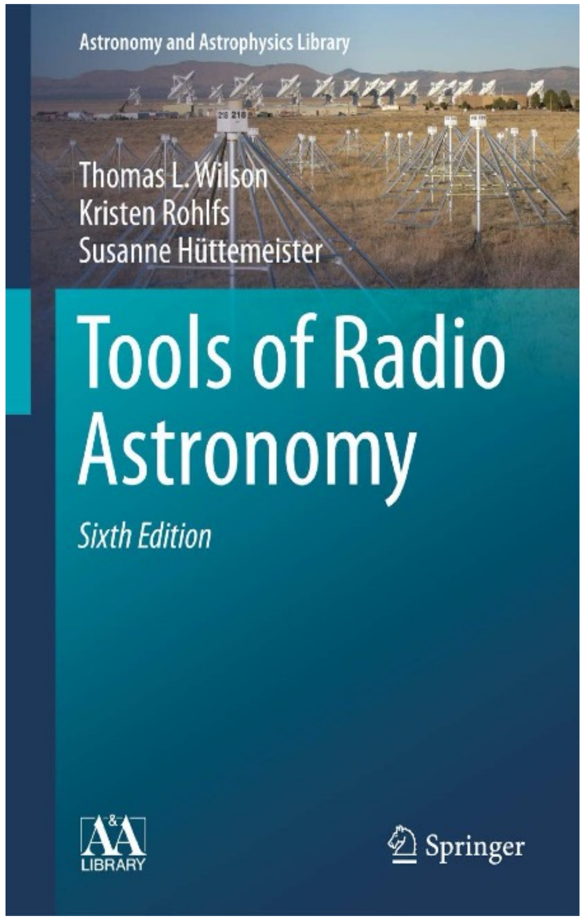
Summary of

Radiative processes relevant to radioastronomy

A solid knowledge of the astrophysics behind radio observables is necessary to achieve a complete understanding of the various phenomena/astrophysical sources that will be investigated in this course.

Continuum emission processes

Line emission/absorption processes



Chapters 1,10, 12, 13, 14, 15, 16

Continuum emission processes

- Black body emission

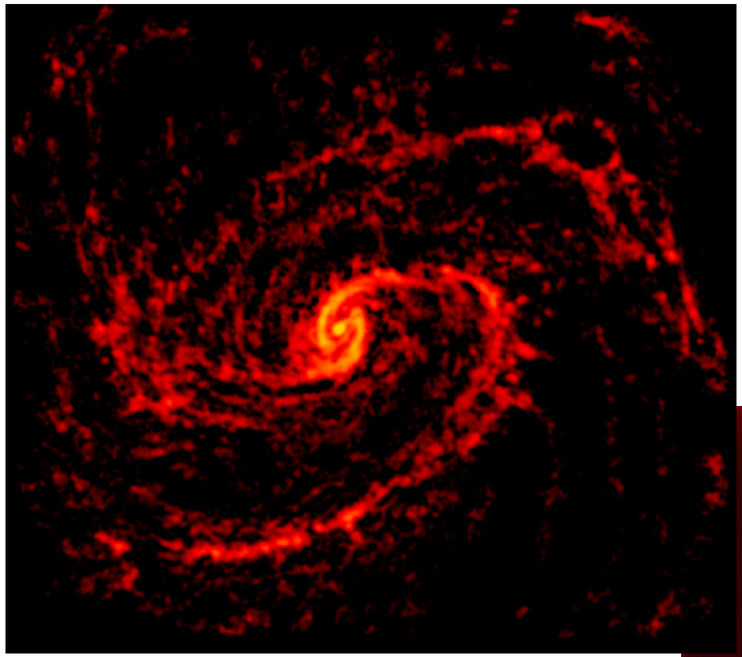
  galaxies, star forming regions

- Synchrotron emission and Inverse Compton

  supermassive black-holes, supernovae, clusters of galaxies

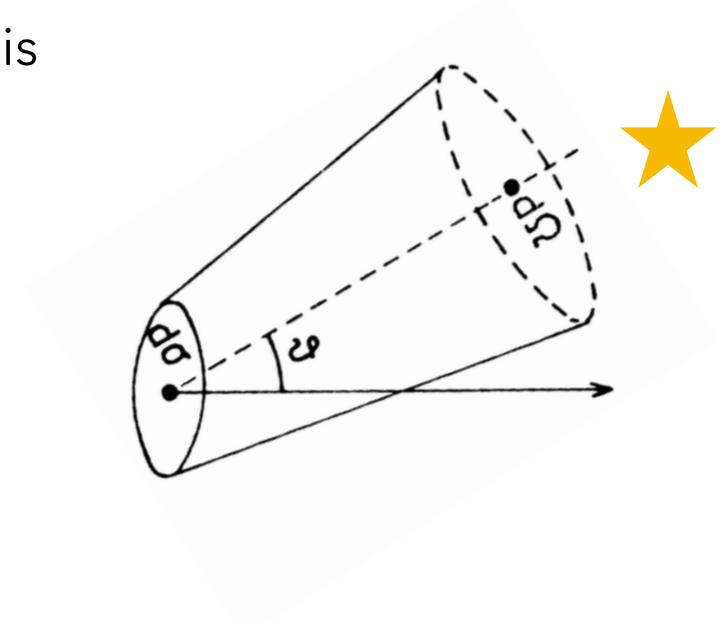
- Bremsstrahlung emission

  clusters of galaxies, star forming regions



Basic definitions

The infinitesimal power dP intercepted by an infinitesimal surface dσ is



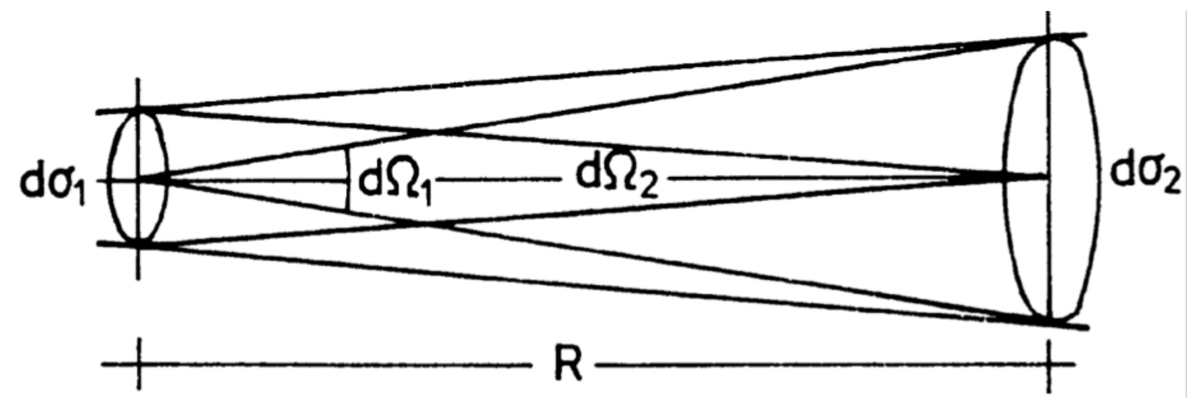
$$dP = I_\nu cos\theta d\Omega d\sigma d\nu$$

I_ν **brightness or specific intensity** [W m^{-2} Hz^{-1} sr^{-1}]

I_ν is conserved along a ray (in free space)

It is the same at the source and at the detector

$$\left.\begin{array}{l} dP_1 = dP_2 \\ dP_i = I_{\nu_i} d\sigma_i d\Omega_i d\nu \\ d\Omega_i = d\sigma_j / R^2 \end{array}\right\} \quad \boxed{I_{\nu_1} = I_{\nu_2}}$$



I_ν changes only if radiation is absorbed or emitted, following the equation of transfer

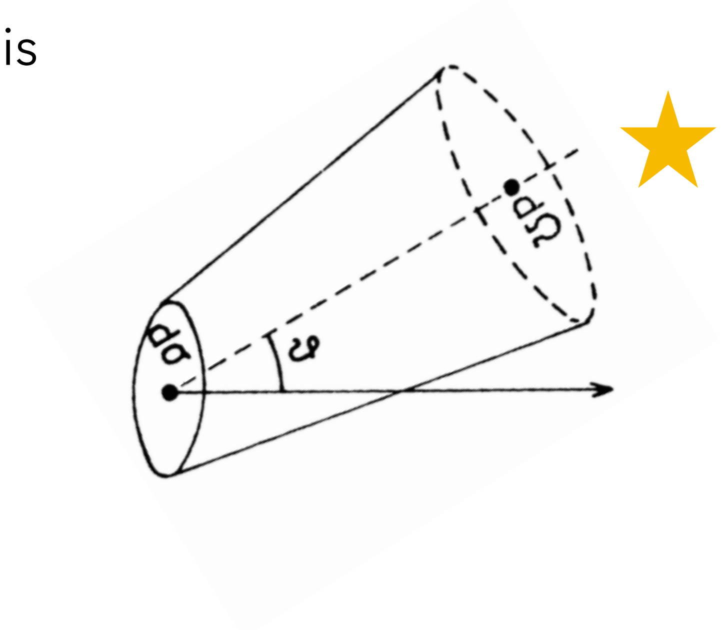
$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

ϵ_ν emission coefficient

k_ν absorption coefficient

Basic definitions

The infinitesimal power dP intercepted by an infinitesimal surface dσ is



$$dP = I_\nu cos\theta d\Omega d\sigma d\nu$$

I_ν **brightness or specific intensity** [W m^{-2} Hz^{-1} sr^{-1}]

$|I_\nu d\nu| = |I_\lambda d\lambda|$ relation between intensity per unit frequency and intensity per unit wavelength

$$\boxed{\frac{I_\lambda}{I_\nu} = |\frac{d\nu}{d\lambda}| = \frac{c}{\lambda^2} = \frac{\nu^2}{c}}$$

Flux per unit frequency

1 Jansky (Jy) = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

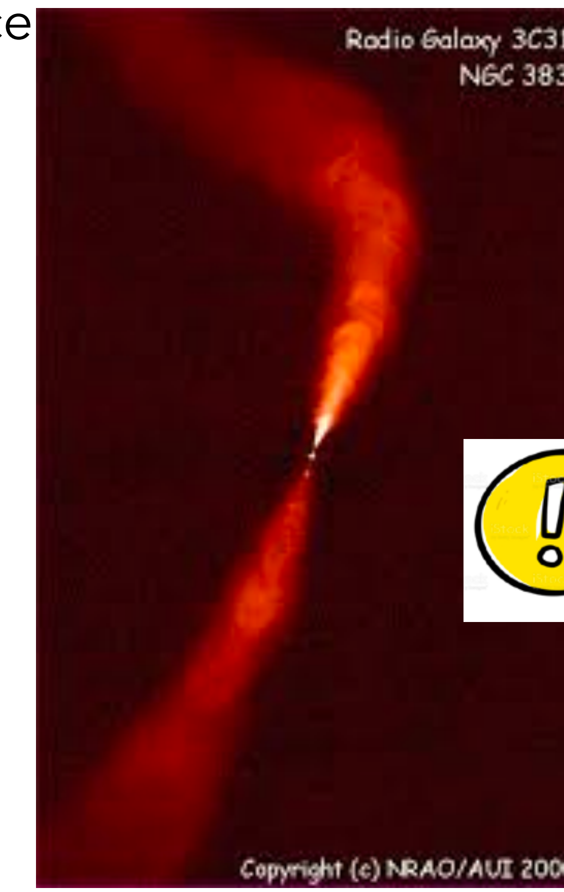
AB magnitude = $-2.5log_{10}\left(\frac{\boxed{S_\nu}}{3631\mathrm{Jy}}\right)$

Basic definitions

$$S_\nu = \int_{\Omega_s} I_\nu(\theta,\phi) cos\theta d\Omega$$ **total flux or flux density** [W m^{-2} Hz^{-1}]

Ω_s solid angle subtended by the source

as $\int_{\Omega_s} d\Omega \propto 1/\mathrm{R}^2$ where R can be interpreted as the source-detector distance



$$\boxed{S_\nu \propto \mathrm{R}^{-2}}$$ **flux density depends on the source distance**

$$\boxed{L_\nu = 4\pi \mathrm{R}^2 S_\nu}$$ **source spectral luminosity** [W Hz^{-1}]

Luminosity distance

(for an isotropic source)

$$\boxed{L = \int_0^\infty L_\nu d\nu}$$ **bolometric luminosity**

radio emission can be highly anisotropic

Basic definitions

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu + \epsilon_\nu$$

ϵ_ν emission coefficient

k_ν absorption coefficient

Emission only: $k_\nu = 0$

$$\frac{dI_\nu}{ds} = \epsilon_\nu$$

Absorption only: $\epsilon_\nu = 0$

$$\frac{dI_\nu}{ds} = -k_\nu I_\nu$$

Thermodynamic equilibrium

$$\frac{dI_\nu}{ds} = 0 \qquad I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

Local thermodynamic equilibrium

$$\frac{dI_\nu}{d\tau_\nu} = I_\nu - B_\nu(T)$$

$$I_\nu(s) = I_\nu(0)e^{-\tau_\nu(s)} + B_\nu(T)(1 - e^{-\tau_\nu(s)})$$

$d\tau_\nu = -k_\nu ds$ optical depth

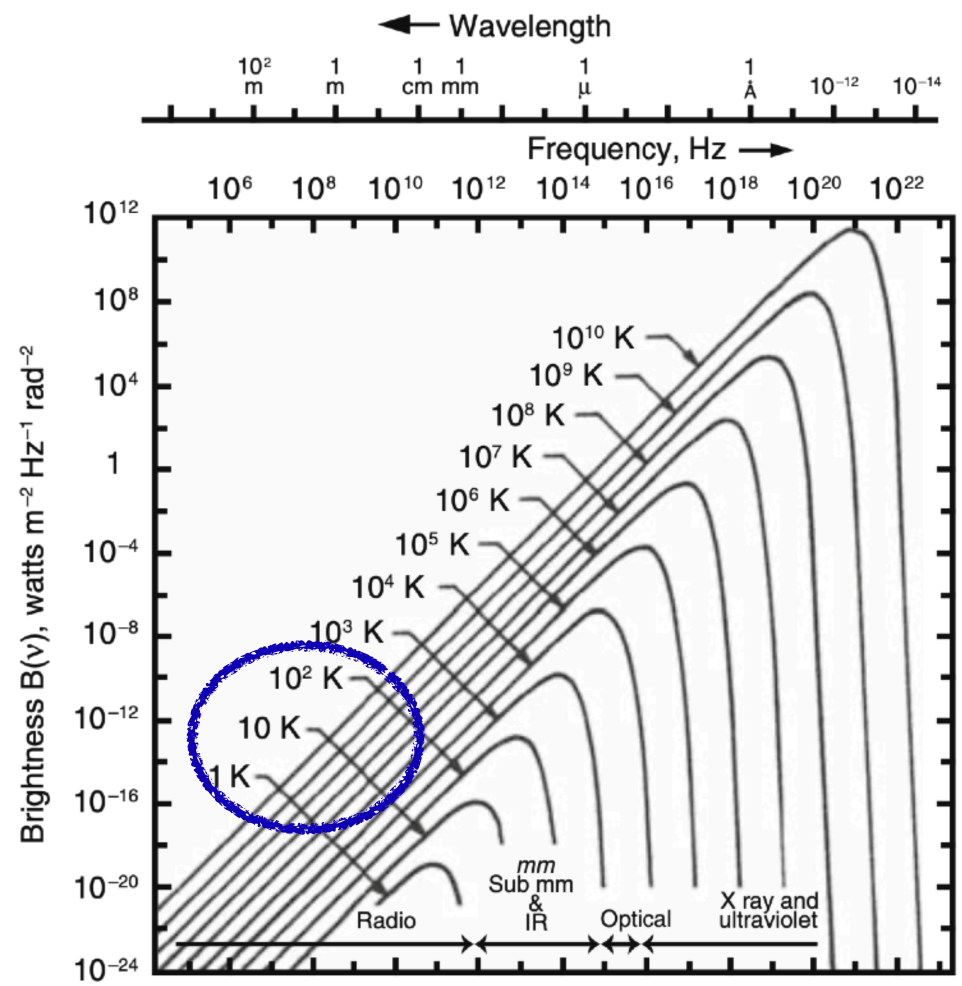
$\epsilon_\nu / k_\nu = B_\nu(T)$ Kirkoff's law

Black body radiation and Brightness Temperature

The spectral distribution for a black body in thermodynamic equilibrium is (previous slide):

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$\boxed{\frac{\nu_{\rm max}}{\rm GHz} = 58.789\left(\frac{T}{K}\right)}$$



Radio astronomy mostly probes BB radiation in the Rayleigh-Jeans regime:

$$B_{RJ}(\nu, T) = \frac{2\nu^2}{c^2}kT \qquad h\nu << kT$$

This implies that brightness and temperature are directly proportional: radio astronomers often measure the brightness of an extended source by its Rayleigh-Jeans Brightness Temperature:

$$T_b(\nu) = \frac{c^2}{2k\nu^2}I_\nu$$ even if $I_\nu \neq B_\nu$ an "equivalent temperature" can be derived

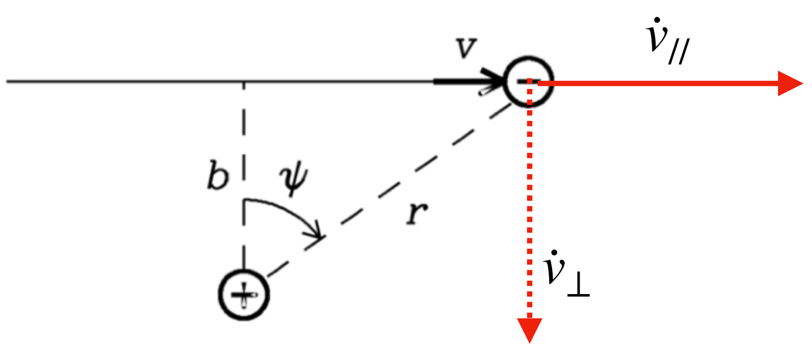
Bremsstrahlung radiation (or free-free)

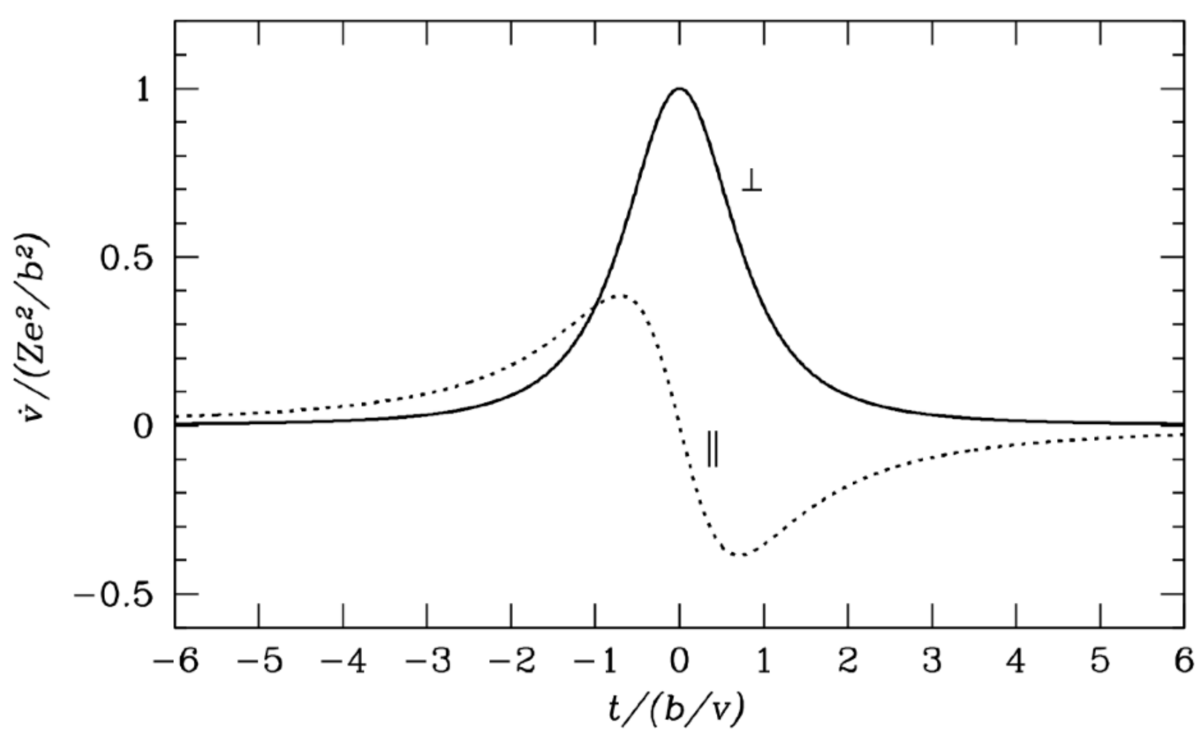
Electromagnetic bremsstrahlung radiation is emitted by an accelerating (or decelerating) charged particle due to an electrostatic force. Thermal emission is produced if the emitting electrons are in LTE (non-thermal if electrons have a power-law energy distribution).

In HII regions, a free electron passing by an ion undergoes an acceleration and emits according to the Larmor formula. The electron is free both before and after the interaction. For reasonable densities of the ionized clouds, electron and photons are in LTE.

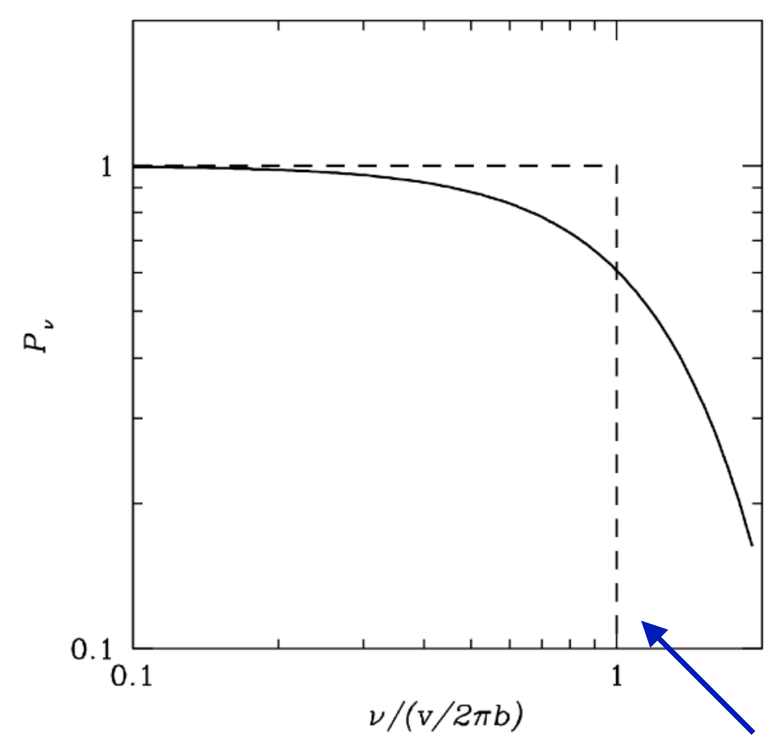
$$P = \frac{2}{2}\frac{e^2\dot{v}^2}{c^3} = \frac{2e^2}{3c^3}\left(\frac{Ze^2}{m_e r^2}\right)^2$$ r ion-electron distance

Coulomb acceleration $\dot{v} \propto \frac{1}{m}$





pulse duration/collision time

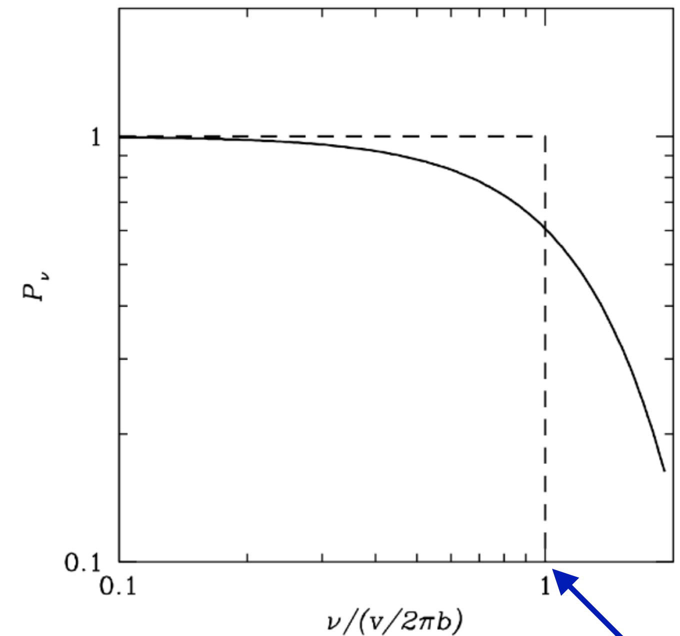


Strong emission at radio frequencies (due to $\dot{v}_{\perp}$) from $\nu \sim 0$ to ν_{max}

ν_{max}~10^{14} Hz in HII regions

Bremsstrahlung radiation (or free-free)

What we have seen so far is valid for optically thin emission



Strong emission at radio frequencies (due to $\dot{v}_\perp$) from $\nu \sim 0$ to $\nu_{\max}$

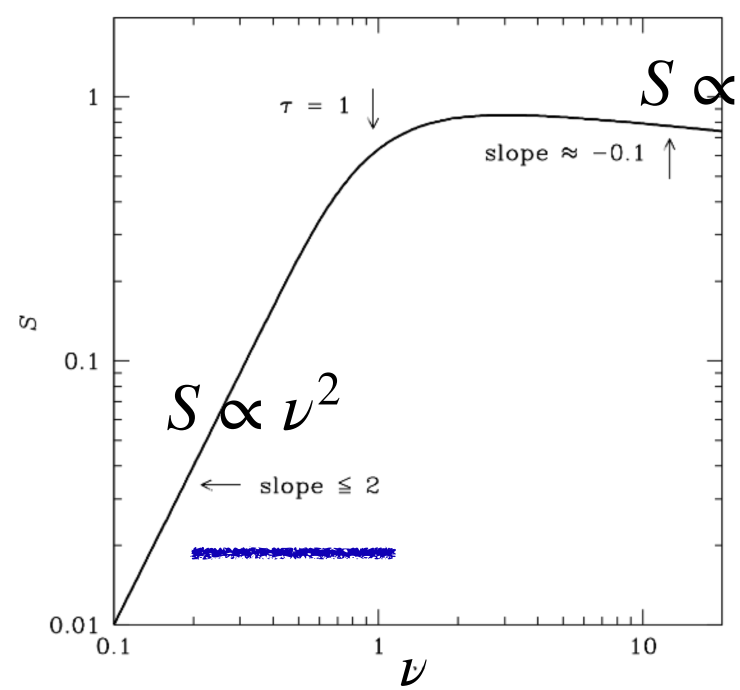
$\nu_{\max}$~10[14] Hz in HII regions

Optical depth of an HII region

$$\tau = -\int_{\mathrm{los}} k ds \propto \int \frac{n_e^2}{\nu^{2.1} T^{3/2}} ds$$

Rayleigh-Jeans + small dependence of k on the maximum impact parameter (and, in turn, on ν)

For frequencies low enough that $\tau >> 1$, the HII region becomes opaque and its spectrum approaches a blackbody in the Rayleigh-Jeans regime

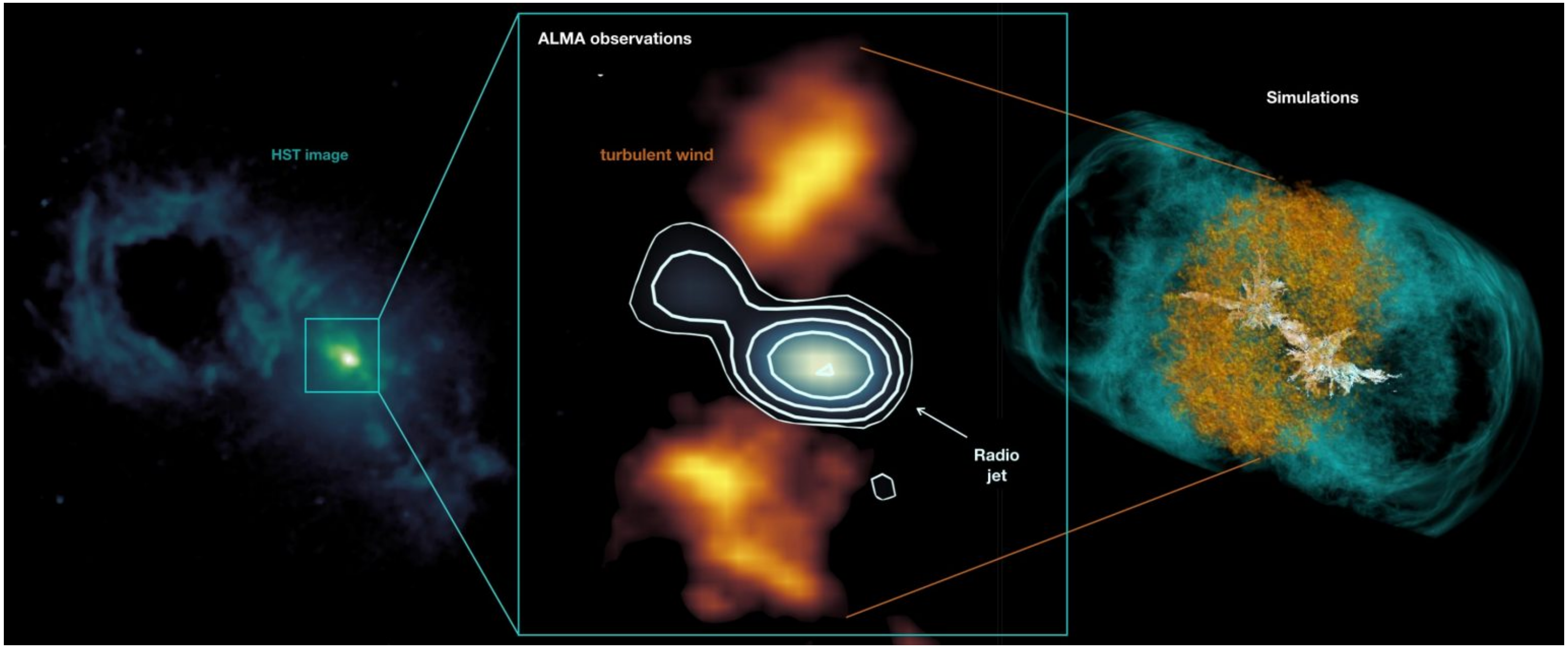


$$S \propto \nu^2 \tau \sim \nu^{-0.1}$$

$$S \propto \nu^2$$

Synchrotron emission

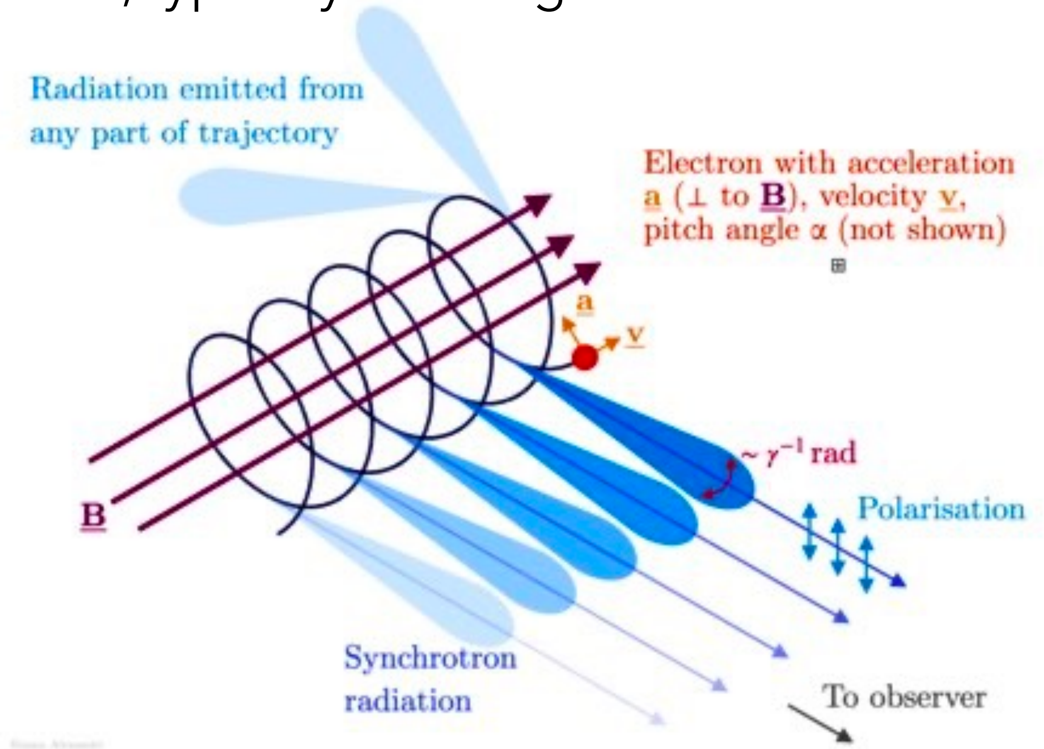
Synchrotron emission is produced when charged particles moving at relativistic speed are subject to an acceleration that is perpendicular to both the direction of motion, typically in a magnetic field.



Audibert et al. 2023, DOI: 10.1051/0004-6361/202345964

Synchrotron emission

Synchrotron emission is produced when charged particles moving at relativistic speed are subject to an acceleration that is perpendicular to both the direction of motion, typically in a magnetic field.



Einstein-Planck relativistic equations

$$\frac{d}{dt}(\gamma m\mathbf{v}) = \frac{e}{c}(\mathbf{v}\times\mathbf{B})$$

$$\frac{d\mathbf{v}_{//}}{dt} = 0$$

$$\frac{d\mathbf{v}_{\perp}}{dt} = \frac{e}{\gamma mc}(\mathbf{v}_{\perp}\times\mathbf{B})$$

$$\frac{d}{dt}(\gamma mc^2) = 0$$

if no electric field
energy conservation

$$\gamma = cost, |\mathbf{v}| = cost$$

$$\downarrow$$

$$\mathbf{v}_{//} = cost$$

$$|\mathbf{v}_{\perp}| = cost$$

uniform rectilinear + uniform circular motion: helix winding around **B**

with pitch angle (between the particle's velocity and the local magnetic field) $tan\alpha = \frac{|\mathbf{v}_{\perp}|}{|\mathbf{v}_{//}|}$

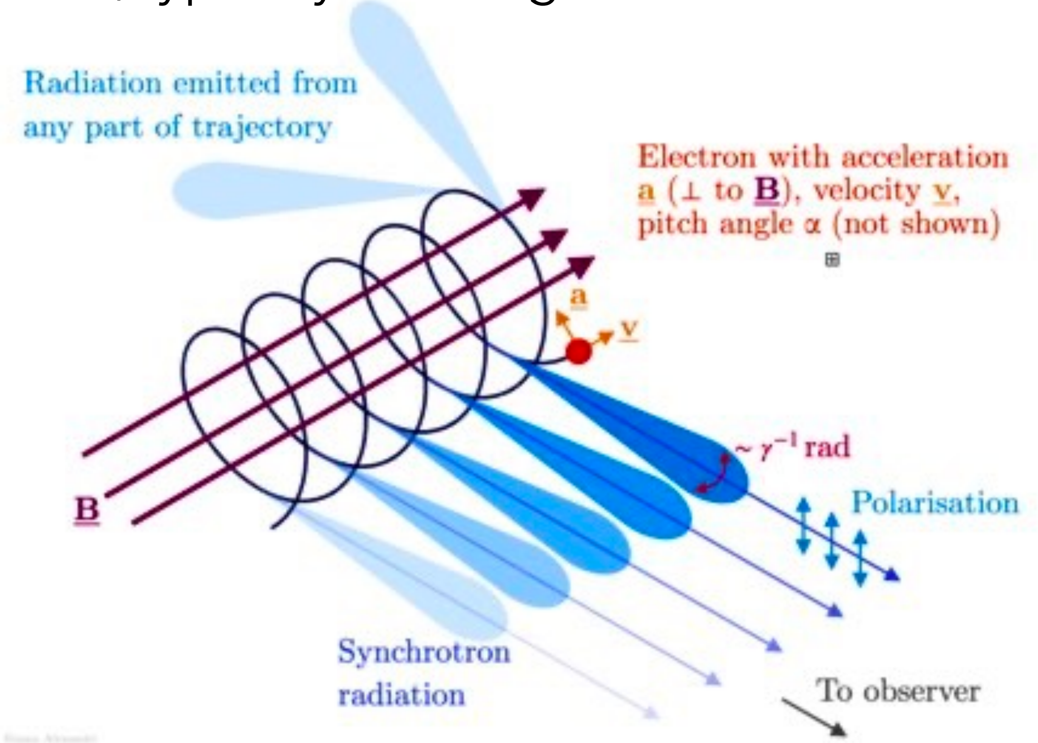
and gyro frequency $\omega_G = \frac{eB}{mc}$

Synchrotron emission

Synchrotron emission is produced when charged particles moving at relativistic speed are subject to an acceleration that is perpendicular to both the direction of motion, typically in a magnetic field.

The gyro frequency $\omega_G = \dfrac{eB}{mc}$ represent the actual orbital frequency if v<<c

$$\frac{\omega_G}{\mathrm{MHz}} = 17.6 \left(\frac{B}{\mathrm{Gauss}} \right) \text{ if } \gamma \simeq 1$$



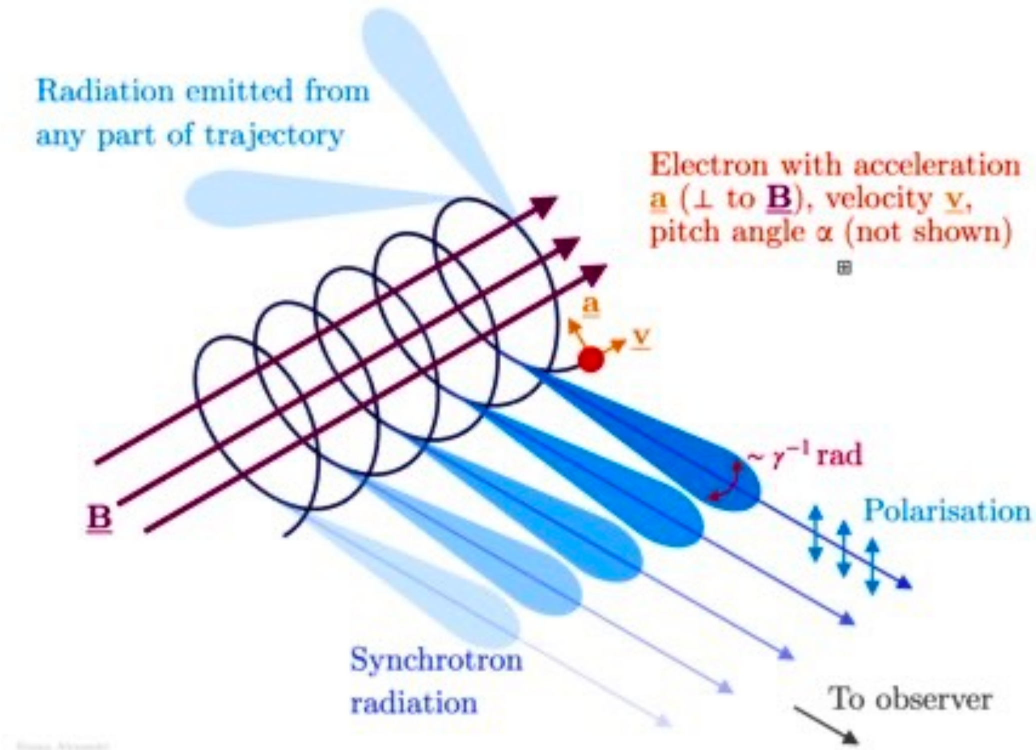
Location	Field strength
Interstellar medium	10^{-6}
Supermassive black-hole	10^{4}
Neutron star	**10^{12}**
This room	0.3
Supernova remnants/Crab nebula	10^{-3}

In typical ISM conditions (e.g. Milky Way) $\omega_G \sim 20$ Hz. The associated gyro radiation cannot propagate as its frequency is lower than the plasma frequency (kHz)

$$\frac{\omega_p}{\mathrm{kHz}} = 28.2 \left(\frac{n_e}{\mathrm{cm}^{-3}} \right)^{1/2} \sim kHz \qquad \text{for } n_e \sim 0.03\ \mathrm{cm}^{-3}$$

Synchrotron emission

In the case of relativistic particles with $\gamma >> 1$, power pattern is that of a dipole modified by relativistic beaming into a cone with angle $\theta \sim \frac{1}{\gamma}$, and the orbital frequency decreases even further



$$\omega_{\text{sync}} = \frac{eB}{\gamma mc} << \omega_G$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

(examples: Comic rays interacting with the interstellar magnetic field produce most of the continuum emission below 30 GHz from our Galaxy)

However, **two compensating relativistic effects can explain the strong synchrotron radiation observed at radio frequencies:**

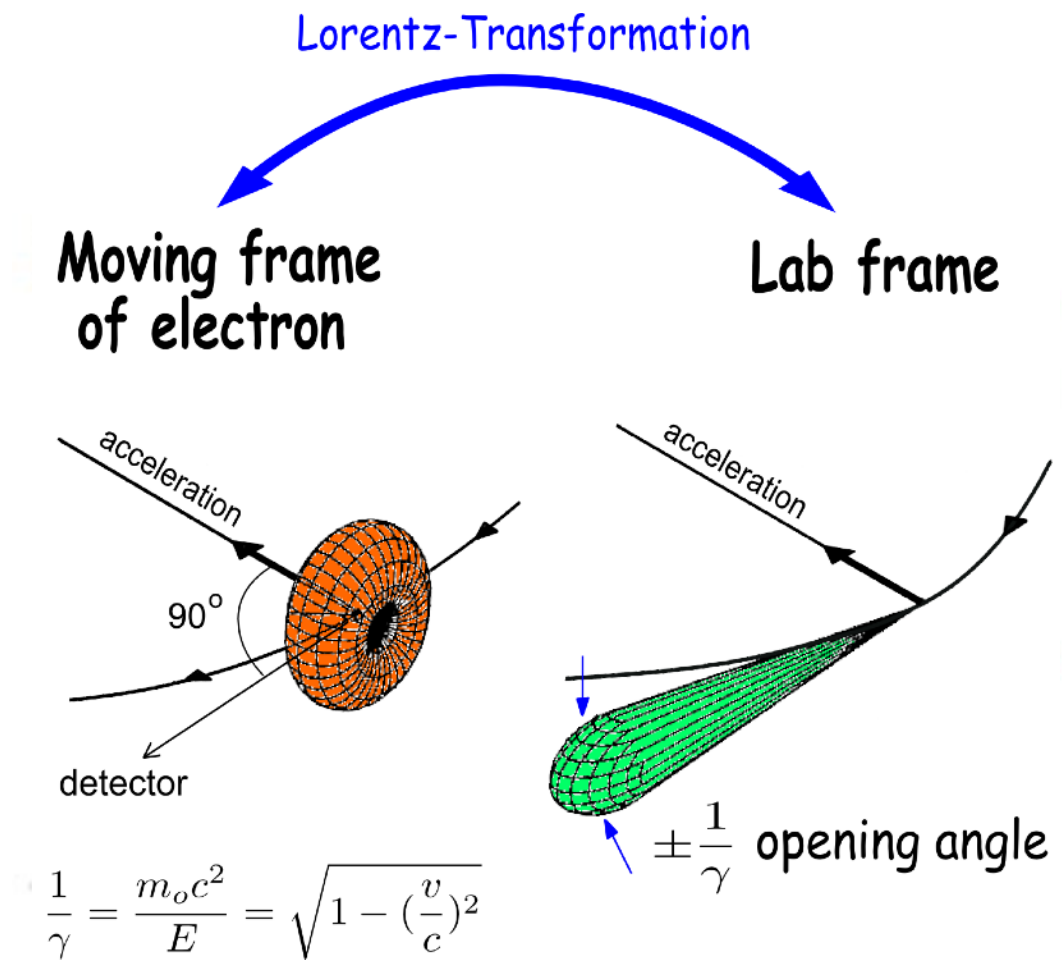
1) the total radiated power in the observer's frame is proportional to γ^2
2) relativistic beaming turns the low-frequency sinusoidal radiation in the electron frame into a series of sharp pulses containing power at much higher frequencies $\propto \gamma^2\omega_G = \gamma^3\omega_{\text{sync}}$ in the observer's frame.

Synchrotron emission

In the case of relativistic particles with $\gamma >> 1$, power pattern is that of a dipole modified by relativistic beaming into a cone with angle $\theta \sim \frac{1}{\gamma}$, and the orbital frequency decreases even further

$$\omega_{\text{sync}} = \frac{eB}{\gamma mc} << \omega_G$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



(examples: Comic rays interacting with the interstellar magnetic field produce most of the continuum emission below 30 GHz from our Galaxy)

However, **two compensating relativistic effects can explain the strong synchrotron radiation observed at radio frequencies:**

1) the total radiated power in the observer's frame is proportional to γ^2
2) relativistic beaming turns the low-frequency sinusoidal radiation in the electron frame into a series of sharp pulses containing power at much higher frequencies $\propto \gamma^2\omega_G = \gamma^3\omega_{\text{sync}}$ in the observer's frame.

Synchrotron emission

For a single relativistic electron

The total emitted power is given by the relativistic Larmor equation $P = \frac{2e^2}{3c^2}\gamma^2\frac{e^2B^2}{m_ec^2}v^2 sin^2\alpha$

Or, equivalently, as $\boxed{P = 2\sigma_T\beta^2\gamma^2cU_B sin^2\alpha}$ where σ_T is the Thomson cross section and $U_B = B^2/8\pi$ is the magnetic energy density

Synchrotron power depends on the electron kinetic energy (via γ^2), the magnetic field strength and the pitch angle

In radio sources $< sin^2\alpha > \sim 2/3$ (multiple scattering by magnetic field fluctuations and other charged particles)

Synchrotron emission

For a single relativistic electron

$\propto q^2 a^2$

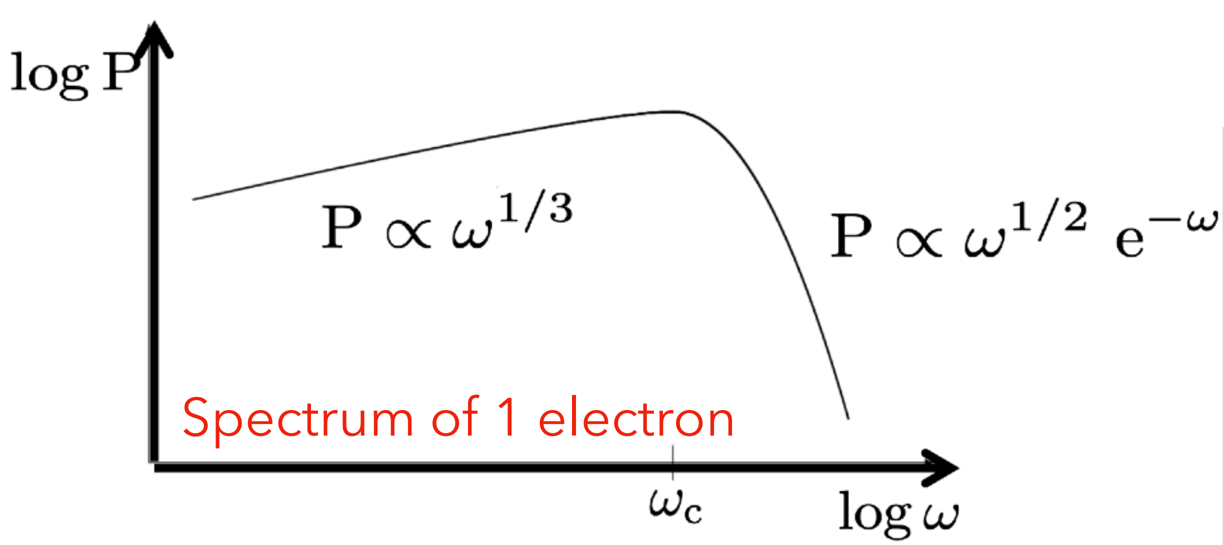
The total emitted power is given by the relativistic Larmor equation $P = \frac{2e^2}{3c^2}\gamma^2\frac{e^2B^2}{m_e c^2}v^2 sin^2\alpha$

Or, equivalently, as $\boxed{P = 2\sigma_T \beta^2\gamma^2 c U_B sin^2\alpha}$ where σ_T is the Thomson cross section and $U_B = B^2/8\pi$ is the magnetic energy density

Synchrotron power depends on the electron kinetic energy (via γ^2), the magnetic field strength and the pitch angle

In radio sources $< sin^2\alpha > \sim 2/3$ (multiple scattering by magnetic field fluctuations and other charged particles)

The spectrum peaks at a critical frequency $\omega_c = \frac{3}{4\pi}\gamma^2\omega_G sin\alpha$



Synchrotron emission

For a single relativistic electron

The total emitted power is given by the relativistic Larmor equation $P = \frac{2e^2}{3c^2}\gamma^2 \frac{e^2B^2}{m_e c^2} v^2 sin^2\alpha$

Or, equivalently, as $\boxed{P = 2\sigma_T \beta^2 \gamma^2 c U_B sin^2\alpha}$ where σ_T is the Thomson cross section and $U_B = B^2/8\pi$ is the magnetic energy density

Synchrotron power depends on the electron kinetic energy (via γ^2), the magnetic field strength and the pitch angle

In radio sources $< sin^2\alpha > \sim 2/3$ (multiple scattering by magnetic field fluctuations and other charged particles)

Electrons in a plasma emitting synchrotron radiation cool down. The cooling time of an electron is given by its energy divided by the energy radiation rate

$$t_{\rm cool} = \frac{E}{P} = \frac{\gamma mc^2}{P} \sim 16 {\rm yr} \left(\frac{1 {\rm Gauss}}{\rm B}\right)^2 \frac{1}{\gamma}$$

Location	t_{cool}
Interstellar medium	10^{10} yrs
Supermassive black-hole	5 days
Neutron star	10^{-19} s

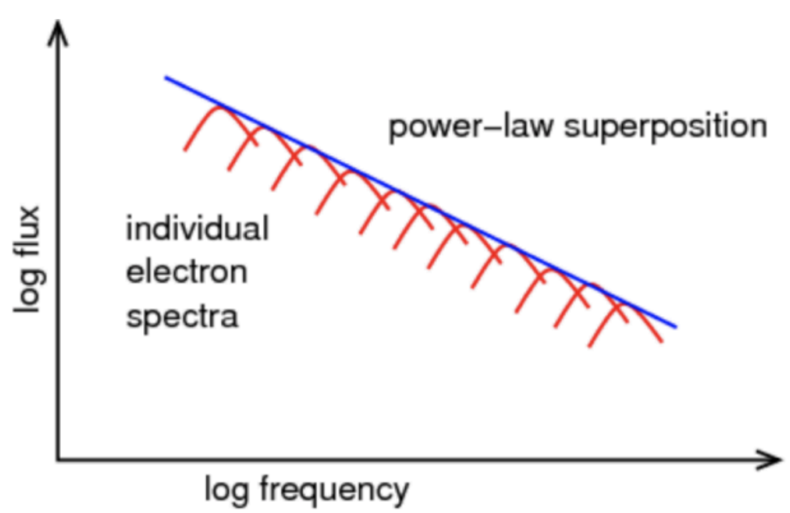
Synchrotron emission

In radio astronomy, we usually observe the radiation from an ensemble of electrons whose energies can be very different. To obtain a realistic synchrotron spectrum, it is necessary to convolve the mono-energetic electron spectrum with an energy distribution function.

For many astrophysical sources, it is reasonable to assume a power law energy distribution, as it would be expected for a stochastic acceleration mechanism: $N(E) \propto KE^{-\delta}$

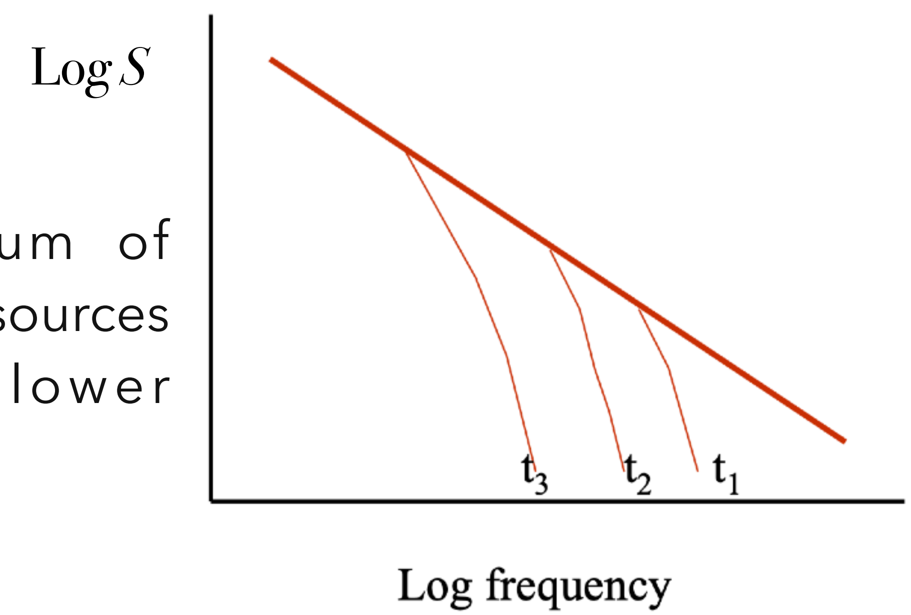
In this case, the total radiated flux is: $S \propto \nu^{-n}$

There is a simple relation between δ and n: $n = \dfrac{\delta - 1}{2}$



The spectrum of older radio sources breaks at lower frequency

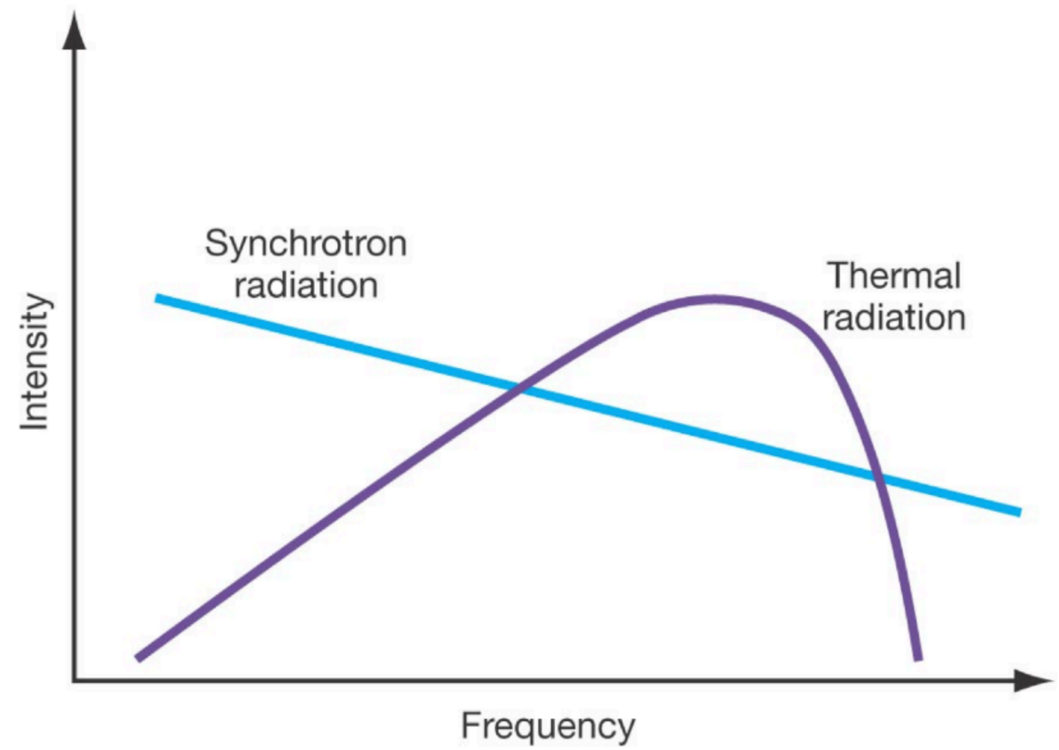
$t_{\text{cool}} \propto \dfrac{1}{\gamma}$



Synchrotron emission

The brightness temperatures of synchrotron sources cannot become arbitrarily large at low frequencies.

In the case of a thermal source (LTE), brightness temperature cannot be greater than the kinetic temperature of the emitting particles. If the energy distribution of relativistic electrons in a synchrotron source were a (relativistic) Maxwellian, the electrons would have a well-defined kinetic temperature, and **synchrotron self-absorption** would prevent the brightness temperature of the synchrotron radiation from exceeding the kinetic temperature of the emitting electrons



What we have seen so far is valid for optically thin emission

If the frequency is low enough, the relativistic electrons in the same field can absorb the synchrotron photons produced by other electrons. Synchrotron self-absorption occurs for any electron energy distribution

Synchrotron emission

Electrons with energy $E = \gamma mc^2$ emit most of their power at nearly the critical frequency $\omega_c \sim \dfrac{\gamma^2 eB}{2\pi m_e c}$

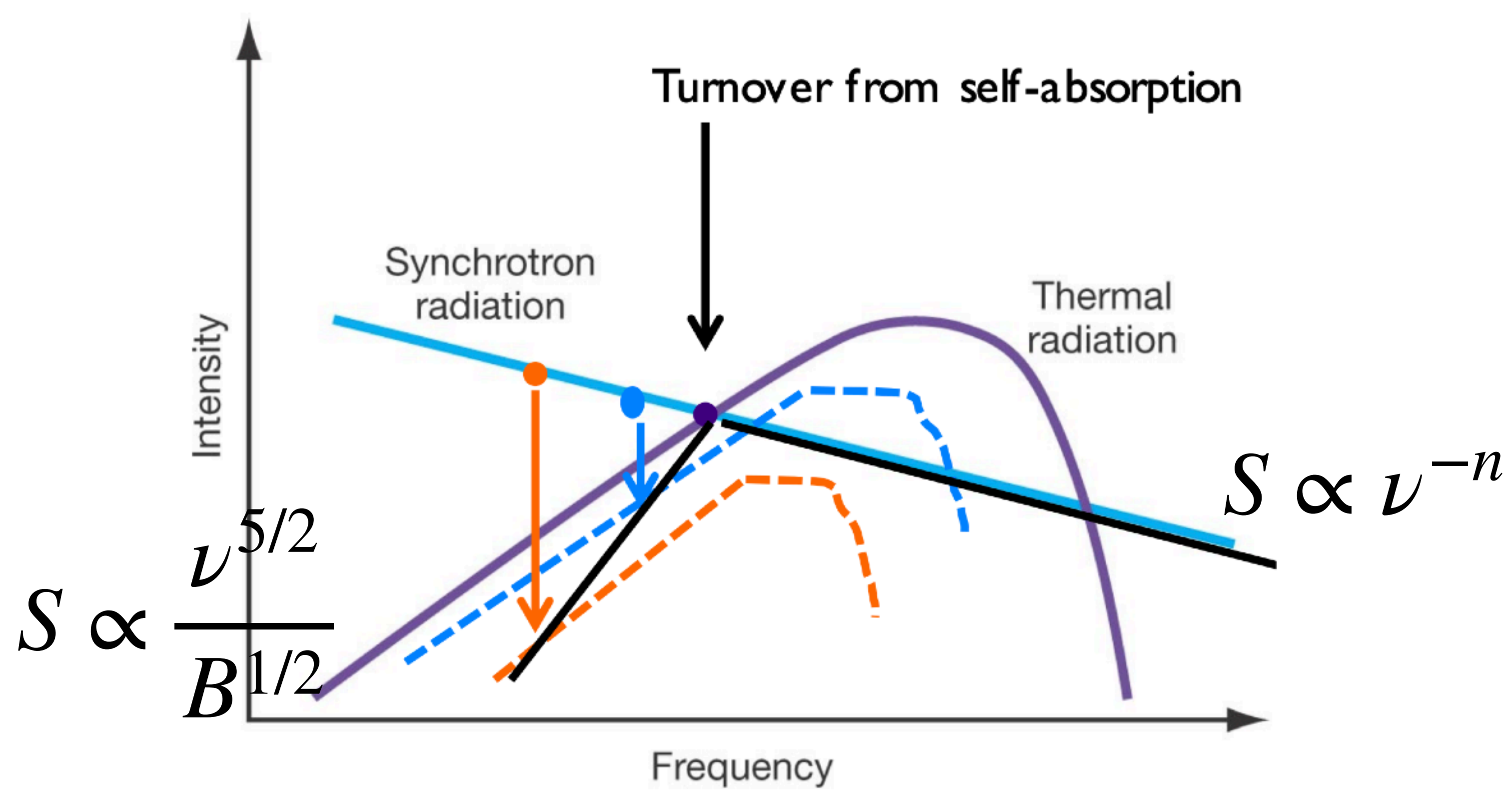
Synchrotron emission at frequency ν thus mostly comes from electrons with Lorentz factor $\gamma \sim \left(\dfrac{2\pi m_e c\nu}{eB}\right)^{1/2}$

As $E = 3kT_e$ the electron effective temperature is $T_e \sim \dfrac{E}{3k} = \dfrac{\gamma m^e c^2}{3k} \sim \left(\dfrac{2\pi m_e c\nu}{eB}\right)^{1/2} \dfrac{m_e c^2}{3k}$

Ultrarelativistic gas

At low frequency, the brightness temperature approaches the effective temperature T_e. From the definition of brightness temperature in the Rayleigh-Jeans regime we have

$$\boxed{I_\nu \sim \frac{2kT_e\nu^2}{c^2} \propto \nu^2\nu^{1/2}B^{-1/2} = \nu^{5/2}B^{-1/2}}$$



Synchrotron emission

Electrons with energy $E = \gamma mc^2$ emit most of their power at nearly the critical frequency $\omega_c \sim \dfrac{\gamma^2 eB}{2\pi m_e c}$

Synchrotron emission at frequency ν thus mostly comes from electrons with Lorentz factor $\gamma \sim \left(\dfrac{2\pi m_e c\nu}{eB}\right)^{1/2}$

As $E = 3kT_e$ the electron effective temperature is $T_e \sim \dfrac{E}{3k} = \dfrac{\gamma m^e c^2}{3k} \sim \left(\dfrac{2\pi m_e c\nu}{eB}\right)^{1/2} \dfrac{m_e c^2}{3k}$

Ultrarelativistic gas

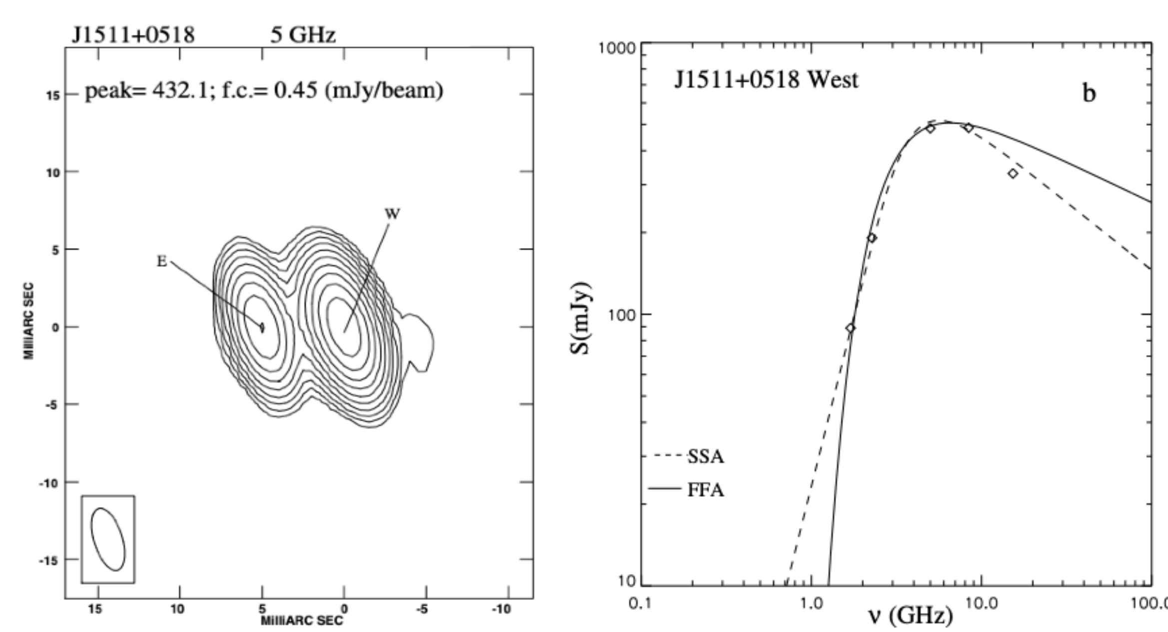
At low frequency, the brightness temperature approaches the effective temperature T_e. From the definition of brightness temperature in the Rayleigh-Jeans regime we have

$$\boxed{I_\nu \sim \frac{2kT_e\nu^2}{c^2} \propto \nu^2\nu^{1/2}B^{-1/2} = \nu^{5/2}B^{-1/2}}$$

For self-absorbed radio sources $B \propto \nu T_b^{-2}$
The magnetic field strength can be therefore measured via a measurement of T_b at a given frequency ν

$$\left(\frac{B}{\mathrm{gauss}}\right) \sim 1.4 \times 10^{12} \left(\frac{\nu}{\mathrm{Hz}}\right)\left(\frac{T_b}{\mathrm{K}}\right)^{-2}$$

e.g. radio active galactic nuclei



Orienti & Dallacasa 2008

Synchrotron emission

The existence of a synchrotron source implies the presence of relativistic electrons with energy density U_e and of a magnetic field whose energy density is U_B.

What is the minimum total energy to produce a synchrotron source of a given radio luminosity?

$$U_e = \int_{E_{\min}}^{E_{\max}} E n(E) dE$$

n(E) number density of electrons with energy between E and E+dE

$$L = \int_{\nu_{\min}}^{\nu_{\max}} L_\nu d\nu = -\int_{E_{\min}}^{E_{\max}} \frac{dE}{dt} n(E) dE$$

From the definition of critical frequency, we know that most electrons emit at a frequency $\nu \propto E^2 B$

Their energy (for a given ν) is therefore $E \propto B^{-1/2}$

$$\omega_c \sim \frac{\gamma^2 eB}{2\pi m_e c}$$

E2

From the Larmor's equation, we know that the synchrotron power emitted per electron is $-\dfrac{dE}{dt} \propto E^2 B^2$

For electrons with a power-law energy distribution, we have $n(E) \propto E^{-\delta}$

The ratio $\dfrac{U_e}{L} \propto \dfrac{E^{2-\delta}\big|_{E_{\min}}^{E_{\max}}}{B^2 E^{3-\delta}\big|_{E_{\min}}^{E_{\max}}} \propto \dfrac{(B^{-1/2})^{2-\delta}}{B^2 (B^{-1/2})^{3-\delta}} = B^{-3/2}$

To produce an observed luminosity, the electron energy has to scale as $\boxed{U_e \propto B^{-3/2}}$

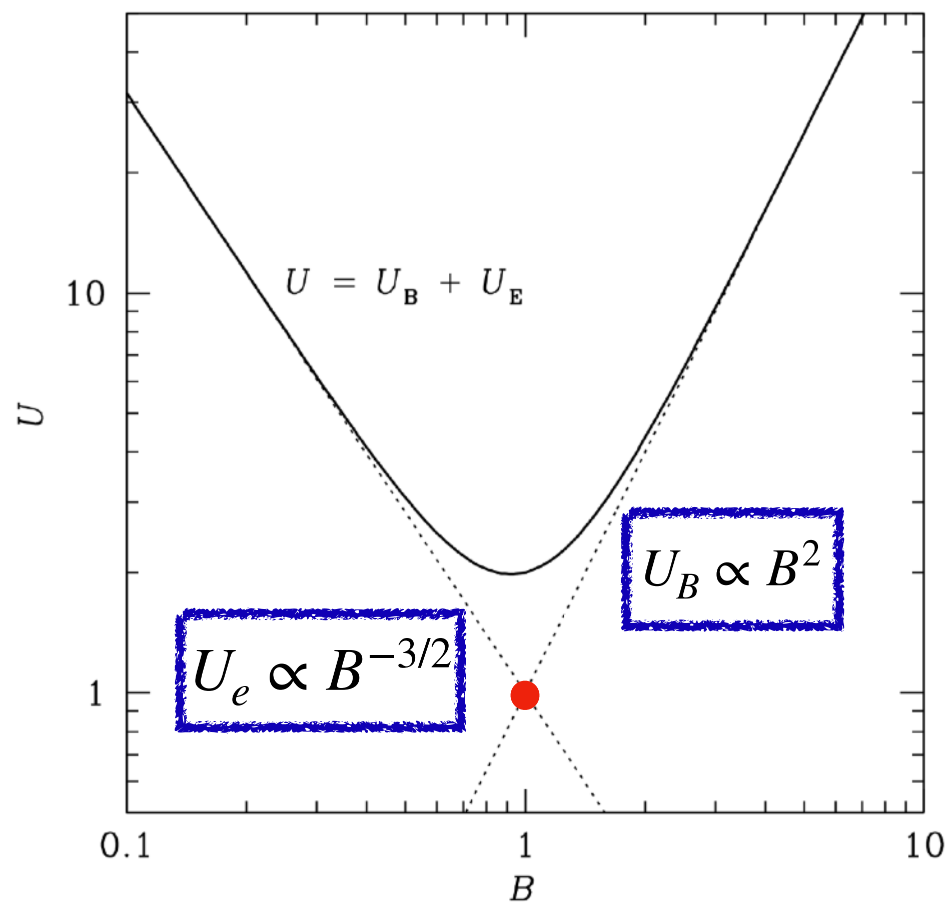
While $\boxed{U_B \propto B^2}$

Synchrotron emission

The total energy density is $U \sim U_e + U_B$

There is a minimum close to equipartition ($U_e \sim U_B$)

$$\frac{dU}{dB} = 0 \longrightarrow \frac{U_e}{U_B} \sim \frac{4}{3}$$



Radio astronomers often assume that synchrotron sources are in equipartition:

- Luminous, extragalactic sources have enormous total energies even near equipartition. A common problem is to understand what is a viable source of energy to these systems.
- Particle energies and magnetic field strength can be measured knowing L and the emitting volume V e.g. for a spherical radio source at distance R

G(*n*)

$$B_{eq} = \left(6\pi \frac{G}{H} \frac{R^2}{V} S_\nu \nu^n \right)^{2/7}$$

H(*n*) *n = observed spectral index*

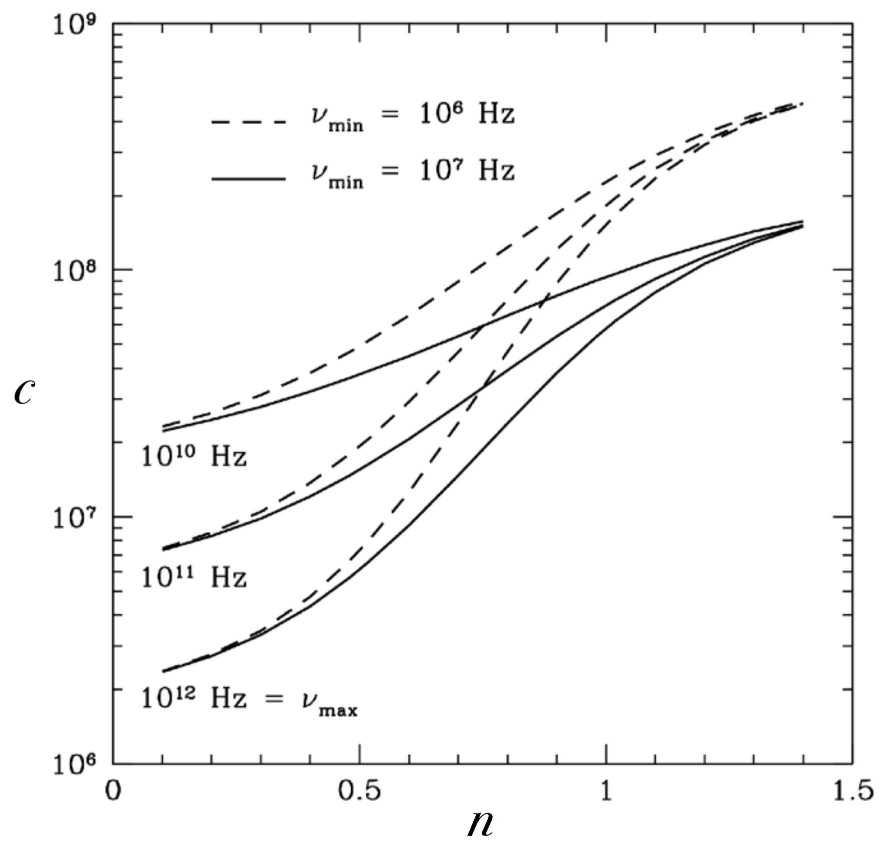
prop to $B^{-3/2} + B^2$

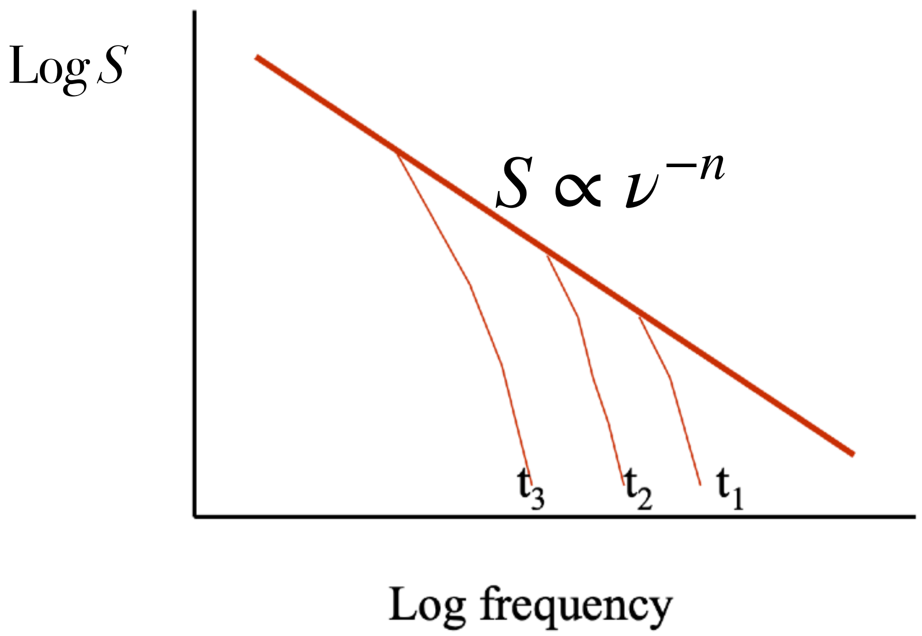
$$U_{eq} = \frac{7}{4}(6\pi)^{-3/7} \left(\frac{G\nu^n}{H} S_\nu \right)^{4/7} R^{8/7} V^{3/7}$$

Synchrotron emission

The synchrotron lifetime of a source with luminosity L is therefore

$$\tau \sim \frac{U_e}{L} = cB^{-3/2}$$





where c accounts for the U_e dependence on n, $\nu_{max}(E_{max})$ and $\nu_{min}(E_{min})$

Steeper spectra (larger n) correspond to older synchrotron sources