STELLAR SPECTROSCOPY II.

DETERMINATION OF STELLAR PARAMETERS AND ABUNDANCES

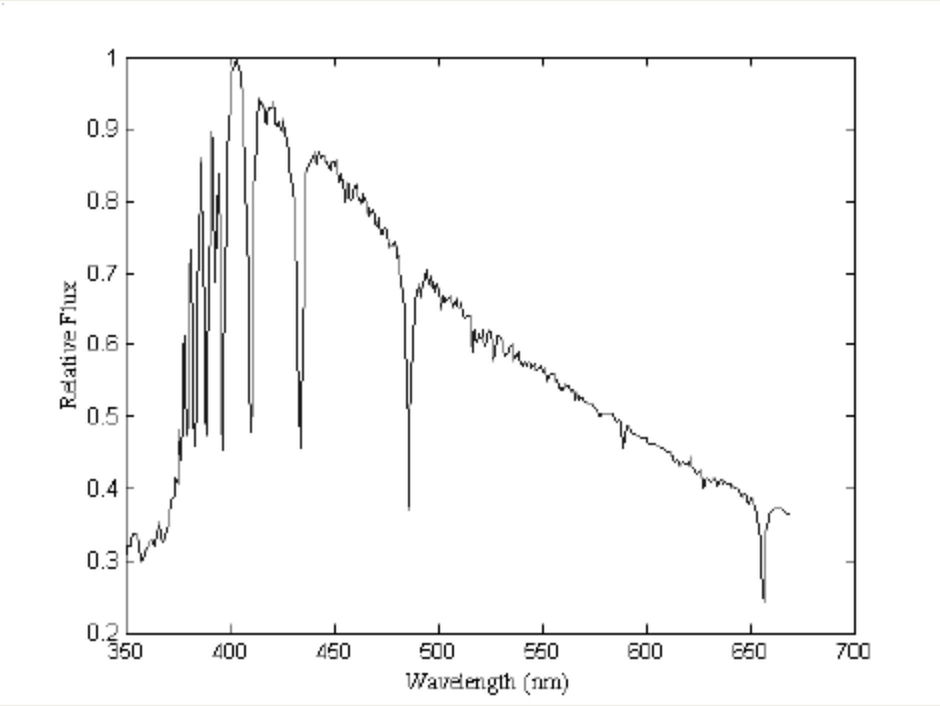
Lezione VII- Fisica delle Galassie
Cap 8-9 Carrol & Ostlie

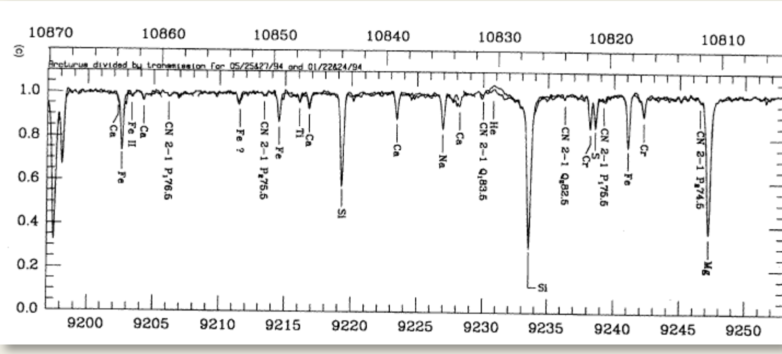
And
For neural network in astronomy:
Leung & Bovy (2018)

Laura Magrini

OUTLINE of the lecture (continuing from the previous lecture):

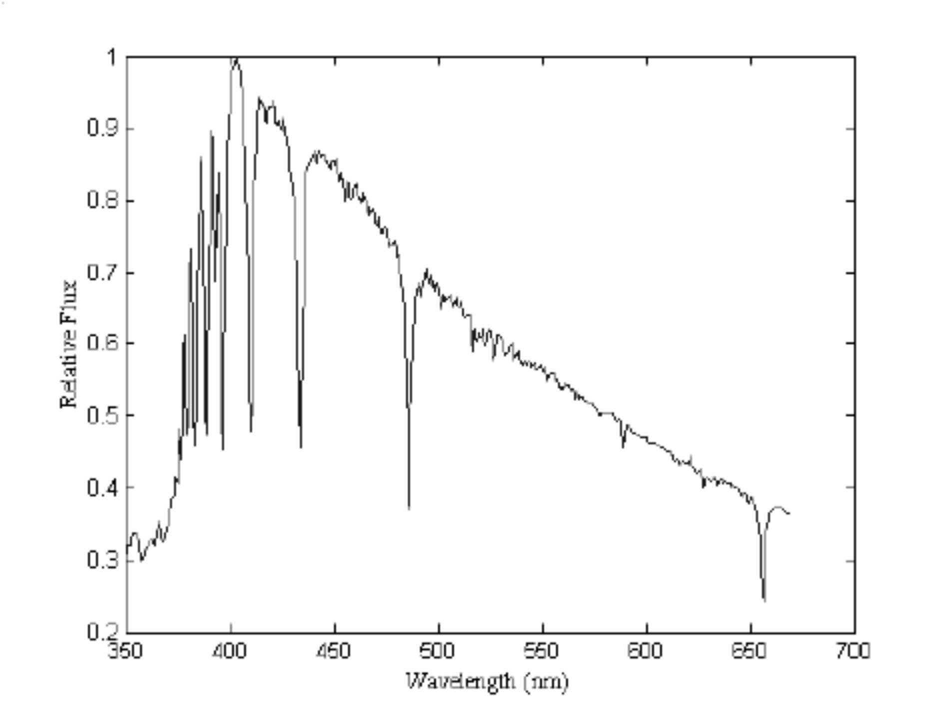
The aim of this lecture is to learn how we can obtain information about the stellar parameters (temperature, surface gravity, global metallicity and individual element abundances) from a stellar spectrum





Globally it is a blackbody, but each small portion of the spectrum contains a large quantity of information

- **The conditions for local thermodynamic equilibrium (LTE) are satisfied, and so, as already seen, the source function is equal to the Planck function, $S_\lambda = B_\lambda$**



The formation of absorption lines

- Absorption lines are created when an atom absorbs a photon with exactly the same energy necessary to make an upward transition from a lower to a higher orbital
- **The energy of photons depends on the types of stars** (their effective Temperature)

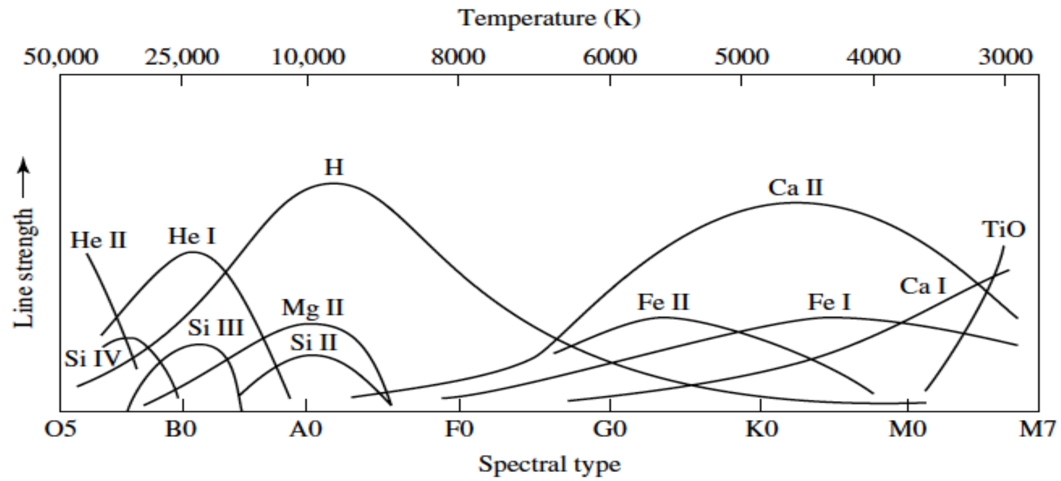


FIGURE 11 The dependence of spectral line strengths on temperature.

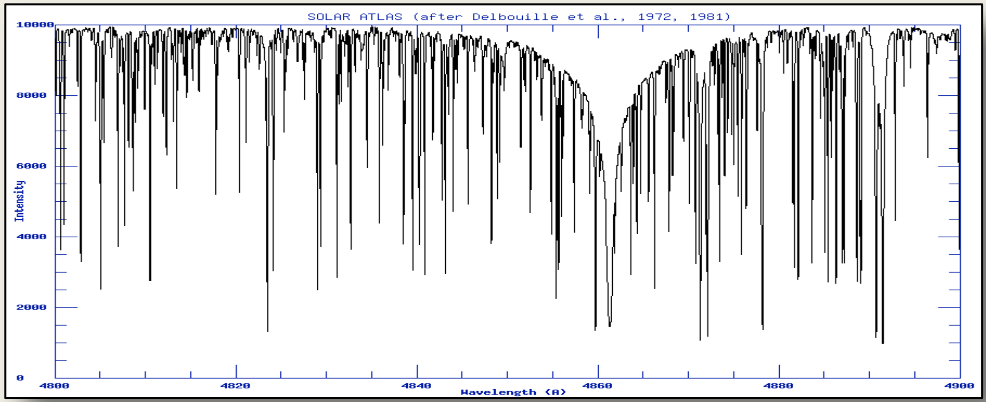
What is an equivalent width?

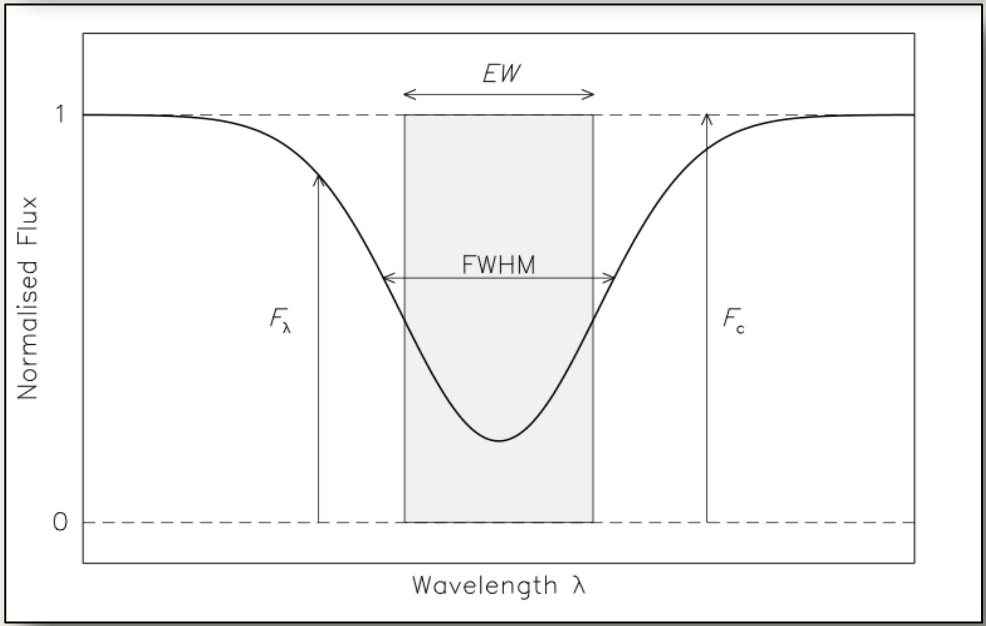
The equivalent width of a spectral line is a measure of the area of the line on a plot of intensity versus wavelength.

It is found by forming a rectangle with a height equal to that of continuum emission, and finding the width such that the area of the rectangle is equal to the area in the spectral line.

It is a measure of the strength of spectral features.

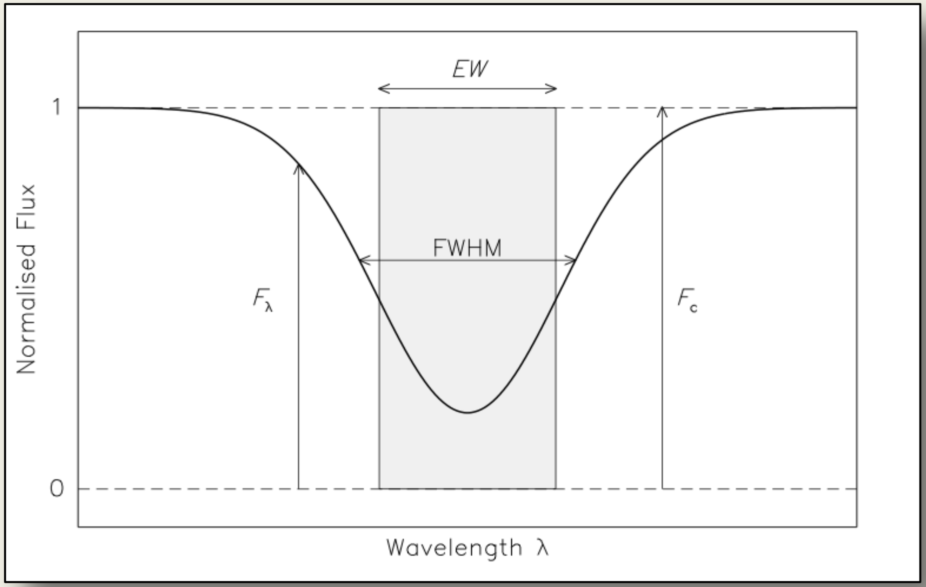
It is usually measured in mÅ.





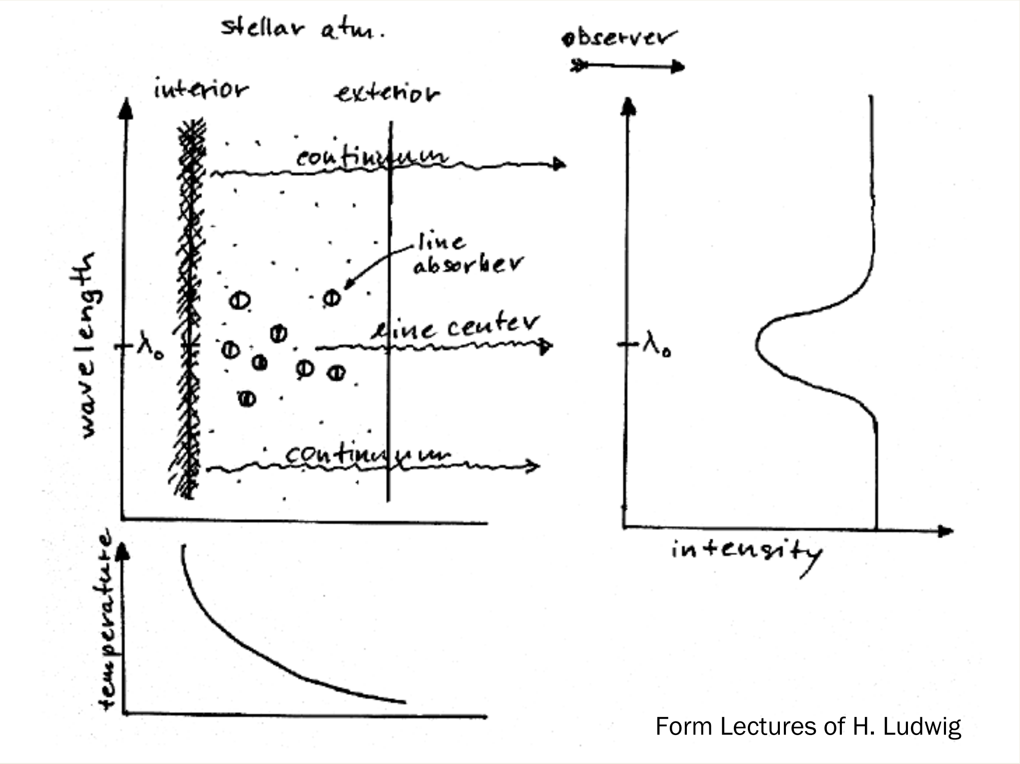
What is an equivalent width?

- The opacity κ_λ of the stellar material is greatest at the wavelength at the center and it decreases moving into the wings.
- The center of the line is formed at lower T, thus in cooler regions of the stellar atmosphere.
- Moving into the wings from , the line formation occurs at higher T
- It merges with the continuum produced at an optical depth of 2/3.



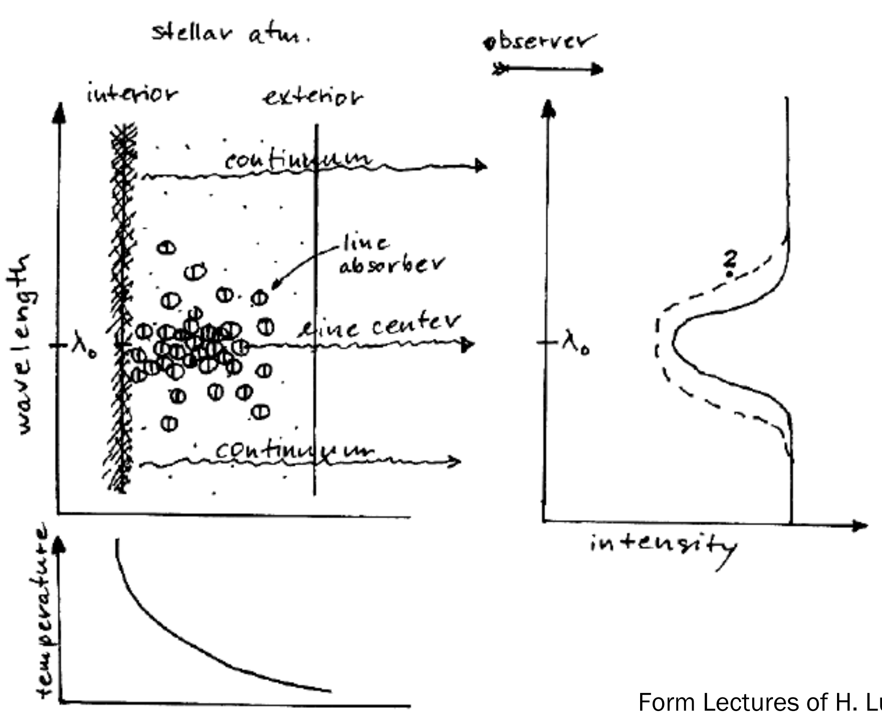
$$W = \int \frac{F_c - F_\lambda}{F_c}\, d\lambda,$$

Formation of an absorption line



- Hot, i.e. bright, continuous spectrum emitted in the deepest layers of the photosphere
- Part of the emitted light is intercepted by absorbing atoms in the higher atmosphere on the way out, forming the absorption line

Formation of an absorption line



Form Lectures of H. Ludwig

- If the number of the absorbers increases, the line becomes deeper, and then it saturates

Line broadening

Natural broadening:

Due to the uncertainty principle of Heisenberg, an electron in an excited states occupies its orbital for a short time, thus the energy cannot be precise

$$\Delta E \simeq \hbar / \Delta t \text{ (uncertainty in the energy)}$$

Considering that the energy of a photon can be expressed as E=hc/λ , the associate uncertainty is

$$\Delta\lambda \simeq \frac{\lambda^2}{c} /(1/\Delta t_{in} + 1/\Delta t_{fin})$$

where Δt_{in} and Δt_{fin} are the lifetimes of the electron in the initial and final states

- The lifetime of an electron in the first and second excited states of hydrogen is about $\Delta t = 10^{-8}$ s.
- The natural broadening of the Hα line of hydrogen, λ = 656. 3 nm, is then $\Delta\lambda \approx 4.57\ 10^{-5}$ nm~ 5 10^{-4} A~0.5 mA

Line broadening

Doppler broadening:

In thermal equilibrium, the velocities of atoms (with mass m) follow the Maxwell-Boltzmann distribution with the **most probable velocity**

$$V = \sqrt{\frac{2kT}{m}}$$

Since the atoms are moving, the wavelengths of the light absorbed or emitted by the atoms in the gas are Doppler-shifted according to $\Delta\lambda/\lambda = \pm|vr|/c$.

$$\Delta\lambda \simeq \frac{2\lambda^2}{c}\sqrt{\frac{2kT}{m}}$$

The Doppler broadening of the Hα line of hydrogen, λ = 656. 3 nm in the Solar photosphere (T~5777 k), is then $\Delta\lambda \approx 4.3\ 10^{-2}$ nm.

→ **The Doppler broadening is ~1000 greater than the natural broadening**

Line broadening

Pressure broadening:

The orbitals of an atom can be perturbed in a collision with a neutral atom or an ion. The results is called collisional or *pressure broadening*.

The general shape of the line is like that found for natural broadening → ***damping profile or Lorentz profile***

An approximate calculation of the broadening can be done estimating **the time between collision**:

$$\Delta t = \frac{l}{v} = \frac{1}{n\sigma\sqrt{2kT/m}}$$

with ***l* mean free path**, that can be expressed with $n\sigma$, in which n number density and σ collision cross section

The corresponding broadening is: $\Delta\lambda \simeq \frac{\lambda^2}{\pi c} 1/\Delta t$

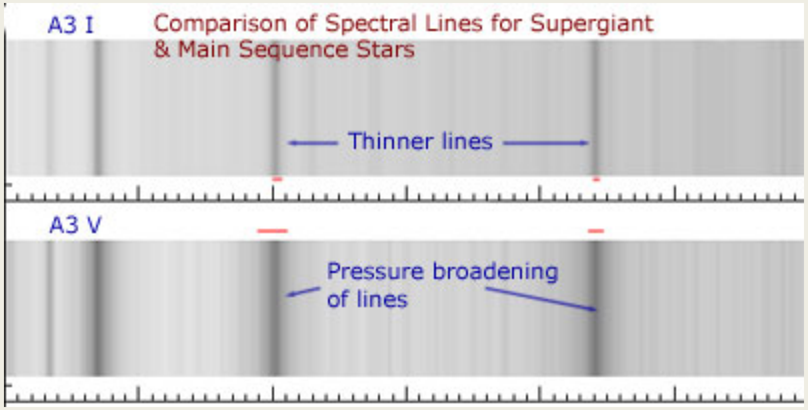
$$\Delta\lambda \simeq \frac{\lambda^2}{\pi c}\, n\sigma\sqrt{\frac{2kT}{m}}$$

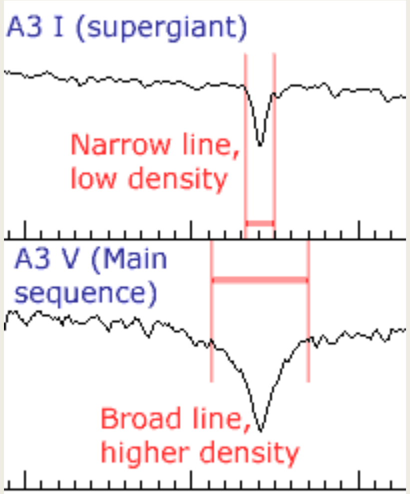
The pressure broadening of the Hα line of hydrogen, λ = 656. 3 nm in the Solar photosphere (T~5777 k), is then $\Delta\lambda \approx 2.6\ 10^{-5}$ nm, similar to the natural broadening (but it depends in the condition of the star, it can be much higher)

Line broadening

Pressure broadening:

The broadening is proportional to the number density: narrower lines observed for the more luminous giant and supergiant stars are due extended low density atmospheres. The typical pressure broadening is similar to natural broadening.





Comparison of spectral line widths for A3 I and A3 V class stars. The broader lines for the V luminosity class star arises due to the denser outer layers in the atmosphere of the main sequence star.

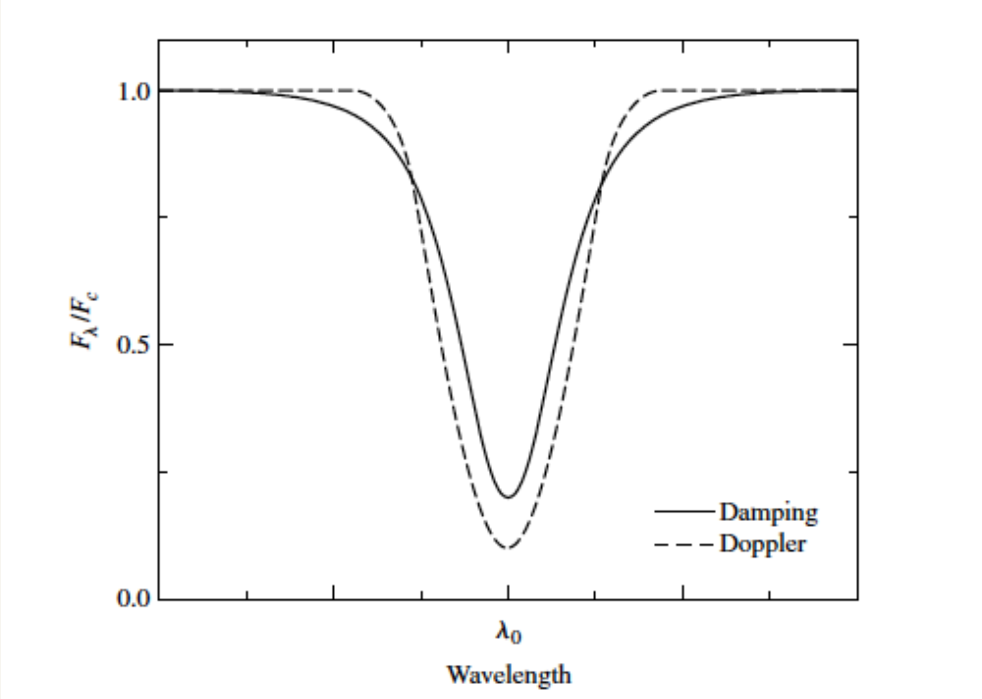
From http://outreach.atnf.csiro.au/education/senior/astrophysics/spectra_astro_types.html.

Line broadening

Voigt profile:

The combination of Damping and Doppler profiles give a total profile called → **Voigt profile**

- Doppler broadening dominates near the center
- Damping dominates on the wings of the line



Calculation of spectral lines

- The shape and intensity of a spectral line is related to the **abundance of the element N_a**, but not only….

- We need to know, how photons interacts with atoms:

  - ➢ Temperature and density to solve the Boltzmann and Saha equations (and to determine the broadening of the lines)

  - ➢ The probability of a given transition within the orbitals

    - The relative probabilities of an electron making a transition from the same initial orbital are given by the f –values or **oscillator strengths** for that orbital.

    - For instance: for hydrogen, f= 0. 637 for the Hα transition and f =0. 119 for Hβ.

Our goal is to determine the value of N_a by comparing the calculated and observed line profiles.

Digression: how abundances are defined in astrophysics

- Abundances by mass, Z

The proportion of the matter made up of elements heavier than helium (Y) and hydrogen (X).

It is denoted by *Z*, which represents the sum of all elements heavier than helium, in mass fraction.

$$X+Y+Z=1$$

Solar *Z* is 0.0134 (Asplund et al. 2009, ARAA 47, 481)

- Abundances by number (gas abundances)

The metallicity of gas is usually expressed by the number ratio of oxygen atoms (or other elements) to hydrogen atoms per unit volume.

Usually a factor 12 is summed to the final value:

$$O/H=\log_{10}(N_O/N_H)+12$$

where N_O and N_H are the numbers of oxygen and hydrogen atoms per unit volume.

- Abundances by number, referred to Solar values (stellar abundances)

The metallicity of stars is usually expressed by the number ratio of iron atoms to hydrogen atoms per unit volume, with respect to the solar values:

$$[Fe/H] = \log_{10}(N_{Fe}/N_H)_{star} - \log_{10}(N_{Fe}/N_H)_{Sun},$$

where N_{Fe} and N_H are the numbers of iron and hydrogen atoms per unit volume.

Digression: how abundances are defined in astrophysics

- From Z to [Fe/H]

$$\frac{N_{Fe}}{N_H} = f_{Fe}(\alpha) \times \frac{Z/m_Z(\alpha)}{(1-Y-Z)/m_H},$$

$f_{Fe}(\alpha)$ is the number fraction of iron with respect to all the elements heavier than helium; m_H is the mass of the hydrogen atom; and $m_Z(\alpha)$ is the average atomic mass of heavy elements weighted by the number of atoms.

It is important to note that $f_{Fe}(\alpha)$ and $m_Z(\alpha)$ depend on the fraction of α-elements ([α/Fe]).

- Here a tool online to convert Y and Z in to [Fe/H]:

http://www2.astro.puc.cl/pgpuc/FeHcalculator2.php

Y~25% by mass, 8% by number of atoms

Going back…from absorption lines to number of absorbers

The curve of growth

The curve of growth is a curve that shows how the equivalent width of an absorption line increases with the number of atoms producing the line.

The EWs of line depends on the number of absorber atoms [three regimes]

- When the density of those atoms is small, the EWs grows linearly with their density (N)

- When the density become higher the EWs tends to saturate, and thus it cannot be used as an indication of the abundances of the atoms

- Finally the EWs is again proportional to the abundance (√N)

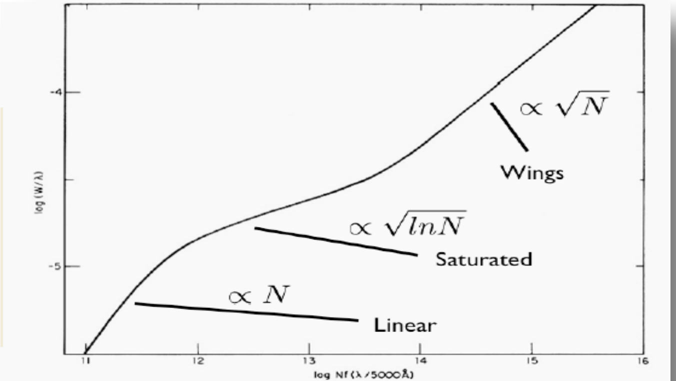
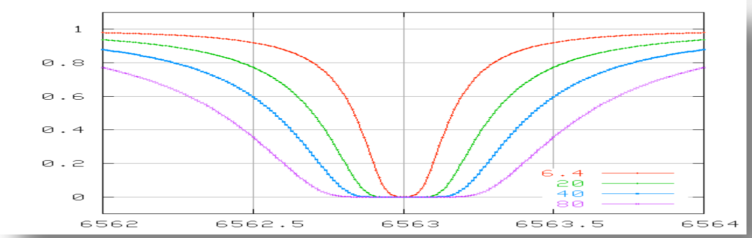
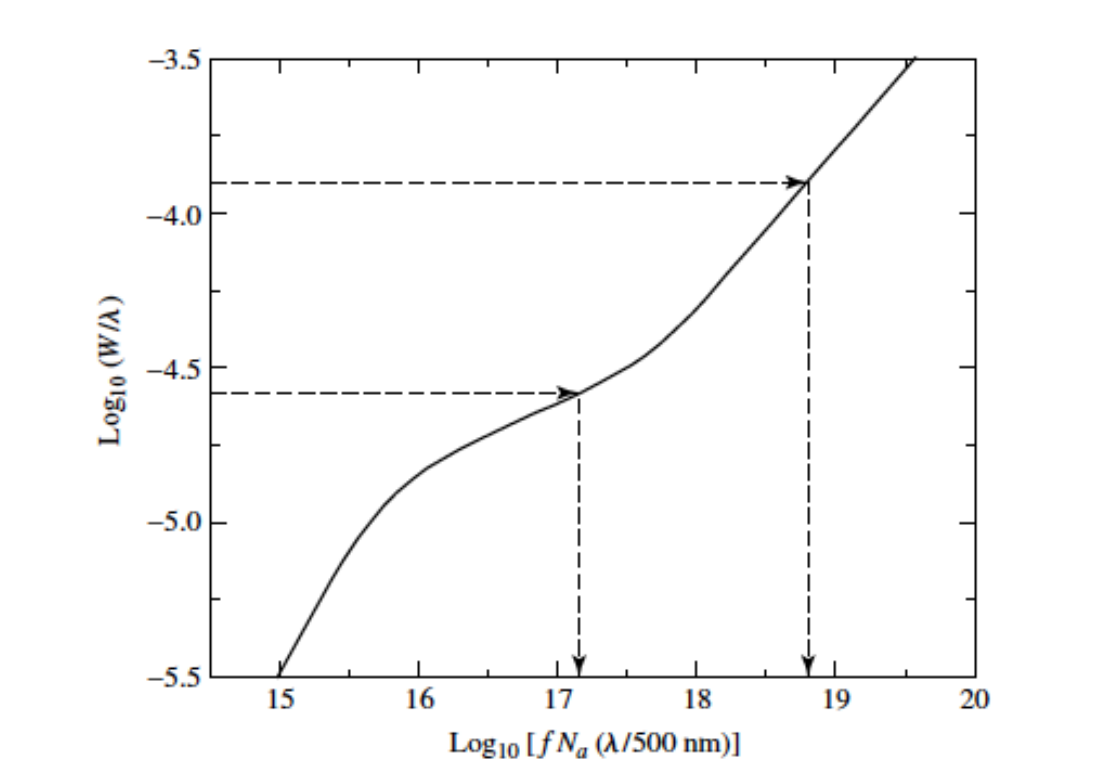


Figure 9.22 A general curve of growth for the Sun. (Figure from Aller, *Atoms, Stars, and Nebulae*, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)



The curve of growth

- The number of absorbing atoms (abundance) can be determined by comparing the equivalent widths from **different absorption lines** produced by the same atom/ion in the same state (and so having the same column density in the stellar atmosphere) with a theoretical curve of growth.



- On a common wavelength scale
- Considering the oscillator strength of each line

The curve of growth

- A curve-of-growth analysis can also be applied to lines originating from atoms or ions in different initial states

1. applying the **Boltzmann equation** to the relative numbers of atoms in different states of excitation allows us to calculate the **excitation temperature → in LTE approximation it is the effective temperature**

2. It is possible to use the Saha equation to find either the **electron pressure** or the ionization temperature (if the other is known) in the atmosphere from the relative numbers of atoms at various stages of ionization.

Determination of stellar parameters from spectral analysis

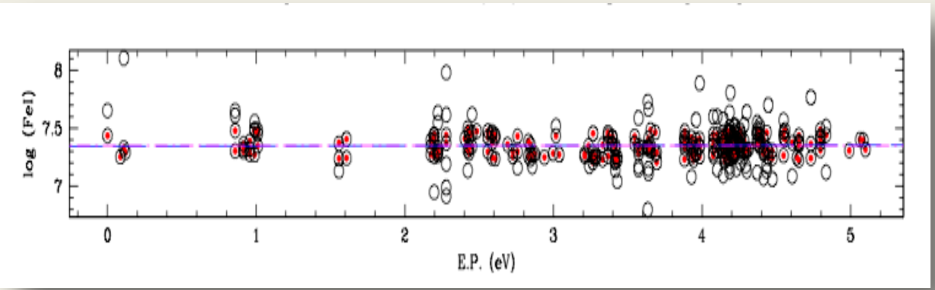
1. Deriving the EFFECTIVE TEMPERATURE (from Boltzmann equation):

To compute the T_{eff} of a star from its spectrum, we require that the abundance of an element is independent of the excitation potential of the individual lines.

This is so because, all the lines of a given element should give the same abundance for the same star.

In practice, there is a (small) scatter around an average value.

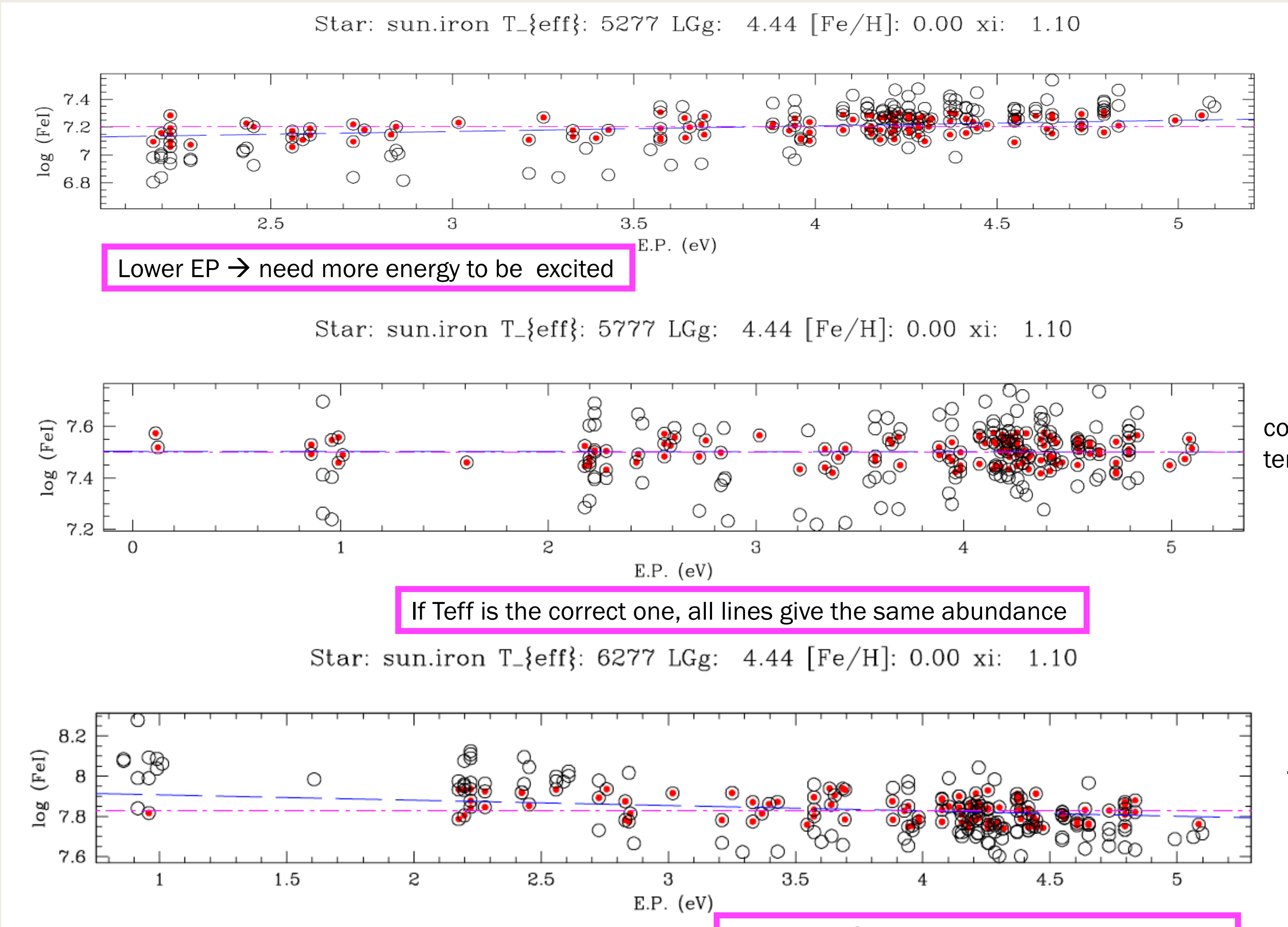
A T_{eff} which is incorrect will affect the weak excitation potentials more than the strong potentials or vice versa.



To use this method, we need many lines of a single element sampling a range of EP, as for example FeI lines in cool stars (F-G-K stars).

Determination of stellar parameters from spectral analysis

1. Deriving the EFFECTIVE TEMPERATURE (from Boltzmann equation):



-500 K

Lower EP → need more energy to be excited

correct temperature

If Teff is the correct one, all lines give the same abundance

+500 K

Higher EP → need less energy to be excited

2. Deriving the Pressure (proxy of surface gravity) (from Saha equation):

The surface gravity of a star of mass $M_\star$ and radius $R_\star$ is defined as $g_\star = GM_\star/R_\star^2$, where G is the gravitational constant.

For stellar types F, G or K, we can easily measure elements in two ionization states, like FeI and FeII.

For a given star and a given element, there should be a single value for the abundance, no matter if the abundance is determined from the neutral or the ionized state.

We can then iterate on the gravity of our model until the abundance of Fe I and Fe II are the same, constraining the stellar gravity.

For the hydrostatic equilibrium condition, gravity is related to the gas pressure.

From the Saha equation, **we found that the ratio between FeI and FeII depends on P_e**

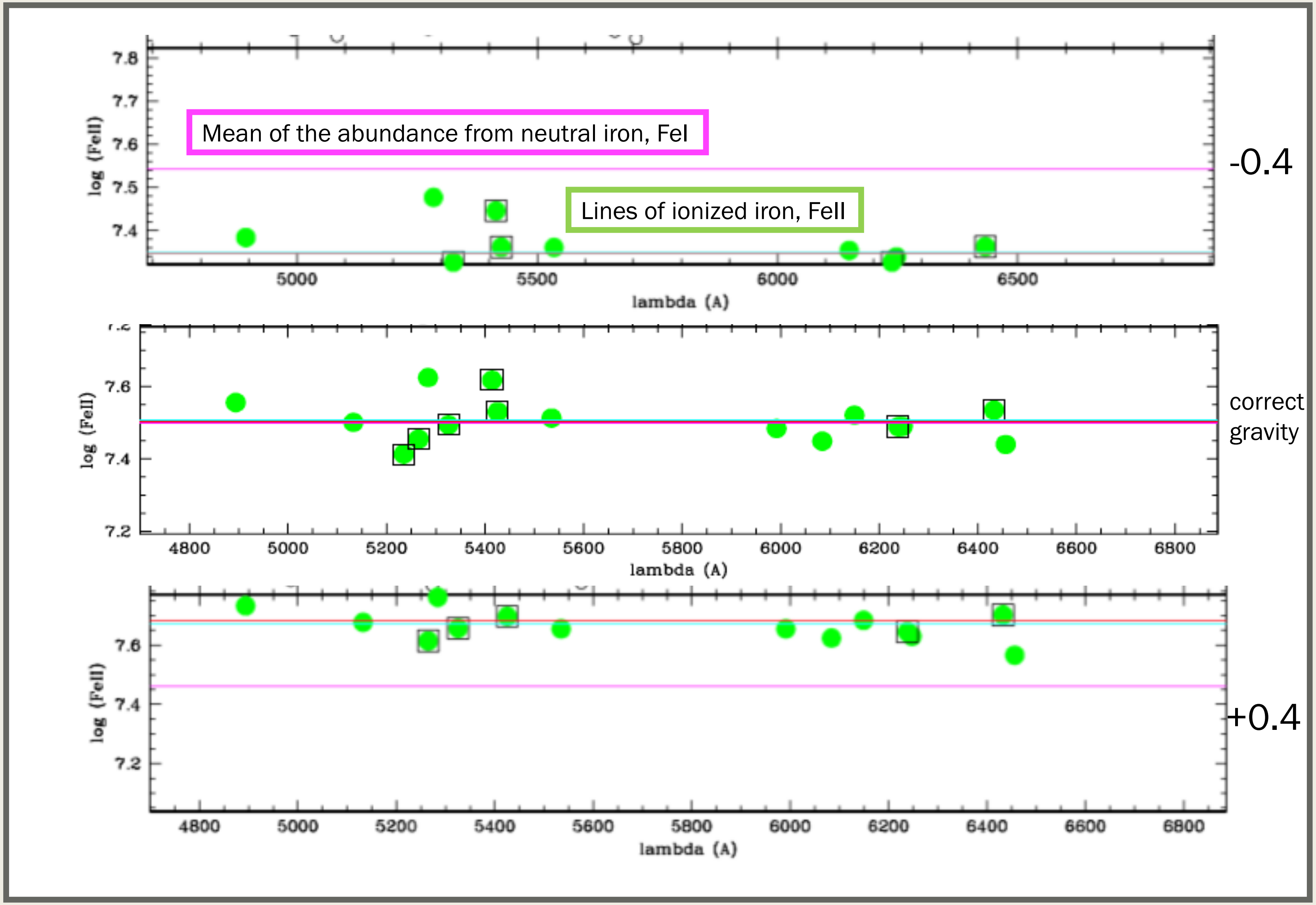
$$N_{FeII}/N_{FeI} \propto 1/P_e$$

Assuming that the star is in ionization equilibrium, we can estimate the pressure, and thus the surface gravity

→ **thus the ionization equilibrium is a good tool to derive gravity.**

Determination of stellar parameters from spectral analysis

2. Deriving the Pressure (proxy of surface gravity) (from Saha equation):



The curve of growth for the abundances

Assuming that we derived T_{eff} and log g –from spectroscopic analysis or for photometric analysis-, we can now use the equivalent widths of different elements/transitions to derive the abundances

For weak lines (first regime) we have:

$$\log\left(\frac{W_\lambda}{\lambda}\right) = \log\left(\frac{\pi e^2}{m_e c^2}\frac{N_i/N}{\mathcal{U}(T)}N_{\rm H}\right) + \log A + \log(gf\,\lambda) - \frac{5040}{T}\chi - \log(\kappa_\nu)$$

Equivalent width of a line

Constant for a given star

Abu nda nce

transition probability

Effective temp

Absorption coefficient

- For weak lines, the equivalent width varies in a linear way with abundances
- The abundance of an element (A) varies with the inverse of **the temperature**
- The abundance depends linearly on the gf -values.

Determination of abundances: the effect of temperature and gravity

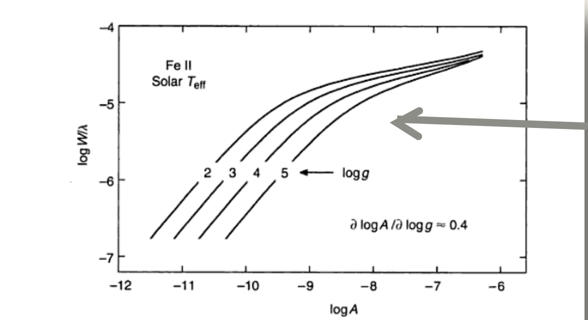
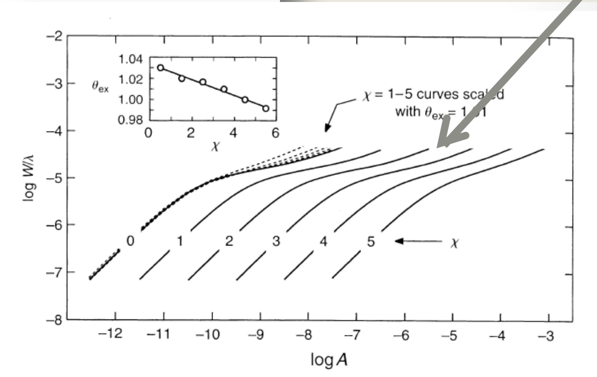


Fig. 16.3. Curves of growth for ions like Fe II are surface-gravity dependent in cool stars.



For instance:

The strength of lines of ionized elements depends on the pressure (log g) and so their curve of growth depends on the pressure/density

→ log g affects ionized species though Kv

The dependence on Teff is even stronger

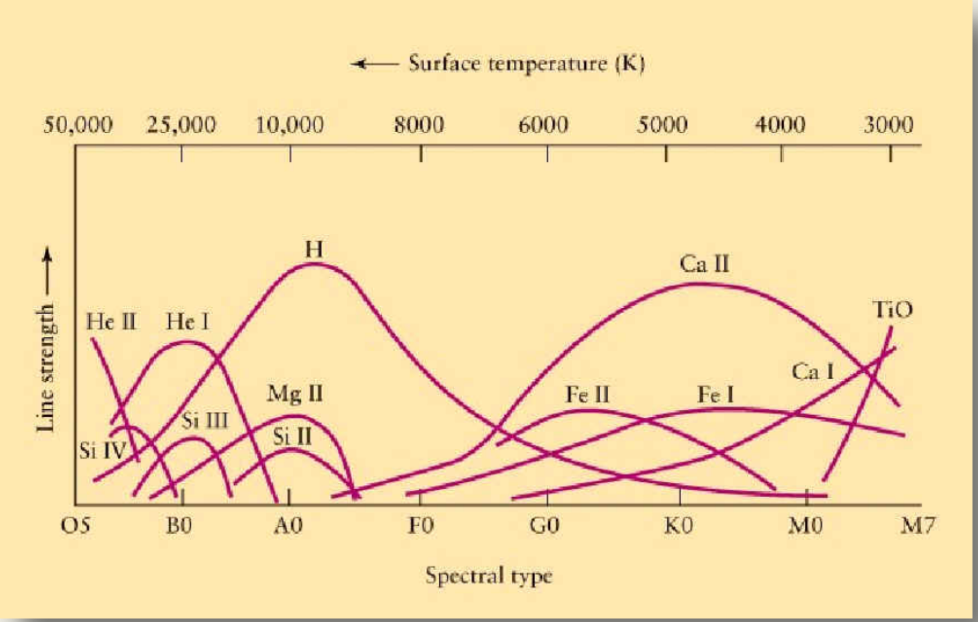
(from Grey, Lecture on spectral-line analysis)

We need to derive T_{eff} and pressure (related to the surface gravity log g) to correctly measure abundances.

Determination of abundances

Once derived Log g and T_{eff}, from the EWs of absorption lines we can derive the abundances using their theoretical curve of growth [in which we have fixed Teff and log g] → the comparison gives us the abundance

Which elements?



Depending on the T_{eff}, there are different dominating species in the atmospheres of stars.

Limiting the parameter space where our approximations are valid

→ Iron I and Iron II dominates and allows us both Teff and Log g determination

Stellar atmosphere models

Thus to solve the Boltzmann and Saha's equations, we must know to the conditions inside the stellar atmospheres, specifically:

- Temperature
- Pressure (or Log g)
- Opacity

The characteristics of the stellar atmospheres are tabulated in several sets of model stellar atmospheres, e.g.:

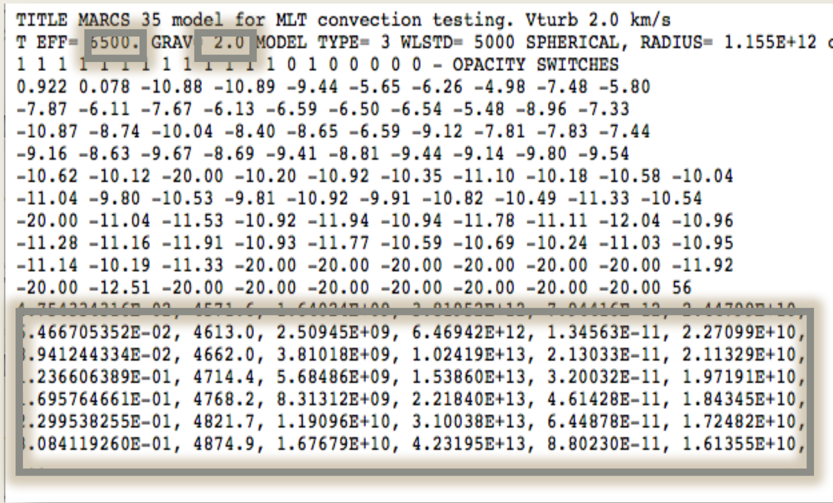
- Kurucz models: http://kurucz.harvard.edu/grids.html
- MARCS models: http://marcs.astro.uu.se/
- ATLAS modes: http://www.stsci.edu/hst/observatory/crds/castelli_kurucz_atlas.html

Stellar atmosphere models

Stellar atmospheric models are a tabulation of physical parameters used to represent the conditions inside an atmosphere.

Models are typically given as the electronic pressure (Pe), the temperature (T) and the optical depth for photons with λ=5000Å for several layers (~50) of a stellar atmosphere.

It is customary in stellar atmosphere work to use **log g** as equivalent for the pressure in the atmosphere (which can be done if **hydrostatic equilibrium is valid**).



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T EFF= 6500. GRAV= 2.0 MODEL TYPE= 3 WLSTD= 5000 SPHERICAL, RADIUS= 1.155E+12 cm
1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 0 0 0 - OPACITY SWITCHES
0.922 0.078 -10.88 -10.89 -9.44 -5.65 -6.26 -4.98 -7.48 -5.80
-7.87 -6.11 -7.67 -6.13 -6.59 -6.50 -6.54 -5.48 -8.96 -7.33
-10.87 -8.74 -10.04 -8.40 -8.65 -6.59 -9.12 -7.81 -7.83 -7.44
-9.16 -8.63 -9.67 -8.69 -9.41 -8.81 -9.44 -9.14 -9.80 -9.54
-10.62 -10.12 -20.00 -10.20 -10.92 -10.35 -11.10 -10.18 -10.58 -10.04
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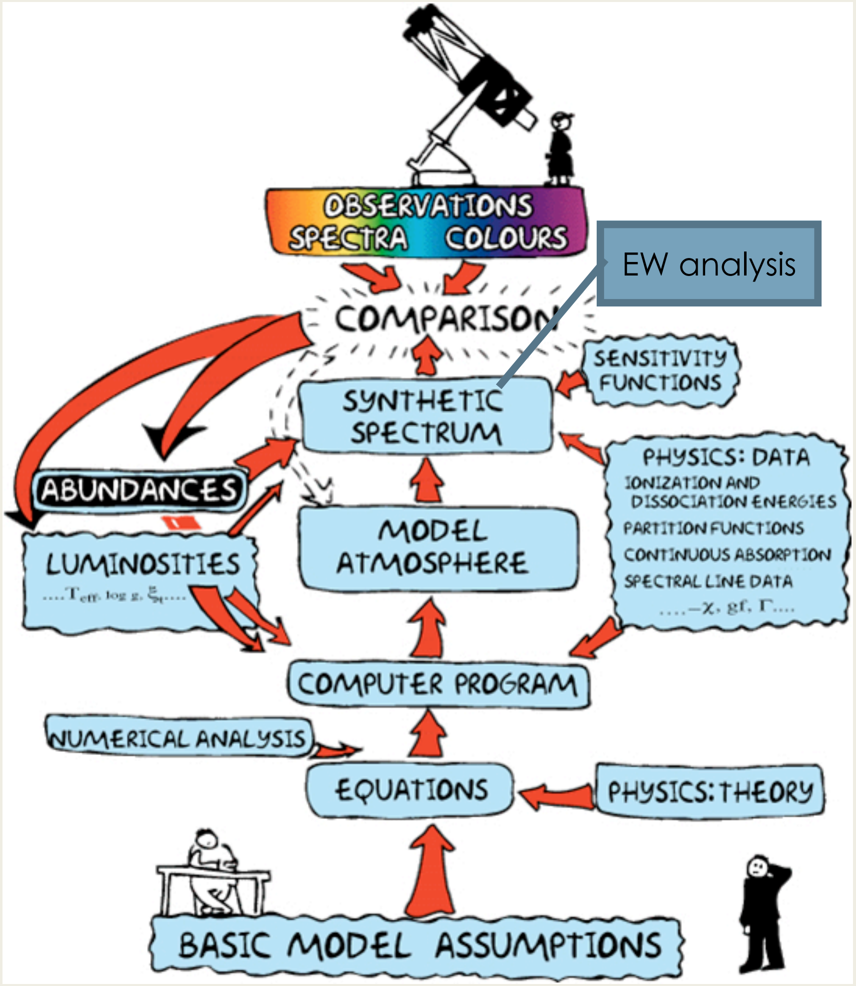
.299538255E-01, 4821.7, 1.19096E+10, 3.10038E+13, 6.44878E-11, 1.72482E+10,
.084119260E-01, 4874.9, 1.67679E+10, 4.23195E+13, 8.80230E-11, 1.61355E+10,
```

- depth in column mass or optical depth at standard wavelength
- temperature in K
- number density of free electrons in cm $^{-3}$
- number density of all other particles (except electrons) in cm $^{-3}$
- column: density in g/cm ^{3}

How-to



EW analysis

Observed spectrum

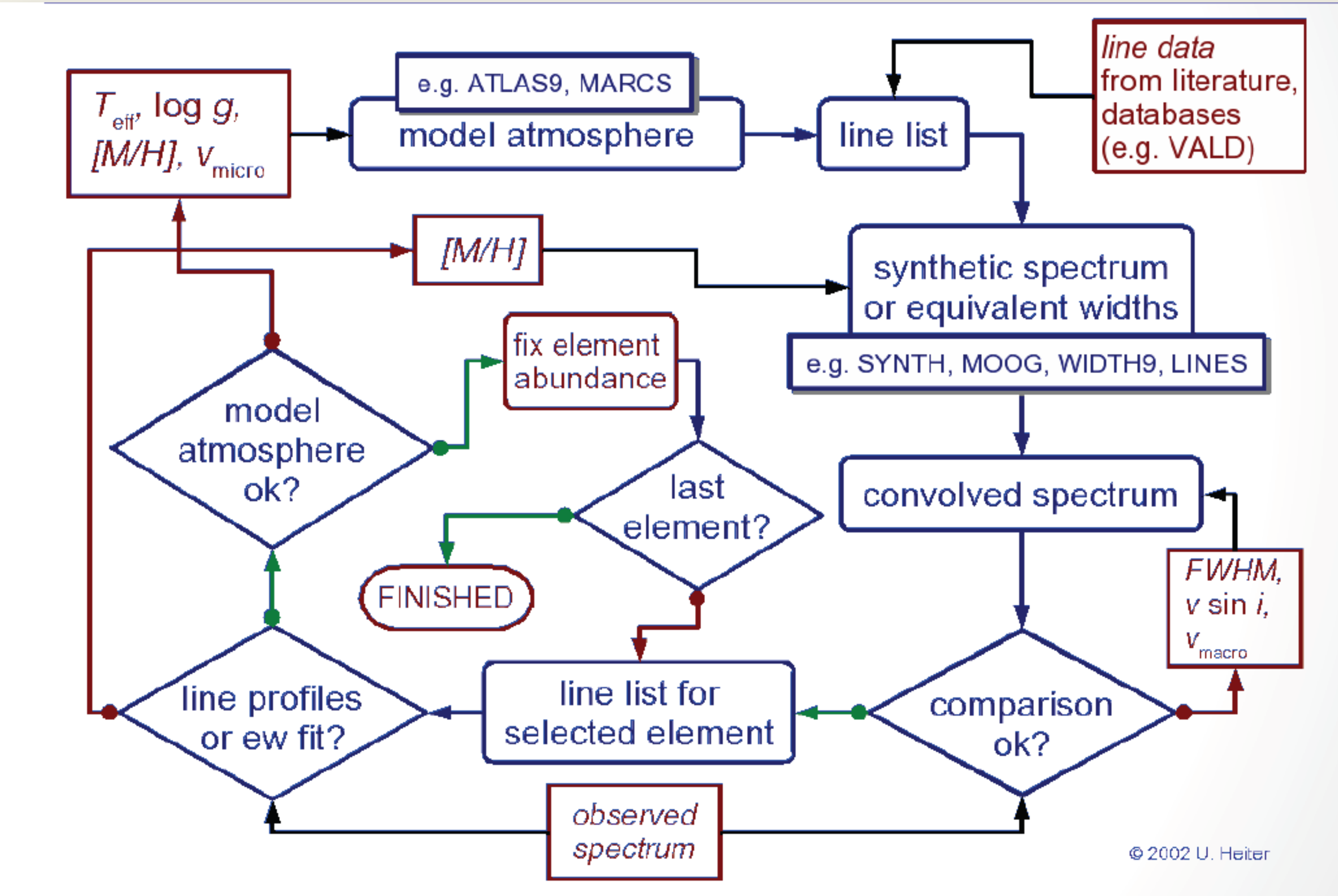
+

Model Atmosphere

↓

Stellar parameters and abundances

Boltzmann and Saha's equations, LTE, pp, etc.


T_{eff}, log g, [M/H], v_{micro}
e.g. ATLAS9, MARCS
model atmosphere
line list
line data from literature, databases (e.g. VALD)
[M/H]
synthetic spectrum or equivalent widths
e.g. SYNTH, MOOG, WIDTH9, LINES
fix element abundance
model atmosphere ok?
last element?
FINISHED
convolved spectrum
FWHM, v sin i, v_{macro}
line profiles or ew fit?
line list for selected element
comparison ok?
observed spectrum
© 2002 U. Heiter

Tools –some examples-

To measure EWs:

- IRAF splot
- Daospec http://www.cadc-ccda.hia-iha.nrc-cnrc.gc.ca/en/community/STETSON/daospec/
- Ares http://www.astro.up.pt/~sousasag/ares/

Model Atmospheres:

- Kurucz models: http://kurucz.harvard.edu/grids.html
- MARCS models: http://marcs.astro.uu.se/
- ATLAS modes: http://www.stsci.edu/hst/observatory/crds/castelli_kurucz_atlas.html

ATOMIC DATA:

- VALD: http://vald.astro.uu.se/
- NIST: http://physics.nist.gov/PhysRefData/ASD/lines_form.html

Tools –some examples-

to derive abundances:

- MOOG http://www.as.utexas.edu/~chris/moog.html
- ATLAS/SYNTHE (Kurucz) http://kurucz.harvard.edu
- (OS)MARCS: http://marcs.astro.uu.se/
- SME http://tauceD.sfsu.edu/Tutorials.html
- Synspec http://nova.astro.umd.edu/Synspec43/synspec.html
- ISPEC: https://www.blancocuaresma.com/s/iSpec
- THE CANNON: https://www.sdss.org/dr14/irspec/the-cannon

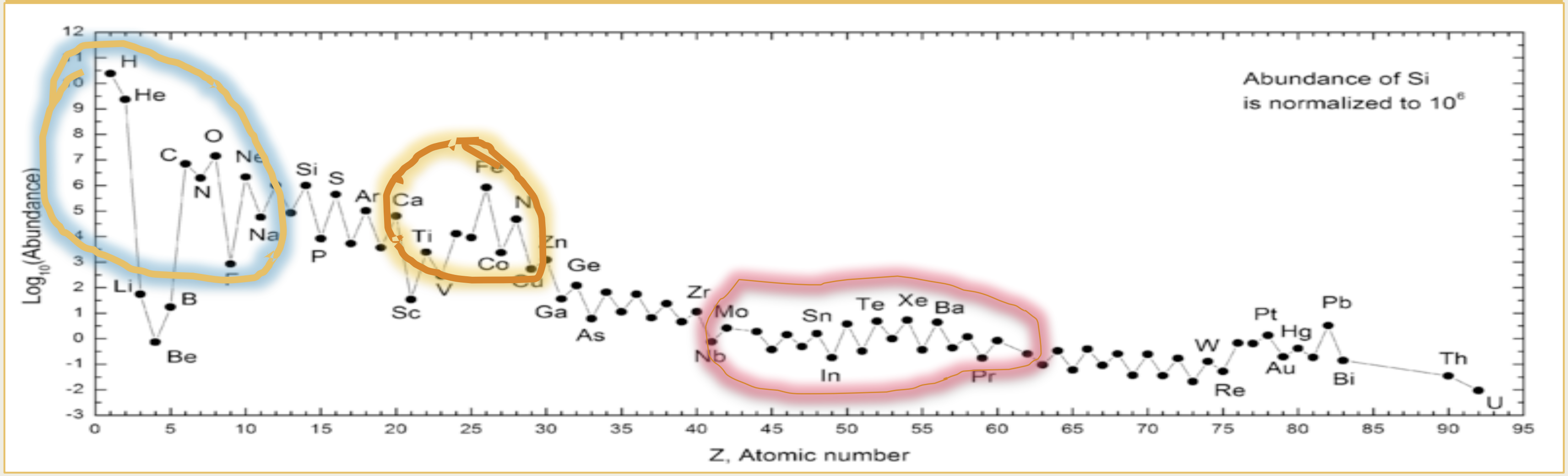
Which elements we can measure:

The main ones are in cool stars:

Alpha-elements: O, Si, Mg, Ca, Ti

Iron-peak elements: V, Sc, Ni, Cr, Fe

Neutron-capture elements: Y, Eu, Ba



Is it enough?

In the previous slides, we have presented the spectral analysis based on several assumptions, in particular LTE, hydrostatic equilibrium, homogeneity of stellar atmospheres.

Reality is much more complex:

- 1D vs 3D results
- LTE vs NLTE