## **GEOSTROPHIC FLOWS**

- **After inertial oscillations, homogeneous geostrophic flows are the 7.1 Homogeneous geostrophic flows second simple case where NSEq. can be solved, and can describe natural GFD** phenomena<br>
Let us consider rapidly rotations where the Coriacceleration strongly dominates the various acceleration terms. Let us further consideration terms. Let us further
- **Hypoteses:**  $h$ omogeneous fluids and ignore fractional effects, by assuming  $h$ 
	- $-$  Coriolis term dominates others (= rapidly rotating flows):  $Ro_T \ll 1$  and  $Ro \ll 1$
	- Homogeneous fluids:  $ρ_0 = cost$  and  $ρ' = 0$ Homogeneous nuids:  $\rho_0 = cost$  and  $\rho_0 = 0$
	- Ignore frictional effect (= far from B.L.): <u>*Ek* ≪ 1</u>
- Primitive equations:  $-\,f v = -\frac{1}{\rho_0}$  $\partial p$  $\overline{\phantom{a}}$   $\overline{\$  $+fu = -\frac{1}{\rho_0}$ ∂p  $\partial y$ 0 =  $-\frac{1}{\rho_0}$  $\partial p$  $\partial z$  $\partial u$  $\partial x$  $+$  $\partial v$  $\partial y$  $+$  $\partial w$  $\partial z$  $= 0,$  $\overline{u}$  and  $\overline{u}$  $0 = -\frac{1}{2}$ ∂  $\frac{\mu}{2}$  $\partial u - \partial v - \partial w$  $\frac{1}{\partial x} + \frac{1}{\partial y} + \frac{1}{\partial z} = 0,$  $\sigma_x$   $\sigma_y$   $\sigma_z$
- $\partial_z$  of x,y-momentum eq.  $\left[\ldots\right]$ :  $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z}$  $= 0$

=> Taylor-Proudman theorem: "horizontal velocity field has no vertical shear and all the particles on the same vertical move in concert"  $i$  is distributed the sense that the horizontal velocity field has no vertical shear and that all particles on the all particles on  $\mu$ 

## **GEOSTROPHIC FLOWS**

 $u = \frac{-1}{4}$ 

 $\rho_0 f$ 

 $\partial p$ 

 $, v =$ 

 $+1$ 

 $\rho_0$ 

 $\partial y$ 

- **Solving the x,y-momentum eqs.:**
- $\bullet$   $(u,v) = \overline{u} \perp \nabla p$  the flow is across-gradient (or isobaric):
	- NO pressure work is performed either on the fluid or by the  $\overline{\textbf{i}}$  initiated the flow can persist without a continuous energy source  $\overline{\textbf{i}}$ initiated the flow can persist without a continuous energy source  $\iota$





http://it.wikipedia.org/wiki/Immagine:Low\_pressure

## **GEOSTROPHIC FLOWS**

- **GEOSTROPHY** comes from  $\gamma \eta = Earth$  and  $\sigma \tau \rho o \varphi \eta = turning$  $\frac{1}{2}$  remaining the direction concerns the direction of  $\frac{1}{2}$  $\bullet$  GEOSTROPHY comes from  $\gamma\eta=Earth$  and  $\sigma\tau\rho o\varphi\eta=tur$ **aming in the SIGN of the signs in expressions (** $\boldsymbol{r}$ **.4)**  $\boldsymbol{r}$  respectively ( $\boldsymbol{r}$  is positive ( $\boldsymbol{r}$ )  $\boldsymbol{r}$  and  $\boldsymbol{r}$ )  $\boldsymbol{r}$  are f  $\boldsymbol{r}$  and  $\boldsymbol{r}$  are f  $\boldsymbol{r}$  and  $\boldsymbol{r}$  are f  $\boldsymbol{r}$  an sphere, counterclockwise ambient rotation), the currents/winds flow with the high pressures
- **Balance between Coriolis force and pressure gradient** sphere, counterclockwise ambient rotation), the currents/winds flow with the high pressures on the their right. Where  $\alpha$  is not the clockwise and  $\alpha$  is negative and  $\alpha$ **• Balance between Coriolis force and pregative Figure**
- **All geostrophic flows are isobaric:** All geostrophic flows are isobaric: here the low pressure in the low pressure in the rotation, but on the rotating a flow  $\alpha$ flow with the high pressures on their left. Physically, the pressure force is directed from the  $\bullet$  - All geostrophic flows are isoparic:
- Northern hemisphere  $f > 0$  : currents flows with H on their right
- $-$  Southern hemisphere  $f < 0$  : currents flows with H on their left If the flow field extends over a meridional span that is not too wide, the variation of the variation of the v 7-2 provides a meteorological example from the Northern Hemisphere.  $-$  southern hermsphere  $j < 0$  : currents in
- IF the flow extends over a meridional span not too wide  $L_y \ll L_x$ :  $\partial f$  $\partial y$  $\rightarrow 0 \Rightarrow f = cost \Rightarrow f - PLANE$  and the horizontal divergence is  $\partial f$  *o is the f-plane* case, the horizontal divergence  $\overline{\partial v}$ reference is the formulation case, the f-planet of the f-planet of the geostrophic of the geostrophic of the g<br>*Af*

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial}{\partial x} \left( \frac{1}{\rho_0 f} \frac{\partial p}{\partial y} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\rho_0 f} \frac{\partial p}{\partial x} \right) = 0
$$

- . Geostrophic flows are naturally non-divergent on the f-plane  $\Rightarrow$   $w = cost$  and, if the fluid is bounded in the vertical by a flat surface:  $w = 0$ , if the fiuld is bounded in the vertic  $\partial w$  $\partial z$  $\mu = 0 \Rightarrow w = cost$  and, if the fluid is
- **Geostrophic flows are 2-dimensional** (see surface for the ocean), this vertical velocity must simply vanish, and the flow is strictly valid velocity must simply valid velocity must simply valid velocity must simply valid velocity  $\alpha$



Figure 7-2 A meteorological example showing the high degree of parallelism between wind velocities and pressure contours (isobars), indicative of geostrophic balance. The solid lines are actually height contours of a given pressure (500 mb in this case) and not pressure at a given height. However, because atmospheric pressure variations are large in the vertical and weak in the horizontal, the two sets of contours are nearly identical by virtue of the hydrostatic balance. According to meteorological convention, wind vectors are depicted by arrows with flags and barbs: on each tail, a flag indicates a speed of 50 knots, a barb 10 knots and a half-barb 5 knots (1 knot = 1 nautical mile per hour =  $0.5144$  m/s). The wind is directed toward the bare end of the arrow, because meteorologists emphasize where the wind comes from, not where it is blowing. The dashed lines are isotherms. (Chart by the National Weather Service, Department of Commerce, Washington, D.C.)

**A simplified schematic (top) of the AMOC. Warm water flows north in the upper ocean (red), gives up heat to the atmosphere (atmospheric flow gaining heat represented by changing color of broad arrows), sinks, and returns as a deep cold flow (blue).** 

Latitude of the 26.5 N AMOC obse[rvations is indicated. The actual flow i](https://marine.copernicus.eu/access-data/ocean-monitoring-indicators/atlantic-meridional-overturning-circulation-amoc-timeseries)s co[nsiderably more complex.](https://agupubs.onlinelibrary.wiley.com/doi/10.1029/2020GL089974) 

(**Bottom**) The 10-year (April 2004 to March 2014) time series of the AMOC strength at 26.5 N in Sverdrups (1 Sv =  $10^6$  m<sup>3</sup> s<sup>-1</sup>). This is the 180-day filtered version of the time series. Visible are the low AMOC event in 2009–2010 and the overall decline in AMOC strength over the 10-year period.

http://www.sciencemag.org/content/348/6241/1255575

See **recent update on AMOC OMI @ CMS**

and **Lobelle et al. (2020), GRL** (statistical significance and projections; is slowing due to natural variability or CC?)



Published by AAAS **M. A. Srokosz, and H. L. Bryden Science 2015;348:1255575**

## **GEOSTROPHIC FLOWS OVER IRREGULAR BOTTOM**

• Same framework, but with no-flat bottom:



**Figure 7-3** Schematic view of a flow over a sloping bottom. A vertical velocity must accompany flow across isobaths.

• Bottom elevation (bathymetry / topography):  $b = b(x, y)$ 

• 
$$
w = \frac{dz}{dt} = \frac{\partial z}{\partial t} + \overline{u} \cdot \nabla z = 0 + u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y} + w \frac{\partial b}{\partial z} = u \frac{\partial b}{\partial x} + v \frac{\partial b}{\partial y}
$$

- But on the f-plane  $w = \text{cost}$  and since  $w(z = H) = 0 \Rightarrow w = 0 \forall z$ the fluid over the cost and since  $w(z = H) = 0 \Rightarrow w = 0 \forall z$
- $\bullet \Rightarrow \overline{u} \cdot \nabla b = 0$  flow is directed to zones of equal depth: FREE geostrophic flows can occur only along closed isobaths princ flows can occur only along closed isobaths
- **ISOBARS = ISOBATHS** coincide with isobaths. These lines are sometimes also called *geostrophic contours*. Note (called *isobaths*). Pressure contours are then aligned with topographic contours, and isobars
- **If bumps or dips exist, the fluid can only go around them: due to** vertical rigidity, fluid particles at all levels must likewise go around: **TAYLOR COLUMNS** are permanent tubes of fluid above bumps or **dips** figiolly, fluid particles at all levels must likewise g only along closed isobaths.

#### **GEOSTROPHIC FLOWS OVER IRREGULAR BOTTOM** of fluids trapped above bumps or cavities are called *Taylor columns* (Taylor, 1923).  $\sum_{i=1}^n$

• Same framework, but with no-flat bottom [...]:  $\blacksquare$  Figure 7-4), the geostrophic flow has no choice but to follow the depth contours to follow the depth contours of  $\blacksquare$ 



**Figure 7-3** Schematic view of a flow **Contract is a sloping bottom.** A vertical ve- $\frac{1}{\sqrt{2}}$  fluid boundaries. The fluid would be required to enter or leave through lateral boundaries. The fluid boundaries is simply distributed to enter or leave through lateral boundaries. The fluid boundaries. The f baths.  $\mathbf{b}$  blocked along the entire length of the entire second of the entire second  $\mathbf{b}$  of the entire second of the entire second of the entries in our second of the entries in  $\mathbf{b}$  of the entries in  $\mathbf{b}$  of t

closed domain and over irregular topography. Solid lines are isobaths (con-

#### • **ISOBARS = ISOBATHS** (called *isobaths*). Pressure contours are then aligned with topographic contours, and isobars coincide with isobaths. These lines are sometimes also called *geostrophic contours*. Note

. If bumps or dips exist, the fluid can only go around them: due to vertical rigidity, fluid particles at all levels must likewise go around: **TAYLOR COLUMNS** are permanent tubes of fluid above bumps or **dips** additional information on the flow. **block along the entire of the entire of the set of the** 

#### **BAROTROPIC FLOWS This part is a direct consequence of the index of the index in any sequence of the index in the index in the i**

- **Generalization to non-geostrophic flows: "second level" of flows 8** Generalization to non-geost
	- **Hypoteses:**  $\blacksquare$ Let us consider rapidly rotating fluids by restricting our attention to situations where the Cori-
- $-$  Coriolis term DOES NOT dominate others:  $Ro_T{\sim}1$  and  $Ro{\sim}1$
- $-$  **Homogeneous fluids:**  $\rho_0 = cost$  and  $\rho' = 0$
- **Ignore frictional effect (SLIP is allowed […]):** ≪  $t_{\text{meas}}$  fristianal effect (CLID is allowed  $\Gamma$ , 1),  $FL \times 4$  $\mathcal{F}$  ignore mundial enect (OLIF is allowed  $\left[\ldots\right]$ ). EX  $\mathcal{N}$  1 no longer dwarfs other acceleration terms. We still continue to suppose that the fluid is  $h_0 = 0$  ignore frictional effect (SLIP is allowed  $\left[\ldots\right]$ ):  $E$

#### **• Primitive equations:** relative acceleration terms:

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}
$$
  

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y}
$$
  

$$
0 = -\frac{1}{\rho_0} \frac{\partial p}{\partial z}
$$
  

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
$$

- IF T.-P. theorem still holds  $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0$  initially, it will hold also at all **future time**  $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z}$  $\,=\,0\,\,$  initially, it will hold al: This result is known as the *Taylor–Proudman theorem* (Taylor, 1923; Proudman, 1953). Phys $v^2$  remain z-independent at all subsequent times. Let us restrict our attention  $v^2$ **to such the in the inner in the integration of**  $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial x} = 0$  **initi.** tions (7.9) then reduce to
	- **Advection, Coriolis and Pressure terms remain z-independent** ically, it means that the horizontal velocity field has no vertical shear and that all particles on  $\tt {uon,}$  Conons and Pressure terms remain z-independent ∂u vectio ∂u  $\mathbf v$ oriolis and Pres<mark>s</mark>  $\mathfrak{f}$  rem

#### **BAROTROPIC FLOWS This part is a direct consequence of the index of the index in any sequence of the index in the index in the i**

- **Generalization to non-geostrophic flows: "second level" of flows 8** Generalization to non-geost
	- **Hypoteses:**  $\blacksquare$ Let us consider rapidly rotating fluids by restricting our attention to situations where the Cori-
- $-$  Coriolis term DOES NOT dominate others:  $Ro_T{\sim}1$  and  $Ro{\sim}1$
- $-$  **Homogeneous fluids:**  $\rho_0 = cost$  and  $\rho' = 0$
- **Ignore frictional effect (SLIP is allowed […]):** ≪  $t_{\text{meas}}$  fristianal effect (CLID is allowed  $\Gamma$ , 1),  $FL \times 4$  $\mathcal{F}$  ignore mundial enect (OLIF is allowed  $\left[\ldots\right]$ ). EX  $\mathcal{N}$  1 no longer dwarfs other acceleration terms. We still continue to suppose that the fluid is  $h_0 = 0$  ignore frictional effect (SLIP is allowed  $\left[\ldots\right]$ ):  $E$
- **Primitive equations:** relative acceleration terms:

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial v}{\partial y} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}
$$
  

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial x} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y}
$$
  

$$
0 = -\frac{1}{\rho_0} \frac{\partial p}{\partial z}
$$
  

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
$$

- IF T.-P. theorem still holds  $\left|\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=0\right|$  initially, it will hold also at all future time  $=$   $0$  initially, it will hold al: This result is known as the *Taylor–Proudman theorem* (Taylor, 1923; Proudman, 1953). Phys- $\overline{v}$  and thus remain z-independent at all subsequent times. Let us restrict our attention  $v$ **to such flows** in the integeorm still holds
	- **Advection, Coriolis and Pressure terms remain z-independent** ically, it means that the horizontal velocity field has no vertical shear and that all particles on  $\tt {uon,}$  Conons and Pressure terms remain z-independent ∂u vectio ∂u  $\mathbf v$ oriolis and Pres<mark>s</mark>  $\mathfrak{f}$  rem

#### **BAROTROPIC FLOWS BAROTROPI**  $\overline{16}$ ∂v ∂p

- **Generalization to non-geostrophic flows: "second level" of flows ∂** Generalization to non-geost  $\mathbf{h}$ ו ו  $\cdot$  m −<br>con-ceostrophic⊾  $\mathsf{fl}$ oure: "coo
- **Hypoteses: with a consideration strongly dominates the various acceleration terms. Let us further consideration terms. Let us further consideration terms. Let us further consideration terms. Let us further considerati** Let us consider rapidly rotating fluids by restricting our attention to situations where the Cori-Pressure still obeys (7.2c), and continuity equation (7.2d) has not changed. If the horizontal flow field is initially independent of depth, it will remain so at all fuel  $\mathcal{L}$
- Coriolis term DOES NOT dominate others:  $Ro_T{\sim}1$  and  $Ro{\sim}1$ independent, and the pressure terms are, too,  $\frac{1}{2}$
- $−$  Homogeneous fluids:  $ρ₀ = cost$  and  $ρ' = 0$
- **Ignore frictional effect (SLIP is allowed […]):** ≪  $t_{\text{meas}}$  fristianal effect (CLID is allowed  $\Gamma$ , 1),  $FL \times 4$  $\mathcal{F}$  ignore mundial enect (OLIF is allowed  $\left[\ldots\right]$ ). EX  $\mathcal{N}$  1 varying and thus remain z-independent at all subsequent times. Let us restrict our attention  $-$  - Ignore frictional effect (SLIP is allowed  $\left[\ldots\right]$ ):  $E$ k  $\ll$

#### **• Primitive equations:** tions (7.9) then reduce to

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -\frac{1}{\rho_0} \frac{\partial p}{\partial x}
$$
  

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -\frac{1}{\rho_0} \frac{\partial p}{\partial y}
$$
  

$$
0 = -\frac{1}{\rho_0} \frac{\partial p}{\partial z}
$$
  

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0,
$$

- IF T.-P. theorem still holds  $\left|\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=0\right|$  initially, it will hold also at all **future time**  $=$   $0$  initially, it will hold al: This result is known as the *Taylor–Proudman theorem* (Taylor, 1923; Proudman, 1953). Phys-**·** IF T.-P. theorem still holds  $\mathbf{r}$  and  $\mathbf{r}$  various variation of  $\frac{\partial z}{\partial x}$   $\frac{\partial z}{\partial x}$  . The flow to support fl
	- **Advection, Coriolis and Pressure terms remain z-independent** ically, it means that the horizontal velocity field has no vertical shear and that all particles on  $\tt {uon,}$  Conons and Pressure terms remain z-independent  $A$  integration of the preceding equation of the entire fluid depth  $\mathcal{A}$  is a set of the entire fluid depth  $\mathcal{A}$

#### **BAROTROPIC FLOWS**

- **Although the flow has NO VERTICAL SHEAR, this remains the only similarity with geostrophic flows: Barotropic flows are not required to be aligned with isobars, neither be non-divergent on the horizontal plane, so they can develop a vertical velocity**  $w \neq 0$ ed with isobars, heither be non-divergent on the local and he is the local and instantaneous and instantaneous ane, so they can develop a vertical velocity  $w\neq 0$
- Integrating continuity eq. over the entire fluid depth [...]:

$$
\int_{b}^{b+h} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} dz = 0
$$



**Figure 7-5** Schematic diagram of unsteady flow of a homogeneous fluid over an irregular bottom and the attending notation.  $\frac{1}{2}$  $\alpha$  diagram of un-

Ø **NEW CONTINUITY EQUATION:** with  $\eta = b + h - H$  and  $\partial_t \eta = \partial_t h$  $f - b + n - n$  and  $v_t \eta - v_t n$  $\partial \eta$  $\mathfrak{c}_1$  and  $\mathfrak{c}_2$  and eliminates the velocity from the formalism.

$$
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0
$$

# **Shallow-water model**

in case of flat bottom  $b(x,y)=0$ 

# 3 unknowns in 3 equations

$$
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial h}{\partial x}
$$
  

$$
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu = -g \frac{\partial h}{\partial y}
$$
  

$$
\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0.
$$

# **Geostrophy and altimetry**

Variation of  $\eta$  with  $x, y$  measured from satellite gives info on geostrophic currents  $x, y$  meas  $\overline{a}$  $\frac{1}{2}$ ≀<br>≀  $\frac{1}{2}$ 





Example of absolute dynamic topography (in cm) of the Mediterranean Sea on 1 June 2009 using the Rio et al. (2007) synthetic mean dynamic topography. http://www.goceitaly.asi.it/GoceIT/index.php?Itemid=94

## **VORTICITY DYNAMICS**

- **Geostrophic flows are non-divergent on the f-plane, with 2ddivergence equal to zero: let's investigate the role of the horizontal divergence in barotropic flows**
- **Subtract y-derivative of x-mom.eq from x-derivative of y-mom.eq of barotropic flow system (or the shallow-water model) […]**
- $\blacksquare$  def. <u>ambient vorticity</u>  $f$

■ def. **relative vorticity**  $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$  (the vertical component of  $\nabla \times \bar{u}$ )

def. total vorticity  $f + \zeta$ 

• 
$$
\frac{d}{dt}(f + \zeta) + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)(f + \zeta) = 0 \Rightarrow
$$
 total vorticity ruled by horiz. div.

- $\frac{dh}{dt} + h \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$  $= 0 \Rightarrow$  fluid column heigth ruled by horiz.div.
- $\frac{d}{dt}(h \cdot dS) = 0 \Rightarrow \text{parcel's volume is conserved in time}$
- Combining the above equations... **[…]**

### **VORTICITY DYNAMICS**

- **Kelvin's theorem for 2-d rotating flows:** *"in barotropic flows without friction the circulation is conserved"*
- **This conservation principle has the same meaning of that of the angular momentum for an isolated system**



Figure 7-6 Conservation of volume and circulation of a fluid parcel undergoing vertical squeezing or stretching. The products h ds and  $(f + \zeta)$  ds are conserved during the transformation. As a corollary, the ratio  $(f + \zeta)/h$ , called the potential vorticity, is also conserved.

#### **VORTICITY DYNAMICS** homogeneous geophysical flows, when a parcel of fluid is squeezed laterally (ds decreasing),

- Kelvin's theorem for 2-d rotating flows: "in barotropic flows without friction the circulation is conserved"
- **This conservation principle has the same meaning of that of the** angular momentum <sup>r</sup>or an isolated system its vorticity must increase (f + ζ increasing) to conserve circulation. ticularly helpful, for it eliminates the parcel's cross-section and thus depends only on local *viservation principle*
- . IF both circulation and volume are conserved, so is their ratio, allowing to eliminate dependency on cross-section re conse in<br>Le d, so is their ratio,

• 
$$
\frac{d}{dt} \left( \frac{f + \zeta}{h} \right) = 0
$$
 where  $q = \frac{f + \zeta}{h} = \frac{f + \partial v / \partial x - \partial u / \partial y}{h}$ 

- **and is called POTENTIAL VORTICITY, or "circulation per volume",** thus obtaining the conservation of potential vorticity is called the *potential vorticity*. The preceding analysis interprets potential vorticity as circus called **PUTENTIAL VONTIGHT**, of circulation per volume,
- For rapidly rotating flows:  $Ro=\frac{U}{\Omega L}\ll 1 \Rightarrow f+\zeta\sim \Omega+\frac{U}{L}\sim \Omega \Rightarrow q=\frac{f}{h}$ directly from (7.18) and (7.20) without recovered to the introduction of the introduction of the variable ds.  $\mathbf{F}$ <u>Let us now go function  $I$  and  $I$  a</u>
- and, IF  $f = cost$  each fluid column must conserve its height  $h$ : and in particular, if the upper boundary is flat, fluid parcels must follow the isobaths => <u>barotropic flows become geostrophic</u> Let us now and return to return to return to return the Coriolis of the Coriolis of the Coriolis of the Coriolis  $\mu = 0$  is the relativity of the relativity ( $\mu$ ,  $\sigma$ u/∂ $\sigma$   $\mu$ ) is negligible. The u/ $\sigma$ cuiar, if the upper boundary is flat, fluid parcels must follow

# **The Ekman models**

**Fridtjof [Nansen](http://en.wikipedia.org/wiki/Vagn_Walfrid_Ekman)** (1861 –1930)

Norwegian scientist, explorer, diplomat.

Nobel Peace Prize 1922

https://en.wikipedia.org/wiki/Fridtjof\_Nansen



**Vagn Walfrid Ekman** (1874 –1954)

Swedish oceanographer.

http://en.wikipedia.org/wiki/Vagn\_Walfrid\_Ekman

- Prandtl hypothesis on Boundary Layers
- $E k \rightarrow 1$  close to the wall  $E k \ll 1$  far from the wall
- study of iceberg's motion (Nansen/Fram  $\rightarrow$  Bjerknes  $\rightarrow$  E

# **The Ekman models**

# **Fridtjof Nansen** (1861 –1930)

## Norwegian scientist, explorer, diplomat.

## Nobel Peace Prize 1922

https://en.wikipedia.org/wiki/Fridtjof\_Nansen



https://frammuseum.no/polar-history/vessels/the-polar-ship-fram/ (VIDEO) https://en.wikipedia.org/wiki/Nansen%27s

https://www.youtube.com/watch?v=pL

#### **EKMAN LAYER**

- **As seen from the scale analysis of the primitive eqs. vertical friction has a very minor role in the balance of forces (** ≪ **) and may be omitted**
- **But we lost something, since the frictional terms have the highest derivative order =>** when  $Ek \ll 1$  not all the BCs can be applied, the **result is that SLIPPING ON THE BOUNDARY is allowed**
- **Prandtl hypothesis:** *the fluid has 2 distinct behaviors:*
	- *far from the boundary (INTERIOR, vertical scale H), friction can be*  ${\sf neglected} \; ({\bm E} {\bm k} \ll {\bm 1}) {\cal :} \, {\bm E} {\bm k} = \frac{v_T}{\Omega H^2} \! \sim \! \frac{10^{-2} m^2/s}{10^{-4} s^{-1} \cdot \! (10^3 m^2)}$  $_{\rm \overline{2}} \sim 10^{-4}$
	- *across a short distance near the boundary (BOUNDARY LAYER, vertical scale d), friction acts to bring the finite interior velocity to zero at the wall*  $(Ek \sim 1)$ :  $Ek = \frac{v_T}{\Omega d^2} \sim 1 \Rightarrow d = \sqrt{\frac{v_T}{\Omega}} \sim 10$   $m \Rightarrow d \ll H$
- **Because of the Coriolis effect, the frictional layer of the geophysical flows, called EKMAN LAYER, greatly differs from the BL in nonrotating flows (), which does not have a thickness and grows downstream** ( $\delta \propto \sqrt{x}$ )

## **THE BOTTOM EKMAN LAYER**

- The bottom exerts a frictional stress against the flow bringing its **interior velocity gradually to zero within a thin layer above the wall**  $d \ll H$
- **Hypotheses:**
	- $\,$  Interior flow is uniform and geostrophic:  $\bm{Ro}_T \ll \bm{1}$  and  $\bm{Ro} \ll \bm{1}$  $\bm{s}$ or flow is uniform and geostrophic:  $\bm{Ro}_T \ll \bm{1}$  and  $\bm{Ro} \ll \bm{1}$  $\sim$   $\frac{1}{2}$
	- $-$  **Homogeneous fluid:**  $ρ₀ = cost$  **and**  $ρ' = 0$
	- **Flat bottom**
- **Primitive equations:**

$$
\begin{array}{rcl}\n\mathbf{M} & \mathbf{S} \mathbf{S} \\
\mathbf{S} \mathbf
$$

• **Boundary conditions:**

**CONDITIONS:** Bottom  $(z = 0)$ :  $u = 0, v = 0,$ Toward the interior  $(z \times d)$ :  $y = \bar{y} - y = 0$   $y = \bar{y}(x, y)$ produce a uniform flow requires a uniform flow requires a uniformly varying pressure  $\alpha$ ,  $\alpha$ Toward the interior  $(z \gg d)$ :  $u = \bar{u}$ ,  $v = 0$ ,  $p = \bar{p}(x, y)$ .

- **Interior flow is uniform, no horizontal gradient**  $\cdots$  to boundary conditions are the boundary conditions are the then  $\cdots$  $\mathcal{B}$  virtue of equation (8.13c), the dynamic pressure p is the same at all depths; the same at all depths; thus, p  $=$
- **[…]**

#### **THE BOTTOM EKMAN LAYER** <sup>v</sup> <sup>=</sup> <sup>e</sup><sup>−</sup>z/d " B cos TTOM E **FIGURE 8-17 THE BOTTOM EKMAN LAYER. THE BOTTOM EKMAN LAYER.**  $\mathcal{L} = \mathcal{L} \times \mathcal{L}$  , and the deflection is to the layer. The reverse holds of the reverse holds is to the reverse holds of the reverse holds of the reverse holds is to the reverse holds of the reverse holds of the re

• **Solutions:** and the application of the remaining boundary conditions (8.14a) yields A = −u¯, B = 0, or

$$
u = \bar{u} \left( 1 - e^{-z/d} \cos \frac{z}{d} \right)
$$
  
\n
$$
v = \bar{u} e^{-z/d} \sin \frac{z}{d}.
$$
  
\n
$$
d = \sqrt{\frac{2\nu_E}{f}}
$$
 **Ekman depth**

- **distance over which it approaches the interior solution is on the interior solution is on the order of d. Thus, expression is on the order of d. Thus, expression is on the order of d. Thus, expression is on the order of d**
- $-$  As expected, the Ekman depth corresponds to  $Ek{\sim}1$ (8.16) gives the thickness of the boundary layer. For this reason, d is called the *Ekman depth*. − As expected, the Ekman depth corresponds to  $Ek∼1$
- Although the driving interior flow is along x, we have a transversal velocity **(along y) which is not negligible** also that the preceding solution also the boundary layer, a flow transverse i have a transversal velocity
- $-$  Close to wall  $z → 0$  or  $\frac{z}{d}$ ≪ 1  $\Rightarrow$   $u~\sim$  $v~\sim$   $\overline{u}z/d$  …the velocity near the bottom is at 45 degree to the left of the interior velocity (with f>0) [...] to the interior flow (v "= 0). Very near the bottom (z → 0), this component is equal to the  $-$  ⊂lose to wall  $z \to 0$   $or$   $\frac{2}{d}$   $\ll$   $1$   $\Rightarrow$   $u$ ∼ $v$ ∼ $\overline{u}z/d$  …the velocity near and the application of the remaining boundary conditions  $\frac{1}{2}$  yields  $\frac{1}{2}$ locity (with f>0) […]<br>'
- Where  $u$  reaches its maximum at  $z=\frac{3\pi}{4}d$  the velocity is  $u=1.07\overline{u}$  that is, **larger than its interior value** the interior flow for f < 0.) Further up, where u reaches a first maximum (z = 3πd/4), the occasionally fool us!) :ity is  $u = 1.07\overline{u}$  that is, This solution has a number of important properties. First and foremost, we notice that the
	- The net transport of fluid transverse to the main flow is  $V=\int_0^\infty vdz=\overline{u}d/2$ while  $U = -\overline{u}d/2$  $\frac{1}{2}$  $\frac{1}{2}$  comparison with  $\frac{1}{2}$

#### **THE BOTTOM EKMAN LAYER** and boundary conditions (8.14a) yields (8.14a) yields a boundary conditions (8.14a) yields A = 0, or B = 0, or



Figure 8-3 Frictional influence of a flat bottom on a uniform flow in a rotating framework.  $\frac{1}{\pi}$  a uniform flow in a folding framework.





Figure 8-4 The velocity spiral in the bottom Ekman layer. The figure is drawn for the Northern ne current above the layer. The reverse for the Southern Hemisphere.

#### THE BOTTOM EKMAN LAYER - GENERALIZED where the pressure position is arbitrary. For a constant  $C$  arbitrary. For a constant  $C$

· Interior geostrophic flow varying on a scale sufficiently large to be interior good. G<sub>P</sub>ino now varying on a scale sufficiently large to be in geostrophic equilibrium: as in the section  $\mathcal{L}_{\mathcal{A}}$  $\cdot$  Interior geostrophic flow varying on a scale sufficiently large to  $\mathsf I$ in geostrophic equilibrium:  $\mathbf{r}$  $\mathbf{v}$   $\mathbf{v}$   $\mathbf{v}$   $\mathbf{v}$  $\partial \bar{r}$ 

$$
-\;f\bar{v}\;=\;-\;\frac{1}{\rho_0}\,\frac{\partial\bar{p}}{\partial x}\;,\quad f\bar{u}\;=\;-\;\frac{1}{\rho_0}\,\frac{\partial\bar{p}}{\partial y}
$$

- for a constant Coriolis parameter (on f-plane) the flow is nondivergent:  $\partial \bar u/\partial x + \partial \bar v/\partial y = 0$ 
	- **The BL equations:**  $\partial^2 v$  $\partial^2 u$ <sup>∂</sup>z<sup>2</sup> , (8.20b)  $\overline{\nu} = \nu_E \frac{\partial z^2}{\partial z^2}$  $\int^{L} \partial z^2$  $- f(v - \bar{v}) = v_E$  $\partial^2 u$  $\frac{\partial}{\partial z^2}$  $f(u - \bar{u}) = \nu_E$  $\partial^2v$  $\frac{\partial}{\partial z^2}$ uons.<br> $- f(v - \bar{v}) = u$  $E \frac{\partial u}{\partial z^2}$  $f(u - \bar{u}) = u$  $E \frac{\partial^2 v}{\partial z^2}$
- …with BCs  $u \to \overline{u}$  and  $v \to \overline{v}$  for  $z \to \infty$  and  $u(z = 0) = v(z = 0) = 0$  $\alpha$ <sup>2</sup> ,  $\alpha$ <sup>2</sup> ,  $\alpha$ <sup>2</sup> ,  $\alpha$ <sup>2</sup> and the solution that satisfies the satisfies the boundary conditions along  $u(x - u) - v(x - u) - v(x - u)$  $\mathbf{F}$  with RCs  $\mathbf{F}$  or  $\overline{v}$  and  $\mathbf{F}$  or  $\overline{v}$  and  $\mathbf{F}$   $\mathbf{F}$  on  $\mathbf{F}$   $\mathbf{F$  $\sum_{i=1}^n a_i$  is and value of  $\sum_{i=1}^n a_i$  is the bottom  $a_i$  sets the bottom  $\sum_{i=1}^n a_i$  sets the independent of  $a_i$  sets the ind
- **and solutions:** and satisfies the solutions along the solutions along (u → u⊃ and v → v∃  $\alpha$ ) and v → v  $\alpha$

$$
u = \bar{u} \left( 1 - e^{-z/d} \cos \frac{z}{d} \right) - \bar{v} e^{-z/d} \sin \frac{z}{d}
$$
  

$$
v = \bar{u} e^{-z/d} \sin \frac{z}{d} + \bar{v} \left( 1 - e^{-z/d} \cos \frac{z}{d} \right).
$$

#### **THE BOTTOM EKMAN LAYER - GENERALIZED** N LAVED CEN

• We can compute the transport related to the Ekman bottom layer: Such a situation and presence of an anticipal interior, and interior, as  $\alpha$ 

$$
U = \int_0^\infty (u - \bar{u}) dz = -\frac{d}{2} (\bar{u} + \bar{v})
$$
  

$$
V = \int_0^\infty (v - \bar{v}) dz = \frac{d}{2} (\bar{u} - \bar{v}).
$$

• this transport is not necessarily parallel to the interior geostrophic **flow and may be divergent […]:**  $\frac{1}{2}$  and more to diverge the  $\frac{1}{2}$ . dz <sup>=</sup> <sup>−</sup> <sup>d</sup>

$$
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \int_0^\infty \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz = - \frac{d}{2} \left( \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} \right) = - \frac{d}{2\rho_0 f} \nabla^2 \bar{p}.
$$

- **The flow in the BL converges/diverges if interior has a relative vorticity**  $\overline{\zeta} = \frac{\partial \overline{v}}{\partial x} - \frac{\partial \overline{u}}{\partial y}$ ≠ **(pos/neg):** converges/diverges if interior has a relative  $\partial \bar{v} = \partial \bar{v} \partial \bar{u}$ orticity  $\zeta = \frac{1}{2x} - \frac{1}{2y} \neq 0$  (position). come, or where does it go, to meet this convergence or divergence? Because of the presence accome, or where the does the presence of a solid bottom, the only possibility is that it be supplied from the interior by means of a supplied from the interior by means of a solid from the interior by means of a solid from the interior by means of a solid fro
	- Divergence in BEL and compensating downwelling in the interior + ACyc gyre
	- Convergence in BEL and compensating upwelling in the interior + Cyc gyre  $\mathcal{L}$ ,  $\mathcal{L}$ ,  $\mathcal{L}$
- ▸ Due to the solid bottom, the only possibility to provide convergence **/ divergence which supports upwelling / downwelling is a vertical** ∂z  ${\sf velocity}\,\,\overline{\!w\!}$  from the interior the fluid velocity must of the fluid. Of the fluid  $\overline{\!w\!}$ te unwelling / downwelling possibility to provide convergence weiling / downweiling is a vertical  $\blacksquare$

### **THE BOTTOM EKMAN LAYER - GENERALIZED**



**Figure 8-5** Divergence in the bottom Ekman layer and compensating downwelling in the interior. Such a situation arises in the presence of an anticyclonic gyre in the interior, as depicted by the large horizontal arrows. Similarly, interior cyclonic motion causes convergence in the Ekman layer and upwelling in the interior.

- Interior is geostrophic and on f-plane  $\partial_z \overline{w} = 0 \Rightarrow \overline{w} = cost \Rightarrow$  the vertical **velocity must occur throughout the depth of the fluid**
- **Since** divergence is  $\propto$  *d*  $\ll$  *H*  $\Rightarrow$  the vertical velocity is very weak […]

• def. EKMAN PUMPING: 
$$
\overline{w} = \frac{d}{2}\overline{\zeta} = \frac{d}{2\rho_0 f} \nabla^2 \overline{p} = -\nabla \cdot (U, V) = -\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)
$$

• **The larger the vorticity from the interior, the greater the upwelling / downwelling, with an effect increasing toward the equator**  $(f \rightarrow 0)$ 

#### **THE BOTTOM EKMAN LAYER OVER UNEVEN TERRAIN** It is not to positive the structure of the structure  $\mathbf{r}_i$  and  $\mathbf{r}_j$  affect the Ekmann structure of the Ekmann struct

- **Irregular topography has an effect over the structure of BEL** Let us now consider a more complex interior flow, namely, a spatially nonuniform flow that  $\cdot$  Irregular topography has an effect over the structure of BEL
- Terrain with elevation  $z = b(x, y)$  above a horizontal reference level a horizontal geostrophic interior flow (u $\sim$  v $\sim$ ), not necessarily spatially uniform, over an uniform, over rrain with elevation  $\mathbf{z} = \mathbf{b}(\mathbf{\mathit{x}}, \mathbf{\mathit{y}})$  above a horizontal reference leve
	- Since GFD flows are almost 2D:  $\nabla b(x, y) = (\partial_x b, \partial_y b) \ll 1$ ρ0 ∂y  $\overline{\phantom{a}}$  $\overline{r}$  $t$  that both both such that the bottom slope  $(\boldsymbol{\theta}_x \boldsymbol{\mu}) \in (\boldsymbol{\theta}_x \boldsymbol{\mu})$  and  $\boldsymbol{\theta}_y$
- $\cdot$  Interior geostrophic flow not uniform in most atmospheric and oceanic situations. Our governing equations are again (8.20), coupled to the continuity equation (4.21d), but
- **The BL equations:** the BL equations:  $\overline{a}$  +  $\overline{b}$  = 0). The BL equations are now are now

$$
- f(v - \bar{v}) = \nu_E \frac{\partial^2 u}{\partial z^2}
$$
  

$$
f(u - \bar{u}) = \nu_E \frac{\partial^2 v}{\partial z^2}
$$

- …with **BCs**  $u \to \bar{u}$  and  $v \to \bar{v}$  for  $z \to \infty$  and  $u(z = b) = v(z = b) = 0$ 
	- …and solutions are the same as previous case with  $z \rightarrow z b$  :

$$
u = \bar{u} - e^{(b-z)/d} \left( \bar{u} \cos \frac{z-b}{d} + \bar{v} \sin \frac{z-b}{d} \right)
$$
  

$$
v = \bar{v} + e^{(b-z)/d} \left( \bar{u} \sin \frac{z-b}{d} - \bar{v} \cos \frac{z-b}{d} \right)
$$

. (8.30b)

# **THE BOTTOM EKMAN LAYER OVER UNEVEN TERRAIN** #

● Computing vertical velocity from continuity eq.: ntinui<mark>t</mark>( dina dia kaominina dia kaom<br>Dia kaominina dia kaominin

$$
\begin{aligned}\n\frac{\partial w}{\partial z} &= -\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \\
&= e^{(b-z)/d} \left\{ \left( \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} \right) \sin \frac{z-b}{d} \right. \\
&+ \frac{1}{d} \frac{\partial b}{\partial x} \left[ (\bar{u} - \bar{v}) \cos \frac{z-b}{d} + (\bar{u} + \bar{v}) \sin \frac{z-b}{d} \right] \\
&+ \frac{1}{d} \frac{\partial b}{\partial y} \left[ (\bar{u} + \bar{v}) \cos \frac{z-b}{d} - (\bar{u} - \bar{v}) \sin \frac{z-b}{d} \right] \right\}\n\end{aligned}
$$

• …then we can integrate from  $z = b$ ,  $w = 0$  to  $z \to \infty$ ,  $w = \overline{w}$  :  $\sum_{i=1}^{n} a_i$  and  $\sum_{i=1}^{n} a_i$  is the bottom  $\sum_{i=1}^{n} a_i$ , where  $\sum_{i=1}^{n} a_i$ the vertical velocity vanishes (w = 0 because u and v are also zero there) into the interior • …then we can integrate from  $z = b, w = 0 \text{ to } z \rightarrow \infty, w = w$  :

$$
\bar{w} = \left( \bar{u} \frac{\partial b}{\partial x} + \bar{v} \frac{\partial b}{\partial y} \right) + \frac{d}{2} \left( \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} \right)
$$

• **The first component was found during the analysis of geostrophic** flow over irregular bottom, and it ensures no normal flow to the **bottom; the second is the Ekman pumping as in the flat bottom** case, which is not affected by the bottom slope ∂x **d** 2 <sup>∂</sup><sup>x</sup> <sup>−</sup> <sup>∂</sup>u¯ ∂y . The first component was found during the analysis of genetraphic flow the bottom bottom, and an enource no normal new c lom, the second **i** how over mogular DOUDIII, THE SECOND IS THE ENTIRE PUTTPING AS IN THE HAT DOUDIN

#### **THE SURFACE EKMAN LAYER**

- **The frictional stress against the flow is exerted by the WIND STRESS** (historically, the 1st problem investigated by Ekman)
- **Hypotheses:**
	- $\;$  Interior flow is uniform and geostrophic:  $Ro_T\ll 1$   $and$   $Ro\ll 1$
	- $-$  **Homogeneous fluid:**  $\rho_0 = cost$  **and**  $\rho' = 0$  $\mathbf{v}$ us nulu.  $\mathbf{p}_0 = \mathbf{v}$ ust dhu  $\mathbf{p}_0 = \mathbf{v}$
	- **-** Presence of wind stress:  $\vec{\tau} = (\tau_x, \tau_y)$
- **Primitive equations + BCs:**  $- f (v - \bar{v}) = v_E$  $\partial^2 u$  $\frac{\partial}{\partial z^2}$  $+ f (u - \bar{u}) = v_E$  $\partial^2v$  $\int f(x-\bar{u}) = \nu_E \frac{\partial^2 v}{\partial z^2}$ Surface  $(z = 0)$  :  $\rho_0 \nu_E$  $\frac{\partial u}{\partial z} \ =\ \tau^x,\ \ \rho_0 \nu_E \frac{\partial v}{\partial z} \ =\ \tau^y$ Toward interior  $(z \rightarrow -\infty)$ :  $u = \bar{u}$ ,  $v = \bar{v}$ . (for  $\alpha$  ), and the surface stress. The surface stress for the  $\partial z^2$
- **Solutions:**

$$
u = \bar{u} + \frac{\sqrt{2}}{\rho_0 f d} e^{z/d} \left[ \tau^x \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) - \tau^y \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) \right]
$$
  

$$
v = \bar{v} + \frac{\sqrt{2}}{\rho_0 f d} e^{z/d} \left[ \tau^x \sin\left(\frac{z}{d} - \frac{\pi}{4}\right) + \tau^y \cos\left(\frac{z}{d} - \frac{\pi}{4}\right) \right]
$$

## **THE SURFACE EKMAN LAYER**

• **The solution has a wind-driven component fully related to the wind stress**  $\vec{\tau}$ , independent by the interior flow but dependent on  $1/d \Rightarrow$ **the wind-driven component can be very large if is very small (for example with almost inviscid flow with very small or near the Equator), and even a moderate wind stress may generate a large wind-driven component**



Figure 8-6 The surface Ekman layer generated by a wind stress on the ocean.

#### **THE SURFACE EKMAN LAYER** The wind-driven horizontal transport in the surface  $E$  in the surface Ekman layer has components given by

• **The wind-driven (Ekman) transport in the SEL has components […]:**



Figure 8-7 Structure of the surface Ekman layer. The figure is drawn for the Northern Hemisphere  $(f > 0)$ , and the deflection is to the right of the surface stress. The reverse holds for the Southern Hemisphere.



## **THE SURFACE EKMAN LAYER**

- **The Ekman transport is perpendicular to the wind stress, to the right in the N. Hemisphere, to the left in the S. Hemisphere, explaining why icebergs, mostly floating underwater, drift right of the [wind](https://people.ucsc.edu/~mdmccar/migrated/ocea1/01_Public/lectures/lect_notes_2/14_SURF_Ocean_Circ_12Fall.pdf) as observed by Fridtjof Nansen**
- The surface velocity  $\overrightarrow{u_0} = \overrightarrow{u}(\overline{z} = 0)$  has an angle of 45° with  $\overrightarrow{u_0}$



#### **THE SURFACE EKMAN LAYER** divergence. In determine the set of the set o<br>Indeed, the set of the **FIGURE SURFAGE ENMAN LATER**

▸ Compute the divergence of the Ekman transport (as done for BEL):<br>│ <sup>∂</sup><sup>y</sup> <sup>=</sup>  $\bf{A}$  the divergence of the  $\bf{F}$ kman transport (as done for  $\bf{RFI}$  ): o moment spend divergence of the boundary of the boundary layers and the boundary set the boundary set of the boundary set of<br>originary set of the boundary set of the boundary set of the set o

$$
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \int_{-\infty}^{0} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz = \frac{1}{\rho_0} \left[ \frac{\partial}{\partial x} \left( \frac{\tau^y}{f} \right) - \frac{\partial}{\partial y} \left( \frac{\tau^x}{f} \right) \right]
$$

• **The divergence is totally independent by and is entirely** dependent on  $\vec{\tau}:\nabla\cdot\vec{U}\propto\nabla\!\times\!\vec{\tau}|_z\rightarrow wind-stress\ curl$  $\,$  ce is totally independent by  $\,v_{E}$  and is entirely idopoliticity. The situation is definitive At constant f, the constant f, the contribution is entirely due to the wind stress since the interior geostrophic  $\bullet$  The divergence is totally independent by  $\boldsymbol{v}_E$  and is entirely content to the turbulent even when the turbulent edge visit  $\epsilon$ dependent on  $\vec{\tau}$  :  $\nabla \cdot \vec{U} \propto \nabla \times \vec{\tau}|_{\tau} \rightarrow wind-stress \ curl$ independent of the value of the viscosity. It can be shown furthermore that the viscosity  $\alpha$ Problem 8-7).

• On f-plane: 
$$
\nabla \cdot \vec{U} = \frac{1}{\rho_0 f} \nabla \times \vec{\tau}|_z
$$

- IF  $\nabla\times\vec{\tau}\vert_z\neq0$  the divergence of the Ekman transport must be **provided by a vertical velocity throughout the interior (as in BEL) […] :** vertical velocity. But, remember (Section 7.1) that geostrophic flows must be characterized must be characteriz by ∂w¯  $\partial_{\eta}$  $\alpha - \frac{1}{\alpha} \int_{-\infty}^{\infty} \left( \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} \right)^{u} \right.$ ∩ri ∂z • IF  $\nabla\times\vec{\tau}\vert_z\neq 0$  the divergence of the Ekman transport must be  $t_{\text{max}}$  is the vertical velocity must of the fluid. Of  $t_{\text{max}}$  $\int$  divergence of the flow in the flow in the Ekman depth, d,  $\int$   $\tau^x$ )  $\frac{1}{\sqrt{2\pi}} \left( \frac{1}{\sqrt{t}} \right) - \frac{1}{\sqrt{2\pi}} \left( \frac{1}{\sqrt{t}} \right) = w_1$ The vertical velocity in the interior, called *Ekman pumping*, can be evaluated by a vertical  $\frac{\partial v}{\partial z}$  =  $\frac{1}{\sqrt{6}}$  $\overline{1}$ ∂y dz  $\int \rho_0 \left[ \partial x \right] f \left[ \partial y \right] f \left[ \int \right]$ provided by a vertical velocity throughout the  $\bar{w} = \ + \int_{-\infty}^{0} \frac{\partial u}{\partial x}$  $+$  $\partial v$  $\partial y$  $\overline{ }$  $dz = \frac{1}{x}$  $\frac{3}{5}$ ×  $\overline{U}$ ∴<br>fh e,  $\rho_0$ \$ ∂  $\partial x$  $\int \tau^y$ f  $\bigg) - \frac{\partial}{\partial y}$  $\int \tau^x$ f  $\Big)$  =  $w_{\text{Ek}}$
- **def. EKMAN PUMPING:**  $\overline{w} = w_{Ek} = \frac{1}{\alpha}$  $\boldsymbol{\rho}_\mathbf{0}$  $\nabla \times \frac{\vec{\tau}}{f} \big|_Z$  $\mathsf{cf.}\ \mathsf{EKMAN}\ \mathsf{PUMPING}\colon \overline{w}=w_{\scriptscriptstyle\it\!EL}}=\dfrac{1}{r}\nabla\times\dfrac{r}{r}|_{\scriptscriptstyle\cal Z}$ integration of the continuity equation (4.21 d), using we can expect the  $\rho_0$ integration of the continuity equation (4.21d), using w(z  $=0$ ), using w(z  $=0$ ),  $\frac{1}{2}$  $\textsf{WAN} \textsf{PUMPING: } w = w_{Ek} = -\nabla \times \frac{1}{\epsilon} | z |$  $\overline{P}$  and  $\overline{P}$  are a downwelling (Figure 8-8a), whereas a downwell  $\overline{a}$  $\frac{1}{\sqrt{2}}$ This vertical velocity is called *Ekman pumping*. In the Northern Hemisphere (f > 0), a This vertical velocity is called *Ekman pumping*. In the Northern Hemisphere (f > 0), a PLIMDING:  $\overline{w} = w_+ = \frac{1}{\sqrt{N}} \nabla \sqrt{\frac{t}{\sqrt{N}}}$ counterclockwise wind  $\rho_0$  is different pattern causes upwelling (Figure 8-8b). The directions are oppositely directly as  $\rho_0$  in the direction oppositely directly are oppositely directly and  $\rho_0$  in the directions
- Ekman pumping on f-plane:  $w_{Ek} = \frac{1}{\rho_0 f} \nabla \times \vec{\tau}|_z$ ou t-b 1e:  $w_{Ek} = \frac{\partial}{\partial \rho} \mathbf{v} \times \mathbf{r}_{ls}$  $\overline{\mathbf{1}}$ pumping on f-plane:  $w_{Ek} = \frac{1}{\epsilon} \nabla \times \hat{\tau}|_{z}$  $\overline{P}$ clockwise wind pattern (negative curl) generates a downwelling (Figure 8-8a), whereas a • Ekman pumping on f-plane:  $w_{\mu\nu} = \frac{1}{2} \nabla \times \vec{\tau}|_{\sigma}$ in the Southern Hemisphere. Exempt is a very effective mechanism by which winds  $\rho_0$ ing on finished  $\mathbf{u}_i = \frac{1}{2} \nabla v_i^2$

## **THE SURFACE EKMAN LAYER**

- Ekman pumping on f-plane:  $w_{Ek} = \nabla \cdot \overrightarrow{U} = \frac{1}{\rho_0 f} \nabla \times \overrightarrow{\tau} |_Z \lessgtr 0$
- **Ekman pumping: a very effective mechanism to drive subsurface ocean currents through the action of winds**



## **THE SURFACE EKMAN LAYER**

- Ekman pumping on f-plane:  $w_{Ek} = \nabla \cdot \overrightarrow{U} = \frac{1}{\rho_0 f} \nabla \times \overrightarrow{\tau} |_Z \lessgtr 0$
- **Ekman pumping: a very effective mechanism to drive subsurface ocean currents through the action of winds => near the coast**



# **Effect of upwelling on biogochemistr the Mediterranean Sea**



# North-Western Med Sea is an area of upw and high productivity due to Ekman pum

https://www.youtube.com/watch

# **Effect of upwelling on biogochemistr the Mediterranean Sea**



North-Western Med Sea is an area of upw and high productivity due to Ekman pum

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## **THE EKMAN LAYER IN REAL GEOPHYSICAL FLOWS**

- **Real geophysical flows are characterized by turbulence and stratification** ⇒ **observations cannot match the highly idealized models of BEL and SEL**
- GFD flows have  $\boldsymbol{Re} \gg \boldsymbol{1} \Rightarrow$  we can replace  $\boldsymbol{v}$  with  $\boldsymbol{v}_F$  to account for **the enhanced momentum transfer in a turbulent flow**
- **Ekman layers are SHEAR FLOWS and turbulence is not homogenous, increasing with the shear and suppressed close to the boundary where size of turbulent eddies is limited** ⇒ **a general theory** of turbulence does not exist, as a minimum  $v_F = v_F(z)$  but **observations do not agree with simple models**
- **The angle between near-boundary velocity / surface velocity and interior in BEL** / SEL is  $<$  45 $^{\circ}$  ranging  $5^{\circ} \div 20^{\circ}$
- Eddy viscosity  $\bm{v_E}$  scales with friction velocity  $\bm{u}^* = \sqrt{|\vec{\bm{\tau}}|/\bm{\rho_0}}$  and  $d$  as **mixing length** ( $\sim$  size of the most turbulent eddies):  $v_F \sim u^* d$
- Ekman depth scales with  $d{\sim}\sqrt{v_E/f}{\sim}\sqrt{u^*d^{\f}} \Rightarrow d{\sim}u^*/f$
- Empirically:  $d = 0.4 u^*/f$

## **THE EKMAN LAYER IN REAL GEOPHYSICAL FLOWS**

- **Real geophysical flows are characterized by vertical density stratification**  $\rho = \rho(z)$  : the gradual change of density with z hinders **vertical movements** ⇒ **reduction of vertical mixing of momentum by turbulence and decoupling motions at separate levels**
- **Stratification reduces the Ekman depth and increases the veering of the velocity vector with**
- **Surface atmospheric layer during daytime over land and above warm currents at sea is frequently in a state of CONVECTION due to the heating from below: the Ekman dynamics is related to convective motions, driven both by the geostrophic flow aloft and by the intensity of the surface heat flux** ⇒ **Atmospheric Boundary Layer (ABL)**
- Ekman depth scales with  $\boldsymbol{d} =$ 1.3  $u^*$  $f(1+\frac{N^2}{c^2})$  $\overline{f^2}$  $\overline{1/4}$

# One of the few cases when  $obs \rightarrow$  theory



*=> surface current*

**Figure 8-9** Comparison between observed currents below a drifting ice floe at 84.3°N and theoretical predictions based on an eddy viscosity  $\nu_E = 2.4 \times 10^{-3} \text{ m}^2\text{/s}$ . (Reprinted from *Deep-Sea Research*, 13, Kenneth Hunkins, Ekman drift currents in the Arctic Ocean, p. 614, ©1966, with kind permission from Pergamon Press Ltd, Headington Hill Hall, Oxford 0X3 0BW, UK)

**observations: angle**  $u_0$  **and**  $u_{INT}$  **< 45°** 



Figure 8-10 Wind vectors minus geostrophic wind as a function of height (in meters) in the maritime friction layer near the Scilly Isles. Top diagram: Case of warm air over cold water. Bottom diagram: Case of cold air over warm water. (Adapted from Roll, 1965)