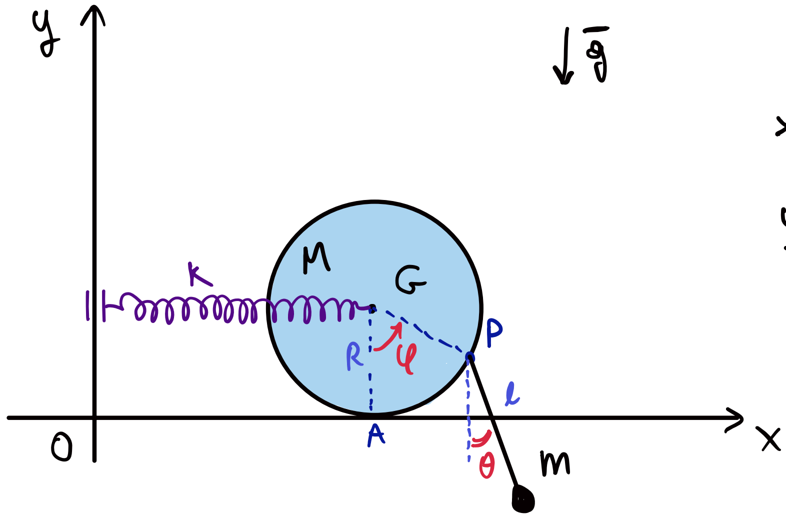


$$x_Q = x_G + R\,\text{sen}\,\varphi + \ell\,\text{sen}\,\theta$$

$$y_Q = R - R\cos\varphi - \ell\cos\theta$$

$$x_G = (\pi - \varphi)R$$

$$\dot{x}_Q = -R\dot{\varphi} + R\dot{\varphi}\cos\varphi + \ell\dot{\theta}\cos\theta \qquad \dot{x}_G = -R\dot{\varphi}$$

$$\dot{y}_Q = R\dot{\varphi}\,\text{sen}\,\varphi + \ell\dot{\theta}\,\text{sen}\,\theta$$

$$\cos\theta\cos\varphi + \text{sen}\,\theta\,\text{sen}\,\varphi = \cos(\theta - \varphi)$$

$$T_m = \frac{m}{2}\Big(2R^2\dot{\varphi}^2\left[1 - \cos\varphi\right] + \ell^2\dot{\theta}^2 + 2R\ell\dot{\varphi}\dot{\theta}(-1 + \cos\varphi)\cos\theta + 2R\ell\dot{\varphi}\dot{\theta}\,\text{sen}\,\varphi\,\text{sen}\,\theta\Big)$$

$$= \frac{m}{2}\Big(2R^2(1 - \cos\varphi)\dot{\varphi}^2 + \ell^2\dot{\theta}^2 + 2R\ell\dot{\varphi}\dot{\theta}\left[-\cos\theta + \cos(\theta - \varphi)\right]$$

$$T_M = \frac{1}{2}M\dot{x}_G^2 + \frac{1}{2}\underbrace{I}_{\frac{MR^2}{2}}\dot{\varphi}^2 = \frac{1}{2}\left(\frac{3}{2}M\right)R^2\dot{\varphi}^2$$

$$V = \frac{1}{2}KR^2(\pi - \varphi)^2 + mg\left(R - R\cos\varphi - \ell\cos\theta\right)$$

1)

$$L = \frac{1}{2}R^2\dot{\varphi}^2\left\{(2 - 2\cos\varphi)m + \frac{3}{2}M\right\} + \frac{1}{2}m\ell^2\dot{\theta}^2 +$$

$$\frac{1}{2}\dot{\varphi}\dot{\theta}\left\{2mR\ell\left(\cos(\theta - \varphi) - \cos\theta\right)\right\}$$

$$- \frac{1}{2}KR^2(\pi - \varphi)^2 - mg\left(R - R\cos\varphi - \ell\cos\theta\right)$$

2) Matrice cinetica

$$a = \begin{pmatrix} 2R^2(1-\cos\varphi)\,m + \frac{3}{2}MR^2 & * \\ mRl(\cos(\theta-\varphi)-\cos\theta) & ml^2 \end{pmatrix}$$

3) Eq. di Lagrange relativa alla variabile θ

$$\frac{d}{dt}\frac{\partial L}{\partial\dot\theta} = \frac{d}{dt}\left[ml^2\dot\theta + \dot\varphi\; mRl(\cos(\theta-\varphi)-\cos\theta)\right] =$$

$$= ml^2\ddot\theta + mRl\ddot\varphi(\cos(\theta-\varphi)-\cos\theta) +$$

$$+ mRl\dot\varphi^2 \operatorname{sen}(\theta-\varphi) + mRl\dot\varphi\dot\theta(\operatorname{sen}\theta - \operatorname{sen}(\theta-\varphi))$$

$$\frac{\partial L}{\partial\theta} = mRl\dot\theta\dot\varphi(\operatorname{sen}(\varphi-\theta)+\operatorname{sen}\theta) - mgl\operatorname{sen}\theta$$

$$l^2\ddot\theta + Rl\ddot\varphi(\cos(\theta-\varphi)-\cos\theta) + Rl\dot\varphi^2\operatorname{sen}(\theta-\varphi) + mgl\operatorname{sen}\theta = 0$$

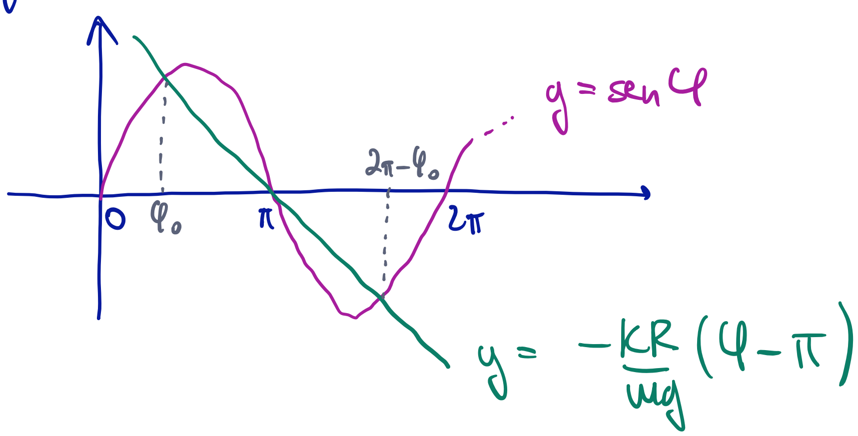
4) Pti di eq. e stab. Si considerino i casi $\frac{KR}{mg} > 1$ e $\frac{2}{5\pi} < \frac{KR}{mg} < 1$.

$$V = \frac{1}{2}KR^2(\pi-\varphi)^2 + mg(R - R\cos\varphi - l\cos\theta)$$

$$\frac{\partial V}{\partial\varphi} = KR^2(\varphi-\pi) + mgR\operatorname{sen}\varphi \rightarrow \operatorname{sen}\varphi = -\frac{KR}{mg}\varphi + \frac{KR}{mg}\pi \quad (*)$$

$$\frac{\partial V}{\partial\theta} = mgl\operatorname{sen}\theta \rightarrow \theta = 0, \pi$$

(*) :



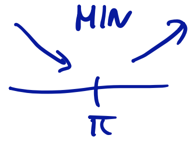
[Rette per $\left(\frac{7}{2}\pi, -1\right)$: $-1 = \frac{KR}{mg}\frac{5}{2}\pi$ $\quad \frac{KR}{mg} > \frac{2}{5\pi} \rightsquigarrow$

$\rightsquigarrow$ pti di intersezione sono 3]

$\frac{KR}{mg} > 1$: 1 pto di intersez. $\varphi = \pi$ $\quad (\partial^2_\varphi V > 0)$

$0 < \varphi < \pi$ $\quad$ sen < retta $\Rightarrow \partial_\varphi V < 0$

$\varphi > \pi$ $\quad$ sen > retta $\Rightarrow \partial_\varphi V > 0$



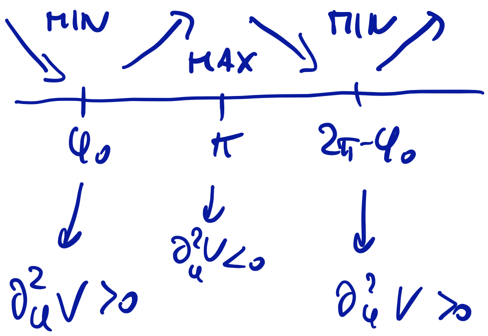
$\frac{2}{5\pi} < \frac{KR}{mg} < 1$: 3 pti di intersez. $\varphi = \varphi_0, \pi, 2\pi - \varphi_0$

$0 < \varphi < \varphi_0$ $\quad$ sen < retta $\Rightarrow \partial_\varphi V < 0$

$\varphi_0 < \varphi < \pi$ $\quad$ sen > retta $\Rightarrow \partial_\varphi V > 0$

$\pi < \varphi < 2\pi - \varphi_0$ $\quad$ sen < retta $\Rightarrow \partial_\varphi V < 0$

$\varphi > 2\pi - \varphi_0$ $\quad$ sen > retta $\Rightarrow \partial_\varphi V > 0$



$\theta = 0, \pi$ $\quad \partial^2_\theta V(0) > 0$

$\quad \partial^2_\theta V(\pi) < 0$

$$\partial^2 U = \begin{pmatrix} \partial^2_\varphi V & 0 \\ 0 & \partial^2_\theta V \end{pmatrix}$$

$\frac{KR}{mg} > 1$

$\theta \backslash \varphi$	π
0	STAB
π	INSTAB

$\frac{2}{5\pi} < \frac{KR}{mg} < 1$

$\theta \backslash \varphi$	φ_0	π	$2\pi - \varphi$
0	STAB	INSTAB	STAB
π	INSTAB	INSTAB	INSTAB

$$\frac{\partial V}{\partial \varphi} = KR^2(\varphi-\pi) + mgR \,\mathrm{sen}\,\varphi \rightarrow \partial^2_\varphi V = KR^2 + mgR\cos\varphi$$

$$\frac{\partial V}{\partial \theta} = mgl\,\mathrm{sen}\,\theta \rightarrow \partial^2_\theta V = mgl\cos\theta$$

5) Frequenza delle piccole oscillazioni. $\varphi=\pi \quad \theta=0$

$$a = \begin{pmatrix} 2R^2(1-\cos\varphi)m + \frac{3}{2}MR^2 & * \\ mRl(\cos(\theta-\varphi)-\cos\theta) & ml^2 \end{pmatrix} \rightarrow A = \begin{pmatrix} 4mR^2 + \frac{3}{2}MR^2 & -2mRl \\ -2mRl & ml^2 \end{pmatrix}$$

$$B = \begin{pmatrix} KR^2 - mgR & 0 \\ 0 & mgl \end{pmatrix}$$

$$\left(\frac{g}{l}-\lambda\right)\left(\left(\frac{K}{m}-\frac{g}{R}\right) - \lambda\left(4+\frac{3}{2}\frac{M}{m}\right)\right) - 4\lambda^2$$

$\cdot R/l$ $\quad -2\lambda$

$$B - \lambda A = R^2 m \begin{pmatrix} \frac{K}{m} - \frac{g}{R} - \lambda\left(4 + \frac{3}{2}\frac{M}{m}\right) & +2\frac{l}{R}\lambda \\ +2\frac{l}{R}\lambda \quad -2\lambda & \frac{g}{R}\frac{l}{R} - \lambda\frac{l^2}{R^2} \end{pmatrix}$$

$R/l\cdot \qquad \frac{g}{l} - \lambda$

$$0 = \det(B-\lambda A) \rightarrow \left(\cancel{4}+\frac{3}{2}\frac{M}{m}\right)\lambda^2 - \left[\left(\frac{K}{m}-\frac{g}{R}\right) + \frac{g}{l}\left(4+\frac{3}{2}\frac{M}{m}\right)\right]\lambda + \frac{g}{l}\left(\frac{K}{m}-\frac{g}{R}\right) - \cancel{4\lambda^2} = 0$$

$$\frac{3}{2}\frac{M}{m}\lambda^2 - \left[\frac{K}{m} - \frac{g}{R}\left(1 - 4\frac{R}{l} - \frac{3}{2}\frac{MR}{ml}\right)\right]\lambda + \frac{g}{R}\frac{K}{m}\frac{R}{l} - \left(\frac{g}{R}\right)^2\frac{R}{l}$$

$\frac{R}{l} = \alpha \qquad \frac{3}{2}\frac{M}{m} = \beta$

$$\beta\lambda^2 - \left[\frac{K}{m} - \frac{g}{R} + \frac{g}{R}(4\alpha + \alpha\beta)\right]\lambda + \alpha\frac{g}{R}\frac{K}{m} - \left(\frac{g}{R}\right)^2\cdot\alpha$$

$$a = \begin{pmatrix} 2R^2(1-\cos\varphi)\,m + \frac{3}{2}MR^2 & * \\ mR\ell\,(\cos(\theta-\varphi)-\cos\theta) & m\ell^2 \end{pmatrix} \rightarrow A = \begin{pmatrix} 4mR^2 + \frac{3}{2}MR^2 & -2mR\ell \\ -2mR\ell & m\ell^2 \end{pmatrix}$$

$$B = \begin{pmatrix} KR^2 - mgR & 0 \\ 0 & mg\ell \end{pmatrix}$$

$$\ell = R, \quad M = \frac{2}{3}m, \quad \frac{KR}{mg} = 2$$

$$A = \begin{pmatrix} 5mR^2 & -2mR^2 \\ -2mR^2 & mR^2 \end{pmatrix} \qquad B = \begin{pmatrix} mgR & \\ & mgR \end{pmatrix}$$

$$\det(B - \lambda A) = \left(\cancel{m}g\cancel{R}/R - 5\cancel{m}\cancel{R^2}\lambda\right)\left(\cancel{m}g\cancel{R}/R - \cancel{m}\cancel{R^2}\lambda\right) - 4\cancel{m^2}\cancel{R^4}\lambda^2 = 0$$

$$\lambda^2 - 6\frac{g}{R}\lambda + \left(\frac{g}{R}\right)^2 = 0$$

$$\lambda_{1,2} = \left(3 \pm 2\sqrt{2}\right)\frac{g}{R}$$

$$A' = \frac{A}{mR^2} = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix} \qquad B' = \frac{B}{mR^2} = \begin{pmatrix} g/R & \\ & g/R \end{pmatrix}$$

$$\lambda = (3 + 2\sqrt{2})\,g/R$$

$$\cancel{\frac{g}{R}} \begin{pmatrix} 1 - 5(3+2\sqrt{2}) & -2(3+2\sqrt{2}) \\ -2(3+2\sqrt{2}) & 1 - (3+2\sqrt{2}) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0$$

e Lagrangiana Linearizzata $\quad M = \frac{2}{3}m$

$$L^* = \frac{1}{2}(\delta\dot\varphi, \delta\dot\theta)\begin{pmatrix} 5mR^2 & -2mR\ell \\ -2mR\ell & m\ell^2 \end{pmatrix}\begin{pmatrix}\delta\dot\varphi \\ \delta\dot\theta\end{pmatrix} -$$

$$-\frac{1}{2}(\delta\varphi, \delta\theta)\begin{pmatrix} mgR\left(\frac{KR}{mg}-1\right) & 0 \\ 0 & mg\ell \end{pmatrix}\begin{pmatrix}\delta\varphi \\ \delta\theta\end{pmatrix}$$

$$A = \begin{pmatrix} 4mR^2 + \frac{3}{2}MR^2 & -2mR\ell \\ -2mR\ell & m\ell^2 \end{pmatrix}$$

$\frac{R}{\ell} = \frac{1}{3} \quad \frac{3}{2}\frac{M}{m} = 1$

$\ell = 3R$

$$B = \begin{pmatrix} KR^2 - mgR & 0 \\ 0 & mg\ell \end{pmatrix}$$

e eq. del moto linearizzate

$$\frac{d}{dt}\frac{\partial L^*}{\partial \delta\dot\varphi} = \left(5mR^2\delta\ddot\varphi - 2mR\ell\delta\ddot\theta\right)$$

$$\frac{\partial L}{\partial \varphi} = -mgR\left(\frac{KR}{mg}-1\right)\delta\varphi$$

$$\begin{cases} 5\delta\ddot\varphi - 2\frac{\ell}{R}\delta\ddot\theta = -\left(\frac{K}{m}-\frac{g}{R}\right)\delta\varphi \\ -2\delta\ddot\varphi + \frac{\ell}{R}\delta\ddot\theta = -\frac{g}{R}\delta\theta \end{cases}$$

$$\frac{d}{dt}\frac{\partial L^*}{\partial \delta\dot\theta} = m\ell^2\delta\ddot\theta - 2mR\ell\delta\ddot\varphi$$

$$\frac{\partial L}{\partial \delta\theta} = -mg\ell\delta\theta$$

Es. 1.

$$\tilde{p} = \frac{1}{2\omega}\left(p^2 + \omega^2 q^2\right) \qquad \tilde{q} = \operatorname{arctg} \frac{\omega q}{p}$$

$$\{\tilde{q}, \tilde{p}\} = \frac{\omega/p}{1 + \frac{\omega^2 q^2}{p^2}} \frac{p}{\omega} - \frac{-\frac{\omega q}{p^2}}{1 + \frac{\omega^2 q^2}{p^2}} \omega q = \frac{1 + \frac{\omega^2 q^2}{p^2}}{1 + \frac{\omega^2 q^2}{p^2}} = 1$$

$m=1$

$$\frac{1}{2}\left(p^2 + \omega^2 q^2\right) = \underset{K}{\underset{\shortparallel}{\omega \tilde{p}}} \Rightarrow \dot{\tilde{q}} = \partial K / \partial \tilde{p} = \omega$$

$$\dot{\tilde{p}} = \frac{\partial K}{\partial \tilde{q}} = 0$$

$$\Rightarrow \tilde{q}(t) = \omega(t) + \varphi_0$$

$$\tilde{p}(t) = \tilde{p}_0$$

$$q = \frac{p}{\omega} \operatorname{tg} \tilde{q}$$

$$\tilde{p} = \frac{p^2}{2\omega}\left(1 + \operatorname{tg}^2 \tilde{q}\right)$$

Funz. generatrice:

$$F(p, \tilde{q})$$

$$\tilde{p} = \frac{\partial F}{\partial \tilde{q}}(p, \tilde{q}) = \frac{p^2}{2\omega}\left(1 + \operatorname{tg}^2 \tilde{q}\right) \rightarrow f(\tilde{q}) = \text{cost}$$

$$q = \frac{\partial F}{\partial p} = \frac{p}{\omega} \operatorname{tg} \tilde{q} \rightarrow F = \frac{p^2}{2\omega} \operatorname{tg} \tilde{q} + f(\tilde{q})$$