Tries and suffix tries

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Tries

A trie (pronounced "try") is a tree representing a collection of strings with one node per common prefix

Smallest tree such that:

Each edge is labeled with a character $c \in \Sigma$

A node has at most one outgoing edge labeled c, for $c \in \Sigma$

Each key is "spelled out" along some path starting at the root

Natural way to represent either a *set* or a *map* where keys are strings

Tries: example

Represent this map with a trie:

Key	Value
instant	1
internal	2
internet	3

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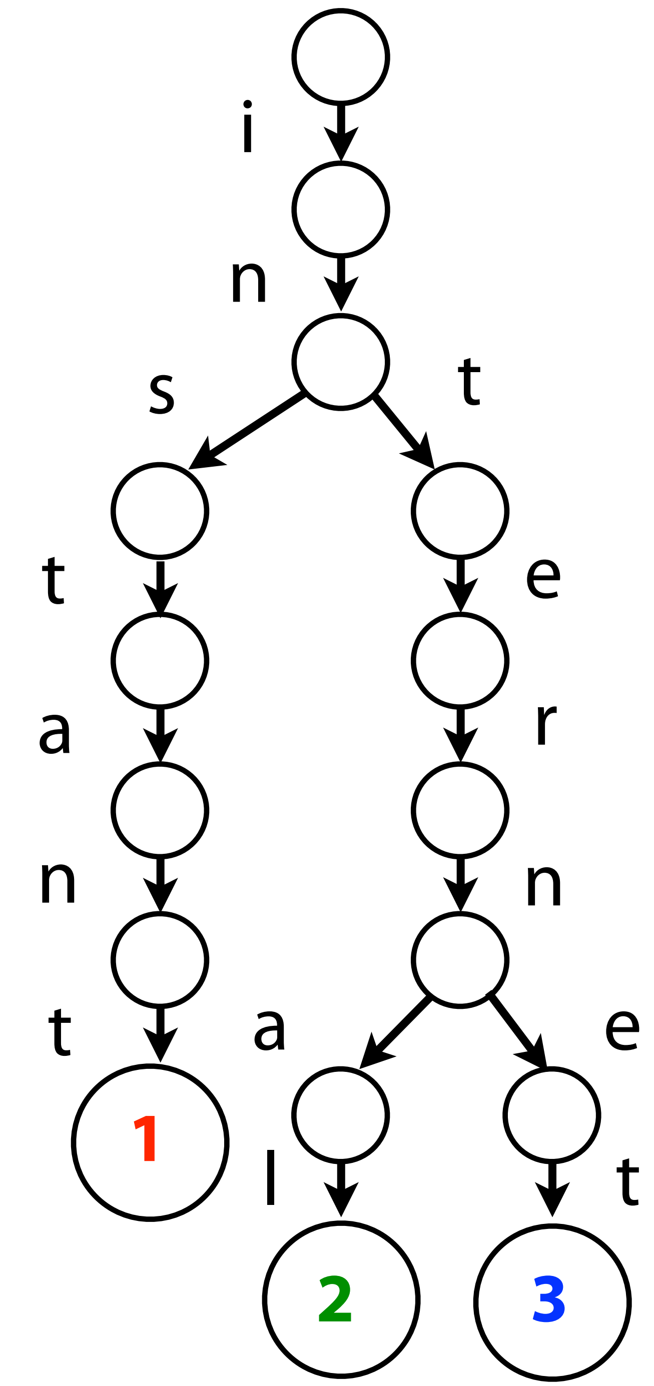
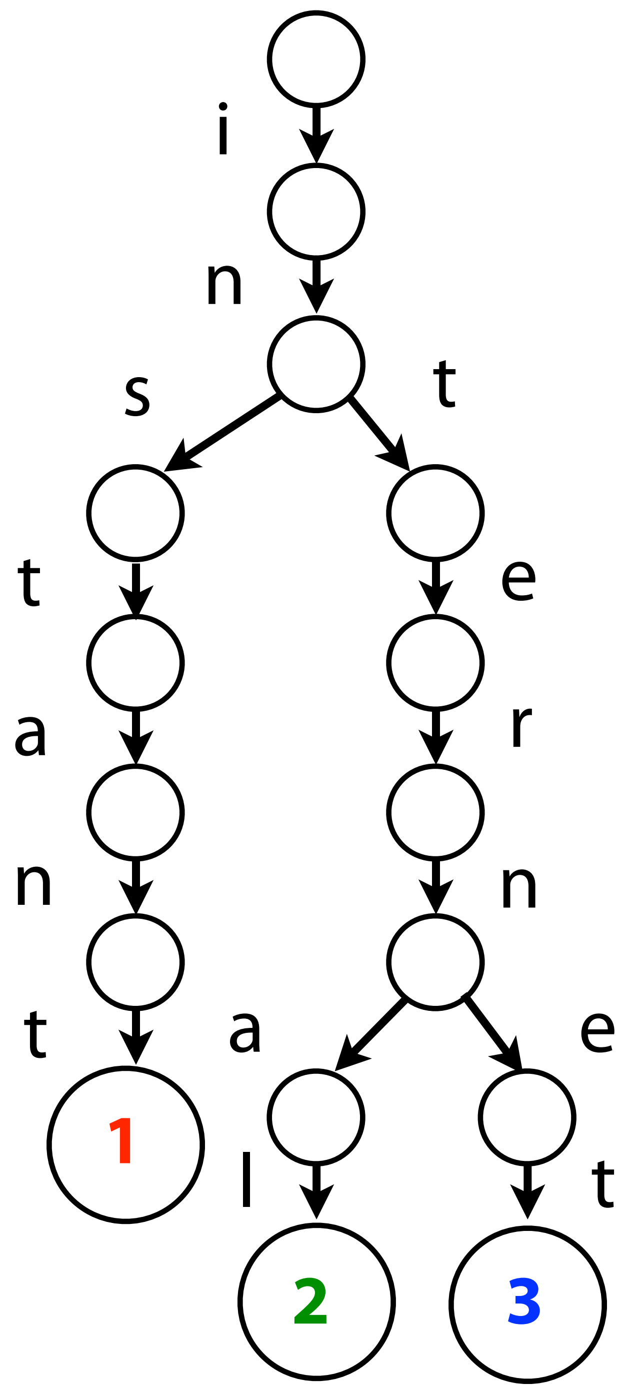


Tries: example



Checking for presence of a key P, where $n = |P|$, is $\boldsymbol{O(n)}$ time

If total length of all keys is N, trie has $\boldsymbol{O(N)}$ nodes

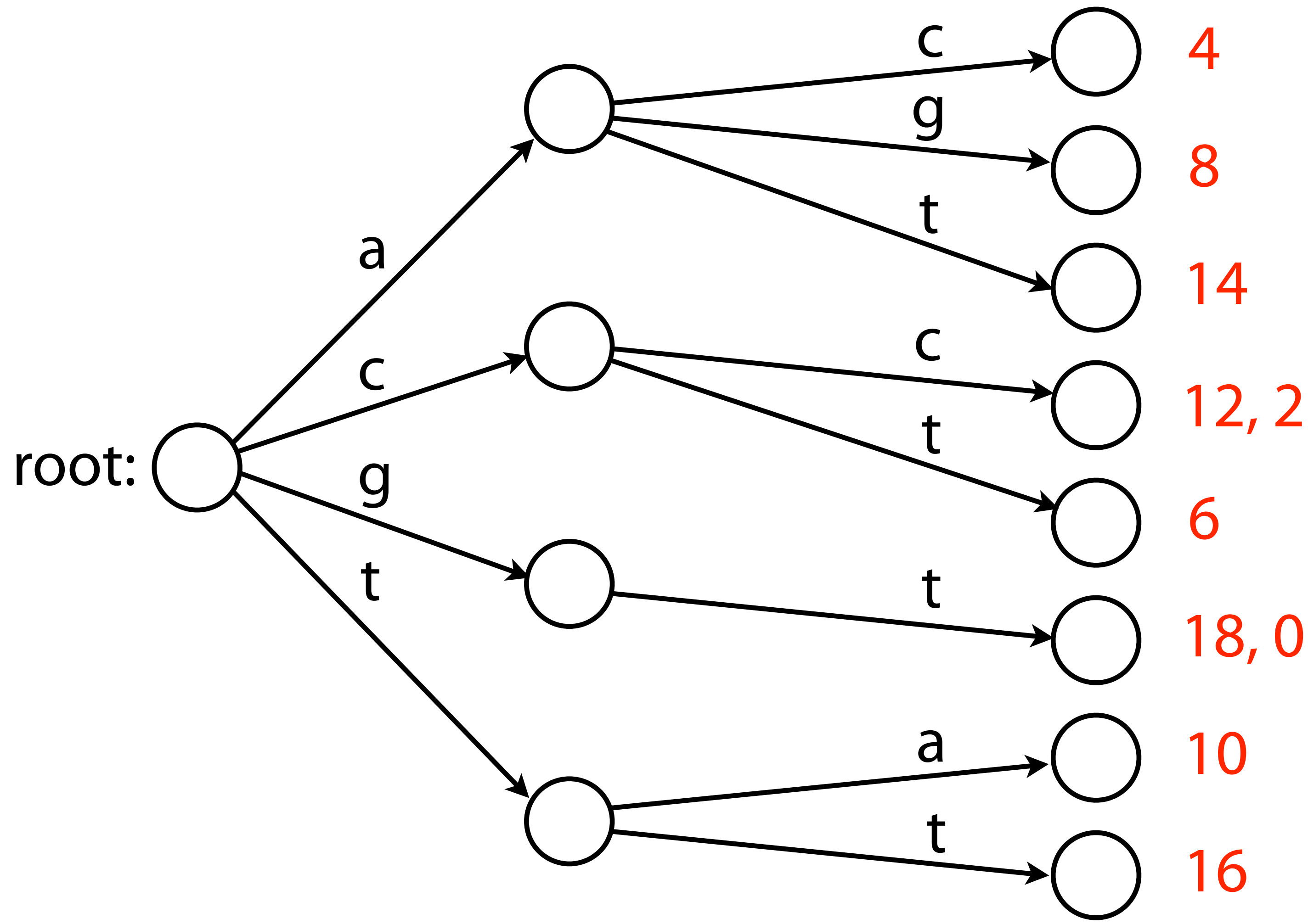
What about $|\Sigma|$?

Depends how we represent outgoing edges. If we don't assume $|\Sigma|$ is a small constant, it shows up in one or both bounds.

Tries: another example

We can index *T* with a trie. The trie maps substrings to offsets where they occur

ac	4
ag	8
at	14
cc	12
cc	2
ct	6
gt	18
gt	0
ta	10
tt	16



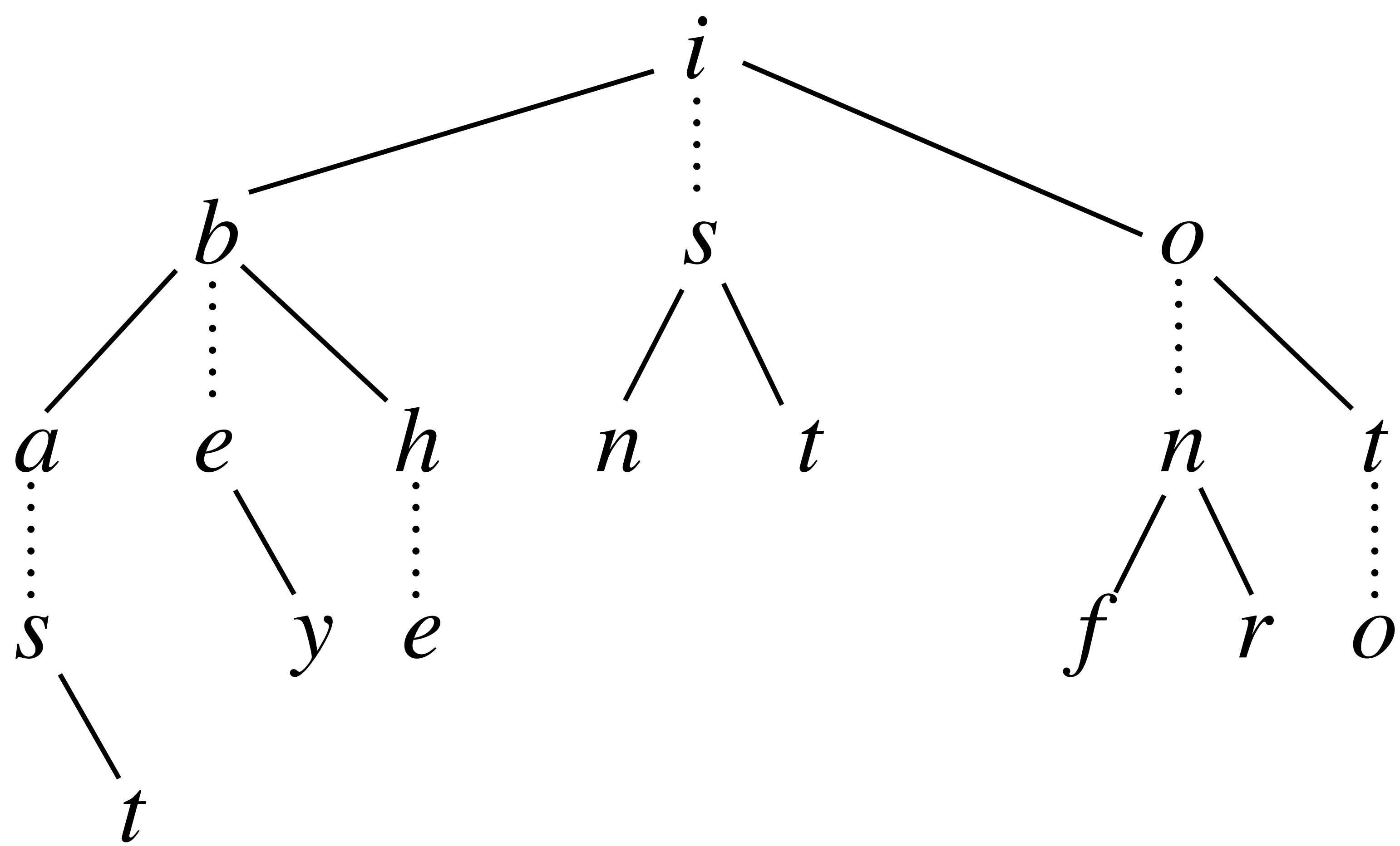
Tries: implementation

```python
class TrieMap(object):
    """ Trie implementation of a map.  Associating keys (strings or other
        sequence type) with values.  Values can be any type. """

    def __init__(self, kvs):
        self.root = {}
        # For each key (string)/value pair
        for (k, v) in kvs: self.add(k, v)

    def add(self, k, v):
        """ Add a key-value pair """
        cur = self.root
        for c in k: # for each character in the string
            if c not in cur:
                cur[c] = {} # if not there, make new edge on character c
            cur = cur[c]
        cur['value'] = v # at the end of the path, add the value

    def query(self, k):
        """ Given key, return associated value or None """
        cur = self.root
        for c in k:
            if c not in cur:
                return None # key wasn't in the trie
            cur = cur[c]
        # get value, or None if there's no value associated with this node
        return cur.get('value')
```

Python example:
http://nbviewer.ipython.org/6603619

Tries: alternatives

Tries aren't the only tree structure that can encode sets or maps with string keys. E.g. binary or ternary search trees.



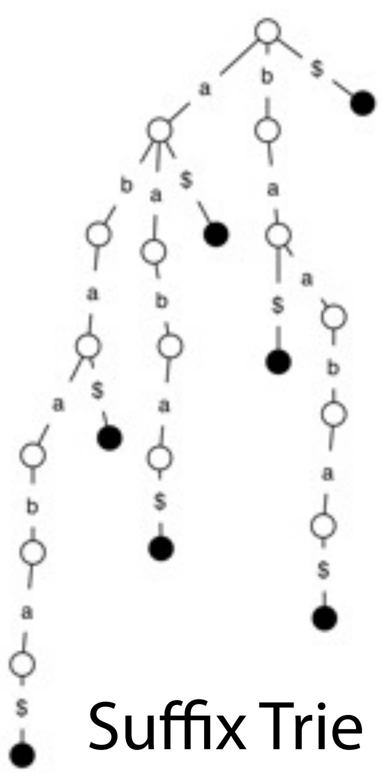
Ternary search tree for as, at, be, by, he, in, is, it, of, on, or, to

Example from: Bentley, Jon L., and Robert Sedgewick. "Fast algorithms for sorting and searching strings." *Proceedings of the eighth annual ACM-SIAM symposium on Discrete algorithms*. Society for Industrial and Applied Mathematics, 1997

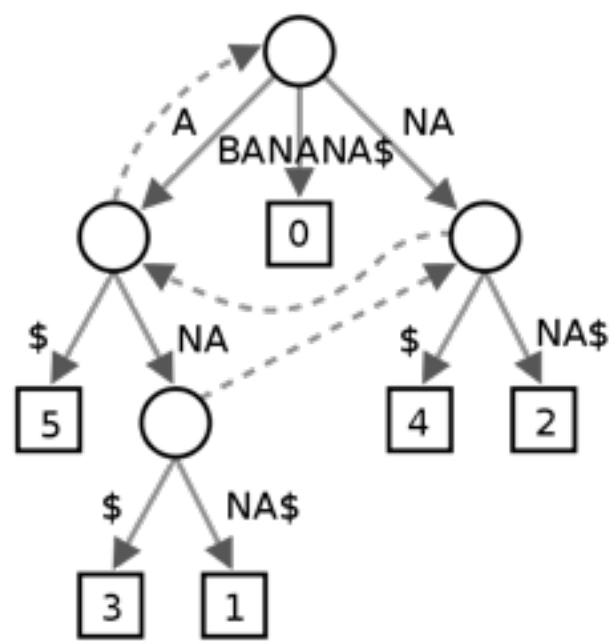
Indexing with suffixes

Until now, our indexes have been based on extracting substrings from *T*

A very different approach is to extract *suffixes* from *T*. This will lead us to some interesting and practical index data structures:



Suffix Trie



Suffix Tree

6	$
5	A$
3	ANA$
1	ANANA$
0	BANANA$
4	NA$
2	NANA$

Suffix Array

$ B A N A N **A**
A $ B A N A **N**
A N A $ B A **N**
A N A N A $ **B**
B A N A N A **$**
N A $ B A N **A**
N A N A $ B **A**

FM Index

Suffix trie

Build a **trie** containing all **suffixes** of a text *T*

```
T: G T T A T A G C T G A T C G C G G C G T A G C G G
   G T T A T A G C T G A T C G C G G C G T A G C G G
     T T A T A G C T G A T C G C G G C G T A G C G G
       T A T A G C T G A T C G C G G C G T A G C G G
         A T A G C T G A T C G C G G C G T A G C G G
           T A G C T G A T C G C G G C G T A G C G G
             A G C T G A T C G C G G C G T A G C G G
               G C T G A T C G C G G C G T A G C G G
                 C T G A T C G C G G C G T A G C G G
                   T G A T C G C G G C G T A G C G G
                     G A T C G C G G C G T A G C G G
                       A T C G C G G C G T A G C G G
                         T C G C G G C G T A G C G G
                           C G C G G C G T A G C G G
                             G C G G C G T A G C G G
                               C G G C G T A G C G G
                                 G G C G T A G C G G
                                   G C G T A G C G G
                                     C G T A G C G G
                                       G T A G C G G
                                         T A G C G G
                                           A G C G G
                                             G C G G
                                               C G G
                                                 G G
                                                   G
```

m(m+1)/2 chars

Suffix trie

First add special *terminal character* **$** to the end of *T*

$ is a character that does not appear elsewhere in *T*, and we define it to be less than other characters (for DNA: **$** < **A** < **C** < **G** < **T**)

$** enforces a rule we're all used to using: e.g. "as" comes before "ash" in the dictionary. **$ also guarantees no suffix is a prefix of any other suffix.

T: G T T A T A G C T G A T C G C G G C G T A G C G G $

G T T A T A G C T G A T C G C G G C G T A G C G G $
T T A T A G C T G A T C G C G G C G T A G C G G $
T A T A G C T G A T C G C G G C G T A G C G G $
A T A G C T G A T C G C G G C G T A G C G G $
T A G C T G A T C G C G G C G T A G C G G $
A G C T G A T C G C G G C G T A G C G G $
G C T G A T C G C G G C G T A G C G G $
C T G A T C G C G G C G T A G C G G $
T G A T C G C G G C G T A G C G G $
G A T C G C G G C G T A G C G G $
A T C G C G G C G T A G C G G $
T C G C G G C G T A G C G G $
C G C G G C G T A G C G G $
G C G G C G T A G C G G $
C G G C G T A G C G G $
G G C G T A G C G G $
G C G T A G C G G $

Tries

Smallest tree such that:

Each edge is labeled with a character from Σ

A node has at most one outgoing edge labeled with c, for any $c \in \Sigma$

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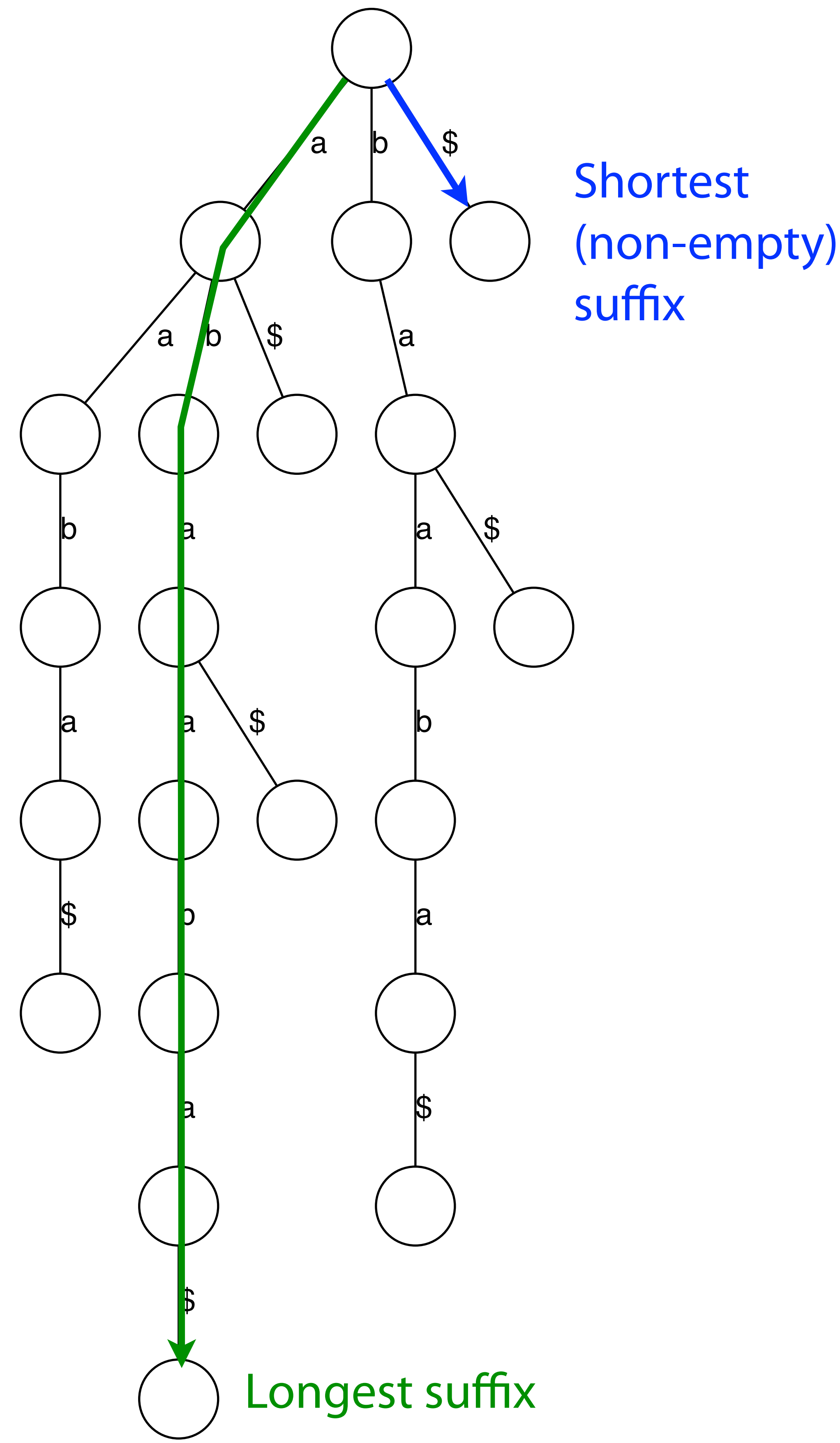
Suffix trie

T: abaaba *T***$***:* abaaba**$**

Each path from root to leaf represents a suffix; each suffix is represented by some path from root to leaf

Would this still be the case if we hadn't added **$**?

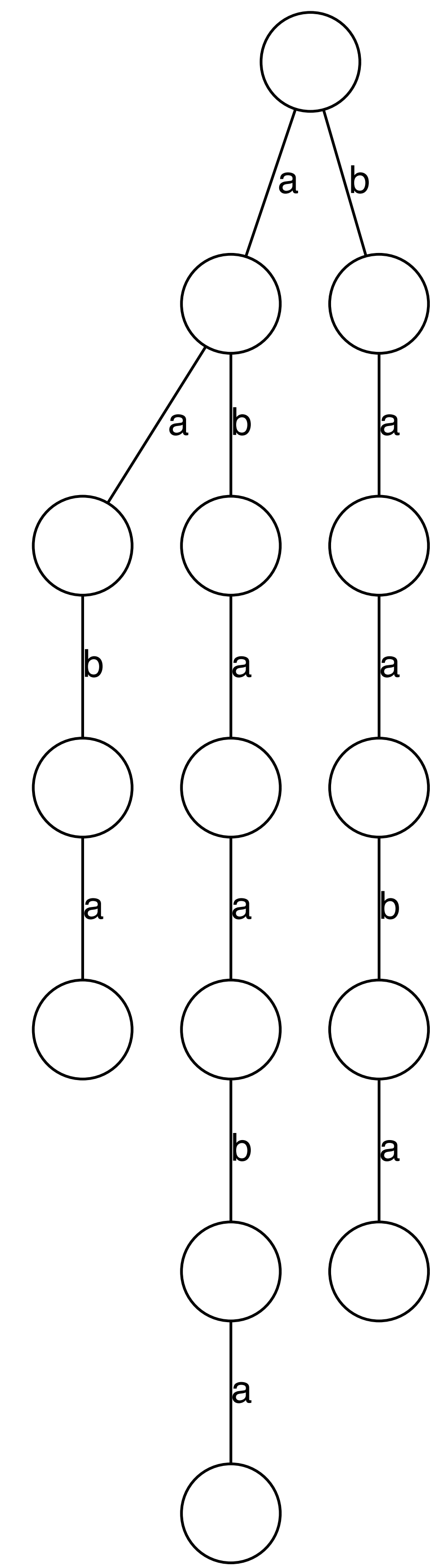


Suffix trie

T: abaaba

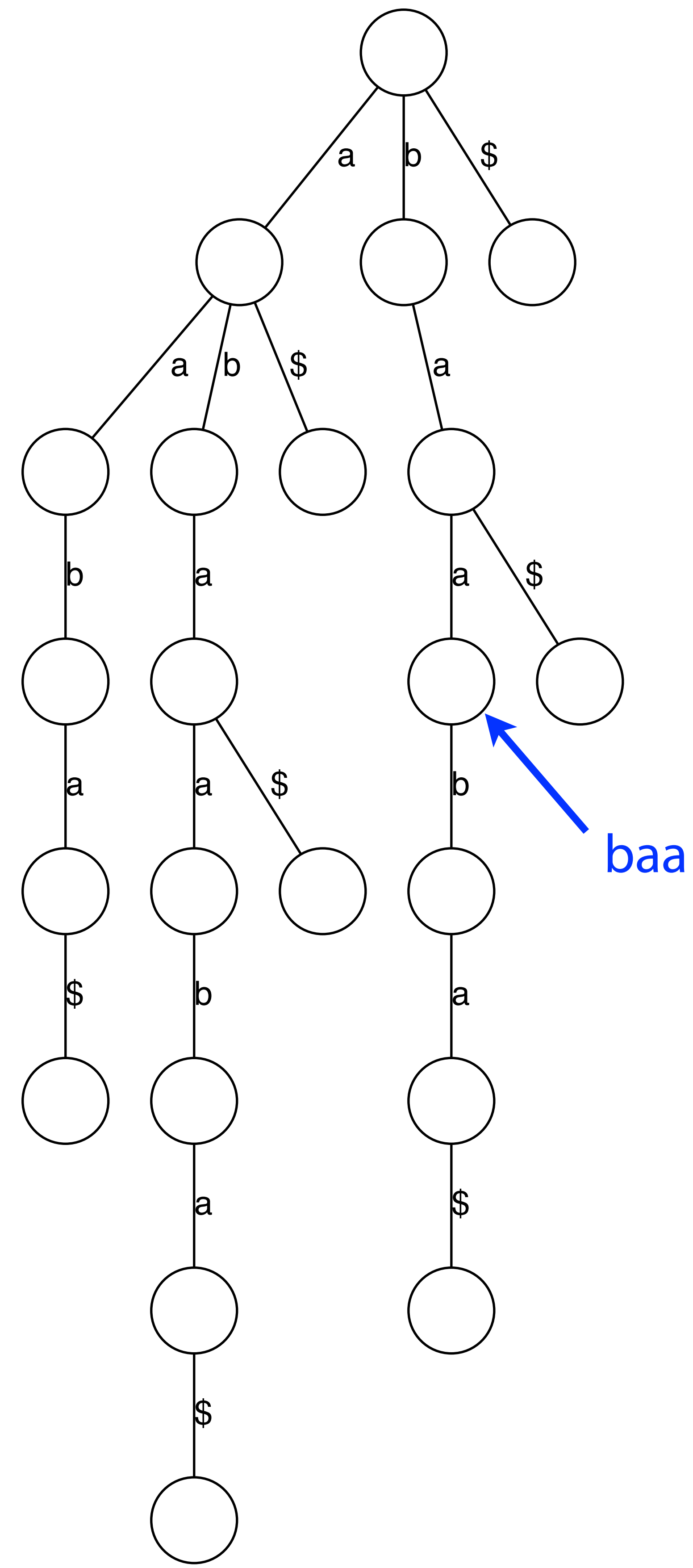
Each path from root to leaf represents a suffix; each suffix is represented by some path from root to leaf

Would this still be the case if we hadn't added **$**? **No**



Suffix trie

We can think of nodes as having **labels**, where the label spells out characters on the path from the root to the node



Suffix trie

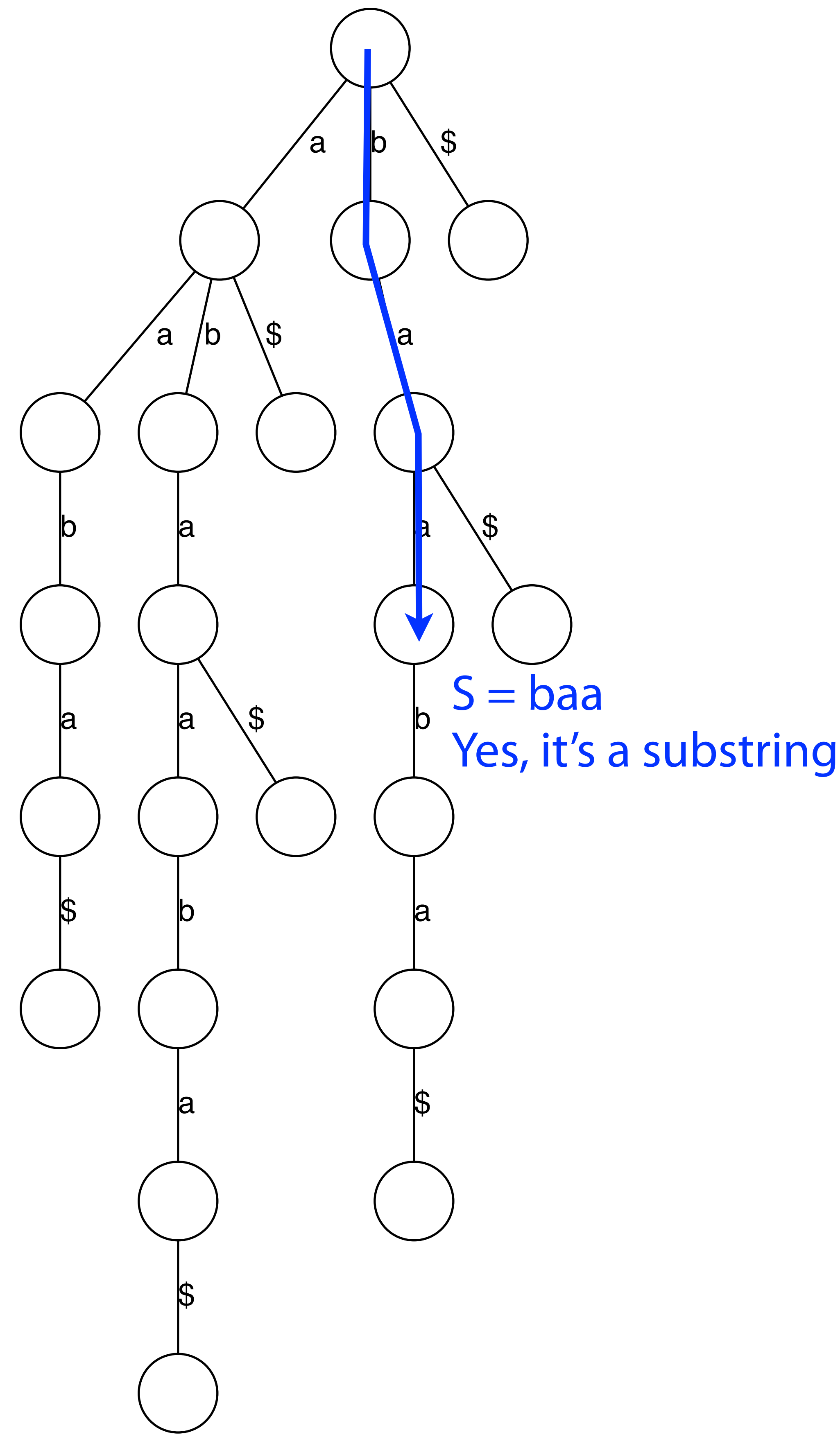
How do we check whether a string *S* is a substring of *T*?

Note: Each of *T*'s substrings is spelled out along a path from the root. I.e., every *substring* is a *prefix* of some *suffix* of T.

Start at the root and follow the edges labeled with the characters of *S*

If we "fall off" the trie -- i.e. there is no outgoing edge for next character of *S*, then *S* is not a substring of *T*

If we exhaust *S* without falling off, *S* is a substring of *T*



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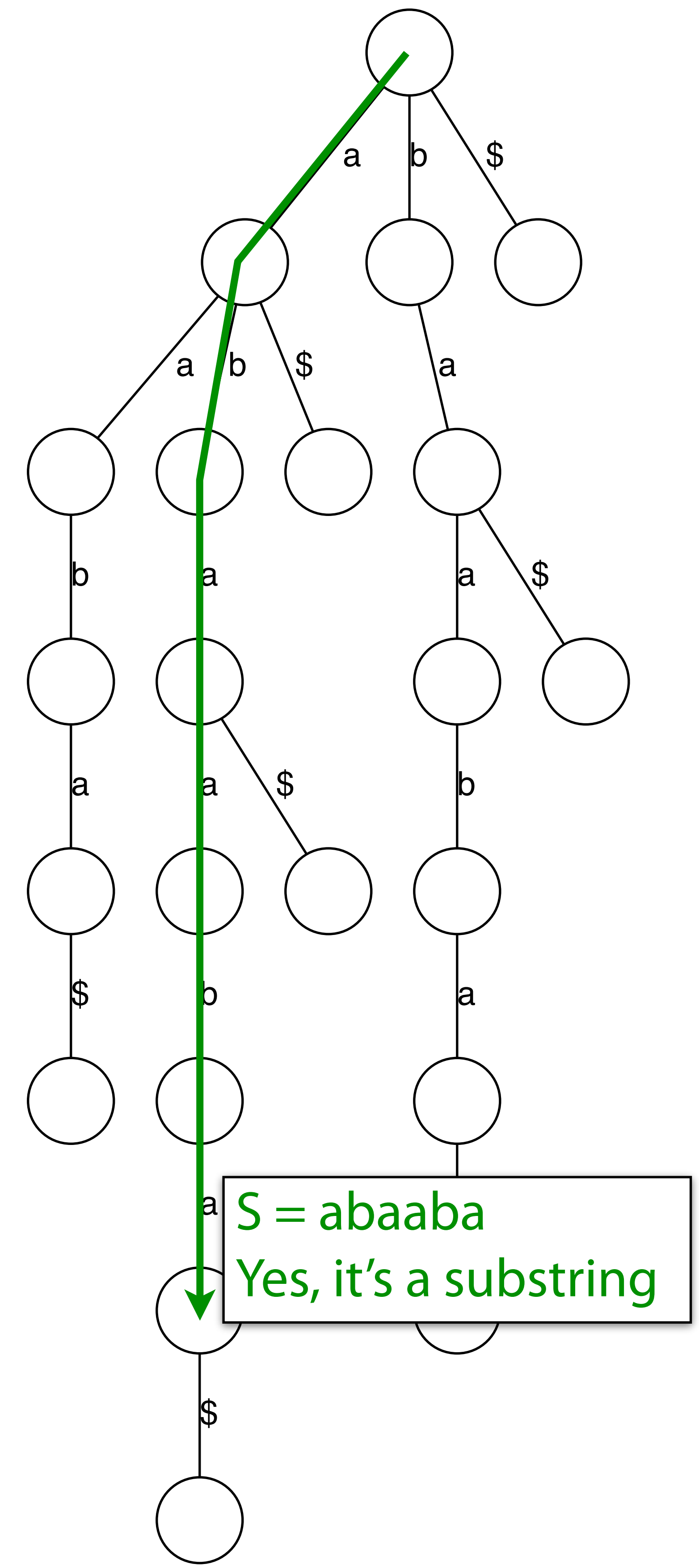

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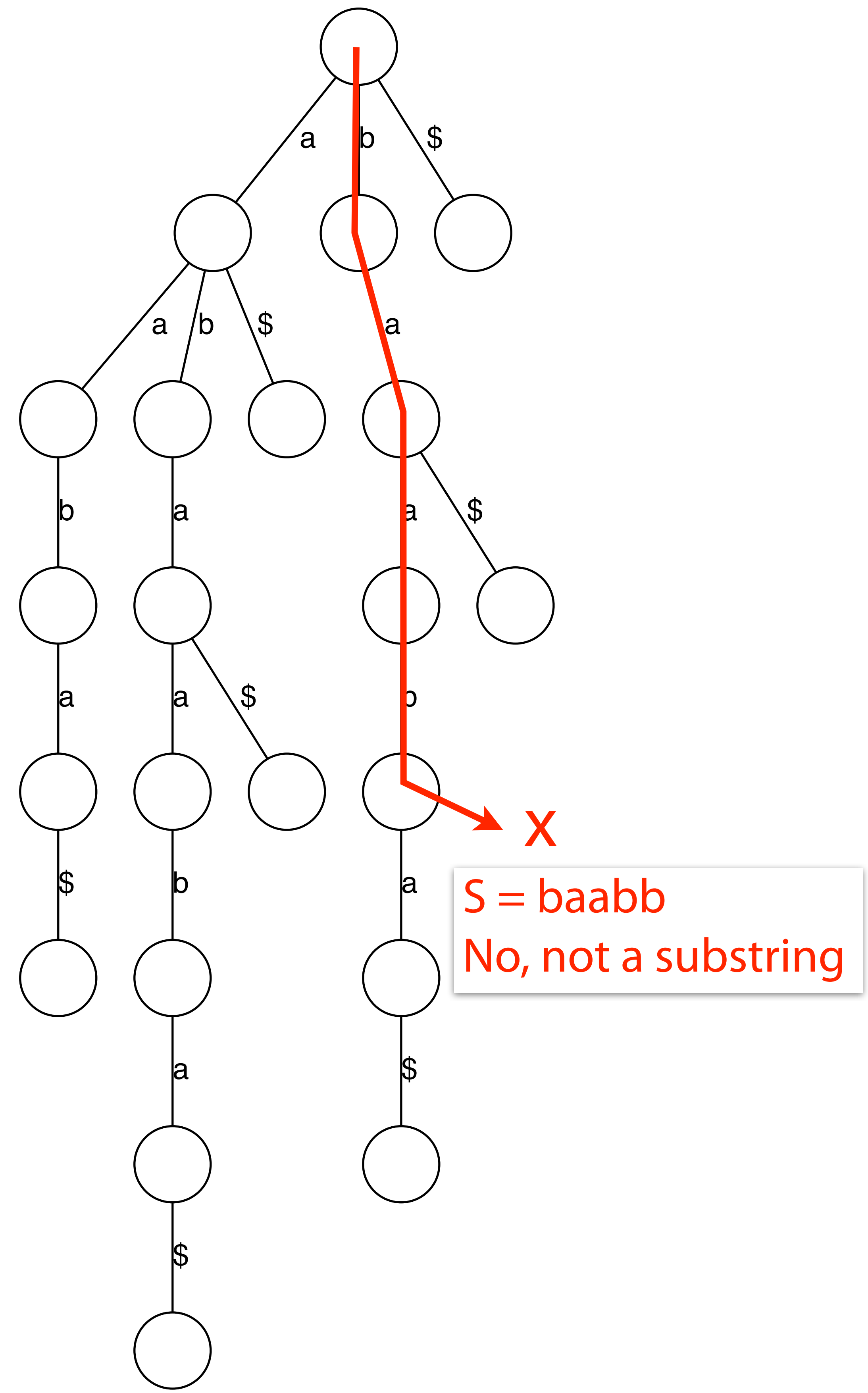
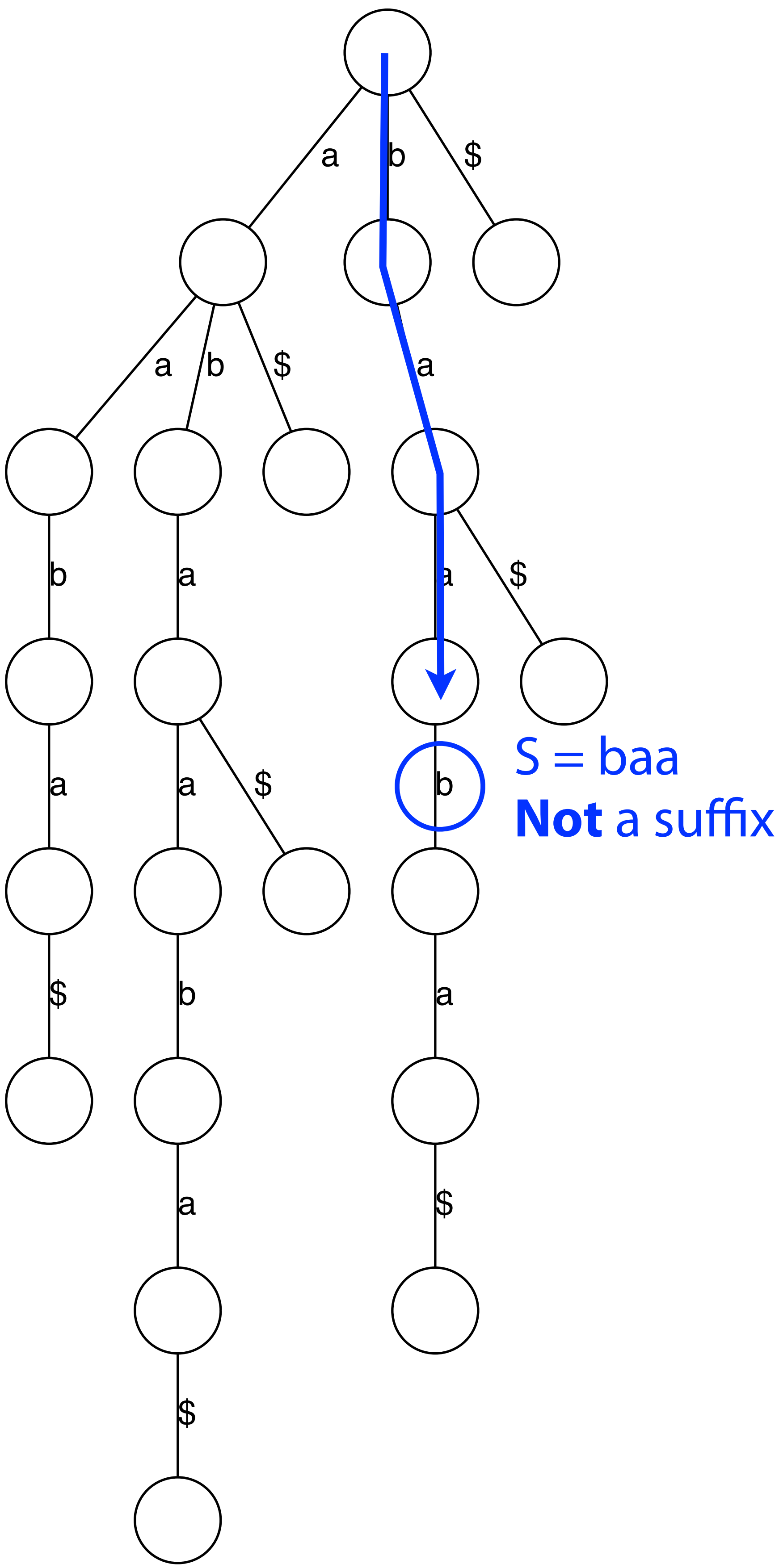


Suffix trie

How do we check whether a string *S* is a **suffix** of *T*?

Same procedure as for substring, but additionally check whether the final node in the walk has an outgoing edge labeled **$**



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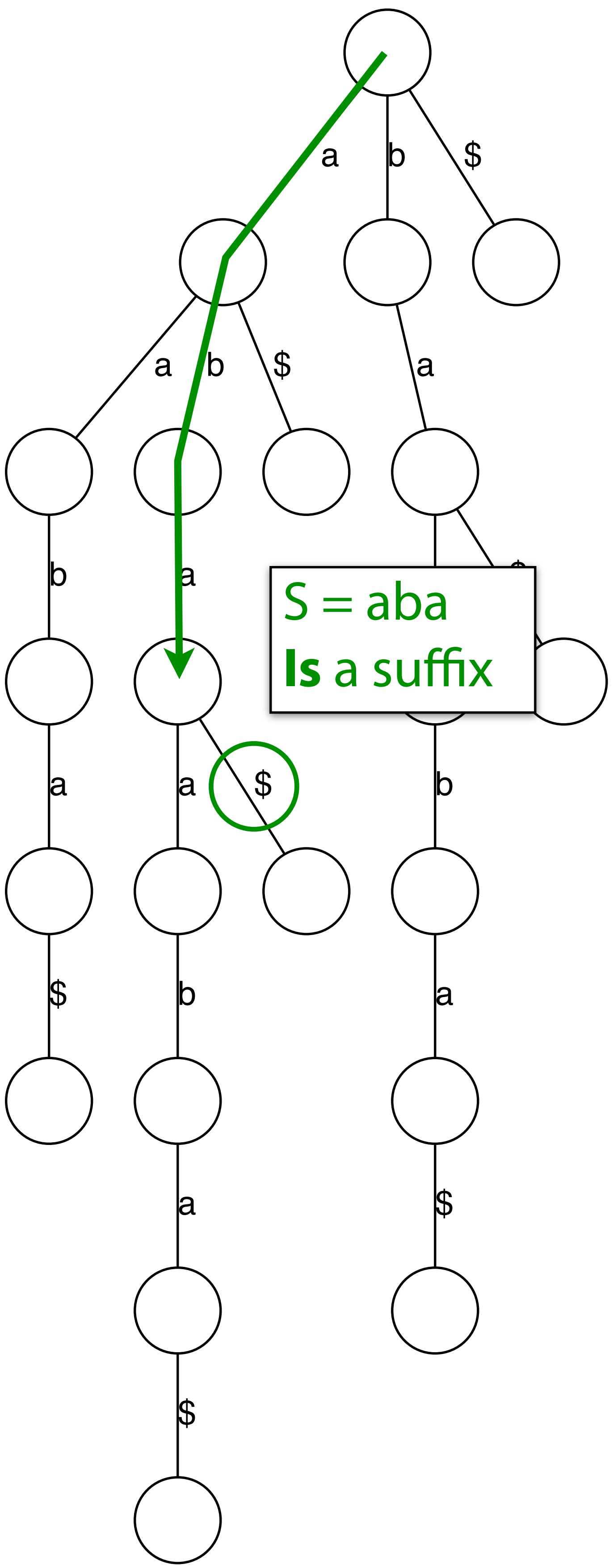
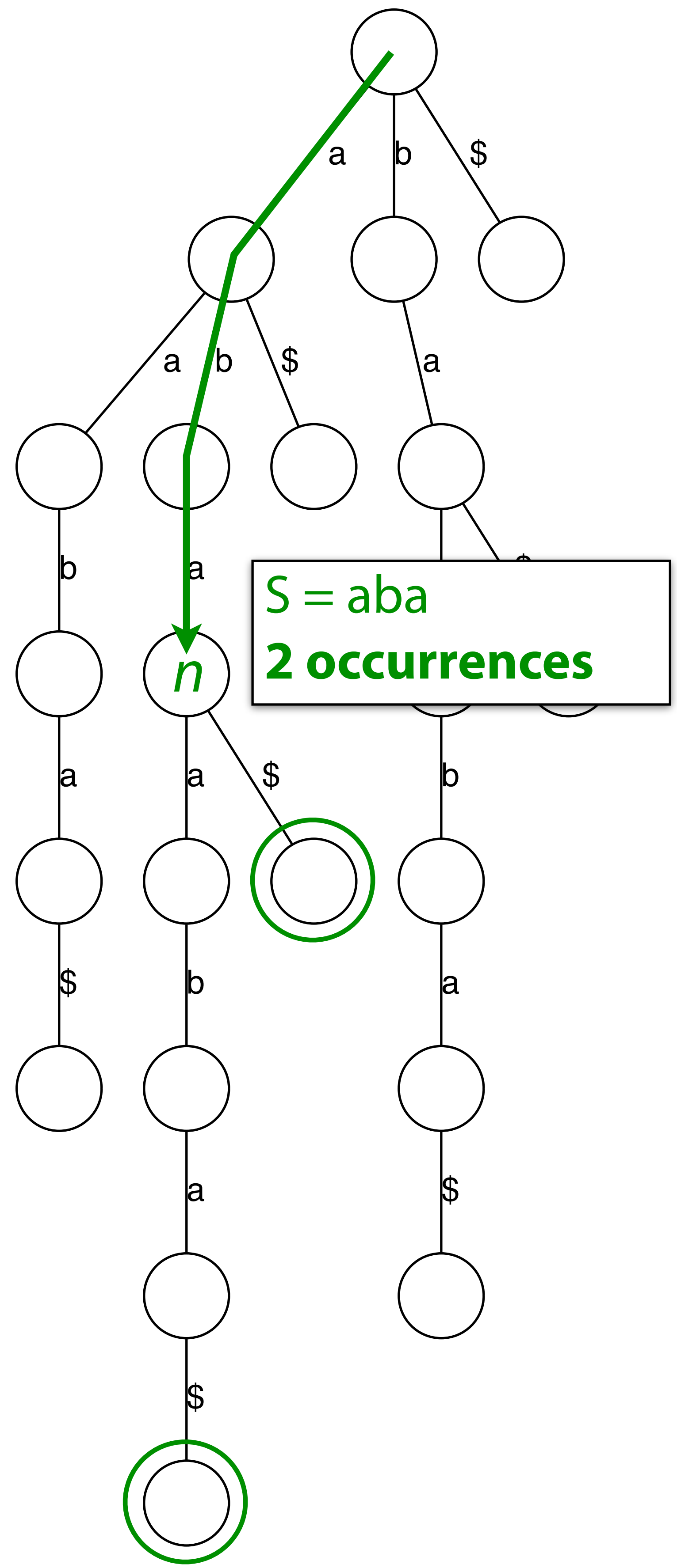


Suffix trie

How do we count the **number of times** a string *S* occurs as a substring of *T*?

Follow path corresponding to *S*. Either we fall off, in which case answer is 0, or we end up at node *n* and the answer = # of leaf nodes in the subtree rooted at *n*.

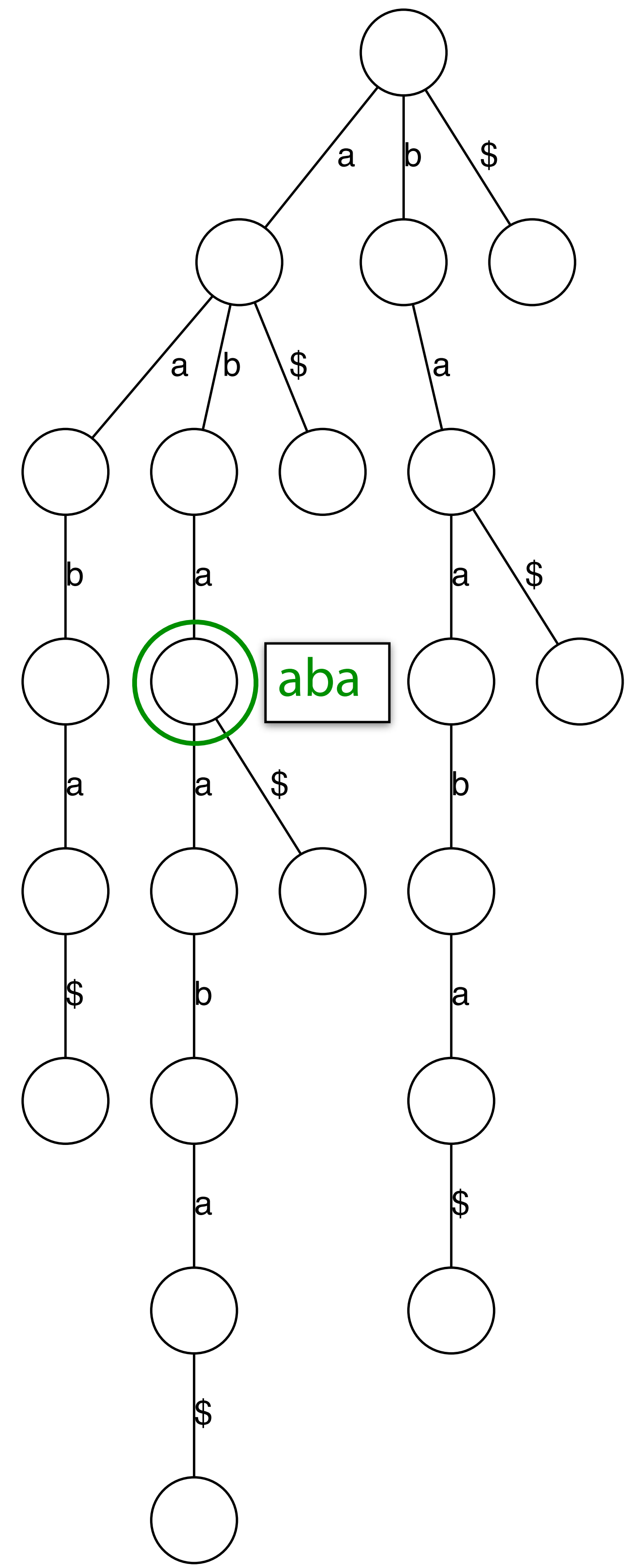
Leaves can be counted with depth-first traversal.



Suffix trie

How do we find the **longest repeated substring** of *T*?

Find the deepest node with more than one child



Suffix trie: implementation

```
class SuffixTrie(object):

    def __init__(self, t):
        """ Make suffix trie from t """
        t += '$' # special terminator symbol
        self.root = {}
        for i in xrange(len(t)): # for each suffix
            cur = self.root
            for c in t[i:]: # for each character in i'th suffix
                if c not in cur:
                    cur[c] = {} # add outgoing edge if necessary
                cur = cur[c]

    def followPath(self, s):
        """ Follow path given by characters of s.  Return node at
            end of path, or None if we fall off. """
        cur = self.root
        for c in s:
            if c not in cur:
                return None
            cur = cur[c]
        return cur

    def hasSubstring(self, s):
        """ Return true iff s appears as a substring of t """
        return self.followPath(s) is not None

    def hasSuffix(self, s):
        """ Return true iff s is a suffix of t """
        node = self.followPath(s)
        return node is not None and '$' in node
```

Python example:
http://nbviewer.ipython.org/6603756

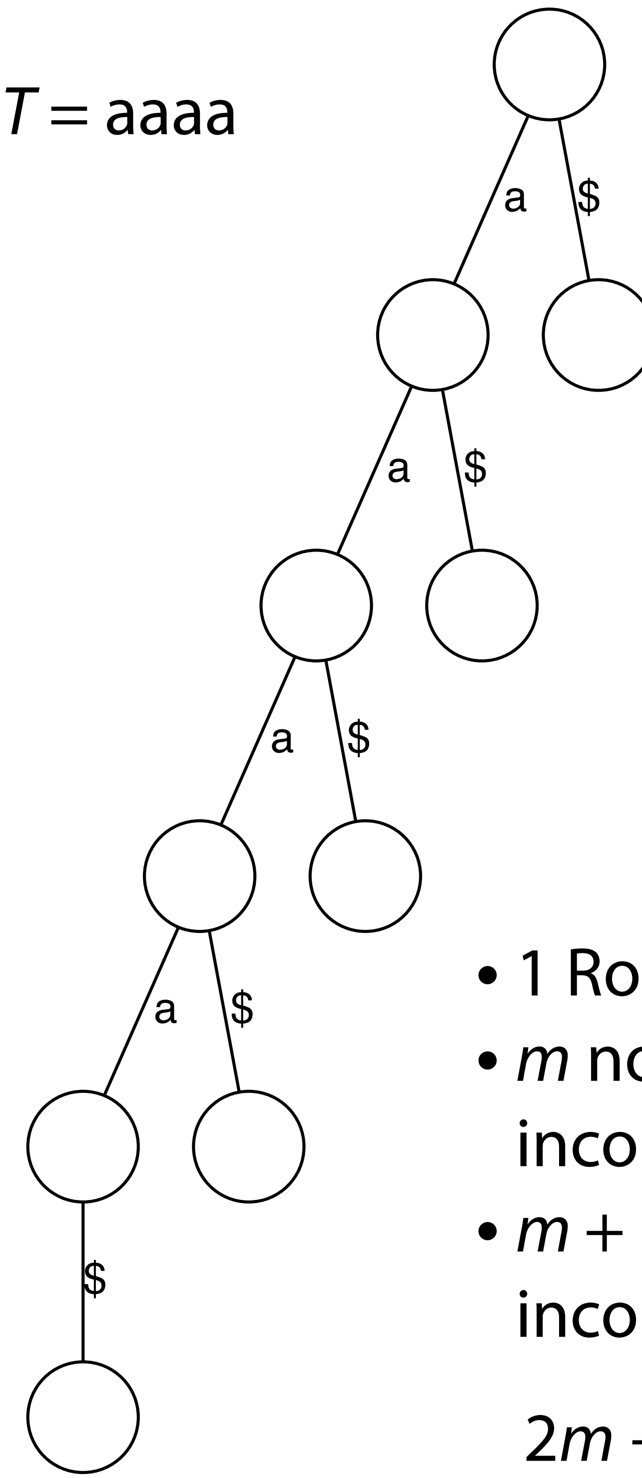
Suffix trie

How many nodes does the suffix trie have?

Is there a class of string where the number of suffix trie nodes grows linearly with m?

Yes: e.g. a string of m a's in a row (a^m)

T = aaaa



- 1 Root
- m nodes with incoming **a** edge
- $m + 1$ nodes with incoming **$** edge

$2m + 2$ nodes

Suffix trie

Is there a class of string where the number of suffix trie nodes grows with m^2?

Yes: $a^n b^n$

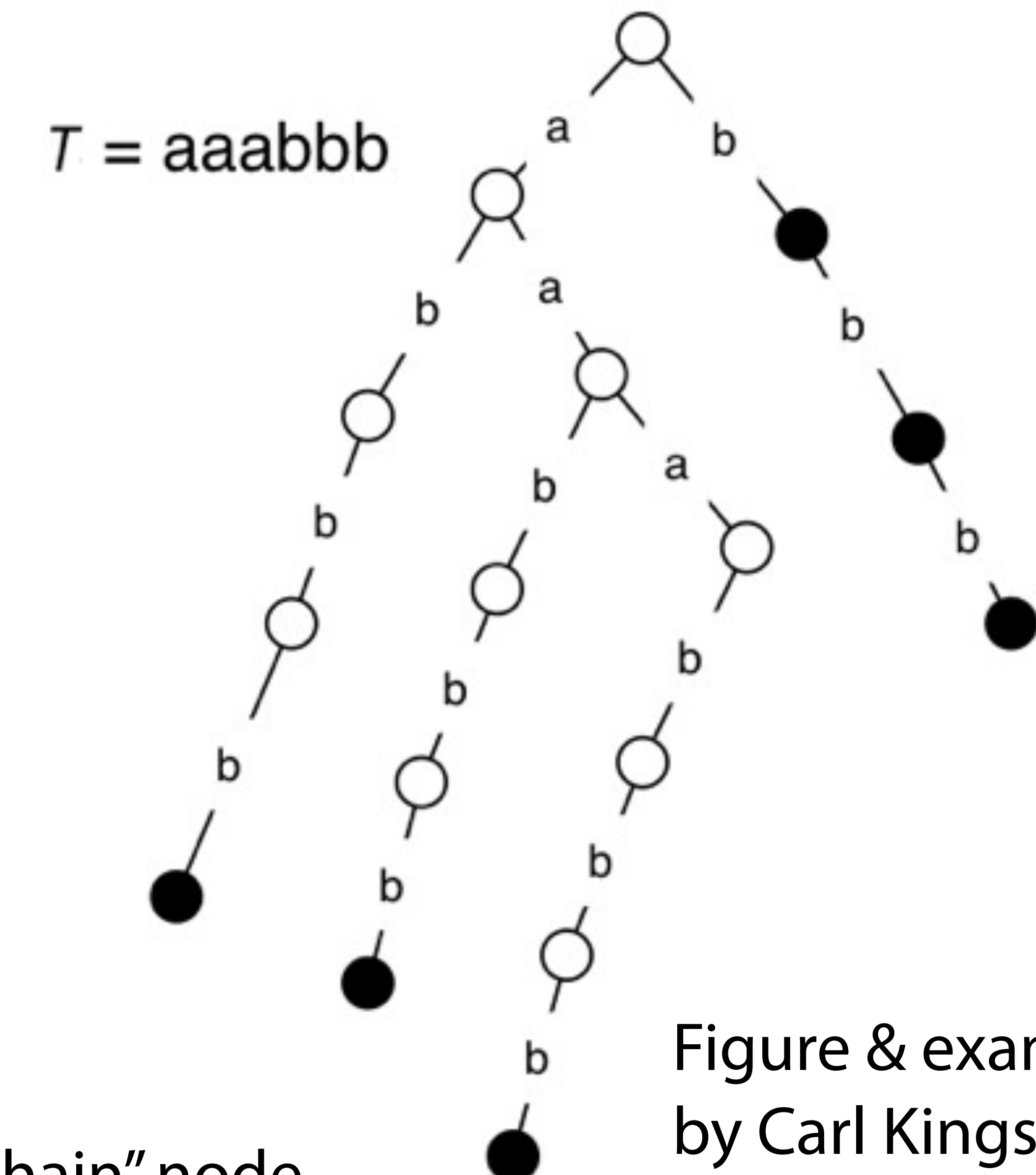


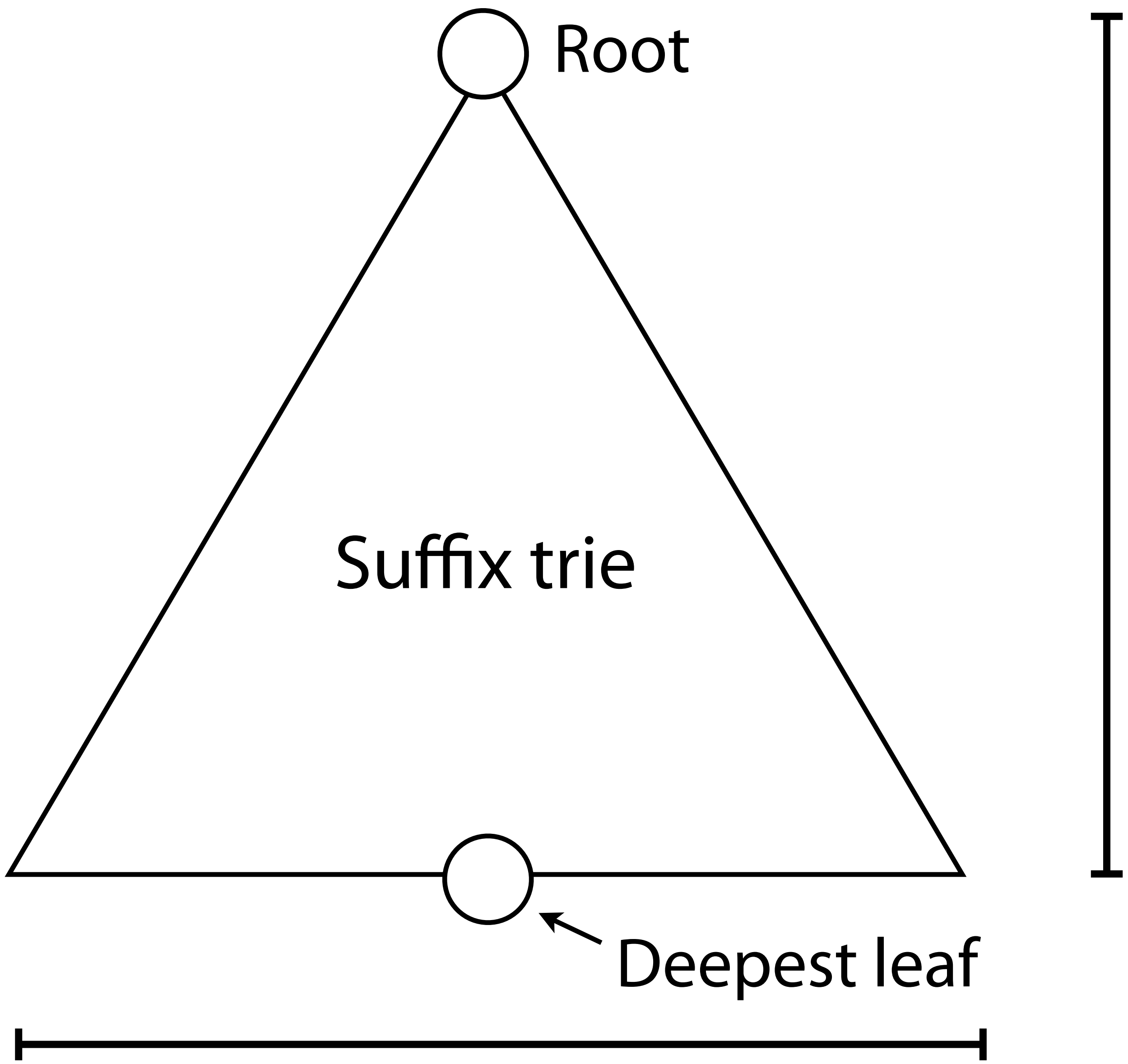
Figure & example by Carl Kingsford

- 1 root
- n nodes along "b chain," right
- n nodes along "a chain," middle
- n chains of n "b" nodes hanging off each "a chain" node
- $2n + 1$ **$** leaves (not shown)

$n^2 + 4n + 2$ nodes, where $m = 2n$

Suffix trie: upper bound on size

Could worst-case # nodes be worse than O(m^2)?



Max # nodes from *top to bottom*
= length of longest suffix + 1
= m + 1

Max # nodes from *left to right*
= max # distinct substrings of any length
≤ m

$O(m^2)$ is worst case

Suffix trie: actual growth

Built suffix tries for the first 500 prefixes of the lambda phage virus genome

Black curve shows how # nodes increases with prefix length