



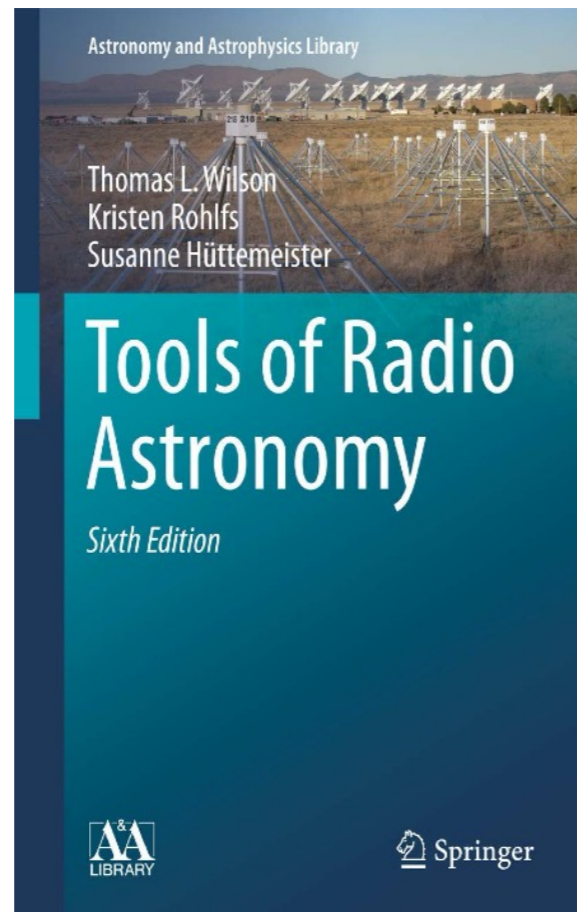
Summary of

Radiative processes relevant to radioastronomy

A solid knowledge of the astrophysics behind radio observables is necessary to achieve a complete understanding of the various phenomena/astrophysical sources that will be investigated in this course.

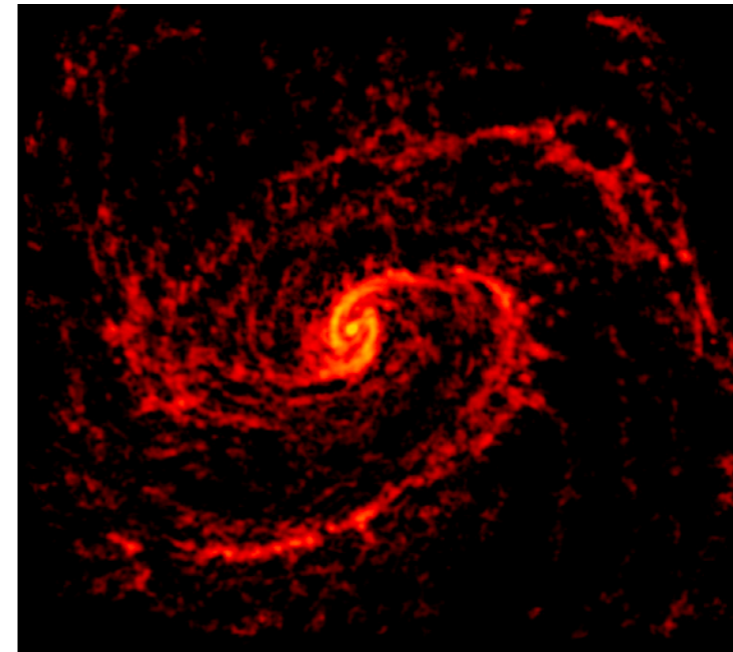
Continuum emission processes

Line emission/absorption processes



Chapters 1, 10, 12, 13, 14, 15, 16

- Black body emission
galaxies, star forming regions
- Synchrotron emission and Inverse Compton
supermassive black-holes, supernovae, clusters of galaxies
- Bremsstrahlung emission
clusters of galaxies, star forming regions



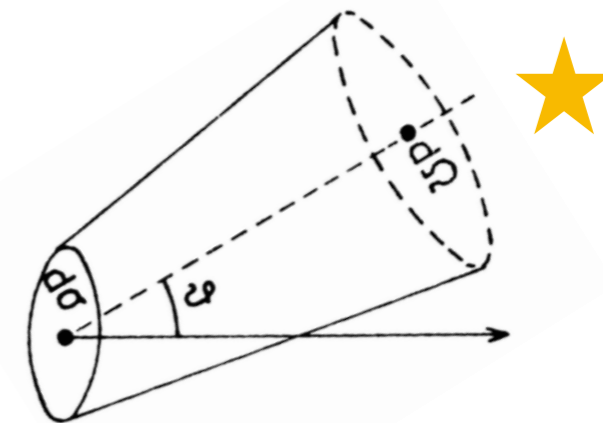


Basic definitions

The infinitesimal power dP intercepted by an infinitesimal surface $d\sigma$ is

$$dP = I_\nu \cos\theta d\Omega d\sigma d\nu$$

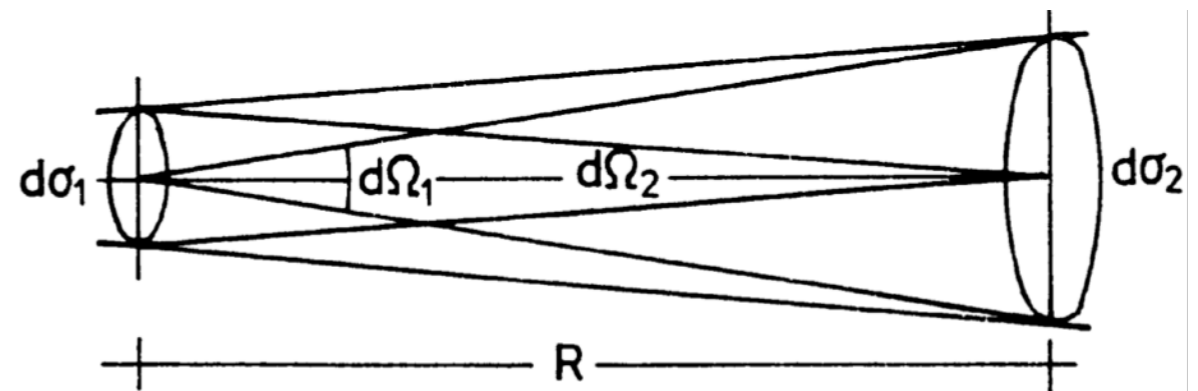
I_ν **brightness or specific intensity** [$\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$]



I_ν is conserved along a ray (in free space)

It is the same at the source and at the detector

$$\left. \begin{aligned} dP_1 &= dP_2 \\ dP_i &= I_{\nu_i} d\sigma_i d\Omega_i d\nu \\ d\Omega_i &= d\sigma_j / R^2 \end{aligned} \right\} I_{\nu_1} = I_{\nu_2}$$



I_ν changes only if radiation is absorbed or emitted, following the equation of transfer

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

ϵ_ν emission coefficient

k_ν absorption coefficient

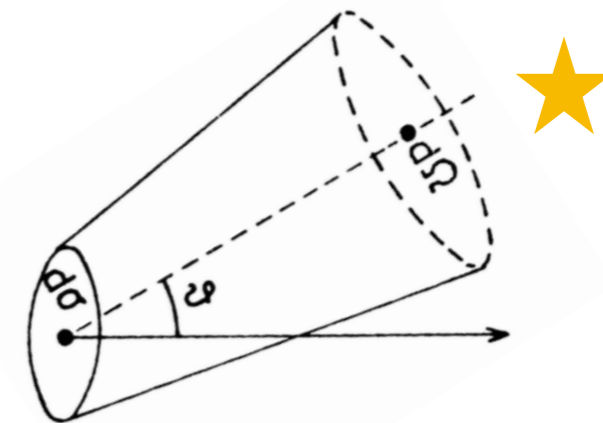


Basic definitions

The infinitesimal power dP intercepted by an infinitesimal surface $d\sigma$ is

$$dP = I_\nu \cos\theta d\Omega d\sigma d\nu$$

I_ν **brightness or specific intensity** [$\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$]



$|I_\nu d\nu| = |I_\lambda d\lambda|$ relation between intensity per unit frequency and intensity per unit wavelength

$$\frac{I_\lambda}{I_\nu} = \left| \frac{d\nu}{d\lambda} \right| = \frac{c}{\lambda^2} = \frac{\nu^2}{c}$$

1 Jansky (Jy) = $10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} = 10^{-23} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$

Flux per unit frequency

↓

$$\text{AB magnitude} = -2.5 \log_{10} \left(\frac{S_\nu}{3631 \text{ Jy}} \right)$$



Basic definitions

$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \phi) \cos\theta d\Omega \quad \text{total flux or flux density} \quad [\text{W m}^{-2} \text{ Hz}^{-1}]$$

Ω_s solid angle subtended by the source

as $\int_{\Omega_s} d\Omega \propto 1/R^2$ where R can be interpreted as the source-detector distance

$$S_\nu \propto R^{-2}$$

flux density depends on the source distance

$$L_\nu = 4\pi R^2 S_\nu$$

source spectral luminosity $[\text{W Hz}^{-1}]$

(for an isotropic source)

Luminosity distance

$$L = \int_0^\infty L_\nu d\nu$$

bolometric luminosity



radio emission can be highly anisotropic



Basic definitions

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

ϵ_ν emission coefficient
 k_ν absorption coefficient

Emission only: $k_\nu = 0$

$$\frac{dI_\nu}{ds} = \epsilon_\nu$$

Absorption only: $\epsilon_\nu = 0$

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu$$

Thermodynamic equilibrium

$$\frac{dI_\nu}{ds} = 0 \quad I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Local thermodynamic equilibrium

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T)$$

$d\tau_\nu = -k_\nu ds$ optical depth

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + B_\nu(T)(1 - e^{-\tau_\nu(s)})$$

$\epsilon_\nu/k_\nu = B_\nu(T)$ Kirkoff's law

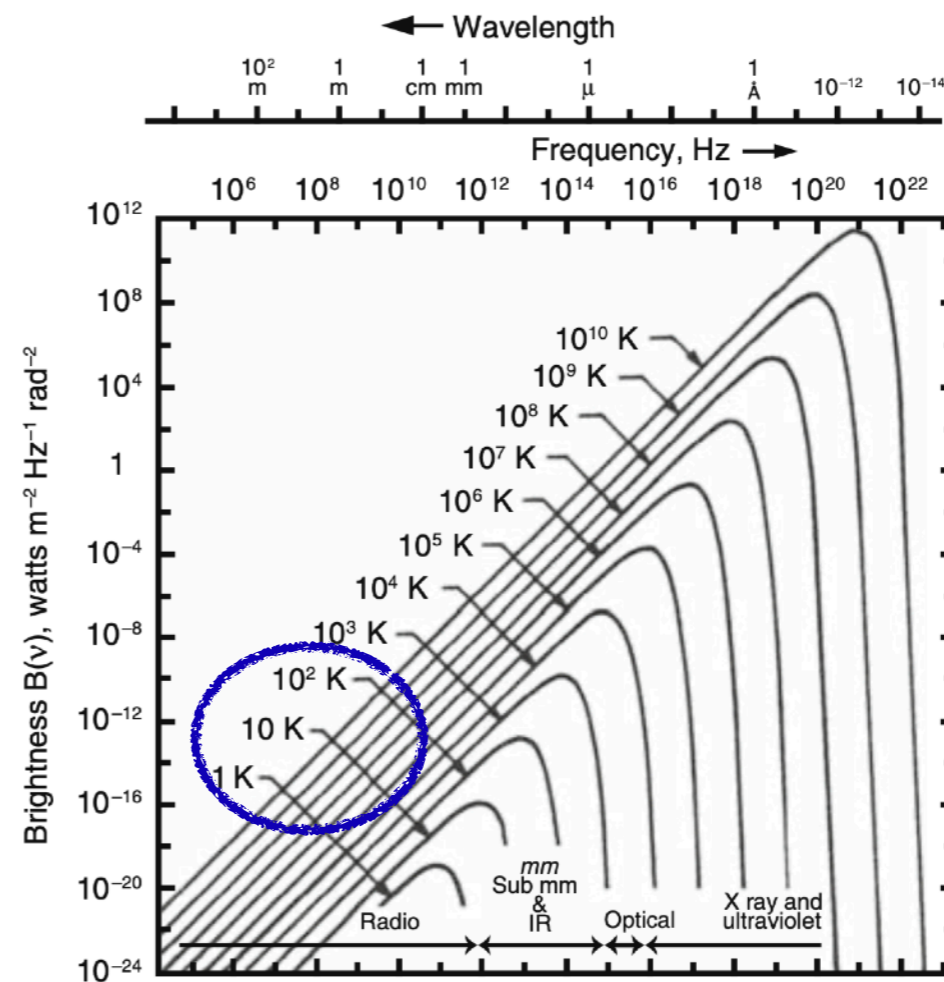


Black body radiation and Brightness Temperature

The spectral distribution for a black body in thermodynamic equilibrium is (previous slide):

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$\frac{\nu_{\max}}{\text{GHz}} = 58.789 \left(\frac{T}{\text{K}} \right)$$



Radio astronomy mostly probes BB radiation in the [Rayleigh-Jeans regime](#):

$$B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2} kT \quad h\nu \ll kT$$

This implies that brightness and temperature are directly proportional: radio astronomers often measure the brightness of an extended source by its Rayleigh-Jeans [Brightness Temperature](#):

$$T_b(\nu) = \frac{c^2}{2k\nu^2} I_\nu \quad \text{even if } I_\nu \neq B_\nu \text{ an "equivalent temperature" can be derived}$$



Black body radiation and Brightness Temperature

At radio frequencies higher than $\nu \sim 1$ GHz, absorption by the Earth's atmosphere may be large enough to affect the accuracy of flux-density measurements and atmospheric emission can increase noise errors. The observed output from a radio telescope can vary with the zenith angle

The amount of atmospheric absorption can be determined by measuring the **amount of atmospheric emission as a function of zenith angle**, to calculate the zenith opacity of the (roughly isothermal) atmosphere ($T_{\text{atm}} \sim 300$ K) The celestial sky above the atmosphere is much colder ($T \sim 3$ K) so the "background" emission above the atmosphere can usually be ignored.

Considering that the atmosphere is in LTE and has a temperature T_{atm} the radiative transfer equation can be written as

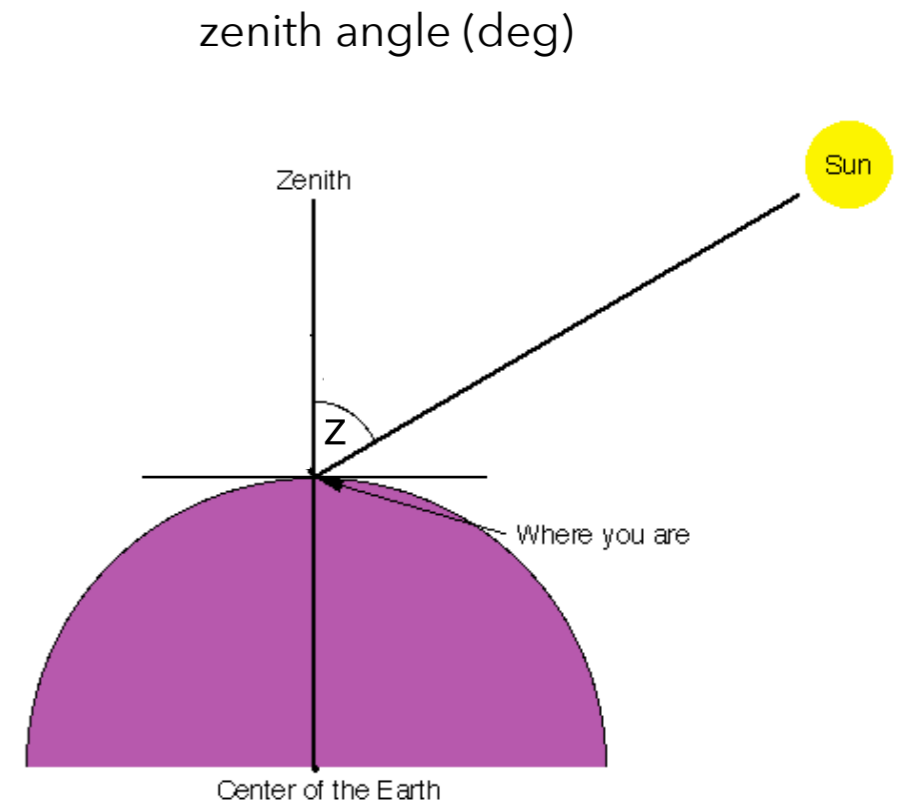
$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T_{\text{atm}})$$

By multiplying both sides for $e^{-\tau}$ and integrating by parts from the top of the atmosphere ($\tau = \tau_A$) to the ground ($\tau = 0$)

$$I_\nu(\tau = \tau_A)e^{-\tau_A} - I_\nu(\tau = 0) = B_\nu(T_{\text{atm}})(e^{-\tau_A} - 1)$$

emission above the atmosphere is small

τ_A optical depth of the atmosphere at a given z





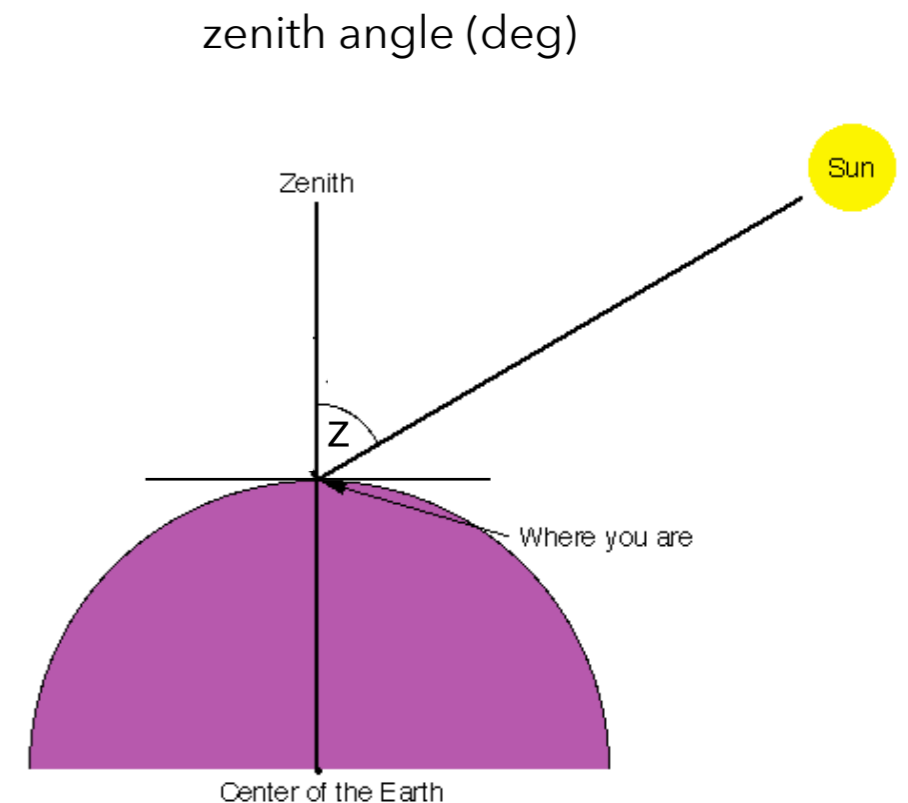
Black body radiation and Brightness Temperature

$$I_\nu(\tau = 0) = B_\nu(T_{\text{atm}})(1 - e^{-\tau_A})$$

The path length through a plane-parallel atmosphere is proportional to $\sec z$ so at any zenith angle z

$$\tau_A \sim \tau_z \sec z$$

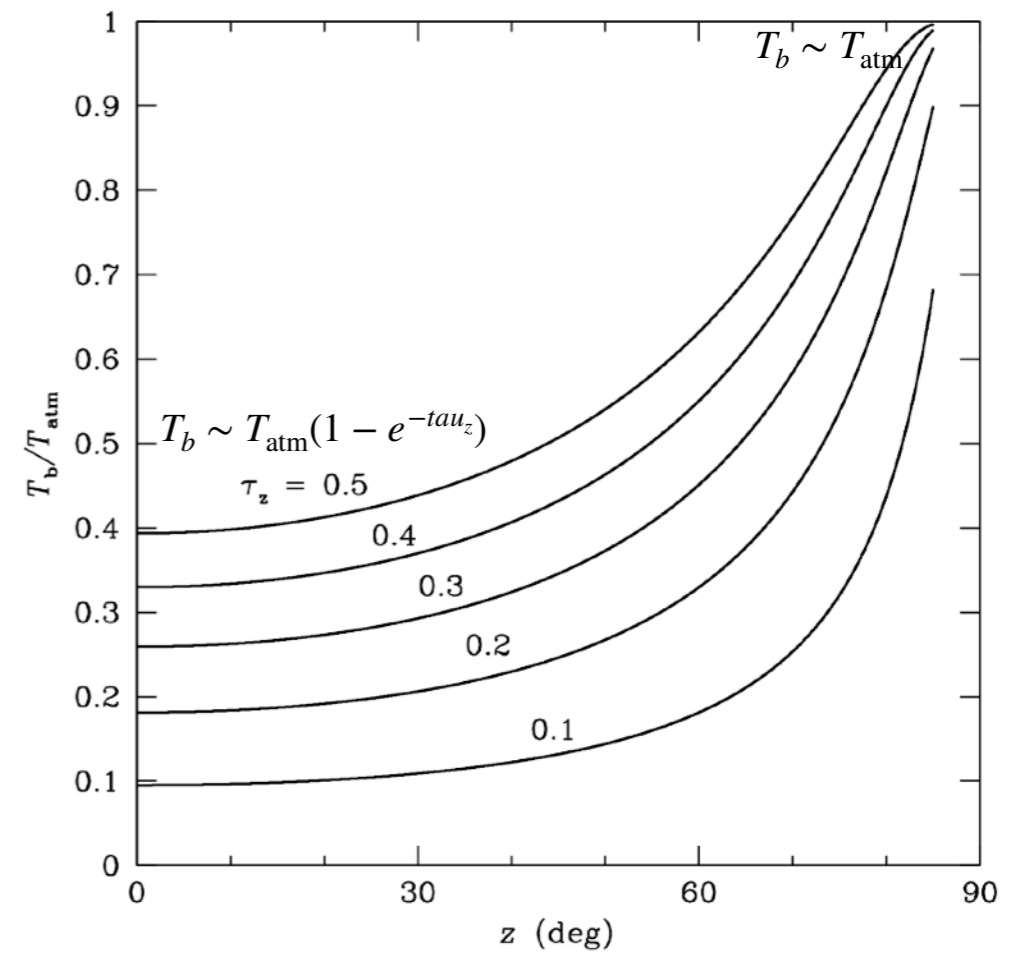
$$\tau_A(z = 0) = \tau_z \quad \text{optical depth at the zenith}$$



$$I_\nu = [1 - \exp(-\tau_z \sec z)] \frac{2kT_{\text{atm}}\nu^2}{c^2}$$

$$T_b = T_{\text{atm}}[1 - \exp(-\tau_z \sec z)]$$

Brightness temperature of the atmosphere as a function of zenith angle





Black body radiation and Brightness Temperature

$$T_b = T_{\text{atm}}[1 - \exp(-\tau_z \sec z)]$$

Brightness temperature of the atmosphere as a function of zenith angle

$$T_{\text{telescope}} \sim T_{\text{source}} + T_{\text{atm}}[1 - \exp(-\tau_z \sec z)]$$

Observations of sources with high z (low elevation) are noisier!

If the zenith opacity is low (radio frequencies)

$$T_b \sim T_{\text{atm}} \tau_z \sec z$$

linear relation with slope τ_z

$$T_{\text{telescope}} \sim T_{\text{source}} + T_{\text{atm}} \tau_z \sec z$$

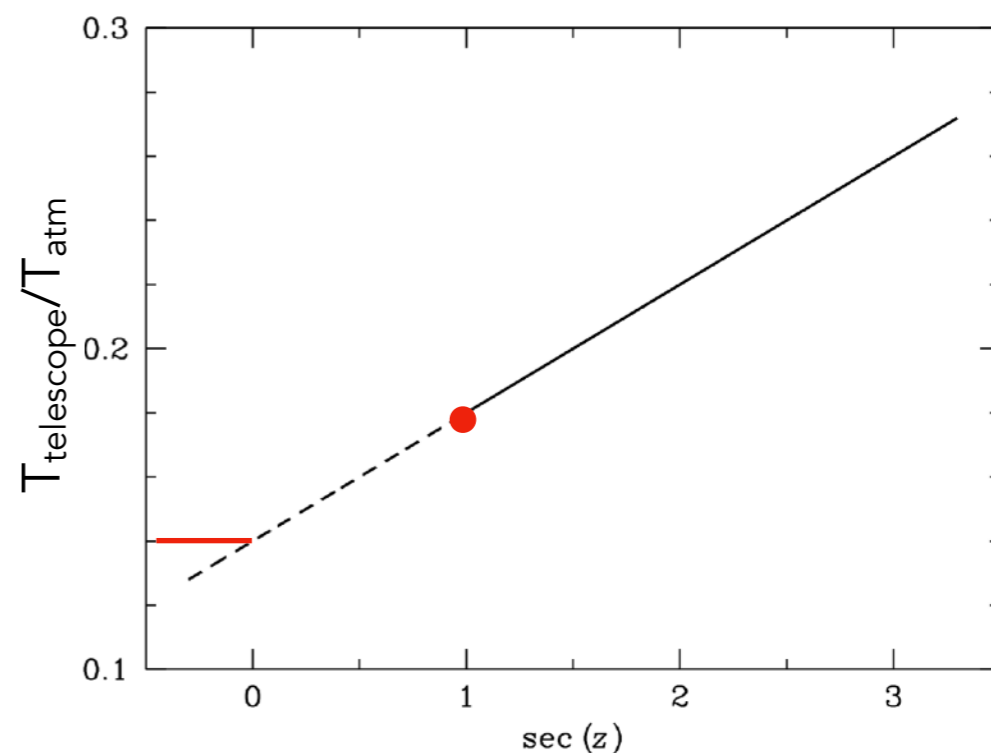
Extrapolating to $\sec z = 0$

$$T_{\text{source}}/T_{\text{atm}} \sim \tau_z$$

$$y = \text{cost} + \tau_z x$$

Extrapolating to $\sec z = 0$

$$y(x = 0) = \text{cost} = T_{\text{source}}/T_{\text{atm}}$$



This is essentially how Penzias and Wilson measured the CMB temperature, corrected for atmospheric emission ($T_{\text{CMB}} \sim 3\text{K}$ at $\nu \sim 4\text{ GHz}$)



Bremsstrahlung radiation (or free-free)

Electromagnetic bremsstrahlung radiation is emitted by an accelerating (or decelerating) charged particle due to an electrostatic force. Thermal emission is produced if the emitting electrons are in LTE (non-thermal if electrons have a power-law energy distribution).



**HII regions
(e.g. Orion nebula)**



(Helix nebula)

The most massive ($M > 15 M_{\odot}$) main sequence stars are big enough ($R \sim 10 R_{\odot}$) and hot enough ($T \sim 30000$ K) to be very luminous of ionizing UV radiation. White dwarfs are compact but hot enough ($T \sim 10^5$ K) to ionize the material expelled during the red-giant phase (planetary nebulae)



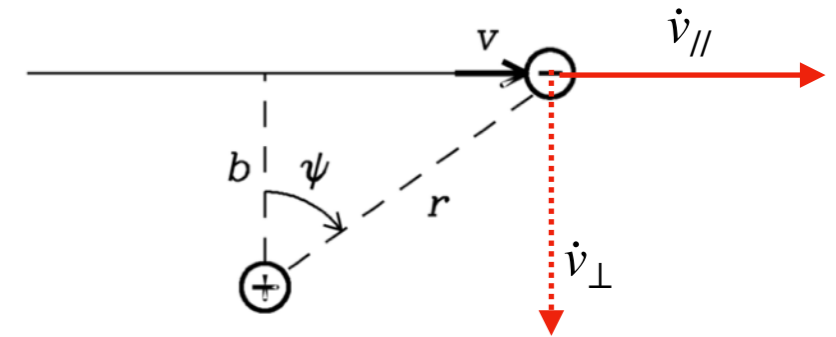
Bremsstrahlung radiation (or free-free)

Electromagnetic bremsstrahlung radiation is emitted by an accelerating (or decelerating) charged particle due to an electrostatic force. Thermal emission is produced if the emitting electrons are in LTE (non-thermal if electrons have a power-law energy distribution).

In HII regions, a free electron passing by an ion undergoes an acceleration and emits according to the Larmor formula. The electron is free both before and after the interaction. For reasonable densities of the ionized clouds, electron and photons are in LTE.

$$P = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3} = \frac{2e^2}{3c^3} \left(\frac{Ze^2}{m_e r^2} \right)^2 r \text{ ion-electron distance}$$

Coulomb acceleration $\dot{v} \propto \frac{1}{m}$



The total power emitted is given by the Larmor's equation (non-relativistic) and is proportional to the square of the acceleration. As $(m_e/m_p)^2 \sim 10^{-6}$, emission from ions can be neglected

Electron-electron encounters can be also ignored (\sim no net electromagnetic field at distances $\gg b$)



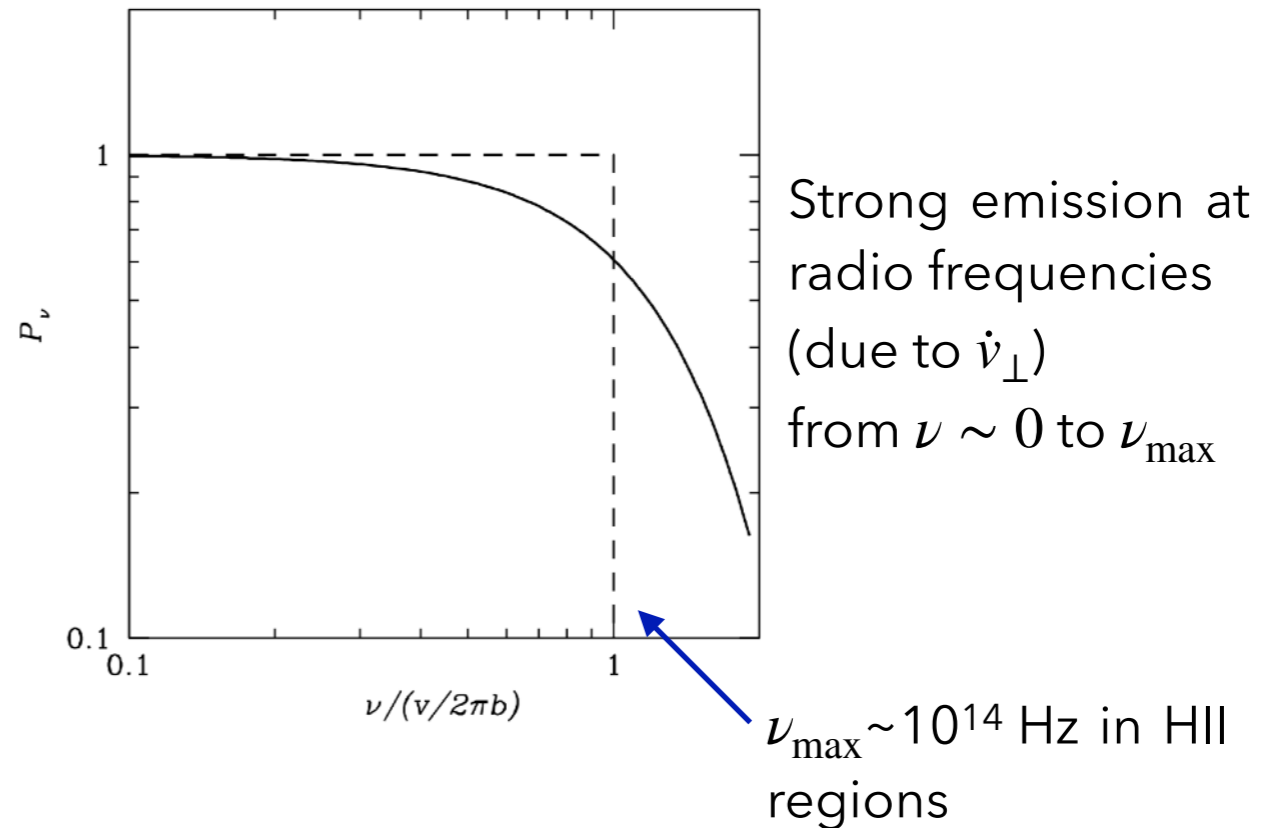
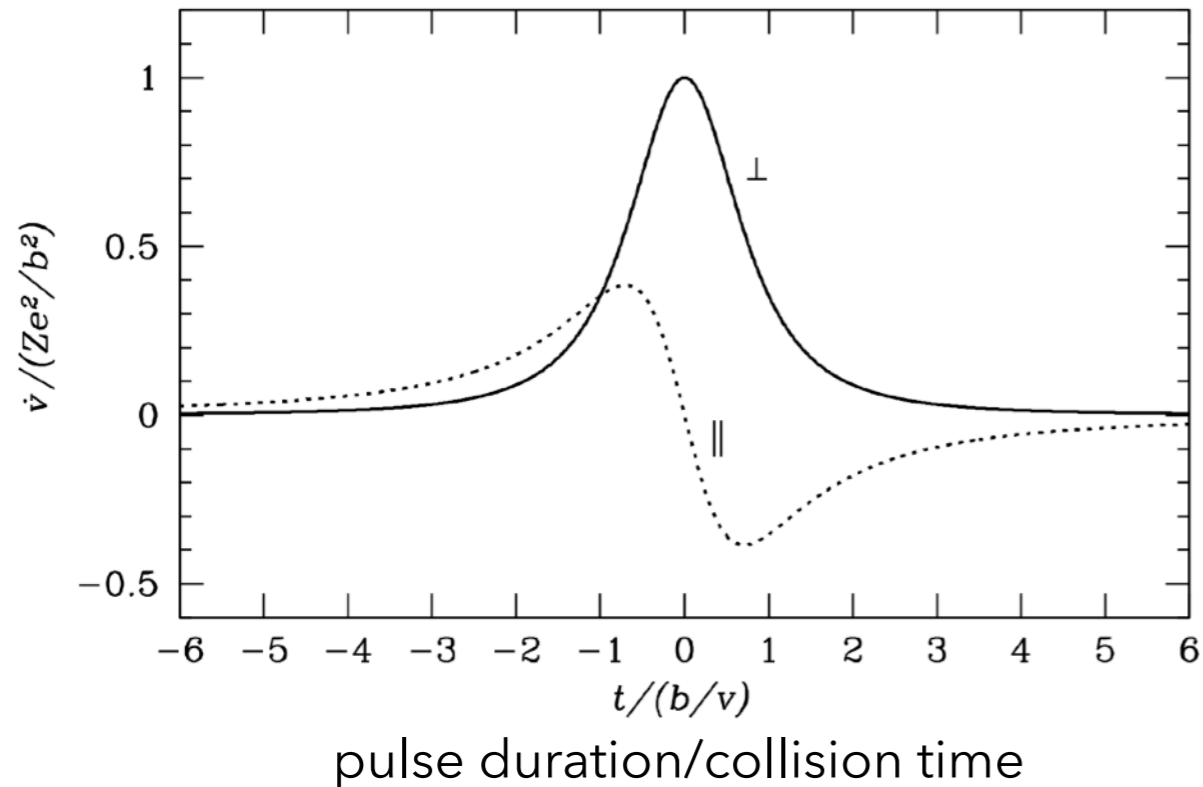
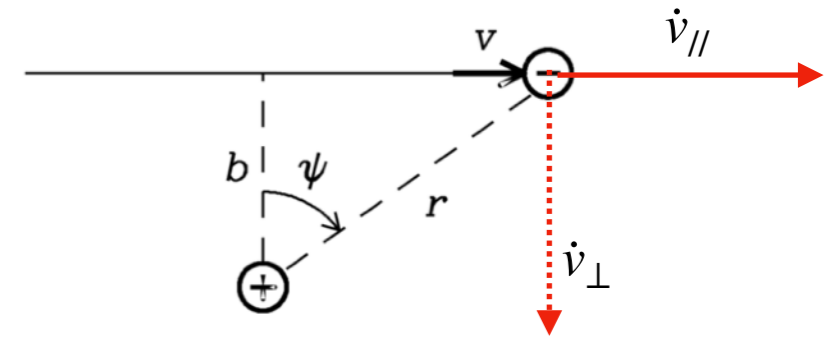
Bremsstrahlung radiation (or free-free)

Electromagnetic bremsstrahlung radiation is emitted by an accelerating (or decelerating) charged particle due to an electrostatic force. Thermal emission is produced if the emitting electrons are in LTE (non-thermal if electrons have a power-law energy distribution).

In HII regions, a free electron passing by an ion undergoes an acceleration and emits according to the Larmor formula. The electron is free both before and after the interaction. For reasonable densities of the ionized clouds, electron and photons are in LTE.

$$P = \frac{2}{3} \frac{e^2 \dot{v}^2}{c^3} = \frac{2e^2}{3c^3} \left(\frac{Ze^2}{m_e r^2} \right)^2 r$$

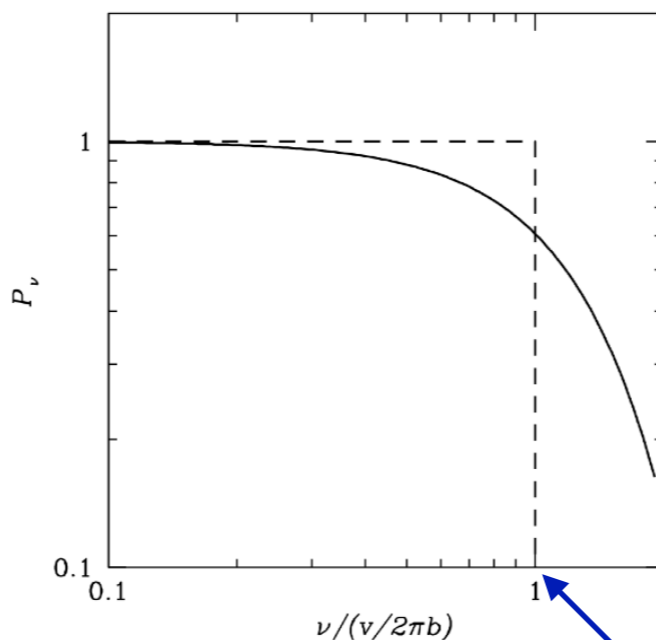
Coulomb acceleration $\dot{v} \propto \frac{1}{m}$





Bremsstrahlung radiation (or free-free)

What we have seen so far is valid for optically thin emission



Strong emission at radio frequencies (due to \dot{v}_\perp) from $\nu \sim 0$ to ν_{\max}

$\nu_{\max} \sim 10^{14}$ Hz in HII regions

Optical depth of an HII region

$$\tau = - \int_{\text{los}} k ds \propto \int \frac{n_e^2}{\nu^{2.1} T^{3/2}} ds$$

Rayleigh-Jeans + small dependence of k on the maximum impact parameter (and, in turn, on ν)

For frequencies low enough that $\tau \gg 1$, the HII region becomes opaque and its spectrum approaches a blackbody in the Rayleigh-Jeans regime

