# Sample Exam

June 10, 2024

**Exam of Algorithmic Design:** sections A and B.

**Exam of Algorithmic Data Mining:** sections B and C.

**Exam of Algorithms for Scientific Computing:** sections A and D.

**Exam of Advanced Algorithms for Scientific Computing:** sections B and D.

## A Basic Algorithms

*Question* A.1.

a) Define what an *in place* algorithm is.

b) Among the sorting algorithms seen in the course, name one which is in place and one which is not. Motivate your answer by analyzing the space required by each of the chosen algorithms.

*Question* A.2.

a) Define max-heaps.

b) What is the worst-case asymptotic time cost of inserting a new element in a max-heap (maintaining the max-heap property)?

c) Draw the visualization for the max-heap $[15, 12, 10, 6, 8, 4]$. Suppose you need to insert the new element 18: explain step-by-step the operations you need to do to insert it and restore the max-heap property, drawing the state of the heap for each step.

*Question* A.3.

a) Define the single-source shortest paths (SSSP) problem.

b) Consider an edge-weighted graph $G = (V, E, W)$ such that the weights of the edges are positive integers, and a source node $s \in V$. Does a breadth-first search (BFS) from $s$ solve the SSSP problem with source $s$? Justify your answer by providing a counterexample in case your answer is NO, or by identifying the property that makes BFS a correct solution for the problem in case your answer is YES.

## B Advanced Algorithms

*Question* B.1.

a) Define the pattern matching problem.

b) State the asymptotic worst-case time complexity of using the KMP algorithm to search for $k$ patterns $P_1, P_2, \ldots, P_k$ in a single text $T$.

c) State the asymptotic worst-case time complexity of using the suffix tree of $T$ to search for $P_1, P_2, \ldots, P_k$ in $T$.

*Question* B.2.

a) Define the *load factor* of a hash table.

b) State the asymptotic worst-case time complexity of searching for an item in a general hash table of size $m$ containing $n$ items where collisions are resolved with chaining. Motivate your answer by identifying the worst-case scenario.

c) State the asymptotic expected time complexity of searching for an item in a general hash table of size $m$ containing $n$ items where collisions are resolved with chaining. Motivate your answer by stating the properties a hash function must satisfy to give the claimed expected time complexity.

# C Data Mining

*Question* C.1.

a) Define the frequent pattern mining problem.

b) State the Apriori property.

c) Draw the set enumeration tree for the set of items $\{1, 3, 5, 7, 9\}$.

*Question* C.2.

a) Describe the insertion procedure into a Bloom filter of size $m$ using $k$ hash functions.

b) Define what is a membership query on a Bloom filter in terms of input and output, specifying in which cases the answer must be exact.

c) Describe the procedure for answering membership queries on a Bloom filter and justify why the procedure is correct.

# D Scientific Computing

Exercises by Prof. Padoan.