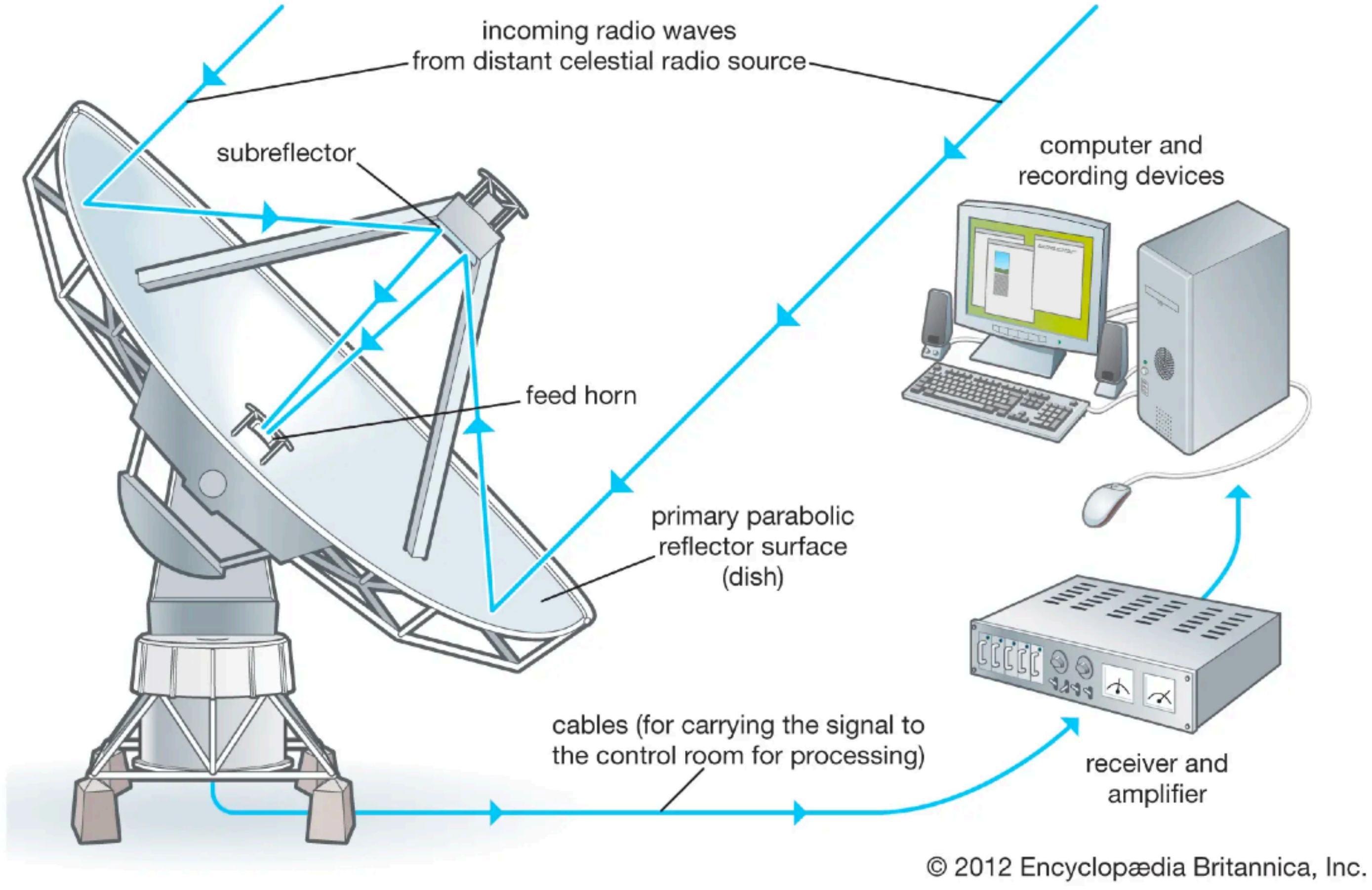
Instruments for Radioastronomy

Elements of a radio telescope

A radio telescope is a specialized antenna and radio receiver, used to detect radio waves from astronomical radio sources in the sky.

An antenna is a passive device that converts electromagnetic radiation in space into electrical currents in conductors, or vice versa, depending on whether it is used for receiving or for transmitting. Radio telescopes are receiving antennas

A radio receiver is an electronic device which receives alternating currents from the antenna and converts the information carried by them into a usable form. It uses electronic filters to select the desired frequencies and an electronic amplifier to increase the power of the signal for further processing.



Antenna fundamentals

The most important characteristic of an antenna is its ability to absorb radio waves incident upon it. This is typically described in terms of antenna effective aperture:

$$A_e = \frac{\text{Power density available at the antenna terminals}}{\text{Flux density of the incident wave}} = [\frac{\mathrm{W/Hz}}{W/m^2/Hz}] = [m^2]$$

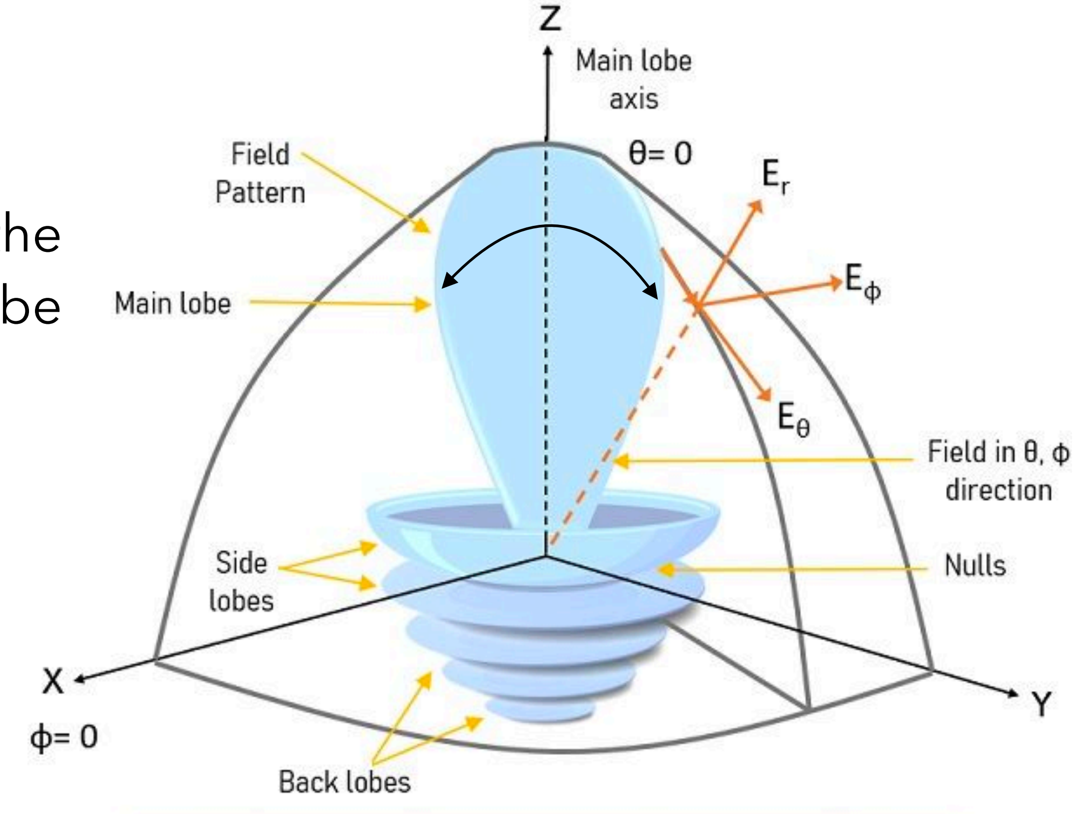
The effective area depends on the direction of the incident wave: the antenna works better is some directions than in others:

$$A_e = A_e(\theta, \phi)$$

This directional property of the antenna is often described in the form of a power pattern, i.e. an effective area normalized to be unity at the maximum

$$\boxed{P(\theta, \phi) = \frac{A_e(\theta, \phi)}{A_e^{\max}}}$$

$$\boxed{\Theta_{\mathrm{HPBW}} \sim \frac{\lambda}{D}}$$



Main lobe: primary maximum of the antenna pattern

Side lobes: subsidiary maxima of the antenna pattern

Half Power Beamwidth Θ_{HPBW}: angular distance between the two points at which $P = P^{max}/2$

Antenna fundamentals

Another patter often used to describe antennas is the gain:

$$G(\theta,\phi) = \frac{\text{Power emitted into } (\theta,\phi)}{(\text{Total power input})/4\pi}$$

For any lossless antenna, energy conservation requires that the gain averaged over all directions is <G>=1, from which

$$\int_{sphere} G(\theta,\phi) sin(\theta) d\theta d\phi = 4\pi$$

The main beam solid angle is defined as the region containing the principal response out to the first zero

$$\boxed{\Omega_{MB} = \frac{1}{G^{\max}} \int_{MB} G(\theta,\phi) sin(\theta) d\theta d\phi}$$

where G^{max} is the maximum gain

And we can define the concept of main beam efficiency as $\eta_{MB} \equiv \frac{\Omega^{\mathrm{MB}}}{\Omega_A}$

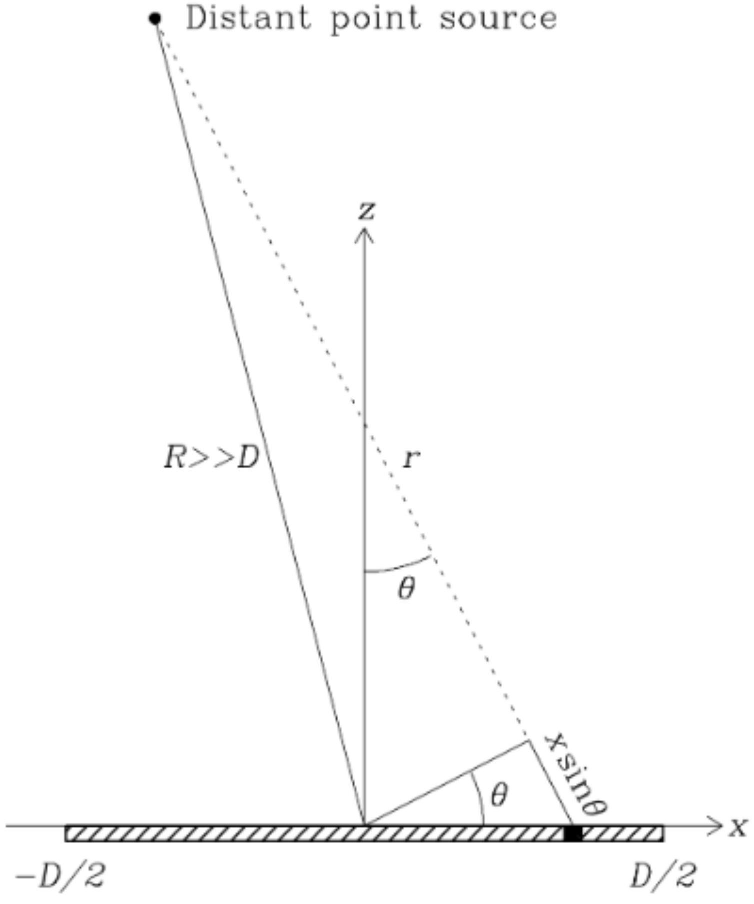
For reflector antennas, the aperture efficiency is defined as:

$\eta_A = \frac{A_e^{\max}}{A_g}$ where A_g is the geometric cross-sectional area of the main reflector

Patterns of Aperture Antennas

How to calculate the beam pattern, or power gain as a function of direction, of an antenna aperture?

Consider the case of a 1-dimensional aperture and for simplicity assume that the antenna is transmitting. We want to calculate the electric field pattern at a large distance R.



R source distance

D aperture size

x distance from aperture center

The antenna feed illuminates the antenna aperture with a sine wave. Illumination induces currents in the reflector. Currents vary with position and time.

$I \propto g(x) exp(-i\omega t)$

$\nu = \omega/(2\pi)$	wave frequency
$g(x)$	electric field strength

Huygens's principle: the aperture is an ensemble of small elements individually acting as small antennas. The electric field produced by the whole aperture at large distances is just the vector sum of the elemental electric fields from these small antennas.

$$df \propto g(x)\frac{exp(-i2\pi r(x)/\lambda)}{r(x)}$$

electric field strength

r(x) distance between the source and aperture element at position x

As R>>R_{ff} the plane wave approximation is valid and $r \sim R + xsin\theta$ usually written as $r \sim R + xl$

$(l = sin\theta)$

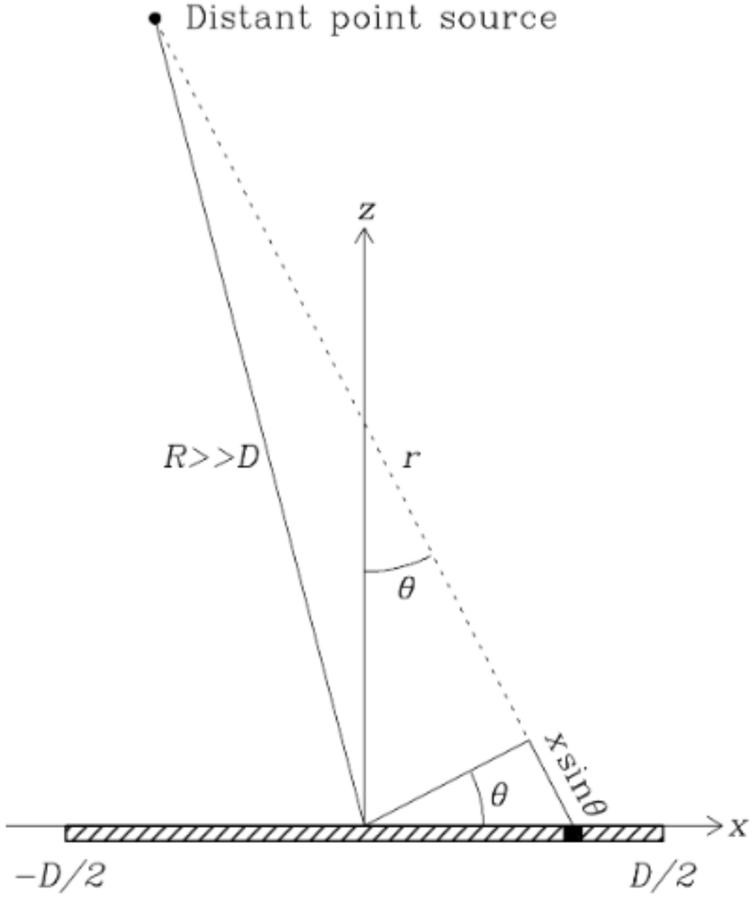
Patterns of Aperture Antennas

$$df \propto g(x)\frac{exp(-i2\pi(\overset{\text{constant}}{R} + xl)/\lambda)}{R}$$

$$\frac{1}{r} \sim \frac{1}{R}$$

$$\boxed{df \propto g(x)\frac{exp(-i2\pi xl)/\lambda)}{R}}$$

The phase $2\pi xl/\lambda$ varies linearly across the aperture. Different parts of the aperture add constructively or destructively to the total electric field.



R source distance

D aperture size

x distance from aperture center

Defining the position along the aperture in units of wavelength $u = x/\lambda$

$$\boxed{f(l) = \int_{\text{aperture}} g(u)e^{-i2\pi lu}du}$$

In the far field, **the electric-field pattern of an aperture antenna is the Fourier transform of the electric field distribution illuminating that aperture**.

Patterns of a uniformly illuminated antenna

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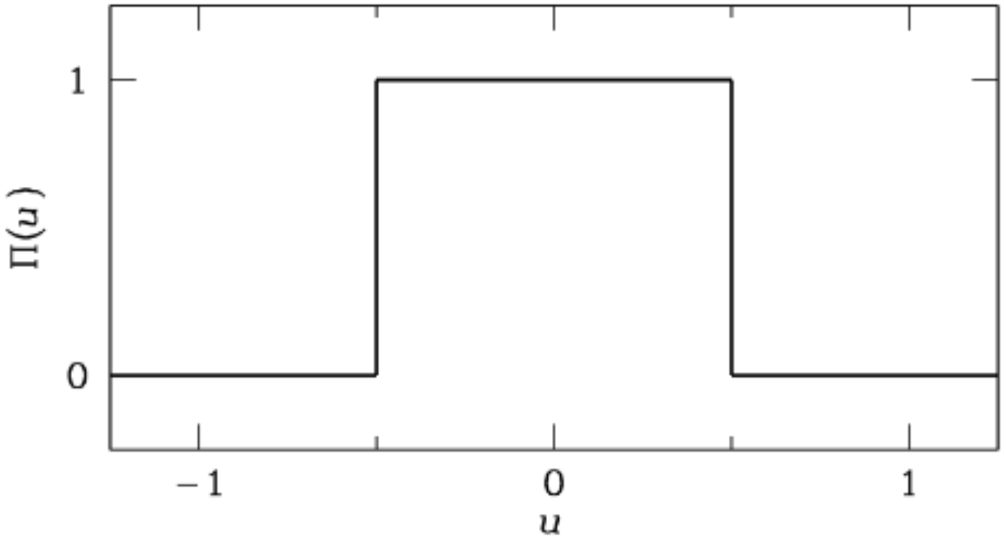
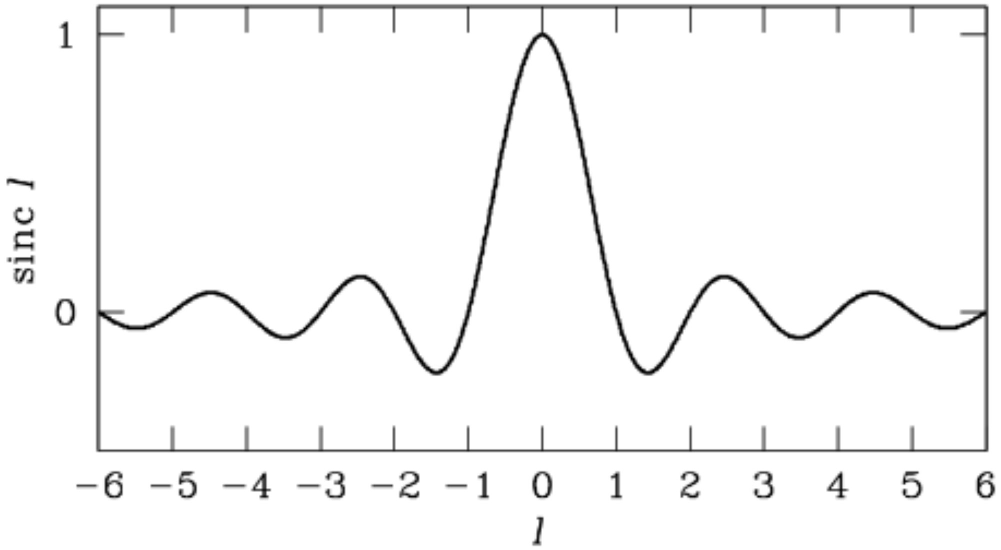


Uniform illumination: $g(u) = \text{constant}$ $\quad \frac{-D}{2\lambda} < u < \frac{D}{2\lambda}$

Unit aperture ($D = \lambda$)



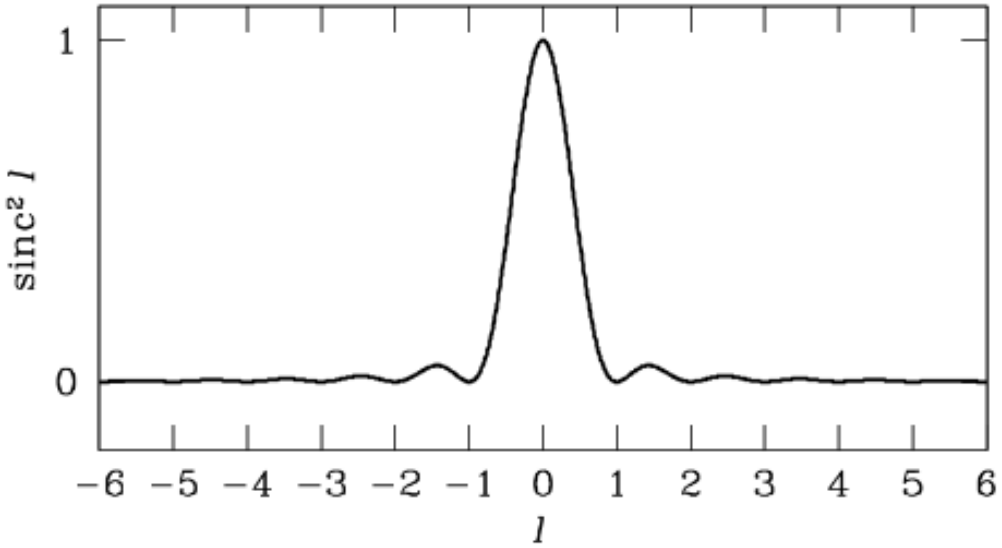
Unit rectangle function: $\Pi(u) = 1$ $\quad -1/2 < u < 1/2$

$$\boxed{f(l)} = \int_{-1/2}^{1/2} \Pi(u) e^{-i2\pi l u} du = \int_{-1/2}^{1/2} e^{-i2\pi l u} du$$

$$= \frac{e^{-i\pi l} - e^{i\pi l}}{-i2\pi l} = \frac{sin(\pi l)}{\pi l} = \boxed{sinc(l)}$$

developing in cos and sin

electric-field pattern of a uniformly illuminated antenna



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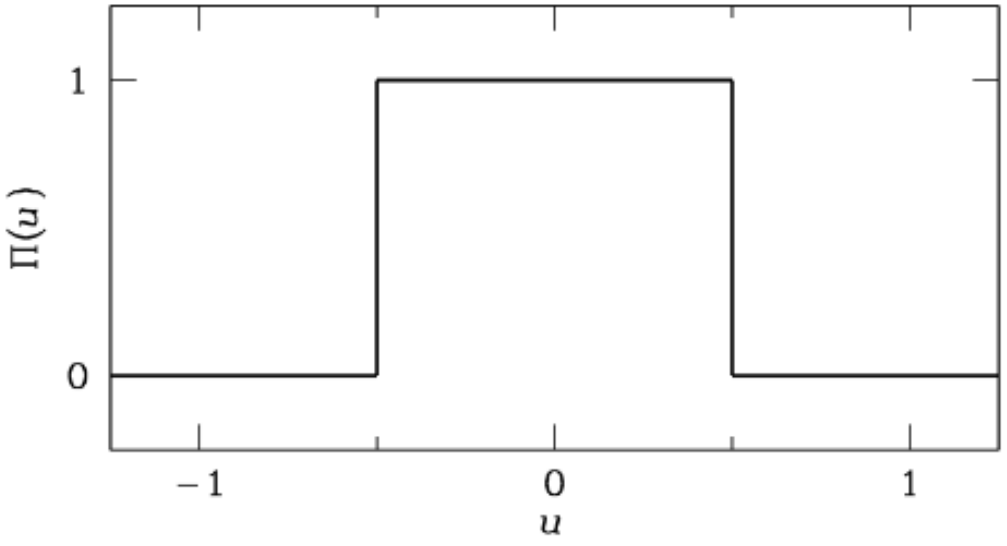

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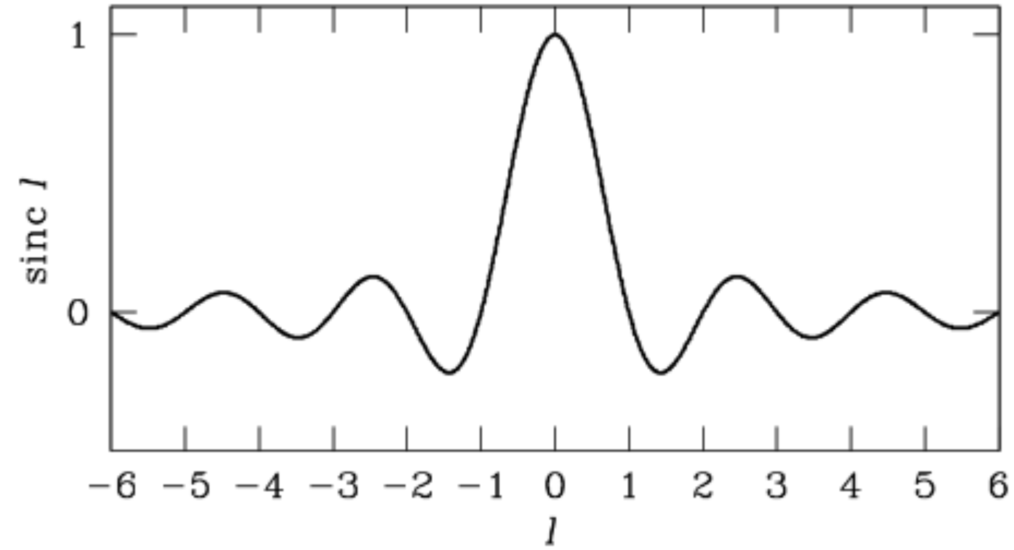
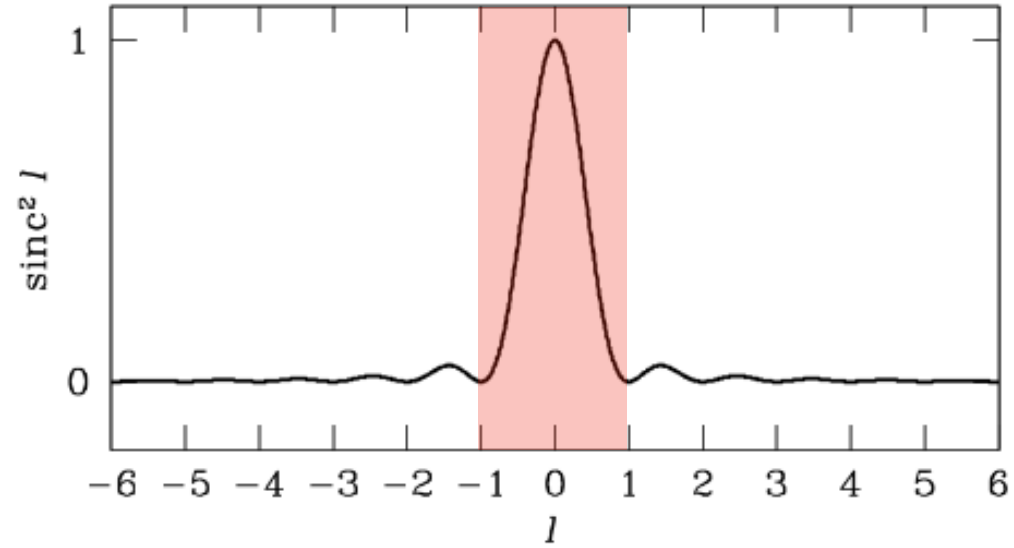


The power pattern is the square of the field pattern

$$\boxed{P(l) = sinc^2(l)}$$

power pattern of a uniformly illuminated antenna

Main beam: peak of the power pattern between the first nulls ($l = \pm 1$)



Patterns of a uniformly illuminated antenna

In general, we can exploit the similarity theorem for Fourier transforms to derive the power pattern for a one-dimensional aperture of size D operating at wavelength λ

$f(l)$ generic function

$g(u)$ Fourier transform of *f*

$$g(au) = \frac{1}{|a|} f(\frac{u}{a})$$

Patterns of a uniformly illuminated antenna

In general, we can exploit the similarity theorem for Fourier transforms to derive the power pattern for a one-dimensional aperture of size D operating at wavelength λ

$$\boxed{P(l) = \left(\frac{D}{\lambda}\right)^2 sinc^2\left(\frac{lD}{\lambda}\right)}$$

P increases with the aperture size D

P increases at smaller λ (higher frequency)

Or, equivalently, as a function of θ

$$\boxed{P(\theta) = \left(\frac{D}{\lambda}\right)^2 sinc^2\left(\frac{\theta D}{\lambda}\right)}$$

For large apertures $D/\lambda >> 1$ the relevant angles are $\theta << 1$ radian, so that $l = sin\theta \sim \theta$

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Recalling the definition of half power beamwidth

$$P(\theta_{\text{HPBW/2}}) = \frac{1}{2} = sinc^2\left(\frac{\theta_{\text{HPBW}}D}{2\lambda}\right)$$

$$\boxed{\theta_{\text{HPBW}} \sim 0.89\frac{\lambda}{D}}$$

the exact constant depends on the illumination taper, that is the variation of the illumination amplitude across the aperture

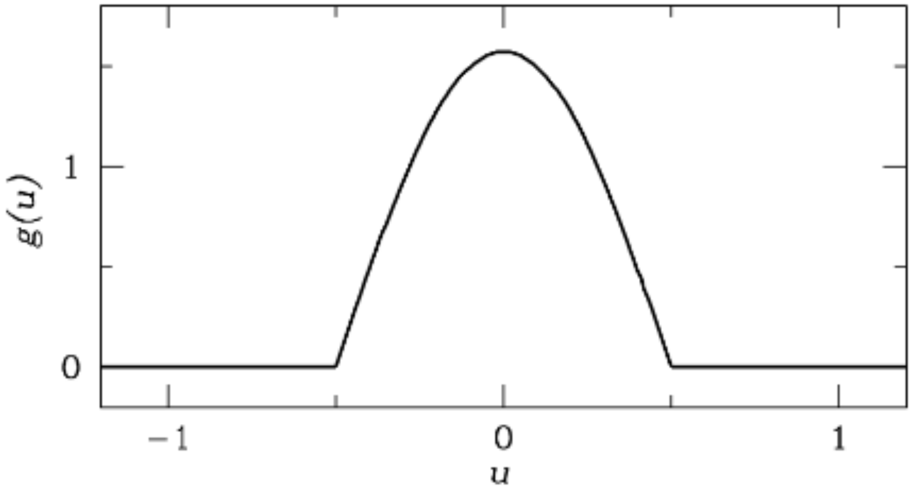
Reciprocity theorem: the derived transmitting power pattern of an aperture antenna also yields its receiving power pattern. In receiving terms, the analog of the power pattern is called the point-source response.

Patterns of an antenna with tapered illumination

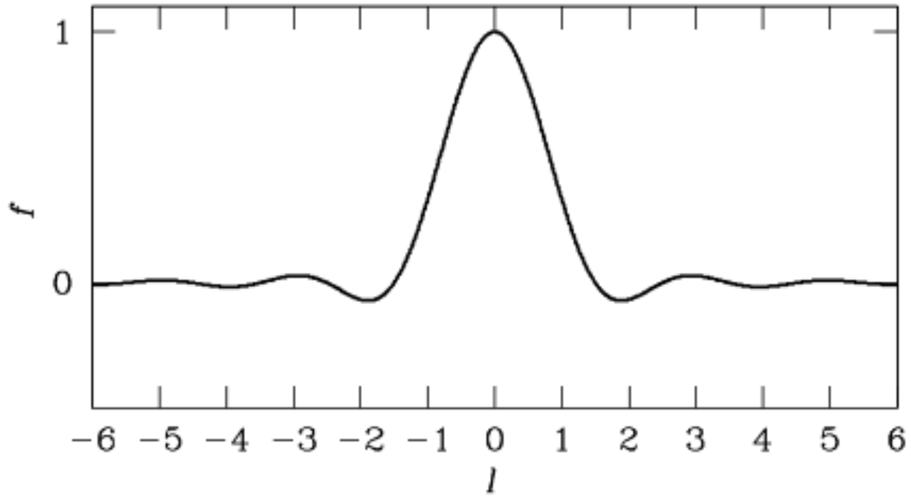
Real antenna feeds cannot illuminate a large aperture uniformly. A better approximation to their illumination is the cosine-tapered field pattern (cosine-squared tapered power pattern)

$$g(u) = \frac{\pi}{2} cos(\pi u) \qquad -1/2 < u < 1/2$$

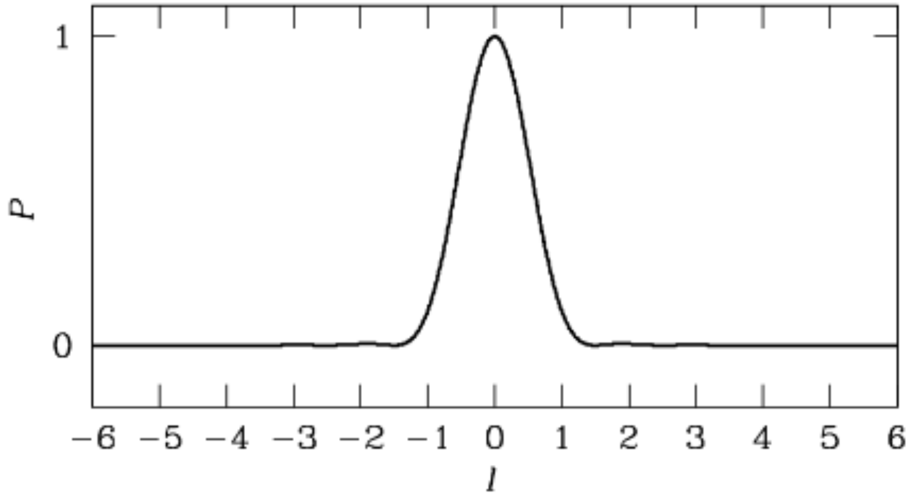
$$g(u) = \int_{-1/2}^{1/2} g(u) du = 1$$



$$f(l) = \frac{cos(\pi l)}{1 - 4l^2}$$



$$P(l) = [\frac{cos(\pi l)}{1 - 4l^2}]^2$$



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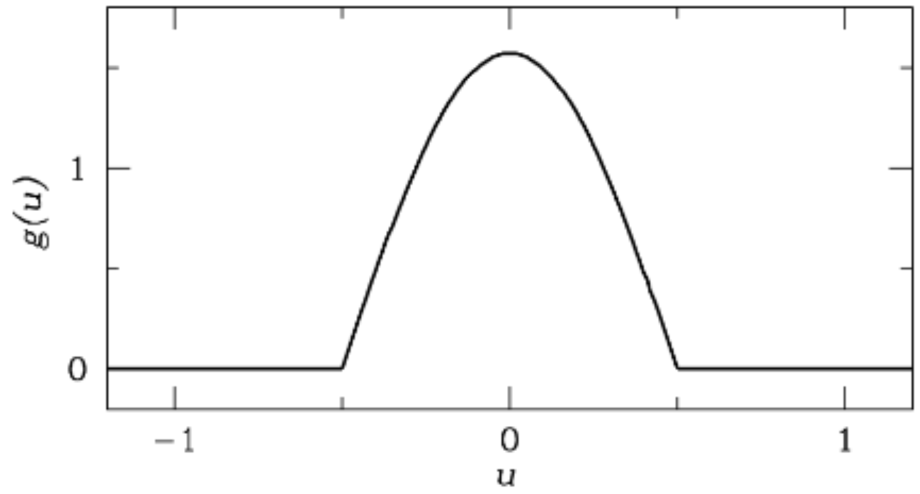

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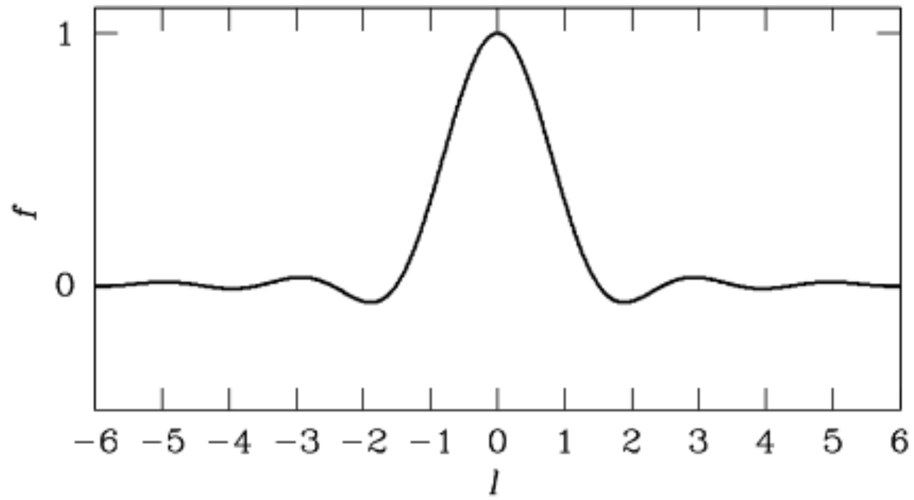
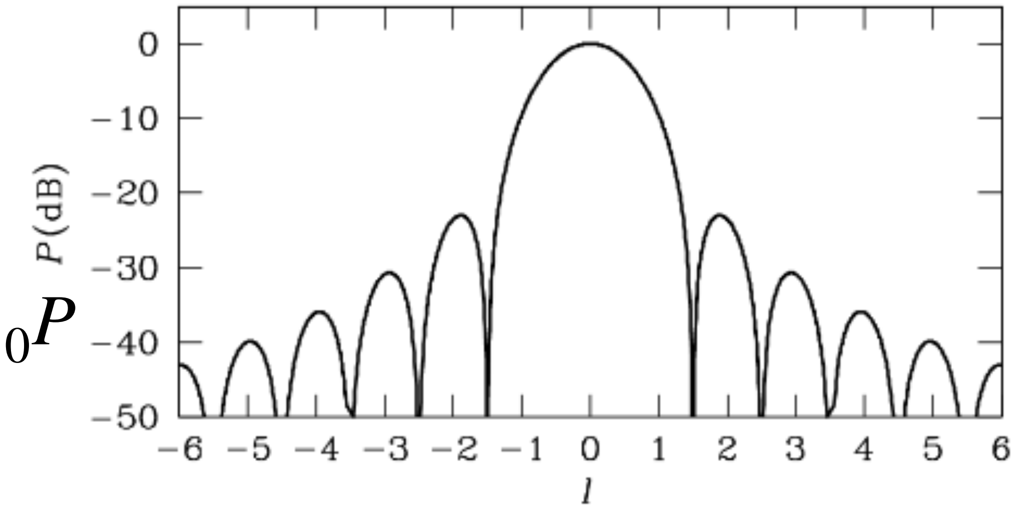


$$P(l) = [\frac{cos(\pi l)}{1 - 4l^2}]^2$$

$$P(dB) = 10log_{10}P$$



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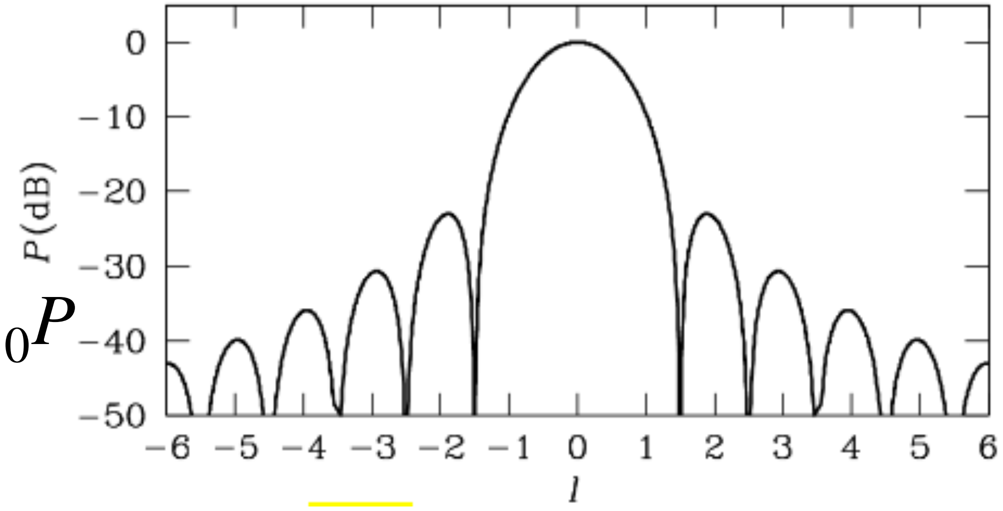


$$P(\theta_{\mathrm{HPBW/2}}) = \frac{1}{2}$$

$$\Big\downarrow \; D/\lambda >> 1$$

$$\boxed{\theta_{\mathrm{HPBW}} \sim 1.2\frac{\lambda}{D}}$$

Tapering increases the half-power beamwidth.

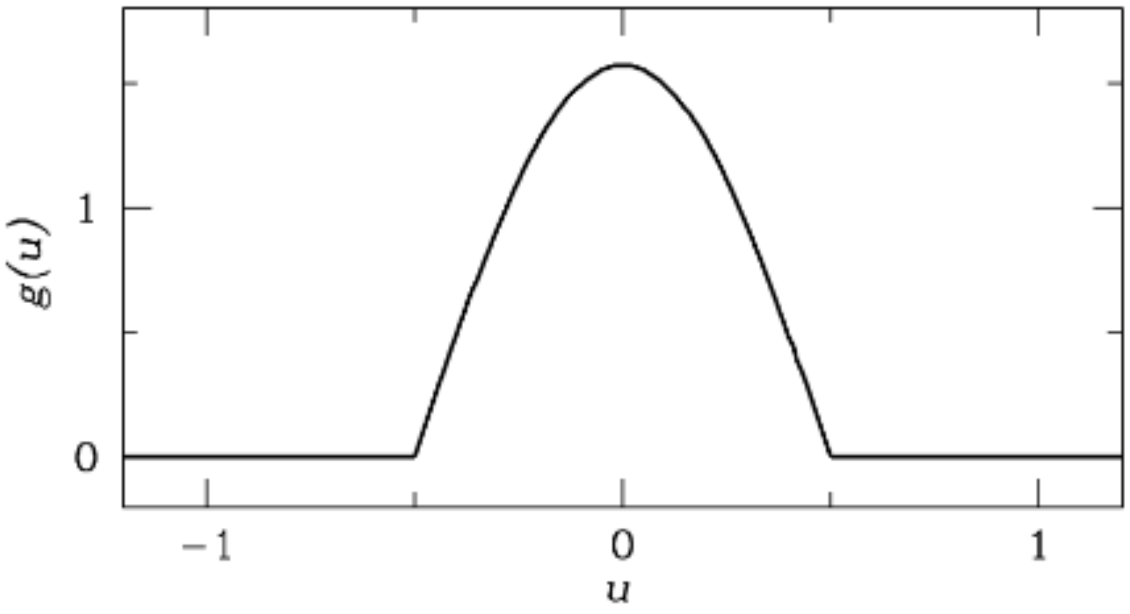
Typical beamwidth of most radio telescopes.

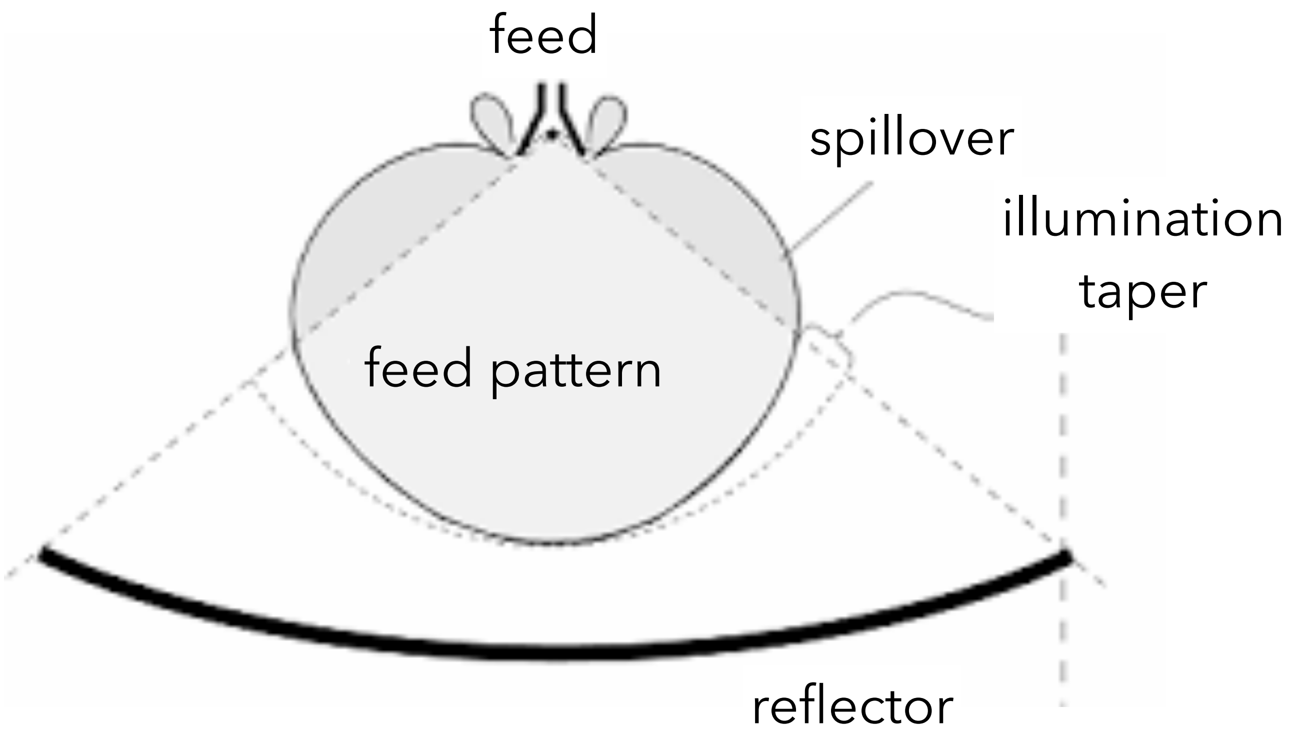
Antenna spillovers

The perfectly sharp cutoff of illumination at the edge of the aperture ($u = \pm 1/2$) cannot be achieved in practice. Any illumination extending beyond the reflector is called spillover.

In the case of a receiving antenna, a prime-focus feed looking down at an aperture also sees spillover radiation from the surrounding ground.

Sols emit blackbody radiation at the ambient temperature T~300K. Ground radiation can therefore add significantly to the system noise temperature of a radio telescope.





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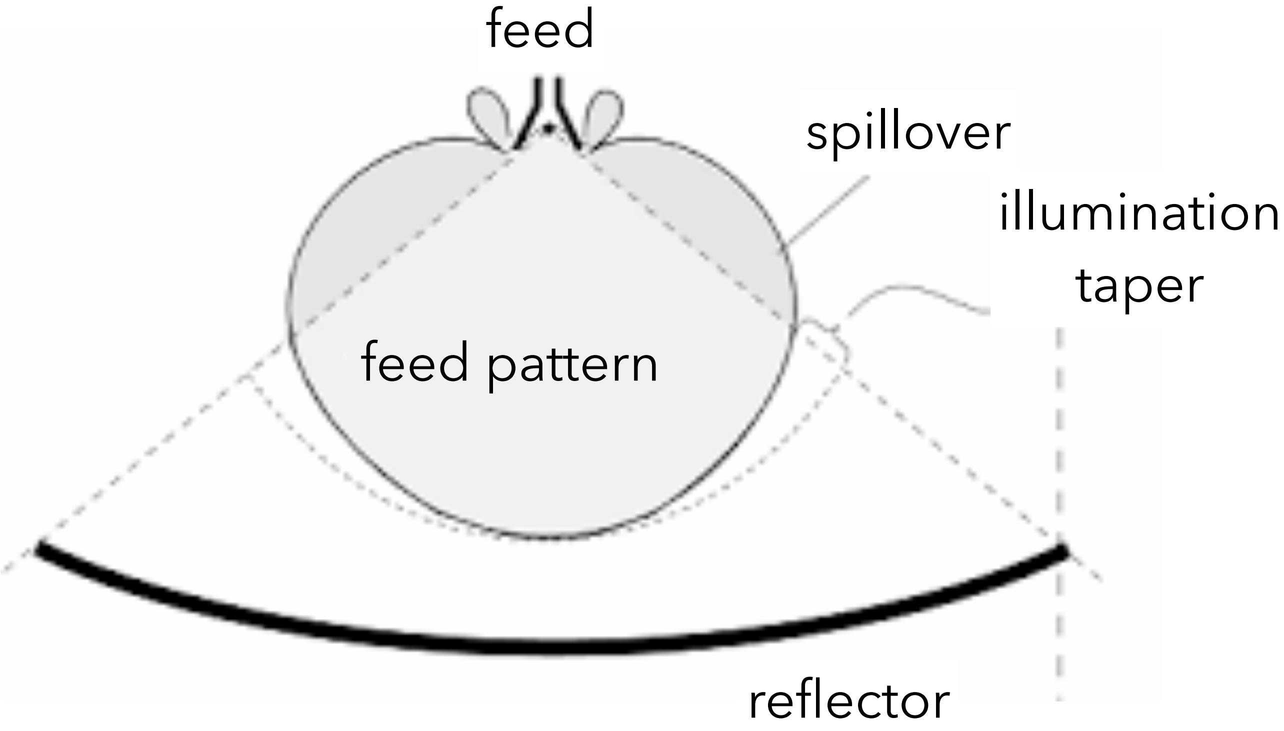
ground screen @Arecibo



Gaussian beam solid angle and beamwidth

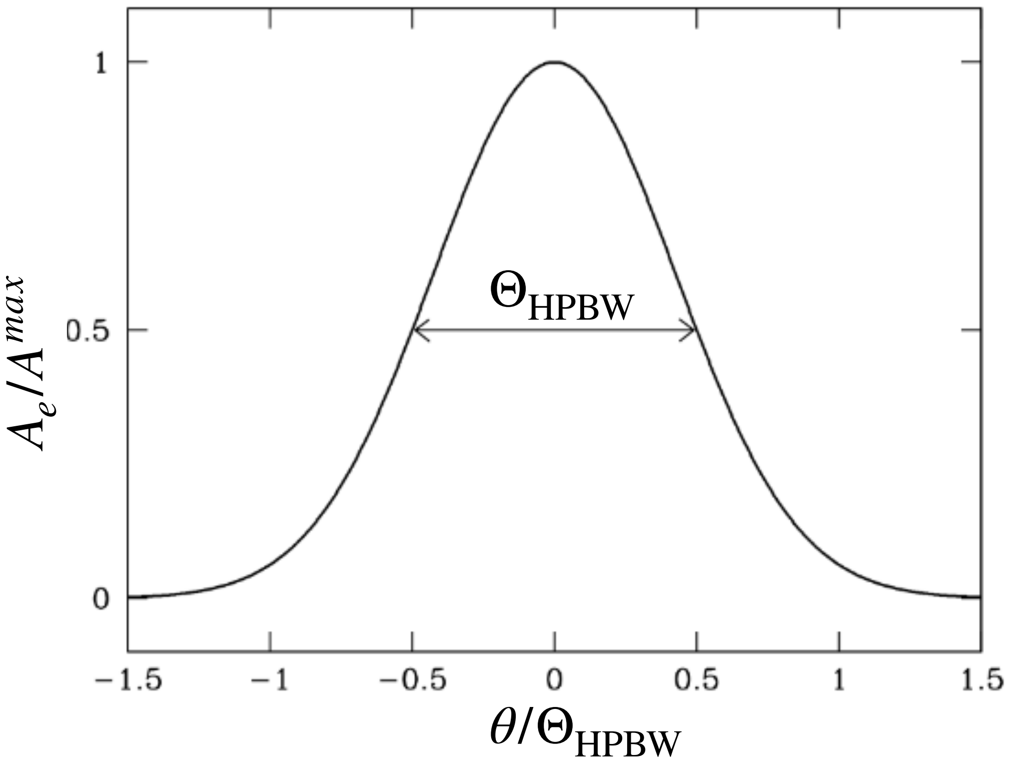
We have introduced the definition of beam solid angle:

$$\Omega_A \equiv \frac{1}{G^{max}} \int_{4\pi} G(\theta, \phi) d\Omega$$ where G is the antenna gain

Or, equivalently, as a function of the antenna effective aperture:

$$\boxed{\Omega_A \equiv \frac{1}{A^{max}} \int_{4\pi} A_e(\theta, \phi) d\Omega}^{*}$$

as the reciprocity theorem implies that $A_e(\theta, \phi) = \frac{\lambda^2 G(\theta, \phi)}{4\pi}$



θ offset from beam center

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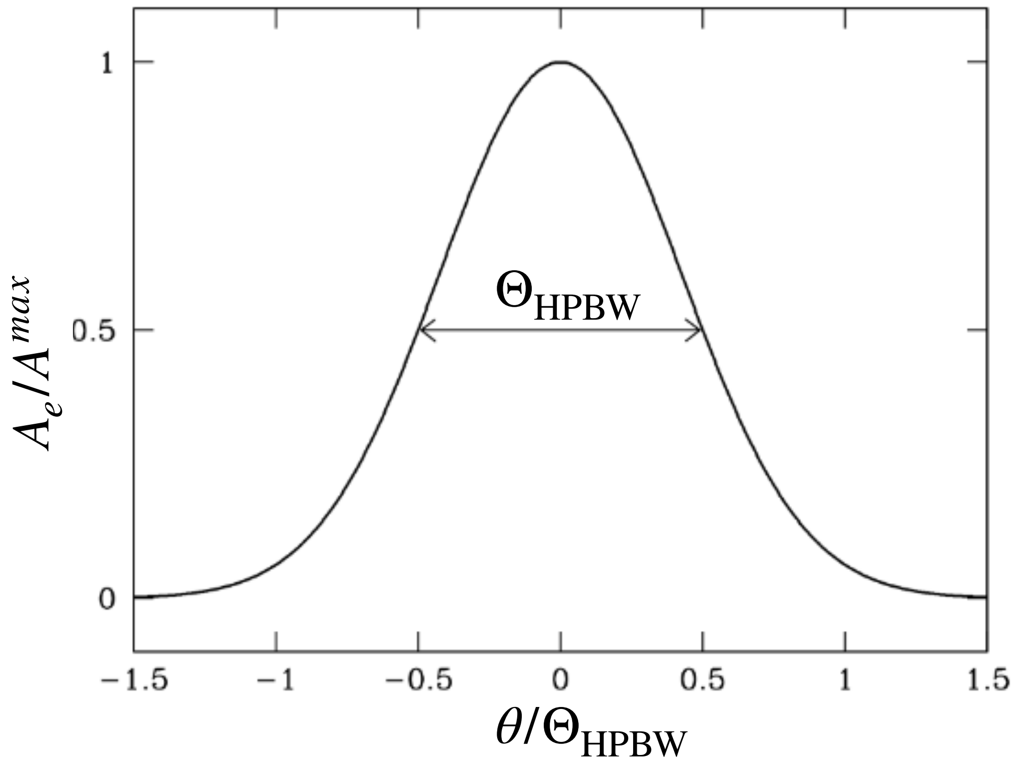


θ offset from beam center

The beams of most radio telescopes (single dish but also interferometers) are nearly Gaussian and can be written as

$$\frac{A_e}{A^{max}} - exp(-x\theta^2)$$ where $x = \dfrac{4ln2}{\Theta^2_{\text{HPBW}}}$ is a scaling factor so that $A_e/A^{max} = 1/2$ when $\theta = \Theta_{\text{HPBW}}$

Substituting into * and integrating, we have that the beam solid angle of a Gaussian beam is

$$\boxed{\Omega_A \equiv \left(\frac{\pi}{4ln2}\right) \Theta^2_{\text{HPBW}} \sim 1.133\Theta^2_{\text{HPBW}}}$$

Reflector accuracy requirements

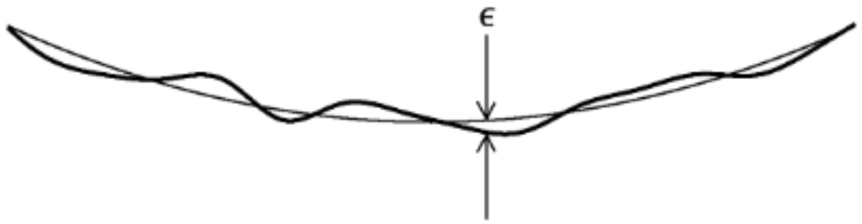
Real radio telescopes don't have perfectly smooth paraboloidal reflectors. Deviations from the paraboloid may be caused by permanent manufacturing errors, changing gravitational deformations as the reflector is tilted, thermal distortions resulting from solar heating, and bending by strong winds.

We have introduced the concept of aperture efficiency, defined as:

$$\eta = \frac{A_e^{\max}}{A_g}$$

actual antenna efficiency

geometric cross-sectional area of the main reflector (perfect paraboloid)



Where the actual reflector surface deviates from the perfect paraboloid by a distance ϵ, the path length of the reflected wave will be in error by ~2ϵ, and the phase error will be

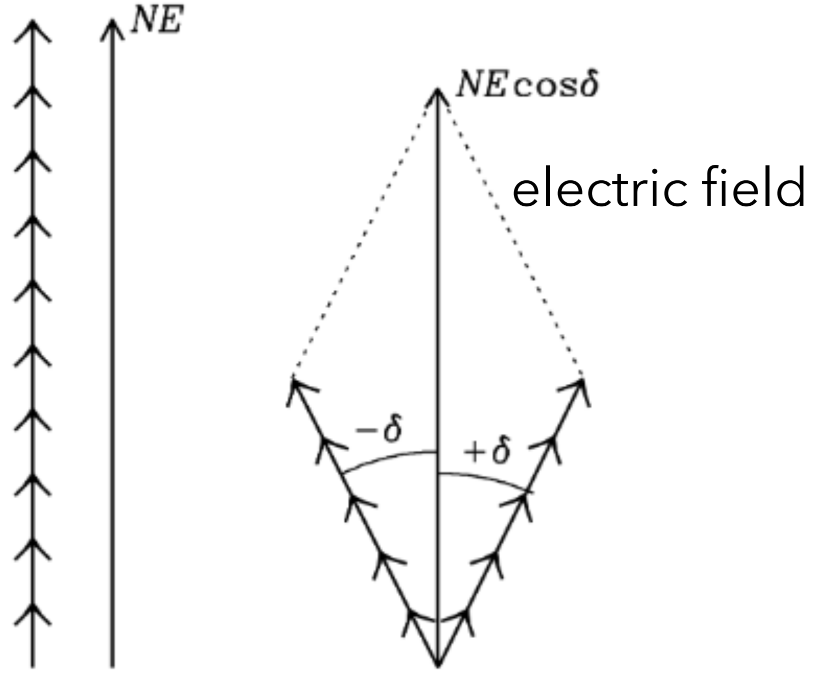
$$\delta \sim \frac{2\pi}{\lambda} 2\epsilon = \frac{4\pi\epsilon}{\lambda}$$

The contribution of each area element to the far electric field will be reduced by a factor

$$\frac{E(\delta)}{E(0)} = cos\delta \sim 1 - \frac{\delta^2}{2} + \dots$$

in the limit $\delta << 1$ rad

$$\frac{A(\delta)}{A_g} = \sim \left(\frac{E(\delta)}{E(0)}\right)^2 \sim 1 - \delta^2 \sim 1 - \left(\frac{4\pi\epsilon}{\lambda}\right)^2$$



perfect aperture imperfect aperture

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in the limit $\delta < < 1$ rad

$$\frac{A(\delta)}{A_g} = \sim \left(\frac{E(\delta)}{E(0)}\right)^2 \sim 1 - \delta^2 \sim 1 - \left(\frac{4\pi\epsilon}{\lambda}\right)^2$$

← surface errors must be << than the shortest usable wavelength

Reflector accuracy requirements

Real radio telescopes don't have perfectly smooth paraboloidal reflectors. Deviations from the paraboloid may be caused by permanent manufacturing errors, changing gravitational deformations as the reflector is tilted, thermal distortions resulting from solar heating, and bending by strong winds.

A more realistic scenario considers that surface errors have a roughly Gaussian probability distribution

$$P(\epsilon) = \frac{1}{\sqrt{2\pi\sigma}} exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) \qquad \boxed{\sigma = \text{ rms}}$$

$$< \frac{E}{E(0)} > = \int_{-\infty}^{\infty} cos\left(\frac{4\pi\epsilon}{\lambda}\right) \cdot \frac{1}{\sqrt{2\pi\sigma}} exp\left(-\frac{\epsilon^2}{2\sigma^2}\right) d\epsilon$$

reduction due to ϵ

distribution of ϵ

This is the Fourier transform of a Gaussian ($e^{ix} = cosx + isinx$, similarity theorem)…which is another Gaussian

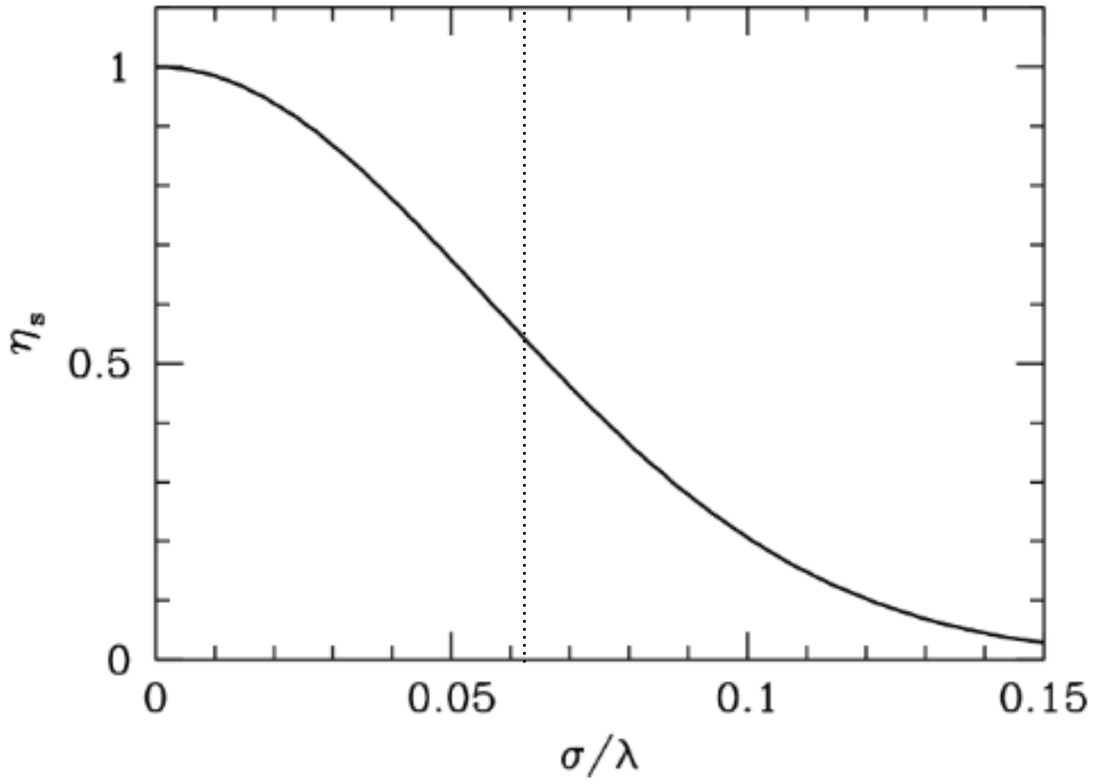
$$\boxed{< \frac{E}{E(0)} > = exp\left(-\frac{8\pi^2\sigma^2}{\lambda^2}\right)}$$

Reflector accuracy requirements

Power is proportional to E^2 so the reflector surface efficiency is

$$\boxed{\eta = exp[-\left(\frac{4\pi\sigma}{\lambda}\right)^2]}$$

Ruze equation



The surface efficiency is closely related to the Strehl ratio used by optical astronomers to specify the peak intensity loss caused by optical aberrations or atmospheric turbulence.

As a rule of thumb the shortest wavelength at which a radio telescope works reasonably well is

$$\sigma \sim \frac{\lambda_{min}}{16}$$

which corresponds to

$$\eta \sim exp[-\left(\frac{\pi}{4}\right)^2] \sim 0.54$$

For example in the case of IRAM 30m, which operates up to ~250 GHz or $\lambda_{min} \sim 1.2$ mm, the deviations from a perfect paraboloid must not exceed $\sigma \sim 75\mu m$, which is about the thickness of a paper sheet.

Pointing accuracy requirements

Real radio telescopes don't have perfectly accurate pointing. Small errors in tracking a target source reduce the aperture efficiency in the source direction and contribute to the uncertainty in flux-density measurements of compact sources. Tracking errors are as important as surface errors in limiting the short-wavelength performance of large radio telescopes.

$$\frac{A_e}{A^{max}} = exp(-\frac{4ln2\rho^2}{\Theta^2_{HPBW}})$$

(ρ offset angle with respect to beam axis)

If the one-dimensional tracking error in each coordinate (e.g., azimuth or elevation angle) has a Gaussian distribution with rms σ

$$P(\rho) = \frac{\rho}{\sigma^2} exp\left(-\frac{\rho^2}{2\sigma^2}\right)$$

Rayleigh distribution

The mean squared tracking error is

$$\boxed{< \rho^2 >} = \int_0^\infty \rho^2 P(\rho) d\rho = \boxed{2\sigma^2}$$

So the average loss in antenna efficiency is

$$< \frac{A_e}{A^{max}} > = \frac{1}{1 + 4ln2\left(\frac{\sqrt{2}\sigma}{\Theta_{HPBW}}\right)^2}$$

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The fluctuating on-source antenna efficiency caused by tracking errors contributes a fractional uncertainty

$$\boxed{\frac{\sigma}{S} = \frac{z}{(1+2z)^{1/2}}}$$

S source flux density

$$z = 4ln2\left(\frac{\sigma}{\Theta_{\rm HPBW}}\right)^2$$

Thus an rms tracking error of $0.2\Theta_{\rm HPBW}$ will contribute to a 10% rms flux density uncertainty. $0.14\Theta_{\rm HPBW}$ will result in 5% uncertainty.

IRAM 30-meter (Pico Veleta, ES)

250 GHz ~1.2×10^{-3}m

$$\Theta_{\rm HPBW} \sim \frac{1.2\times10^{-3}}{30} \sim 4\times10^{-5}\ \mathrm{rad} \sim 8.3''$$

To achieve rms flux uncertainties <~5%, the total tracking error must be smaller than 1.1 ".

For a steel antenna a differential temperature variation of 1° corresponds to a pointing shift of ~2". Pointing has to be calibrated frequently!

Instruments for Radioastronomy

Why Interferometers?

In (radio)astronomy, we aim to know the angular distribution of the electromagnetic emission coming from a source in the sky. This means that we are interested in the brightness of the emission

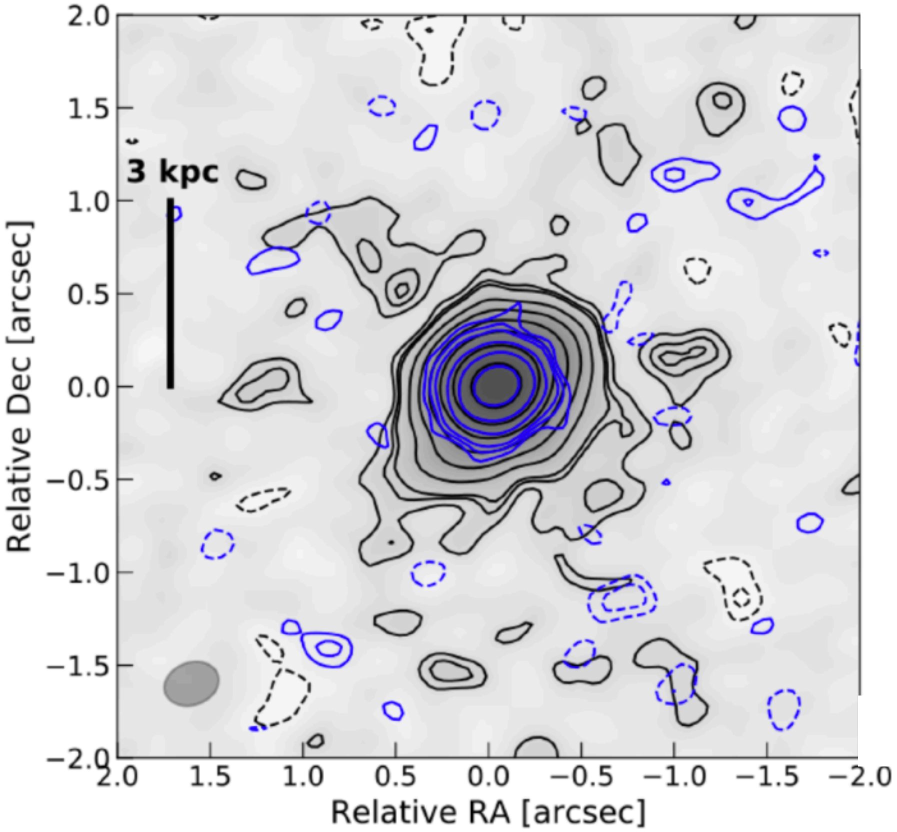
I_ν [W m^{-2} Hz^{-1} **sr^{-1}**]

and not only in its total flux

$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \phi) cos\theta d\Omega$$ [W m^{-2} Hz^{-1}] Ω_s solid angle subtended by the source

Measuring the brightness of a source means making a "map" of the source

Bischetti et al. 2019a



ALMA map of **CO(3-2)** and **blackbody continuum emission** in the galaxy of PDS456

z =0.185

$\Theta_{\rm HPBW} \sim 0.2$ arcsec

$\nu \sim 290{\rm GHz}(\lambda \sim 1{\rm mm})$

Instruments for Radioastronomy

Why Interferometers?

In (radio)astronomy, we aim to know the angular distribution of the electromagnetic emission coming from a source in the sky. This means that we are interested in the brightness of the emission

I_ν [W m^{-2} Hz^{-1} **sr^{-1}**]

and not only in its total flux

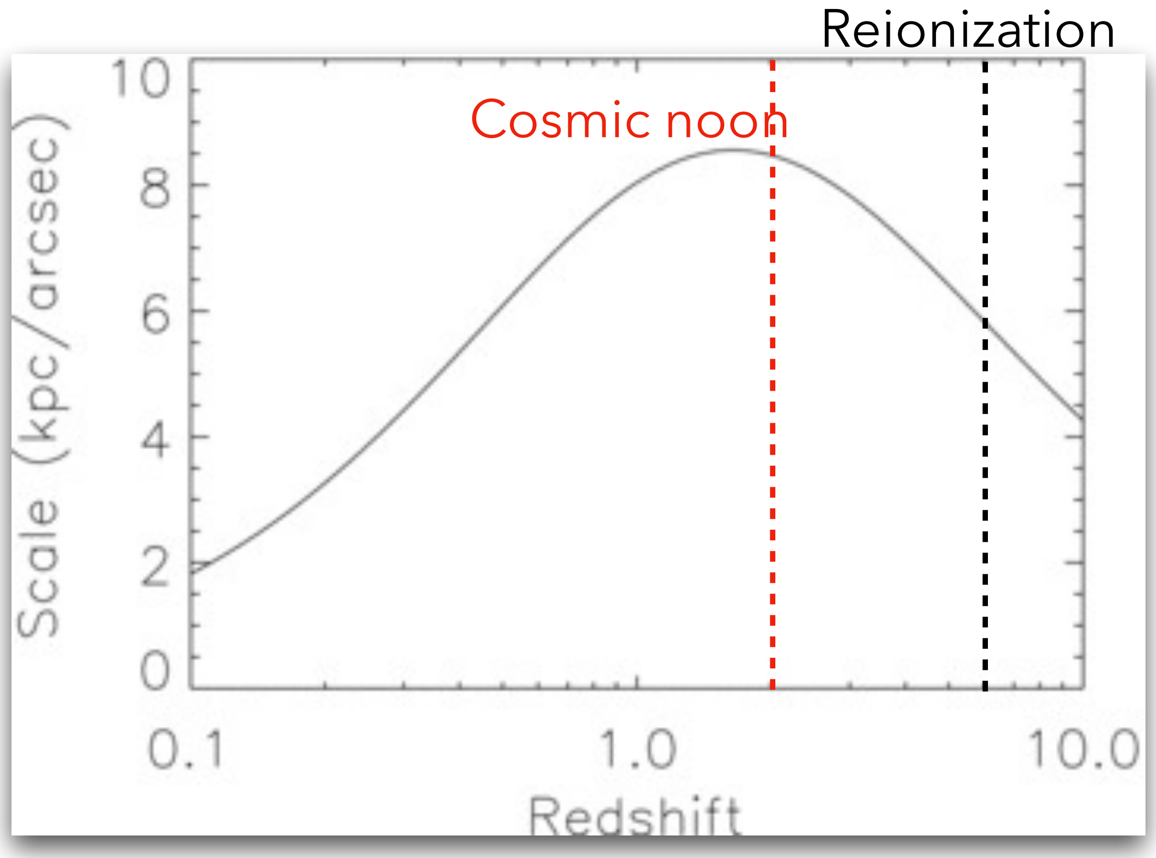
$$S_\nu = \int_{\Omega_s} I_\nu(\theta, \phi) cos\theta d\Omega$$

[W m^{-2} Hz^{-1}] Ω_s solid angle subtended by the source

Measuring the brightness of a source means making a "map" of the source

Because targets are distant, emission is extremely weak and of small angular size.

High-angular resolution (<< arcsec) observations are needed to map high-redshift sources!



Aperture synthesis

We have seen that even the largest single-dish radio telescopes have relatively low angular resolution, especially at low frequency:

$$\Theta_{\rm HPBW} \sim \frac{\lambda}{D} \quad \text{[rad]}$$

Impossibly large diameters would be needed to achieve (sub-)arcsecond resolution at radio wavelengths.

As an example, to observe the H 21 cm emission with 1 arcsec resolution, an aperture of 42 km would be needed! The currently largest, fully-steerable apertures are only 100m (Effelsberg telescope DE, Green Bank telescope US).

We have also seen that it is problematic to satisfy reflector requirements (deviations from a perfect paraboloid of $\lesssim 100\ \mu$m) and pointing/tracking accuracy requirements (at best ~arcsec) in the case of large single dishes.

Can we synthesize an aperture of km size with pairs of antennas?

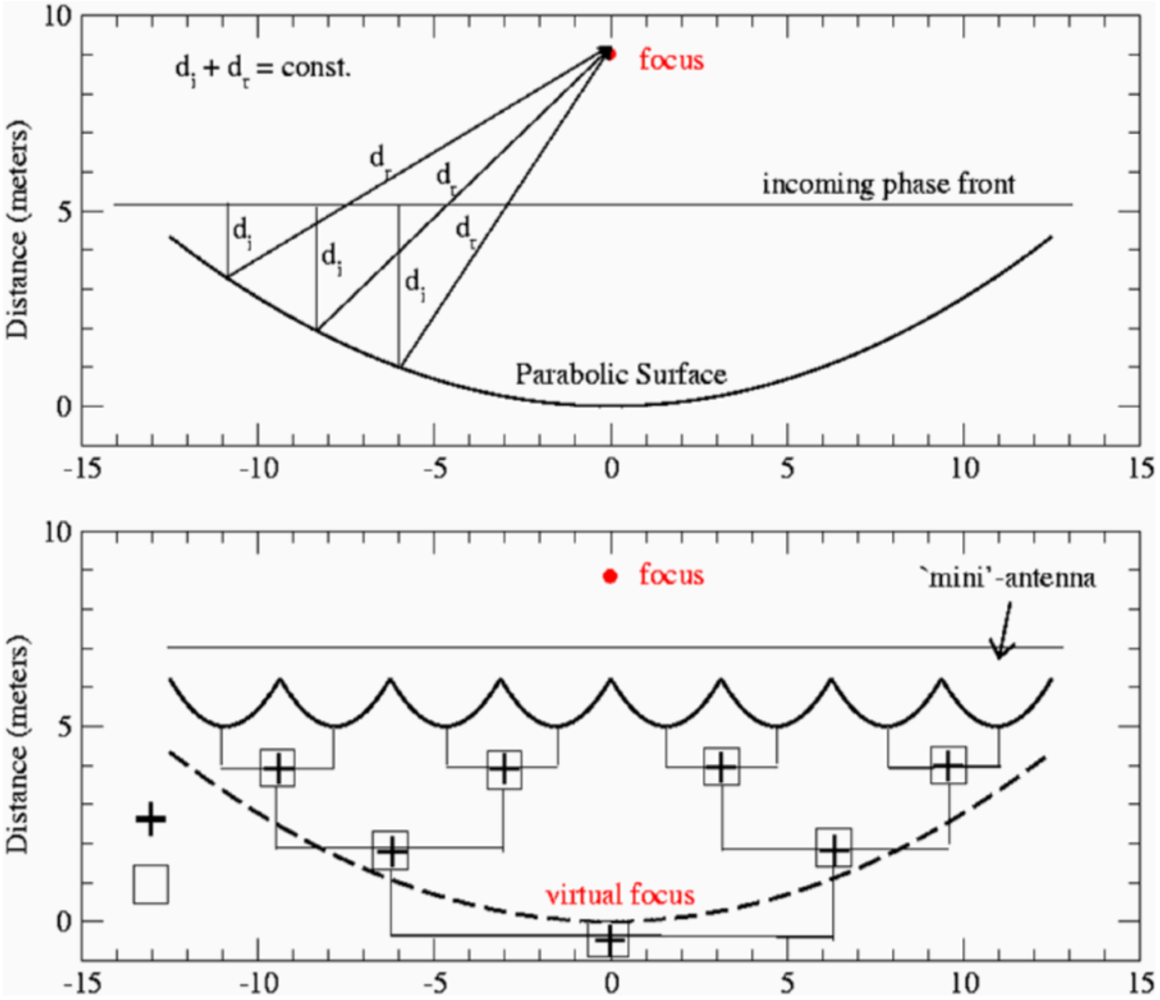
YES: the technique of synthesizing a larger aperture through combination of separated pairs of antennas is called **aperture synthesis**

Interferometry: basic concepts

We have seen that a parabolic dish coherently sums all electromagnetic fields at the focus

The same result can be achieved by adding in a network voltages from individual antennas

This is the basic concept of interferometry. Aperture synthesis is an extension of this concept.



Interferometer: ensemble of $N \geq 2$ relatively small (easier to build and operate) dishes.

✳The collecting area of an interferometer is $N\pi D^2/4$ and can be arbitrarily increased as N is the # of antennas.

✳The angular resolution is $\Theta_{\mathrm{HPBW}} \sim \lambda / b_{max}$ where b_{max} is the longest baseline, i.e. the largest distance between two antennas in the array.

NOEMA (Plateau de Bure, FR)

Interferometry: basic concepts

✳The angular resolution is $\Theta_{\rm HPBW} \sim \lambda / b_{max}$ where b_{max} is the longest baseline, i.e. the largest distance between two antennas in the array.

Table 2: Planned 12-m Array Configuration

Start date	Configuration	Longest baseline
2022 October 1	C-3	0.50 km
2022 October 20	C-2	0.31 km
2022 November 10	C-1	0.16 km
2022 November 30	C-2	0.31 km
2022 December 20	C-3	0.50 km
2023 January 10	C-4	0.78 km
2023 February 1	*No observations d*	
2023 March 1	C-4	0.78 km
2023 March 20	C-5	1.4 km
2023 April 20	C-6	2.5 km
2023 May 20	C-7	3.6 km
2023 June 20	C-8	8.5 km
2023 July 11	C-9	13.9 km
2023 July 30	C-10	16.2 km
2023 August 20	C-9	13.9 km
2023 September 10	C-8	8.5 km

ALMA
interferometer

54 antennas

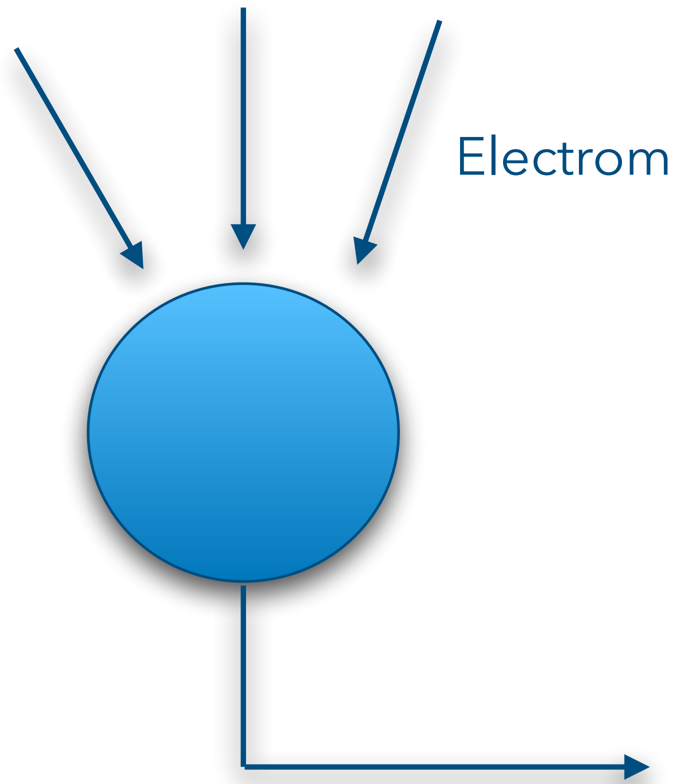
$$\Theta_{\rm HPBW} \sim \frac{\lambda}{b_{max}} \lesssim \frac{\overset{\text{band 1}}{0.007\text{m}}}{14000\text{m}} \sim 0.1''$$

Interferometry: basic concepts

Coherent interferometry is based on the ability to correlate electric fields measured at spatially separated locations (i.e. from different antennas)

Doing this (without mirrors) requires a conversion of the electric field $E(\mathbf{r}, \nu, t)$ at a given location $\mathbf{r}$ to a voltage $V(\nu, t)$ that can be conveyed to the receiver for processing

To this purpose, the sensor ("antenna") is simply a device with is sensitive to the electric field at a given location and converts it into a voltage retaining the amplitude and phase of electric field.



Electromagnetic waves in

Voltage out
(amplitude and phase of input fields are preserved)

The two-element quasi-monochromatic interferometer

The simplest radio interferometer is a pair of radio telescopes whose voltage outputs are correlated. Even the most elaborate interferometers with $N >> 2$ antennas can be treated as N(N-1)/2 independent two element interferometers.

antenna pairs

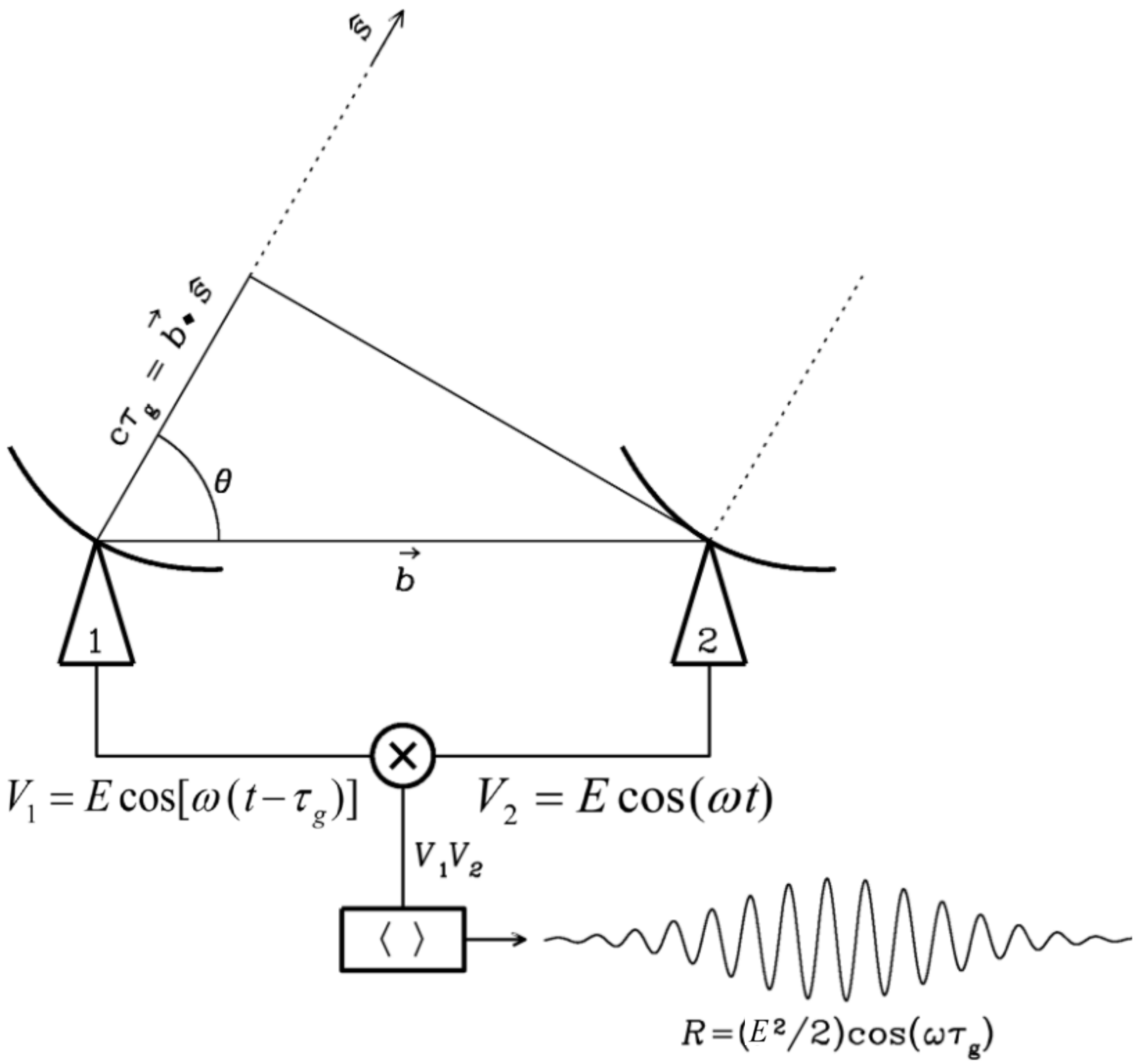
For simplicity, we consider a quasi-monochromatic interferometer that responds to radiation in a narrow band $\Delta\nu << \nu$ centered on frequency $\nu = \omega/(2\pi)$

Consider two dishes, separated by the baseline vector $\overrightarrow{b}$. Both dishes point the same direction $\hat{s}$.
θ is the angle between $\overrightarrow{b}$ and $\hat{s}$

Plane waves from a distant point source have to travel an extra distance $\overrightarrow{b} \cdot \hat{s} = bcos\theta$ to reach antenna 1, with respect to antenna 2.

The output of antenna 1 is therefore the same of antenna 2 but it lags in time by the geometric delay

$$\tau_g = \frac{\overrightarrow{b} \cdot \hat{s}}{c} = \frac{bcos\theta}{c}$$



The two-element quasi-monochromatic interferometer

The output voltages of antennas 1 and 2 at time t can be written as

$$V_1 = Ecos[\omega(t - \tau_g)]$$

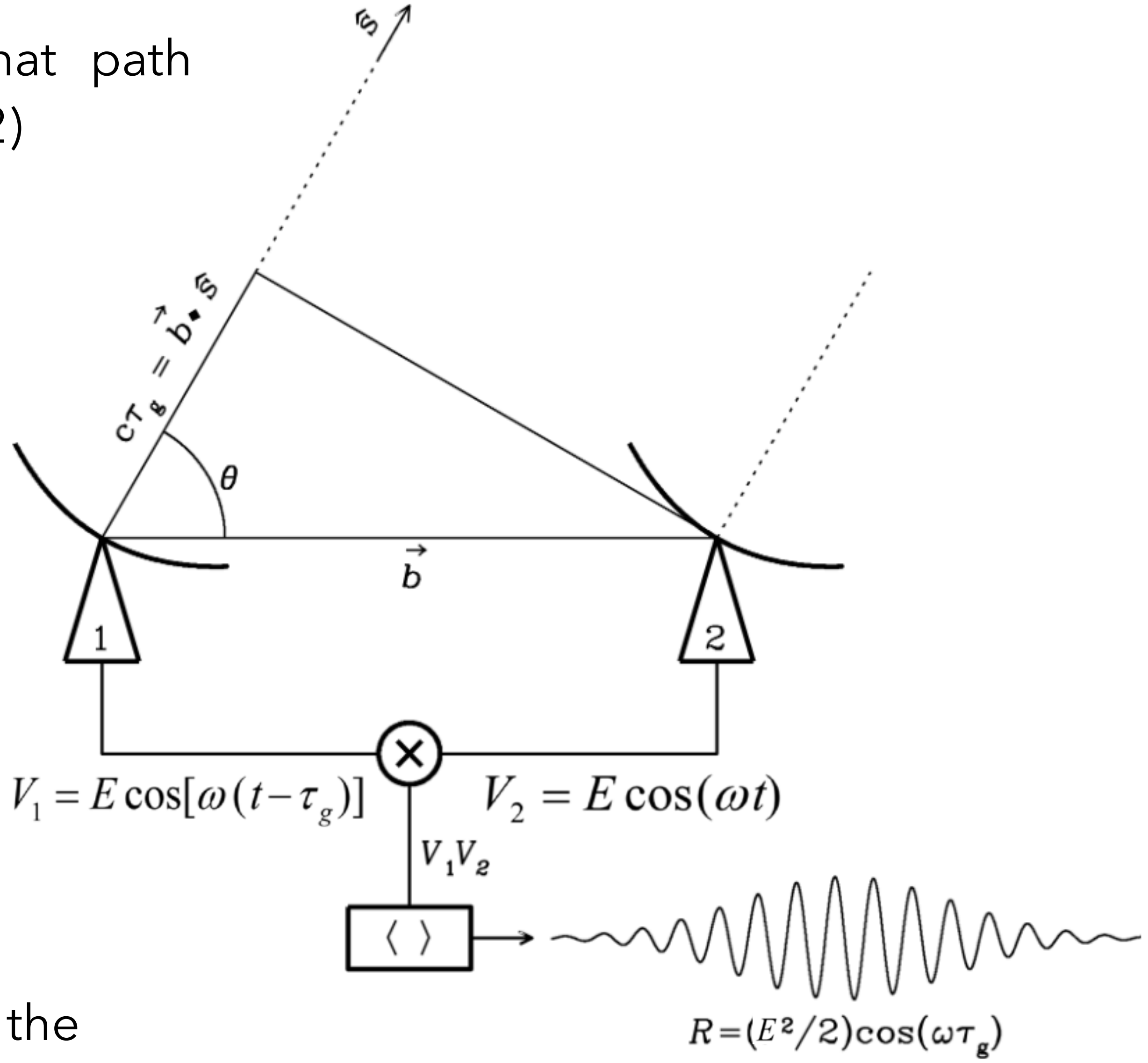
$$V_2 = Ecos(\omega t)$$

A correlator multiplies the two voltages (assuming that path lengths from sensors to the multiplier are equal for 1 and 2)

$$V_1V_2 = E^2cos[\omega(t - \tau_g)]cos(\omega t)$$

cosxcosy = [cos(x+y)+cos(x-y)]/2

$$= \frac{E^2}{2}[cos(2\omega t \underbrace{- \omega\tau_g)}_{\text{rapidly varying}} + \underbrace{cos(\omega\tau_g)}_{\text{slowly varying}}]$$



✳ the minimum step to have independent time samplings is $t \sim 1/(2\Delta\nu) \lesssim 10^{-3}$ s

✳τ_g varies slowly with time as Earth rotation changes the source direction relative to the baseline vector. $\tau_g = bcos\theta/c$ where θ varies on ~10^5 s

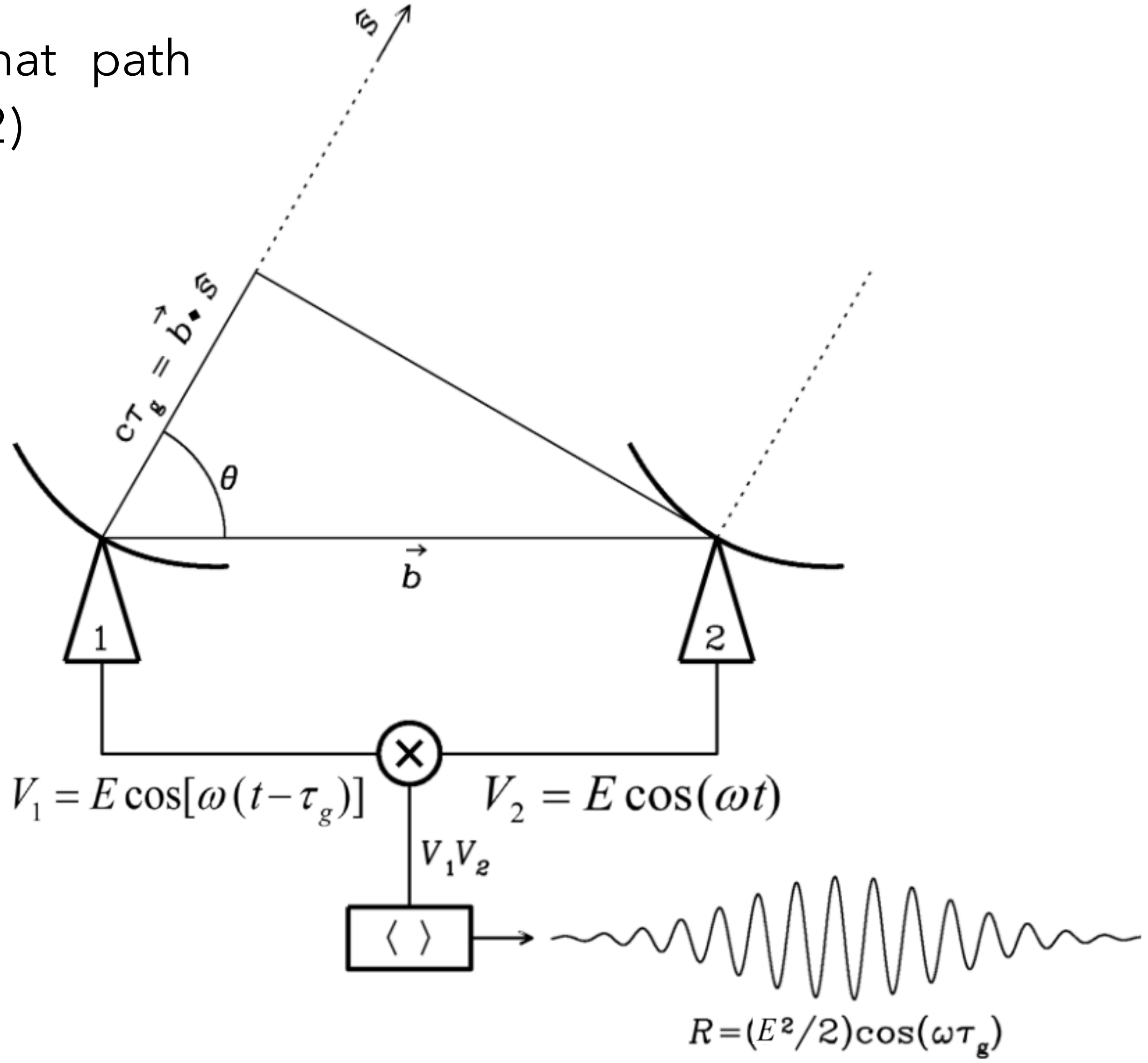
The two-element quasi-monochromatic interferometer

The output voltages of antennas 1 and 2 at time *t* can be written as

$$V_1 = Ecos[\omega(t - \tau_g)]$$

$$V_2 = Ecos(\omega t)$$

A correlator <u>multiplies</u> the two voltages (assuming that path lengths from sensors to the multiplier are equal for 1 and 2)



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cosxcosy = [cos(x+y)+cos(x-y)]/2

$$= \frac{E^2}{2}[cos(2\omega t \underbrace{- \omega\tau_g)}_{\text{rapidly varying}} + \underbrace{cos(\omega\tau_g}_{\text{slowly varying}})]$$

The correlator takes a time average long enough to remove the high-frequency term from the <u>correlator response</u> (output voltage)

$$\boxed{R = < V_1V_2 > = \frac{E^2}{2}cos(\omega\tau_g) = \frac{E^2}{2}cos\phi}$$

The correlator response is a measurement of the spatial correlation of the signal

The two-element quasi-monochromatic interferometer

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$$V_1 = Ecos[\omega(t - \tau_g)]$$

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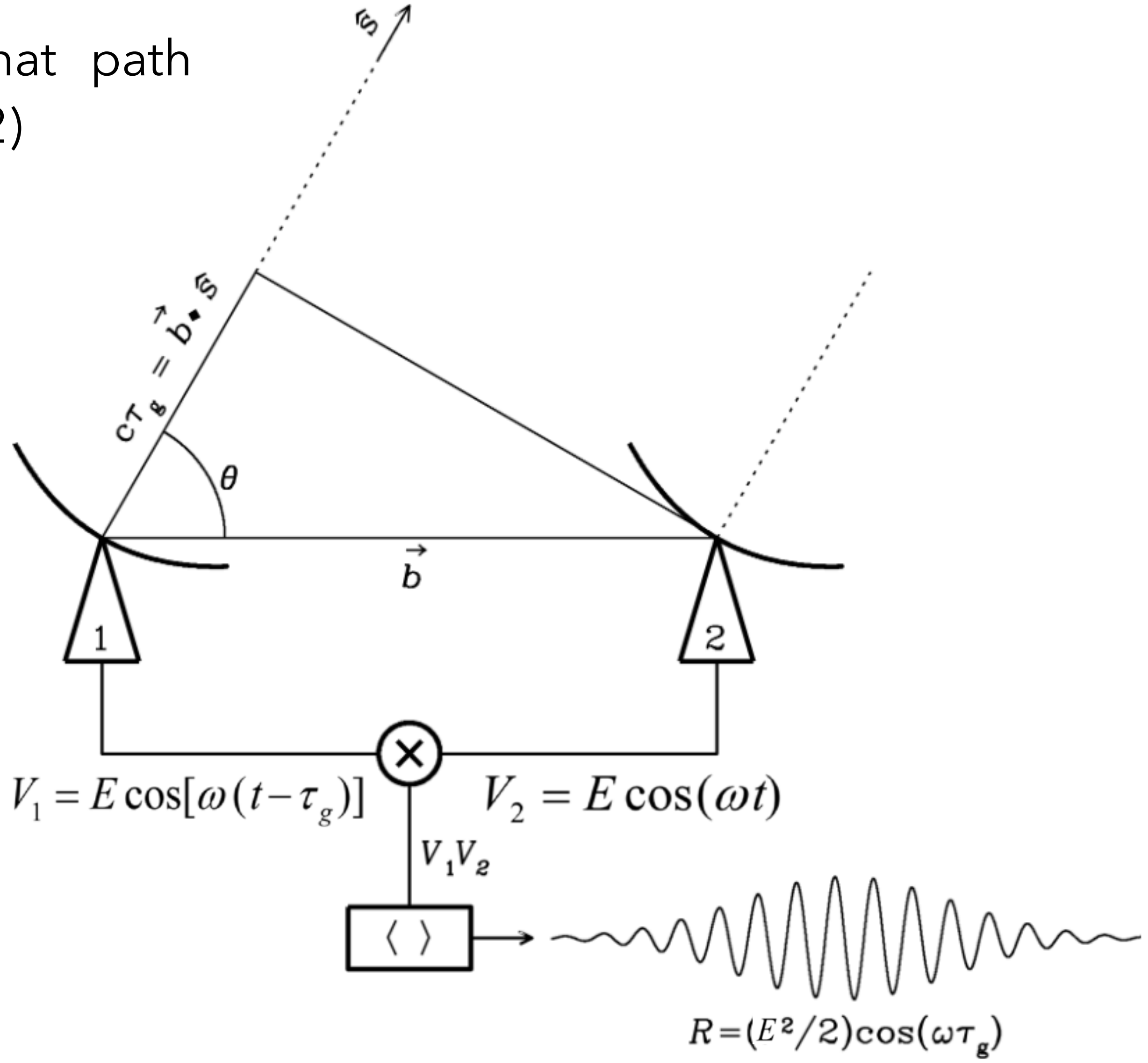

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The correlator response varies sinusoidally. This sinusoids are called fringes and ϕ is the fringe phase

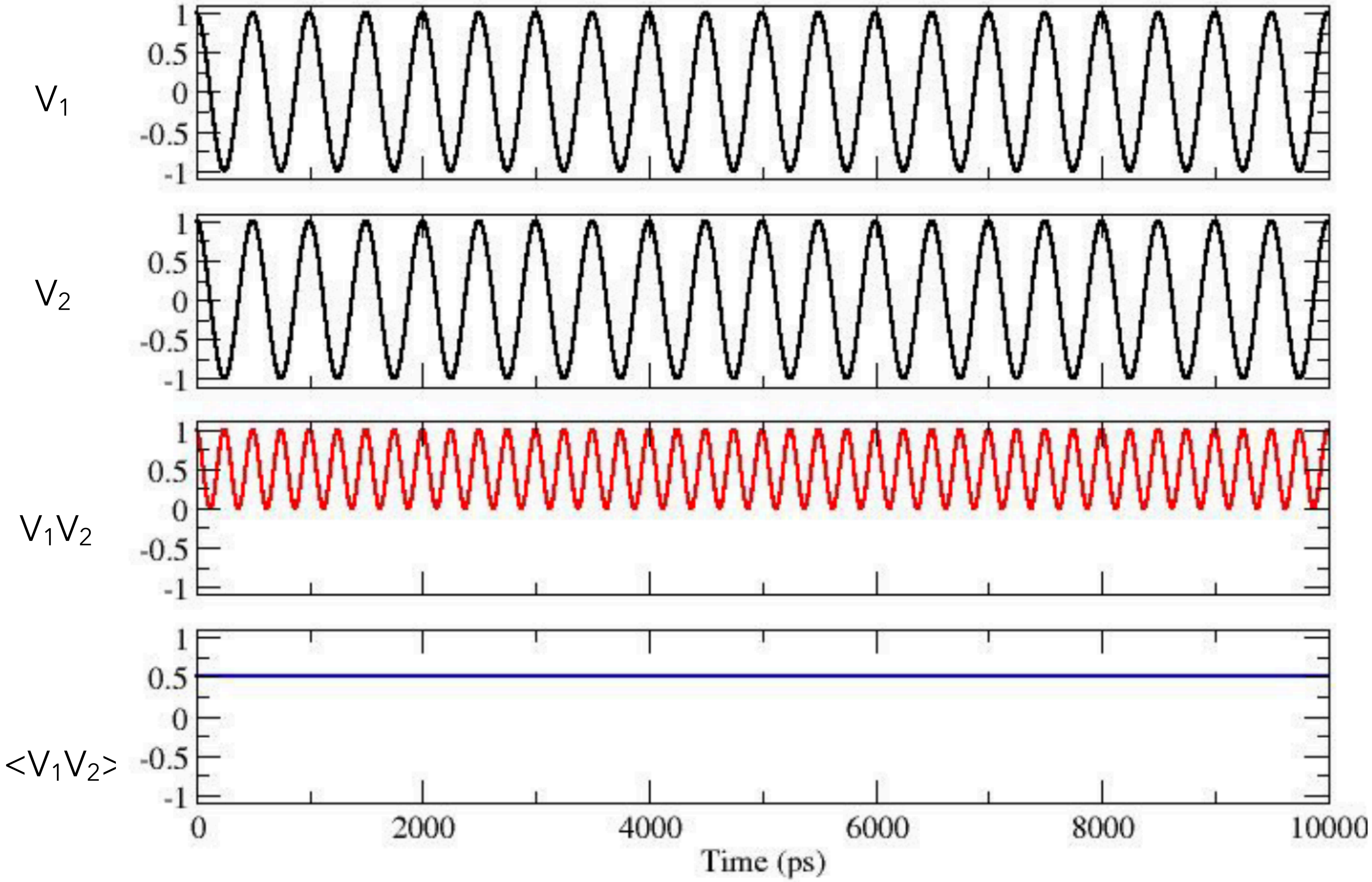
Example: signals in phase

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega \tau_g) = \frac{E^2}{2} cos\phi}$$

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}$$

Example: voltages in phase

$b \cdot s = n\lambda, \tau_g = n\nu$



$\nu = 2\text{GHz}$

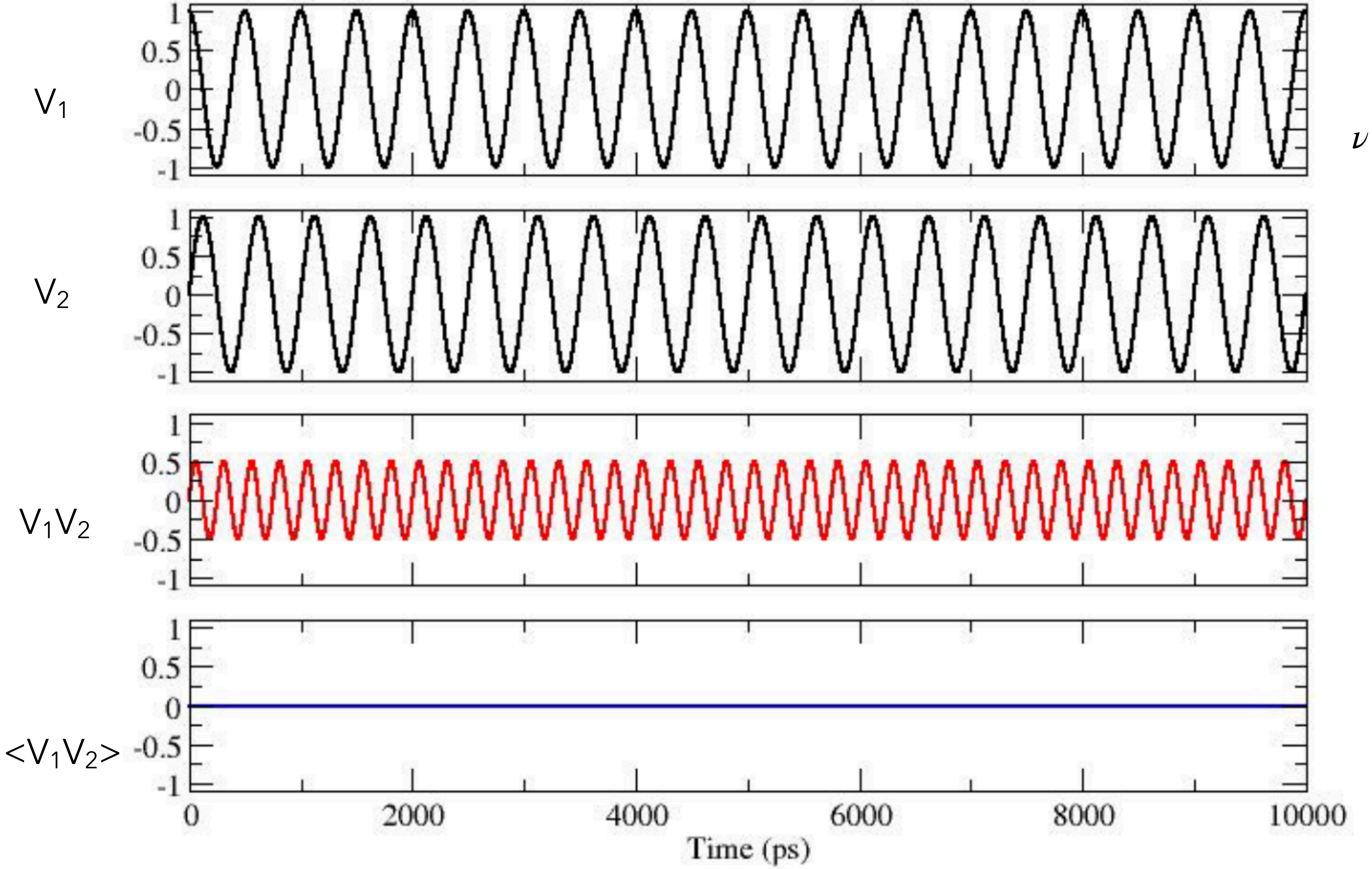
Example: signals in quadrature phase

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega\tau_g) = \frac{E^2}{2} cos\phi}$$

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}$$

Example: voltages in quadrature phase

$b \cdot s = (n \pm 1/4)\lambda, \tau_g = (4n \pm 1)/4\nu$



$\nu = 2\text{GHz}$

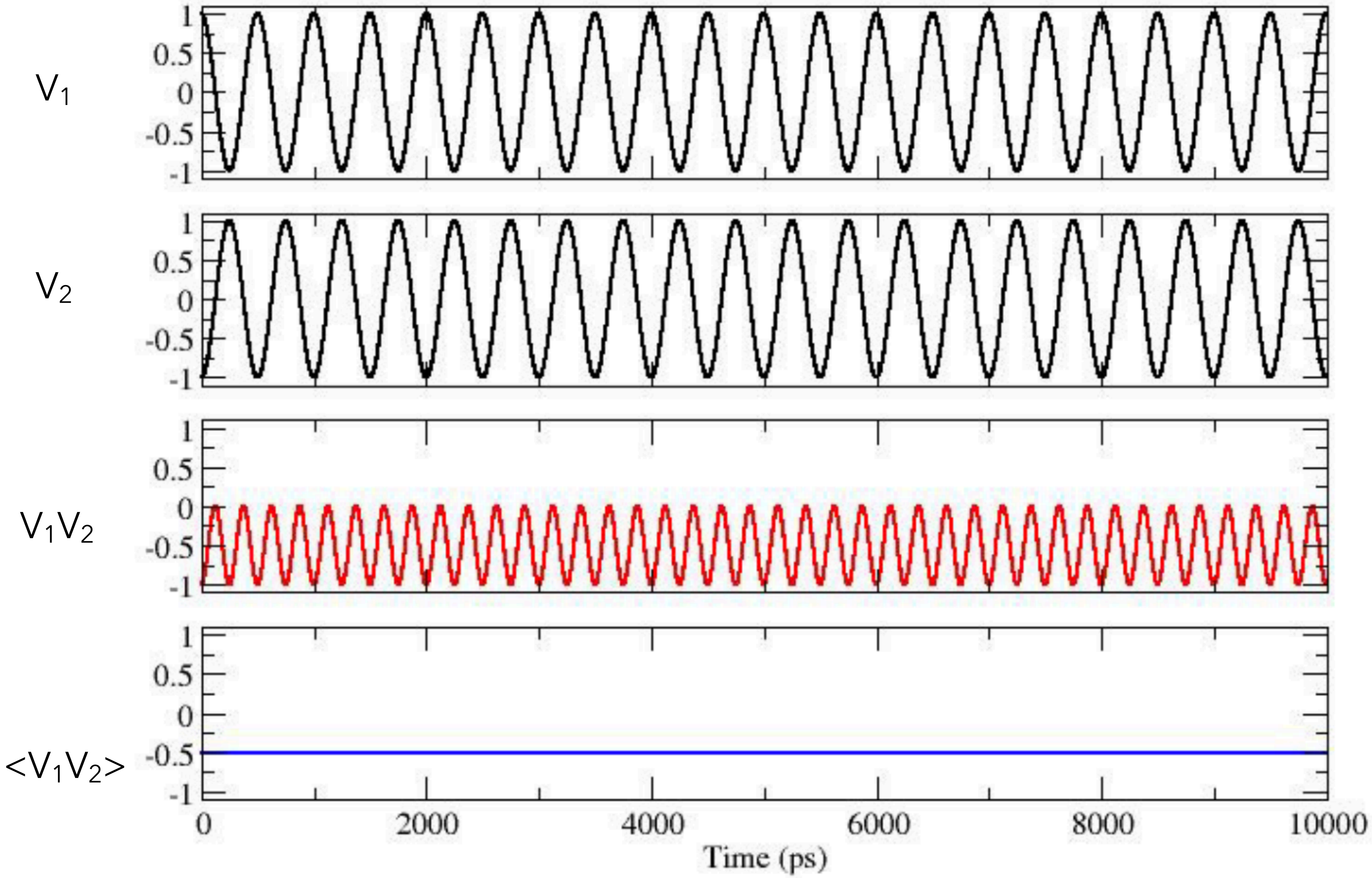
Example: signals out of phase

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega \tau_g) = \frac{E^2}{2} cos\phi}$$

$$\tau_g = \frac{\overrightarrow{b} \cdot \hat{s}}{c}$$

Example: voltages out of phase

$b \cdot s = (n \pm 1/2)\lambda, \tau_g = (2n \pm 1)/2\nu$



$\nu = 2\text{GHz}$