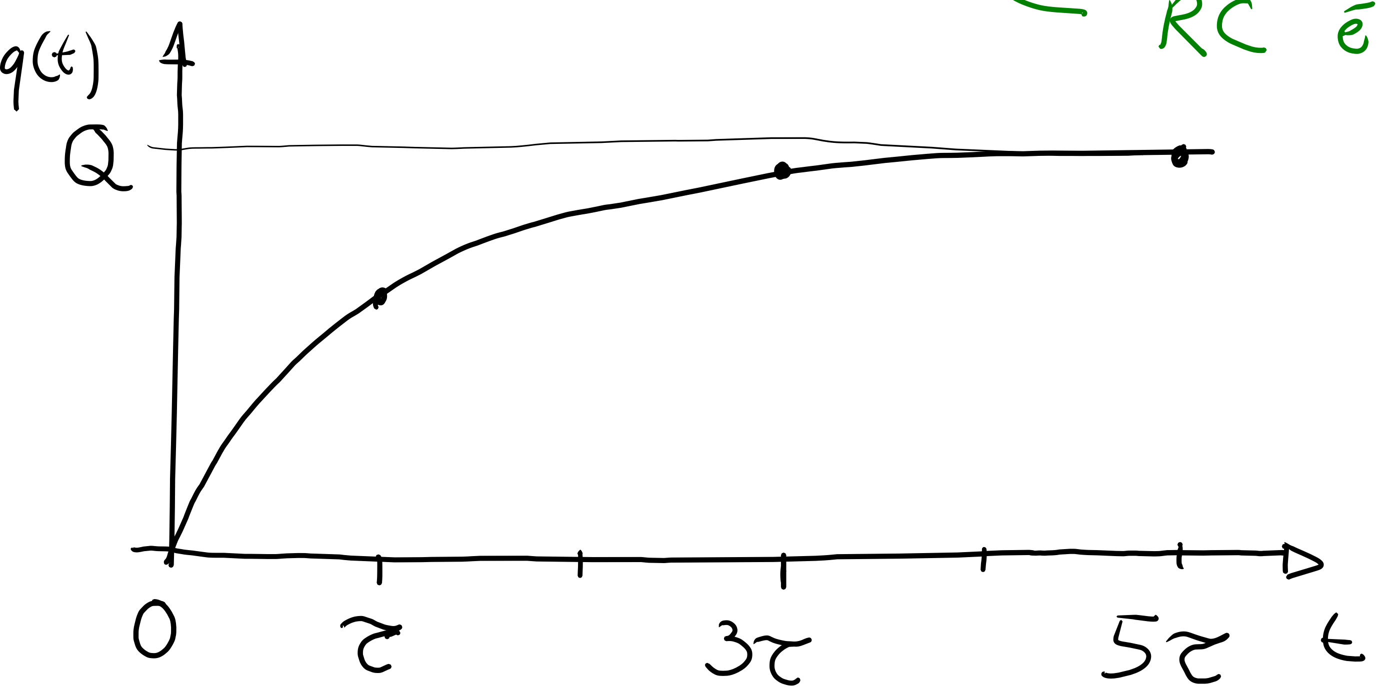
$Q = C \cdot \Delta V$ condensatore carico dopo tempo lungo

Quanto vale $q(t)$, la carica in funzione del tempo?

$$q(t) = Q\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\lim_{t \to \infty} q(t) = Q = C \cdot \Delta V$$

RC è un tempo! $\tau = RC$



$e^{-1} = 0,37$

$e^{-3} = 0,05$

$e^{-5} = 0,007$

$$I = \frac{dq}{dt} = \frac{d}{dt} Q\left(1 - e^{-\frac{t}{RC}}\right)$$

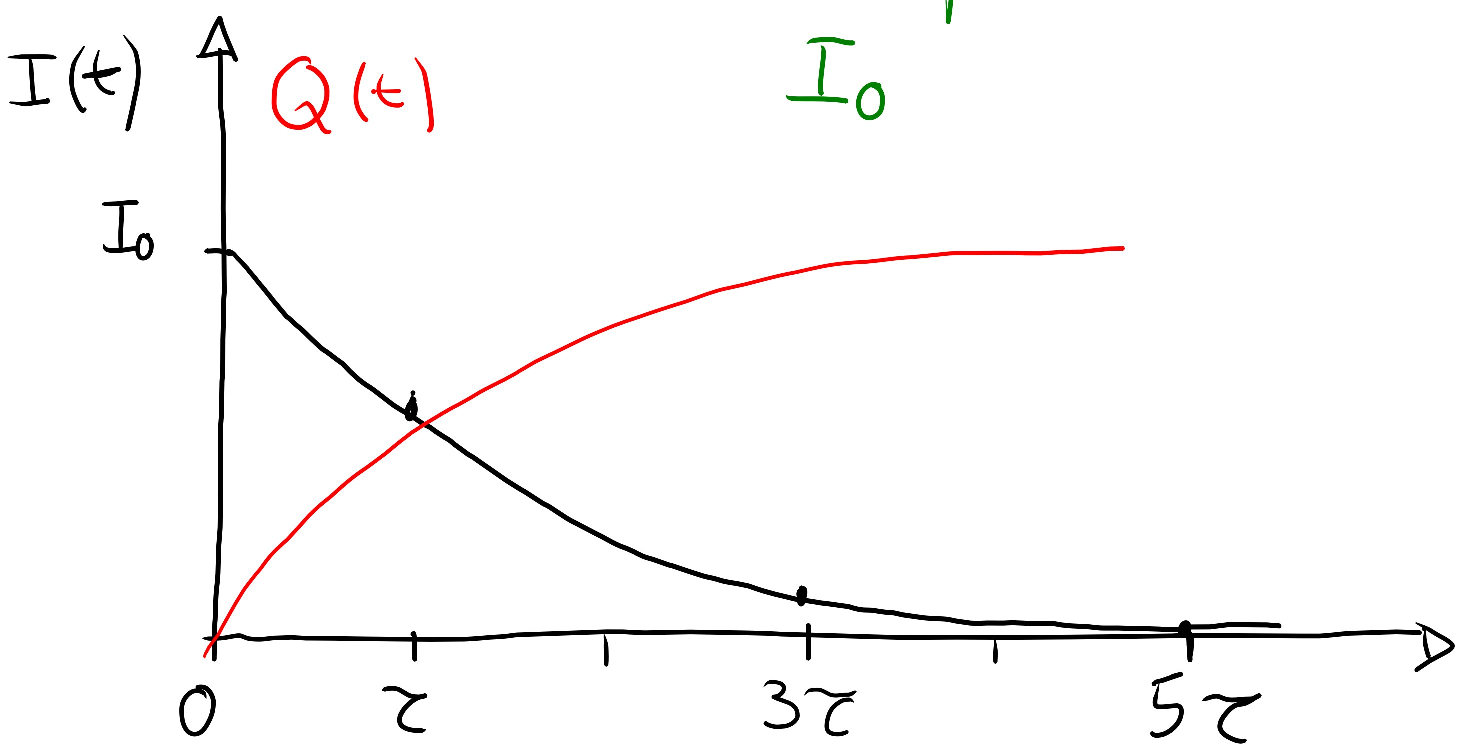
$$= -\frac{d}{dt}\left(e^{-\frac{t}{RC}}\right) Q$$

$$= -Q e^{-\frac{t}{RC}} \cdot \frac{d}{dt}\left(-\frac{t}{RC}\right) = \frac{Q e^{-\frac{t}{RC}}}{RC} = \frac{Q}{RC} e^{-\frac{t}{RC}}$$

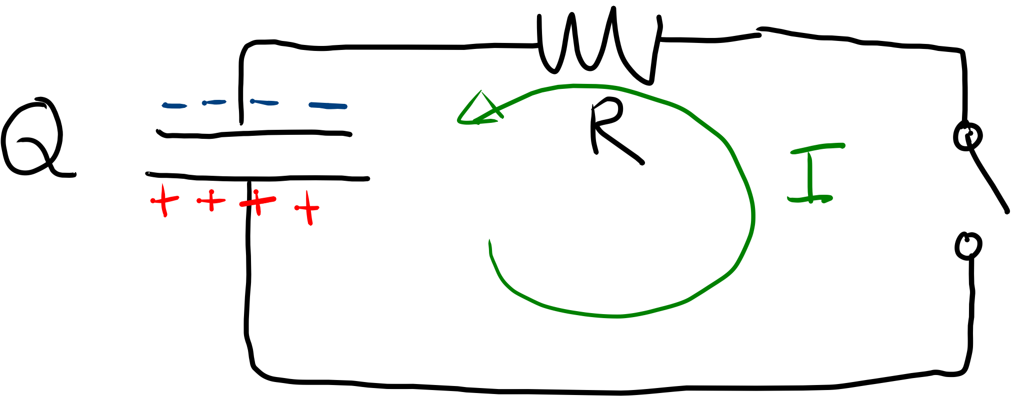
$$I(t) = \frac{Q}{RC} e^{-\frac{t}{RC}} = \underline{I_0 e^{-\frac{t}{RC}}}$$

corrente per $t=0$

I_0



CIRCUITO RC (scarica)



$$q(t) = Q\, e^{-\frac{t}{RC}}$$

$$\frac{d\,q(t)}{dt} = I(t) = Q\,\frac{d}{dt}\,e^{-\frac{t}{RC}} = Q\,e^{-\frac{t}{RC}} \cdot \left(-\frac{1}{RC}\right)$$

$$I(t) = -\underbrace{\frac{Q}{RC}}_{I_0}\,e^{-\frac{t}{RC}}$$

$$I(t) = -\,I_0\,e^{-\frac{t}{RC}}$$

la corrente circola in verso opposto!