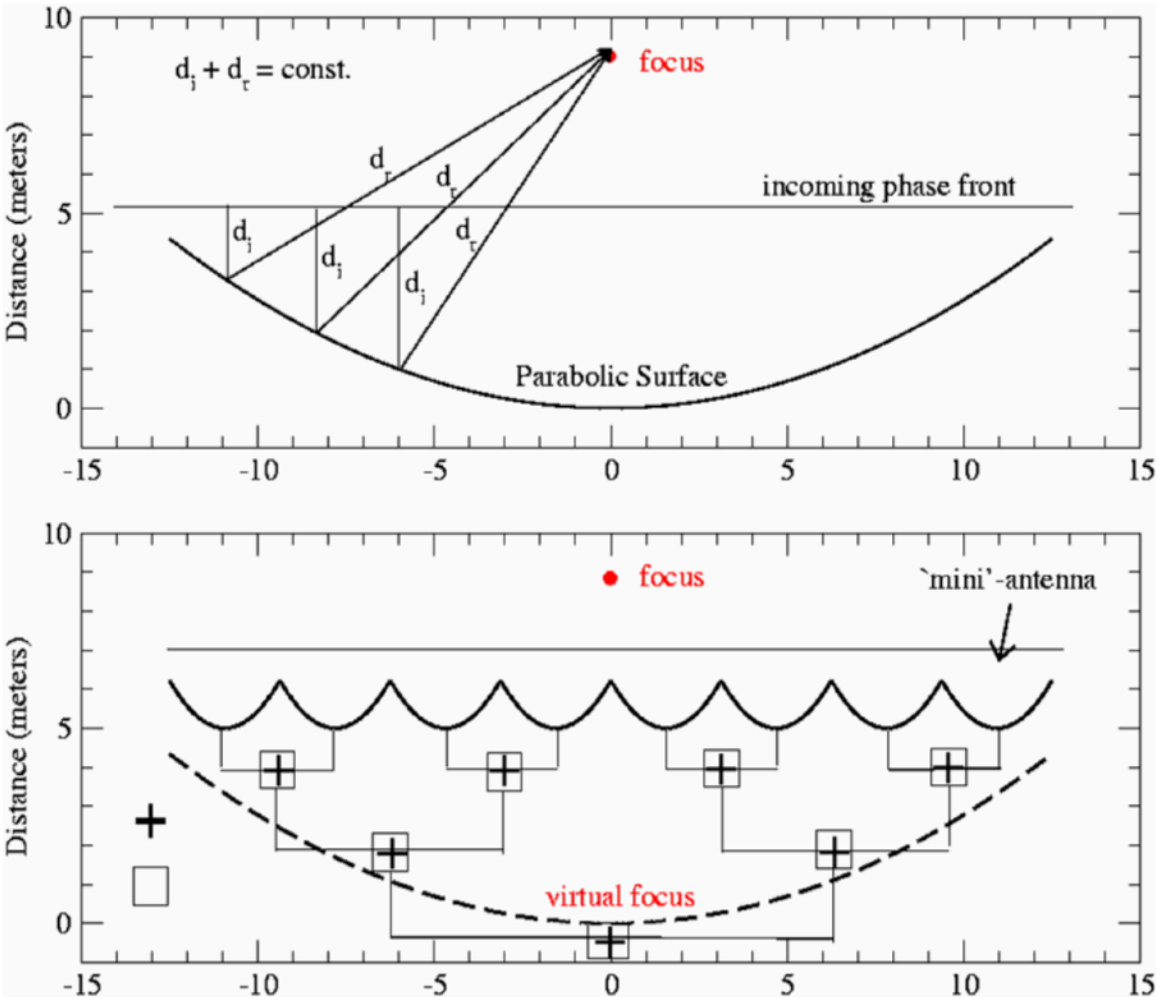
Instruments for Radioastronomy

Why Interferometers?

We have seen that a parabolic dish coherently sums all electromagnetic fields at the focus. The same result can be achieved by adding in a network voltages from individual antennas: interferometry.

Interferometer: ensemble of $N \geq 2$ dishes.



∗The collecting area of an interferometer is $N\pi D^2/4$ and can be arbitrarily increased as N is the # of antennas.

∗The angular resolution is $\Theta_{\text{HPBW}} \sim \lambda / b_{max}$ where b_{max} is the longest baseline, i.e. the largest distance between two antennas in the array.

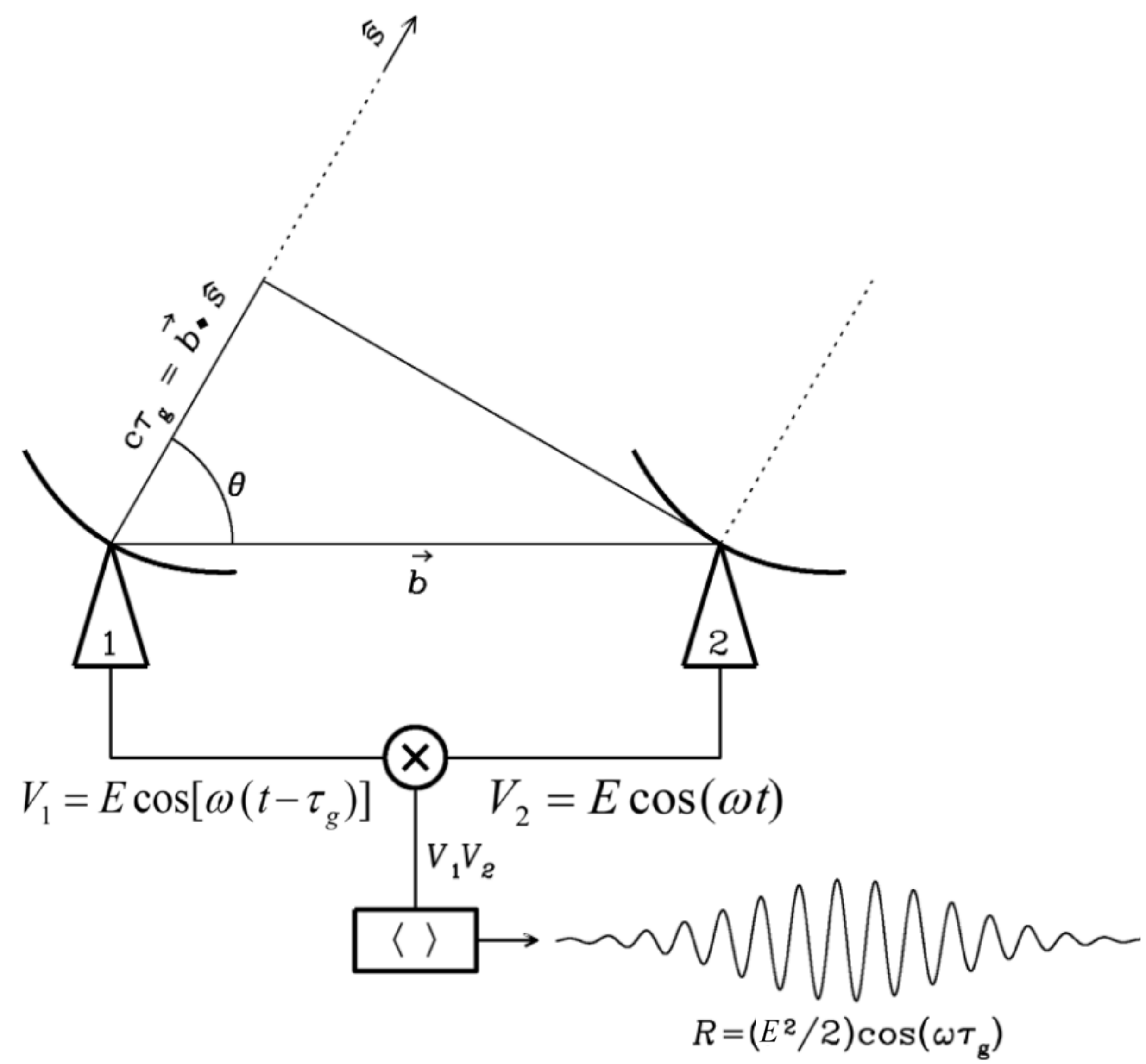
NOEMA (Plateau de Bure, FR)

The two-element quasi-monochromatic interferometer

The simplest radio interferometer is a pair of radio telescopes whose voltage outputs are correlated. More elaborate interferometers with $N >> 2$ antennas can be treated as N(N-1)/2 independent two element interferometers.

antenna pairs

We considered a quasi-monochromatic interferometer that responds to radiation in a narrow band $\Delta\nu << \nu$ centered on frequency $\nu = \omega/(2\pi)$.



The output of antenna 1 is therefore the same of antenna 2 but it lags in time by the geometric delay

$$\tau_g = \frac{\vec{b}\cdot\hat{s}}{c} = \frac{b cos\theta}{c}$$

Correlator response: spatial correlation of the signal

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega\tau_g)}$$

Sinusoidal variation: fringes

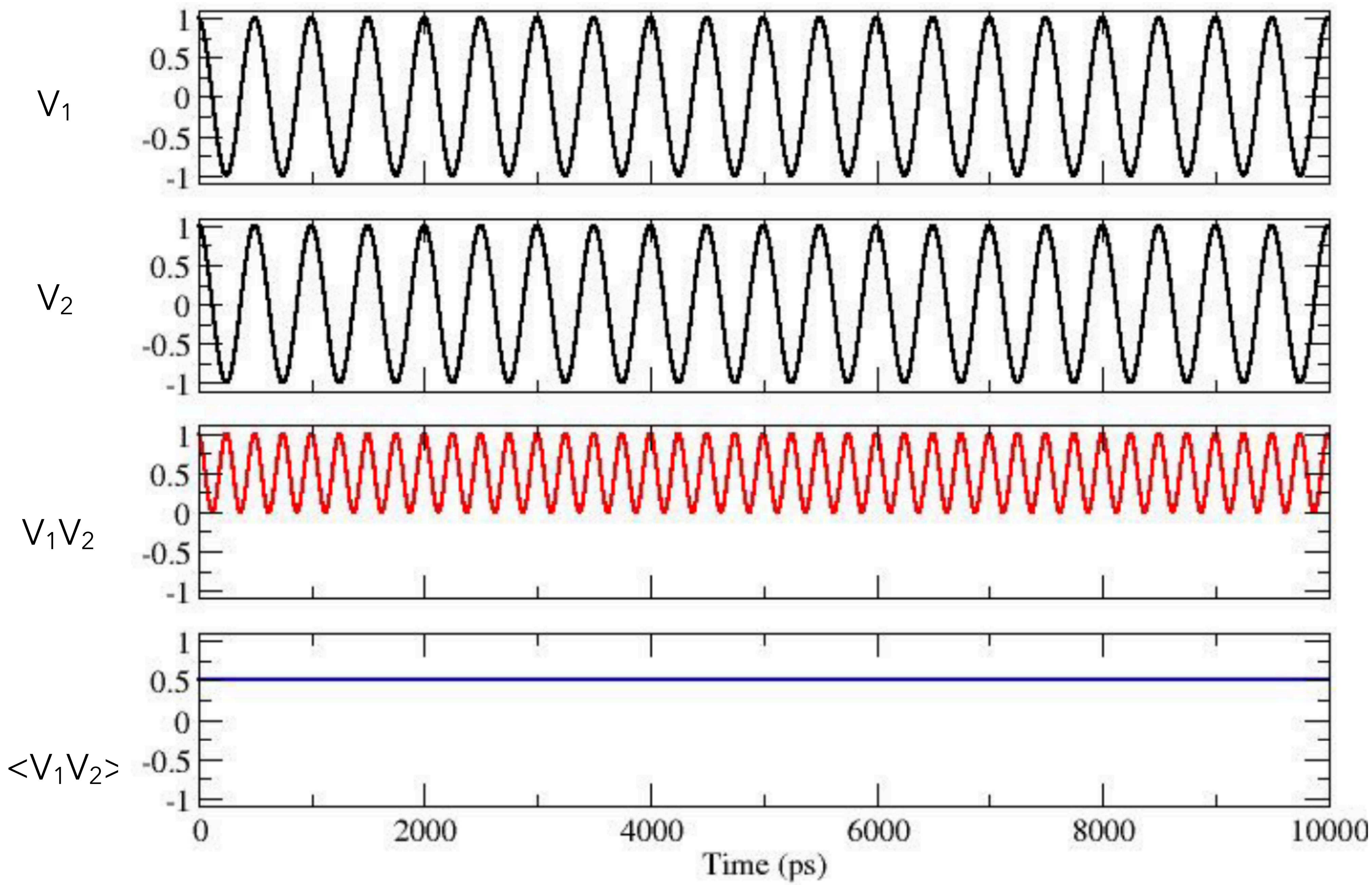
Example: signals in phase

$$\boxed{R = <V_1V_2> = \frac{E^2}{2}cos(\omega\tau_g)}$$

$$\tau_g = \frac{\vec{b}\cdot\hat{s}}{c}$$

Example: voltages in phase

$$b \cdot s = n\lambda, \tau_g = n\nu$$



$\nu = 2\text{GHz}$

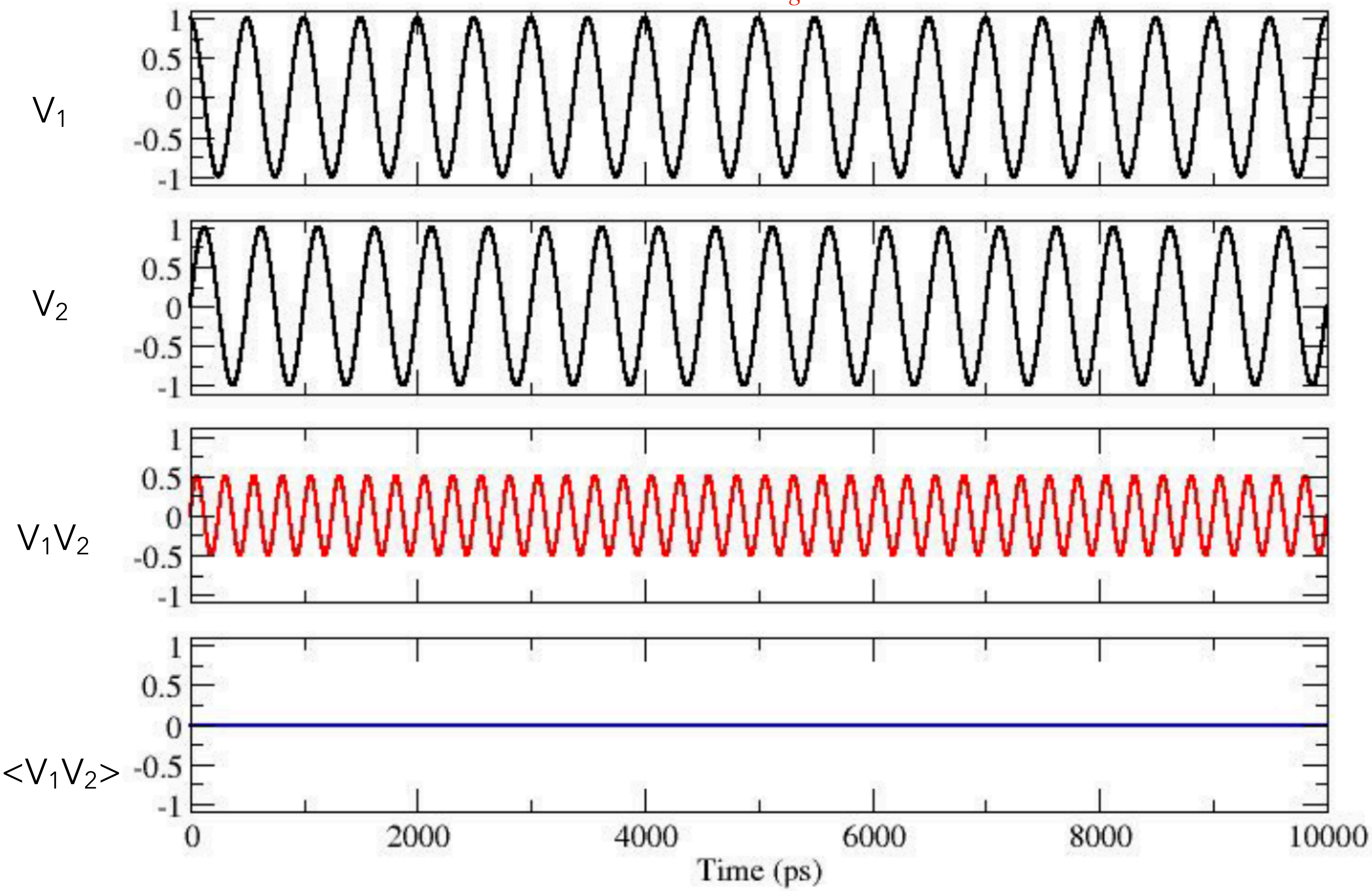
Example: signals in quadrature phase

$$\boxed{R = \langle V_1 V_2 \rangle = \frac{E^2}{2} cos(\omega\tau_g)}$$

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}$$

Example: voltages in quadrature phase

$b \cdot s = (n \pm 1/4)\lambda, \tau_g = (4n \pm 1)/4\nu$



$\nu = 2\text{GHz}$

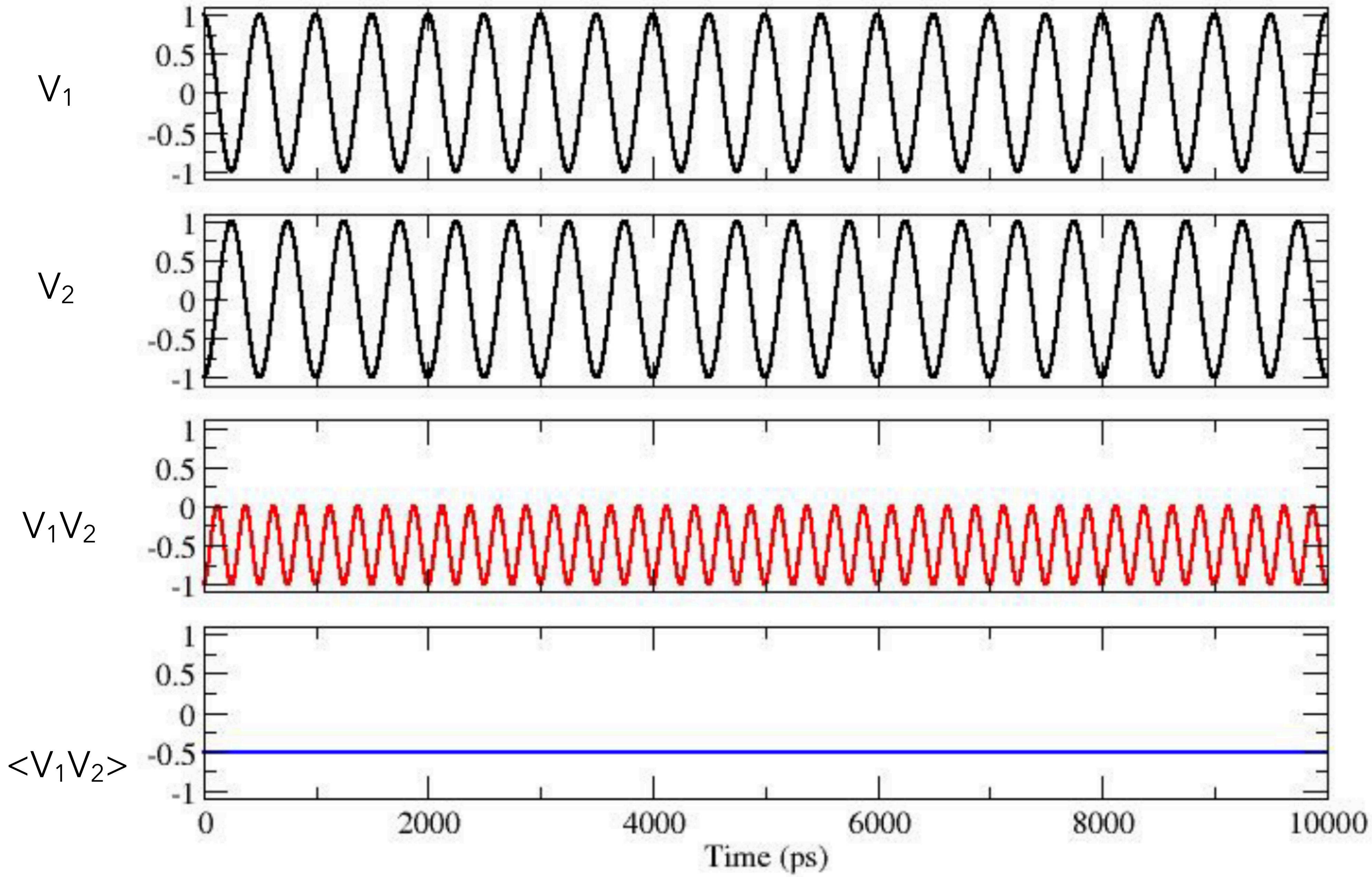
Example: signals out of phase

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega \tau_g)}$$

$$\tau_g = \frac{\overrightarrow{b} \cdot \hat{s}}{c}$$

Example: voltages out of phase

$b \cdot s = (n \pm 1/2)\lambda, \tau_g = (2n \pm 1)/2\nu$



$\nu = 2\mathrm{GHz}$

Fringe pattern: whole sky perspective

We can rewrite the correlator response as

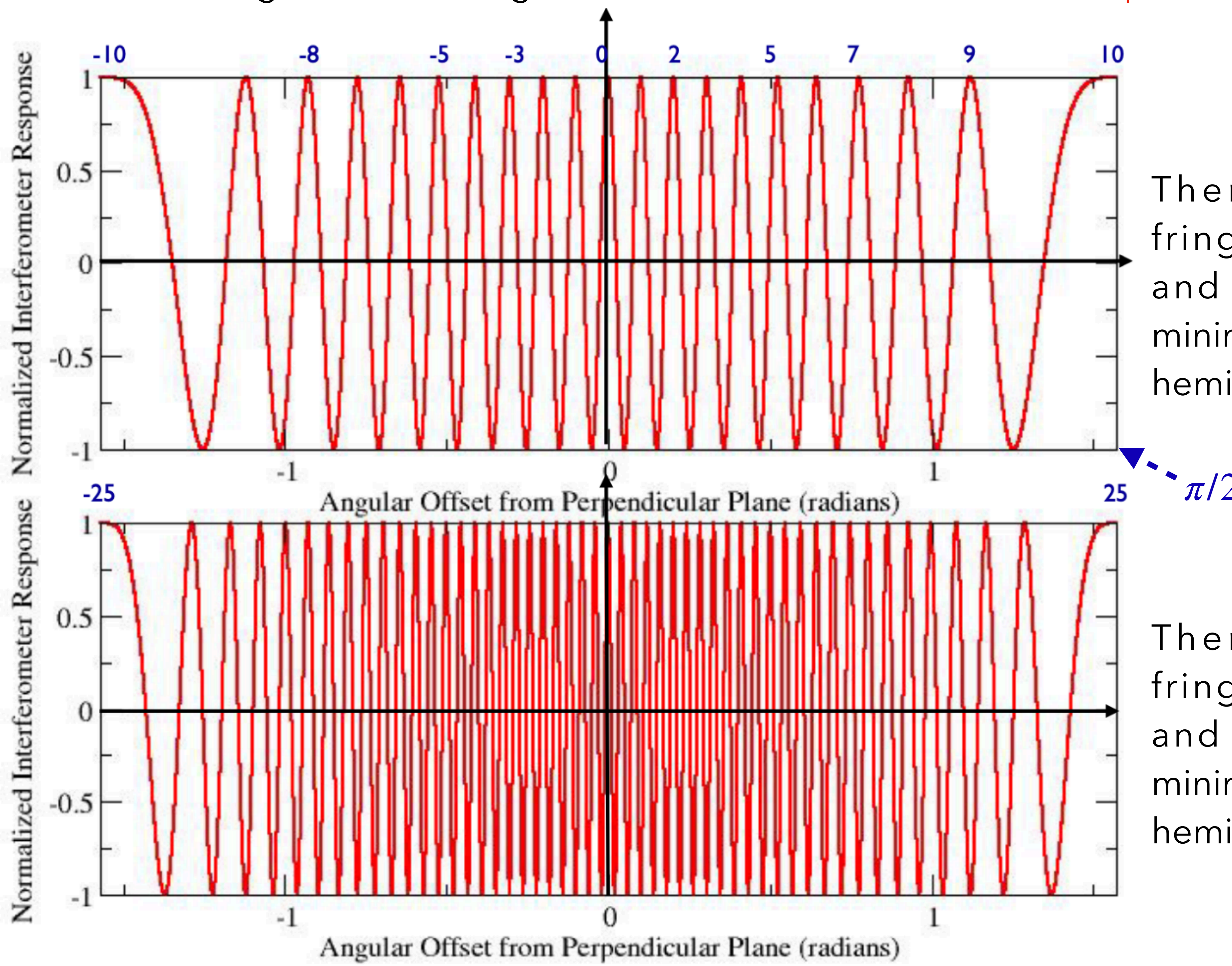
$$R = \frac{E^2}{2} cos(2\pi ul)$$

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c}$$

where $u = b/\lambda$ is the baseline lengths in wavelength units and $l = cos\theta$. How does this pattern look on the sky?

$u = 10$

$u = 25$



There are 21 fringe maxima and 20 fringe minima over the hemisphere

There are 51 fringe maxima and 50 fringe minima over the hemisphere

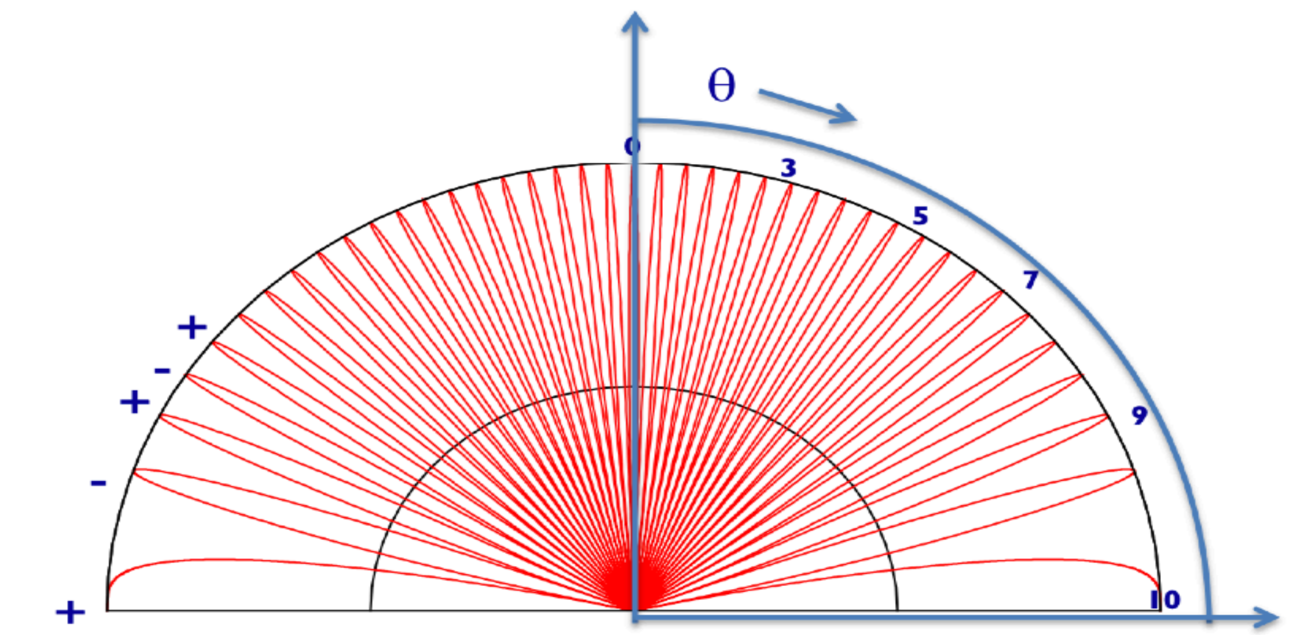
Fringe pattern: angular perspective

We can rewrite the correlator response as

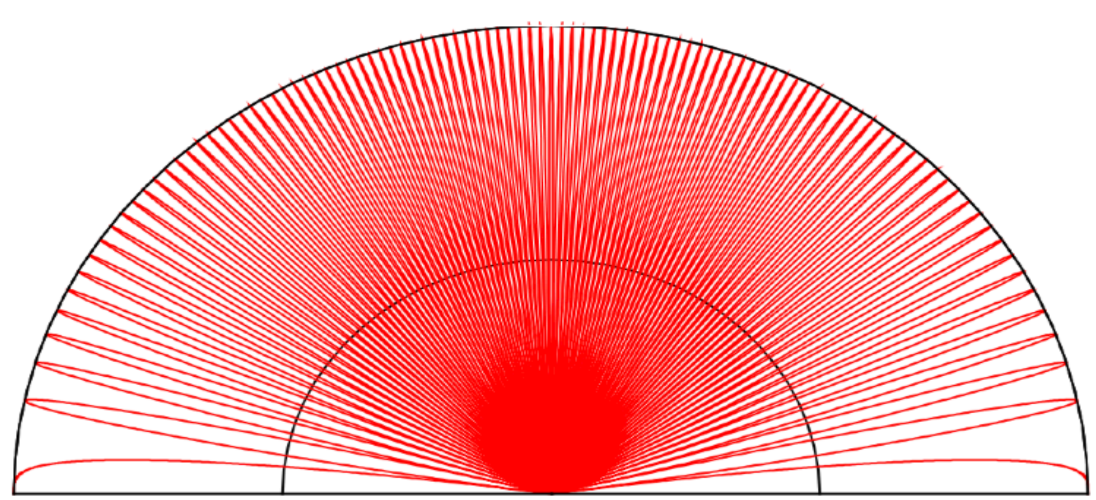
$$R = \frac{E^2}{2} cos(2\pi u l)$$

where $u = b/\lambda$ is the baseline lengths in wavelength units and $l = cos\theta$. How does this pattern look on the sky?

$u = 10$



$u = 25$



These patterns assume that the power pattern of the antennas was isotropic..but this is not true!

The separation between lobes of the same sign is $\delta\theta \sim 1/u = \lambda/b$ rad

Correlator response: properties

$$\boxed{R = < V_1 V_2 > = \frac{E^2}{2} cos(\omega\tau_g) = \frac{E^2}{2} cos\phi}$$

$$\tau_g = \frac{\vec{b} \cdot \hat{s}}{c} = \frac{bcos\theta}{c}$$

In general, the correlator response depends on:

- the received power $P \propto E^2$
- the geometric delay τ_g and hence on the baseline orientation and source direction

It does not depend on:

- the time of the observation, assuming that the source is not variable
- the location of the baseline, assuming the far field approximation
- the phase of the incoming signal (i.e. the distance of the source), assuming the far field approximation

We have seen that the electric field (or power) pattern depends on the antenna size and aperture efficiency, but these factors can be calibrated for.

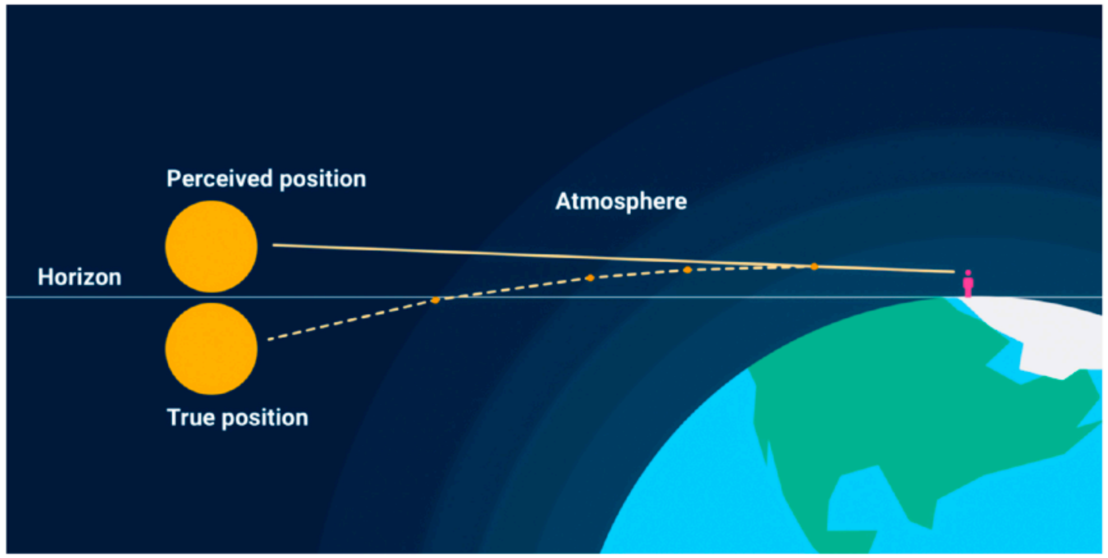
Fringe pattern and source position

The fringe phase $\phi = \omega\tau_g = \frac{\omega}{c}bcos\theta$ depends on θ as follows

$$\frac{d\phi}{d\theta} = -\frac{\omega}{c}bsin\theta = -2\pi\left(\frac{bsin\theta}{\lambda}\right)$$

The fringe period $\Delta\phi = 2\pi$ corresponds to an angular shift $\Delta\theta = \lambda/(bsin\theta)$. Therefore, the fringe phase is an accurate measurement of the source position if the projected baseline $bsin\theta >> \lambda$.

The fringe phase is not affected by tracking errors of individual antennas. It depends on time, and times can be measured with much higher accuracy than angles.



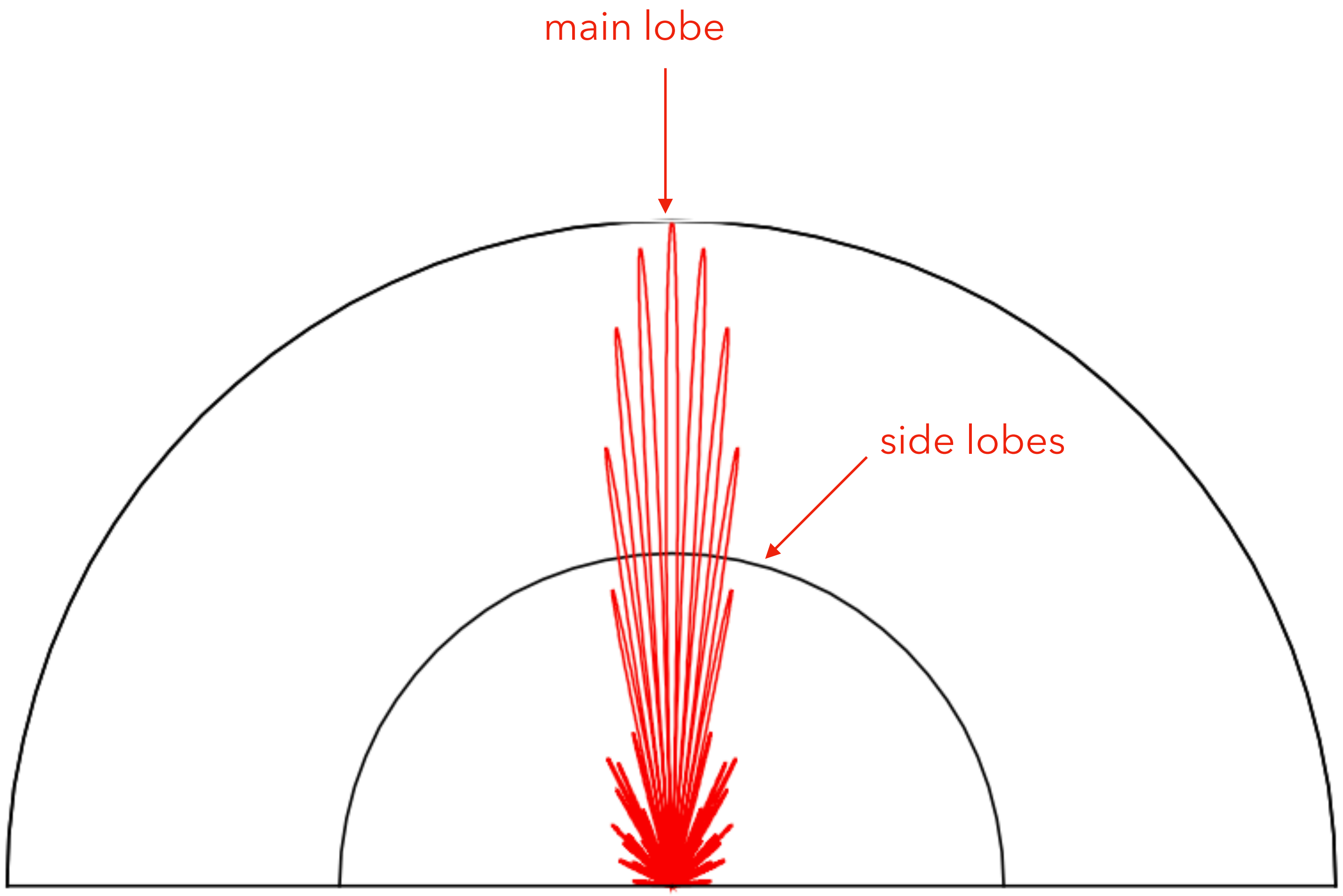
Also, an interferometer whose baseline is horizontal is not affected by the plane-parallel component of atmospheric refraction (Both V_1 and V_2 are delayed equally)

Interferometers can determine absolute positions with errors as small as 10^{-3} arcsec and differential positions down to 10^{-5} arcsec

Fringe pattern in the case of realistic antenna power patterns

In a more realistic case, the response a two-element interferometer with directive antennas is the *cos* sinusoid multiplied by the product of the voltage patterns of the individual antennas.

Normally the two antennas are identical, so this product is the power pattern of the individual antennas, i.e. the **primary beam** of the interferometer. The **synthesized beam**, that is the response obtained by averaging the output of all two-elements interferometers, rapidly approaches a Gaussian.



Fringes are modulated by the
~Gaussian response of an interferometer
(N>>1 antennas)

Fringe pattern in the case of realistic antenna power patterns

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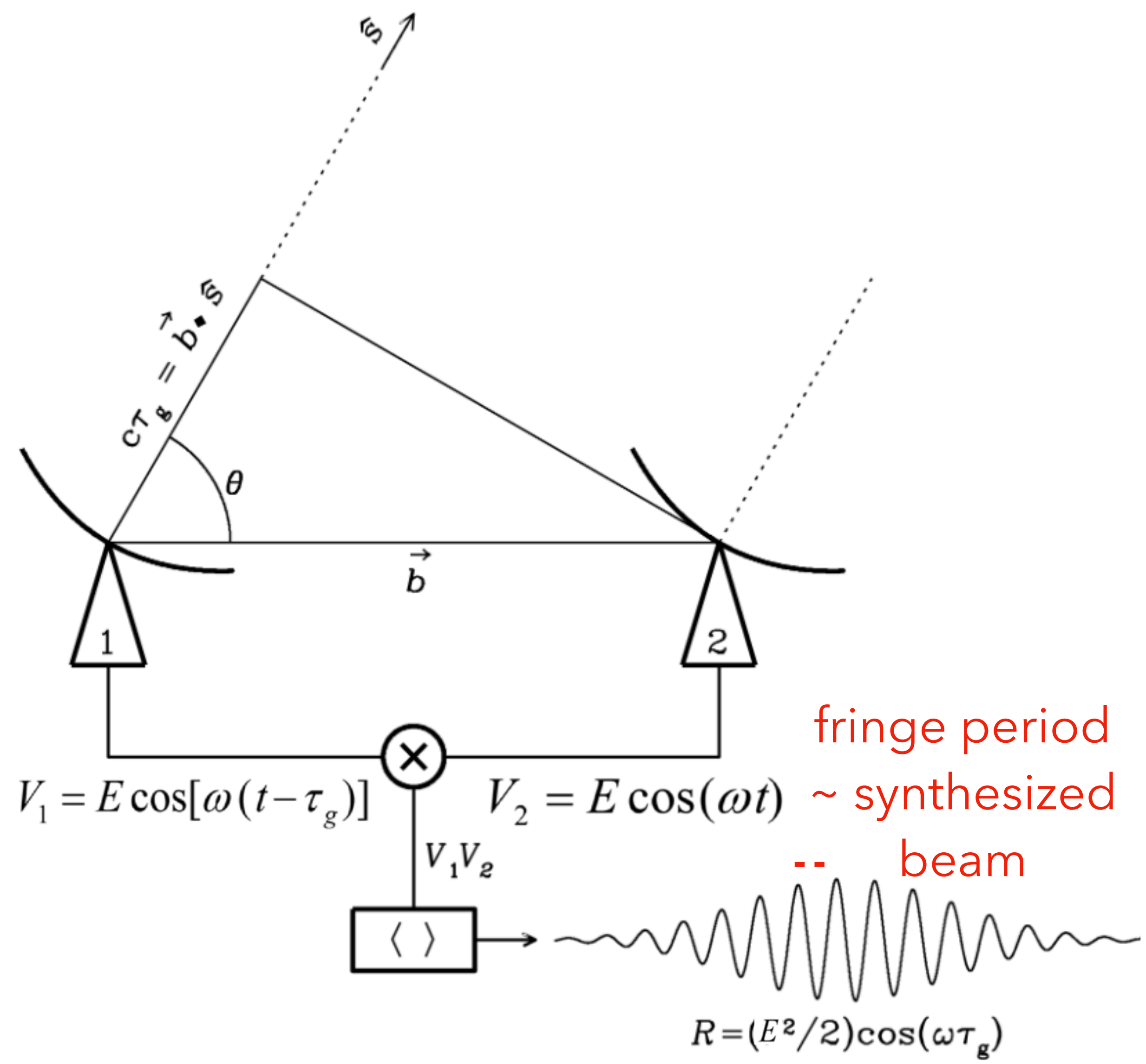
An interferometer with N antennas contains N(N-1)/2 pairs of antennas, each of which is a two-element interferometer.

As N increases, the **synthesized beam**, that is the point source response obtained by averaging the output of all two-elements interferometers, rapidly approaches a Gaussian.



fringe period
~ synthesized beam

beam width
~primary beam

Fringe pattern in the case of realistic antenna power patterns

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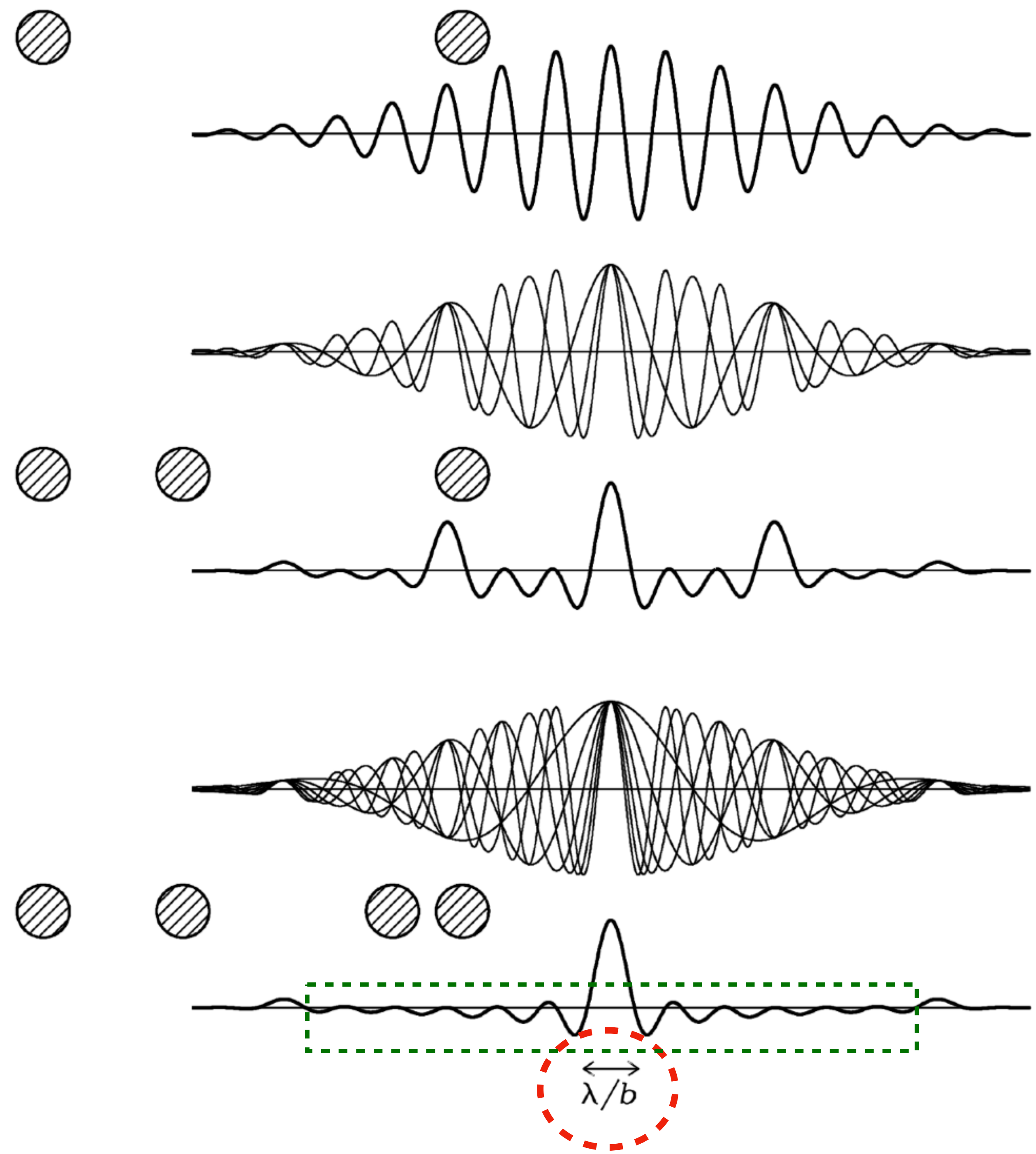
$\Theta_{\mathrm{HPBW}} \sim \lambda / b_{max}$ synthesized beam

$\Theta_{\mathrm{FOV}} \sim \lambda / D$ primary beam

due to the lack of spacings < *D*, where *D* is the diameter of an individual antenna (**zero spacing**)



Response for an extended source

We have so far derived the cosinusoidal correlator response for the quasi-monochromatic two-element correlator in the case of a point source. For simplicity, we assume again a uniform antenna response.

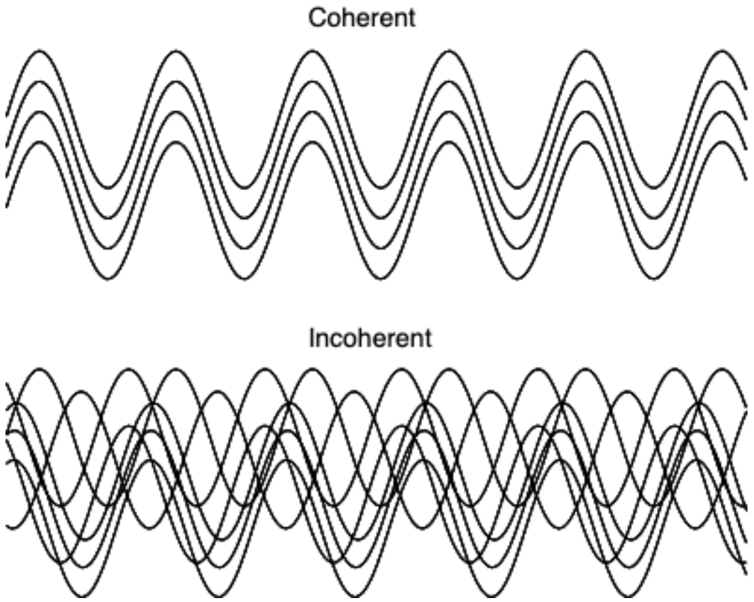
The response to an extended source, that is larger than the synthesized beam (but still smaller than the primary beam) can be derived as the same of many independent point sources.

$$R = < \int_{\Omega\text{source}} V_1 d\Omega_1 \int_{\Omega\text{source}} V_2 d\Omega_2 >$$

That can be written using the definition of brightness as (for an *incoherent source* $\cdot$ and $\int$ are interchangeable)

$$\boxed{R = \int_{\Omega\text{source}} I_\nu(\hat{s}) cos(2\pi\nu \overrightarrow{b} \cdot \hat{s}/c) d\Omega}$$

$$\tau_g = \frac{\overrightarrow{b} \cdot \hat{s}}{c}$$ geometric delay

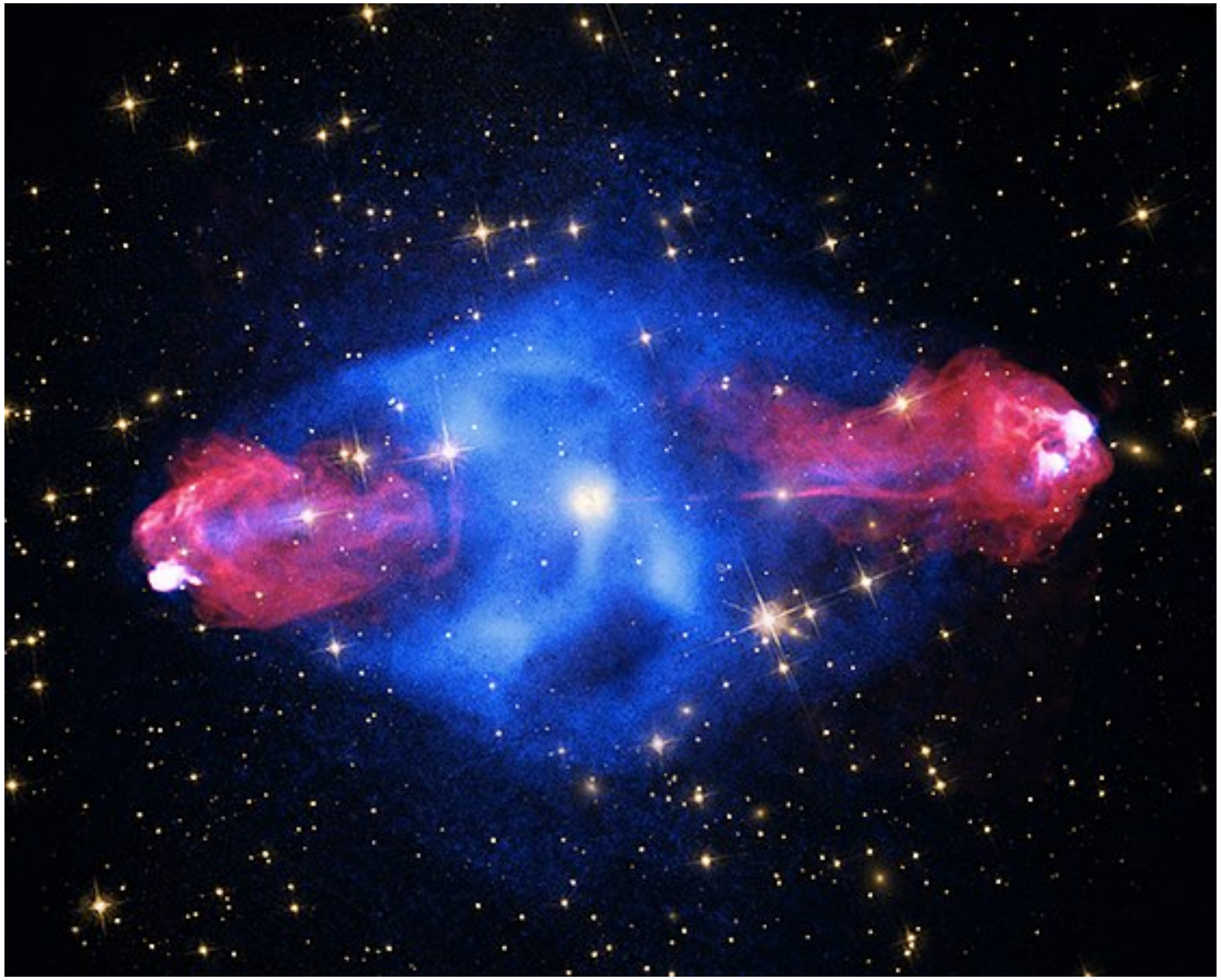


The response is the integral of the source brightness, modulated by the cosinusoidal interferometer pattern.

This relation links what we can measure (*R*) to what we would like to know ($I_\nu(\hat{s})$).

Can we recover $I_\nu(\hat{s})$ from *R*?

Example of fringes: Cygnus A

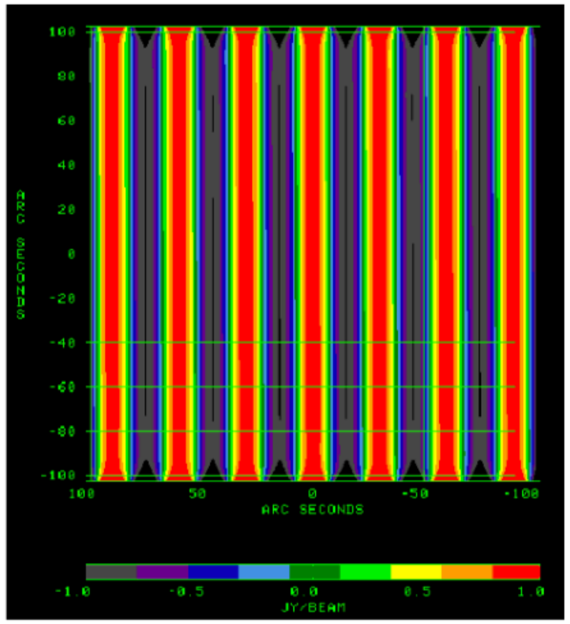


Composite image: Chandra (X-ray) + Hubble (Optical) + VLA (Radio)

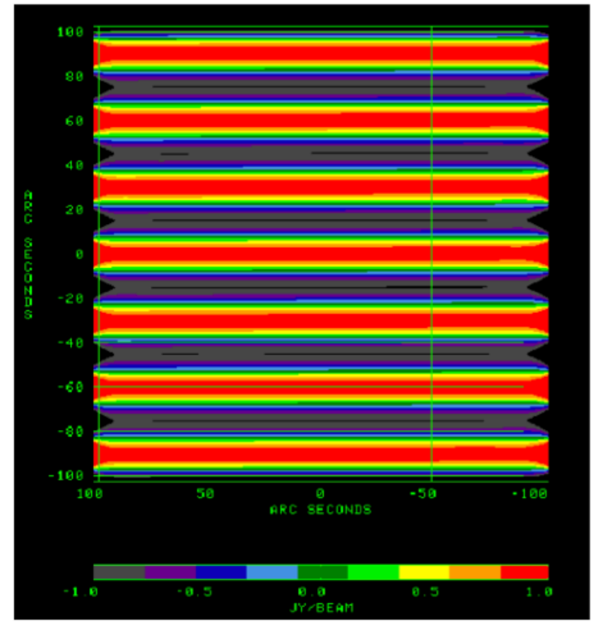
Example of fringes: Cygnus A seen by the VLA (uniform response)

$$\nu \sim 2.5\text{GHz}(\lambda \sim 15\text{cm})$$

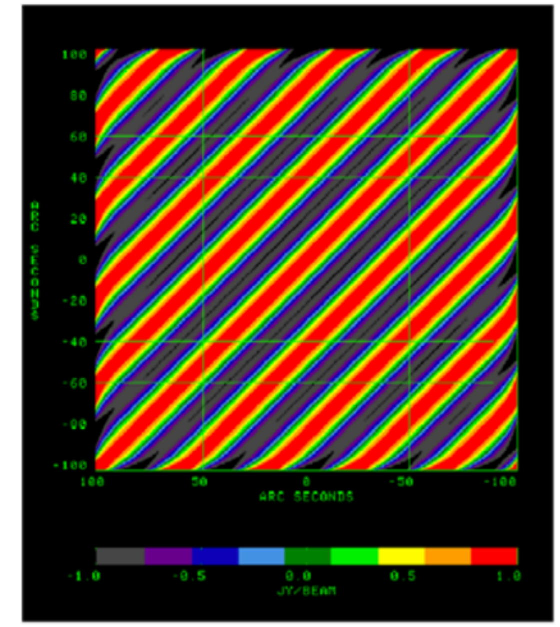
same baseline (1km) different orientations



East-West baseline makes vertical fringes



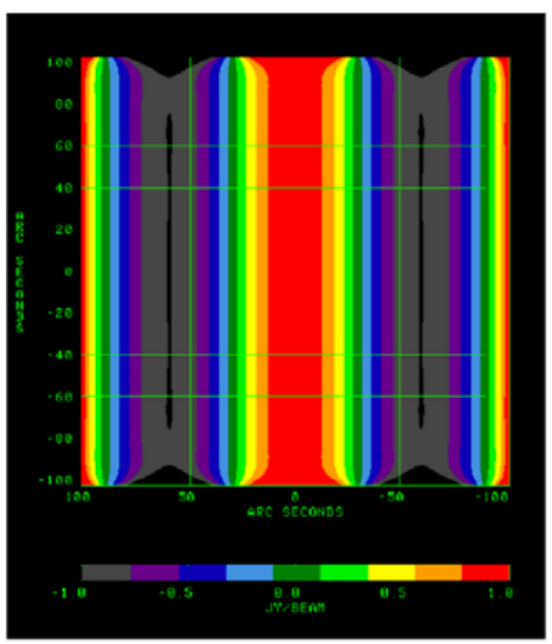
North-South baseline makes horizontal fringes



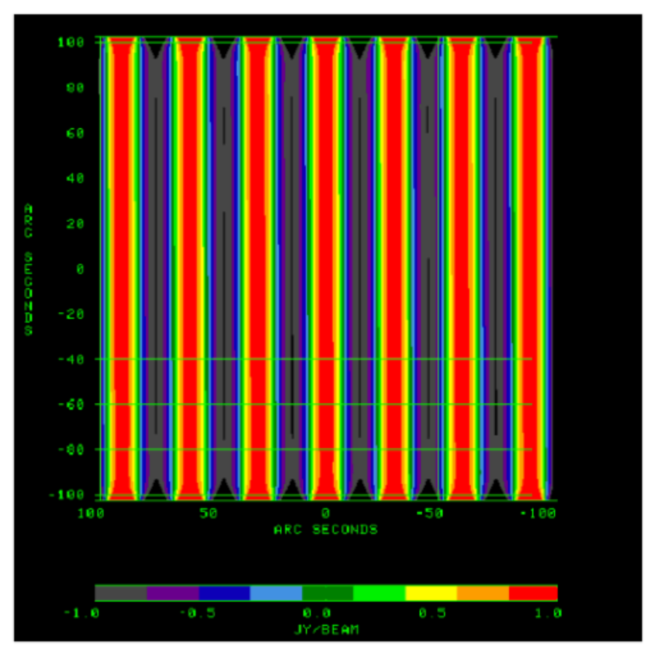
Rotated baseline makes rotated fringes

Different baselines, different fringes width

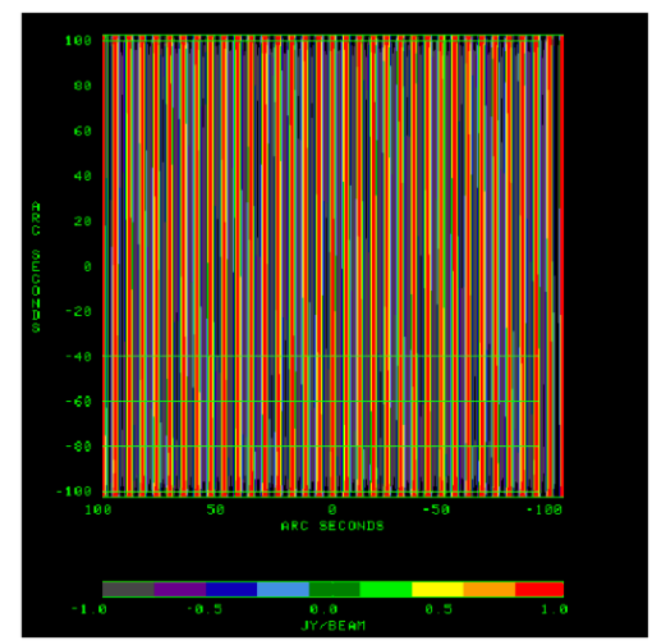
$$\theta \sim \lambda / b$$



250 meter baseline
120 arcsecond fringe



1000 meter baseline
30 arcsecond fringe

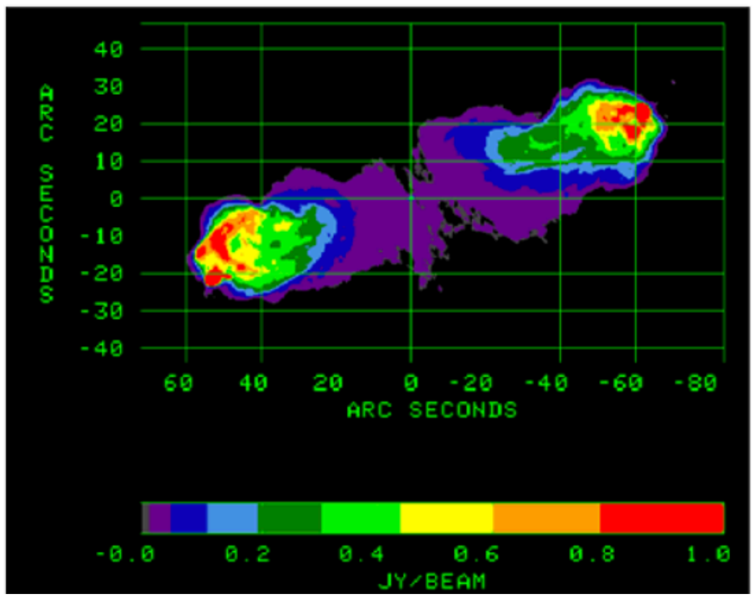


5000 meter baseline
6 arcsecond fringe

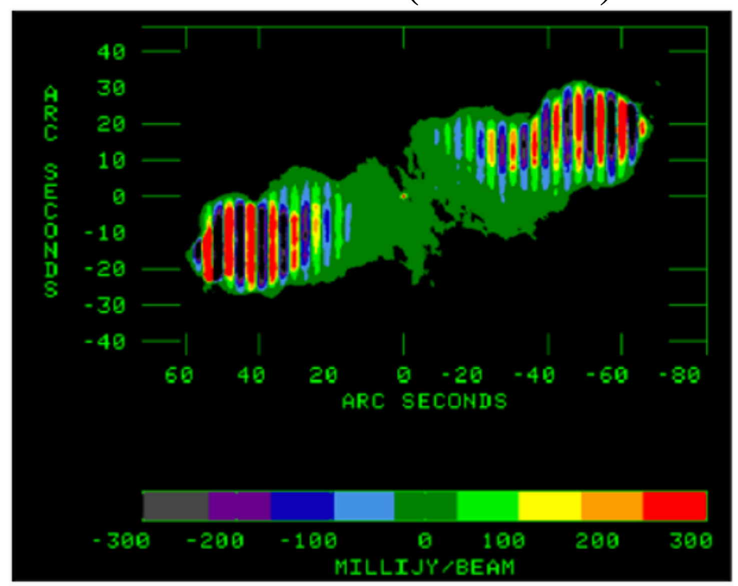
Example of real fringes: Cygnus A seen by the VLA

The interferometer casts a cosinusoidal pattern on the sky, with the result that we obtain a response which is some function of the source brightness and the fringe separation and orientation. How does that get us to our goal of determining the actual brightness?

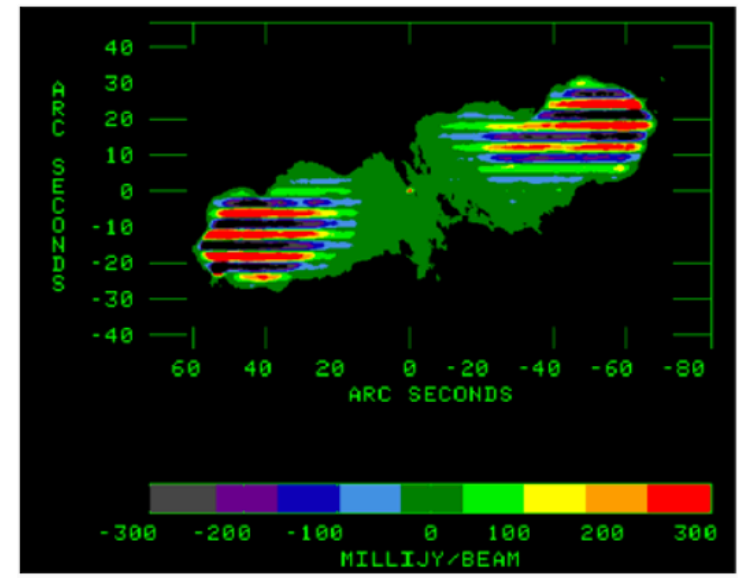
$\nu \sim 2.5\mathrm{GHz}(\lambda \sim 15\mathrm{cm})$



Zero-Spacing Image
Sum = 999 Jy



5 km EW spacing
Sum = 61 Jy



5 km NS spacing
Sum = -16 Jy

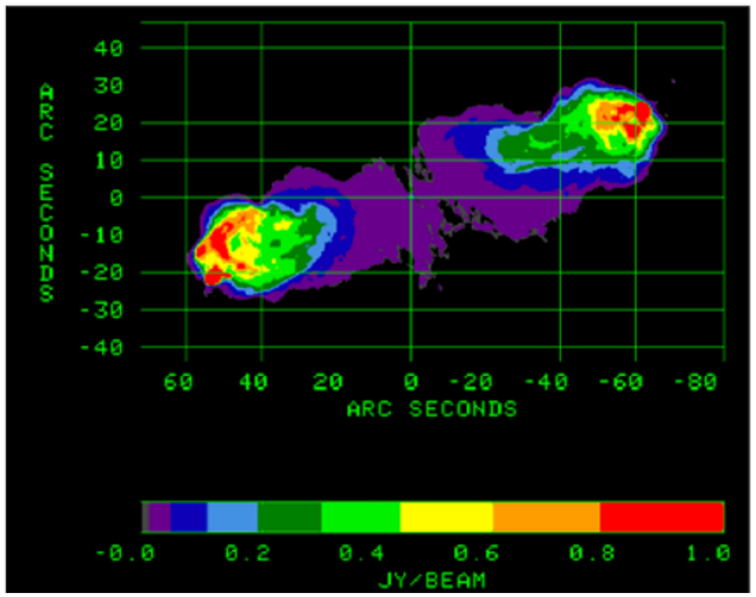
Actual brightness
of the source
(single dish)

How a 5-km baseline interferometer “sees” the source

Example of real fringes: Cygnus A seen by the VLA

The interferometer measures the integral (sum) of the product of this pattern with the source brightness

$\nu \sim 2.5\text{GHz}(\lambda \sim 15\text{cm})$



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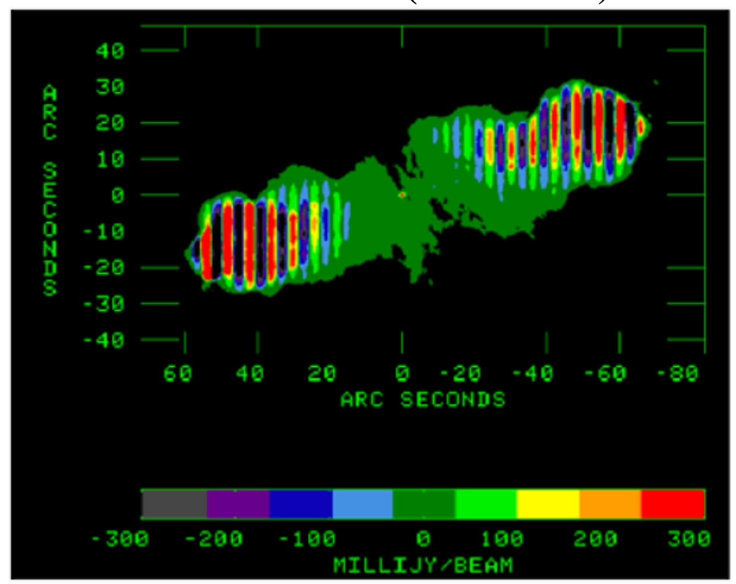

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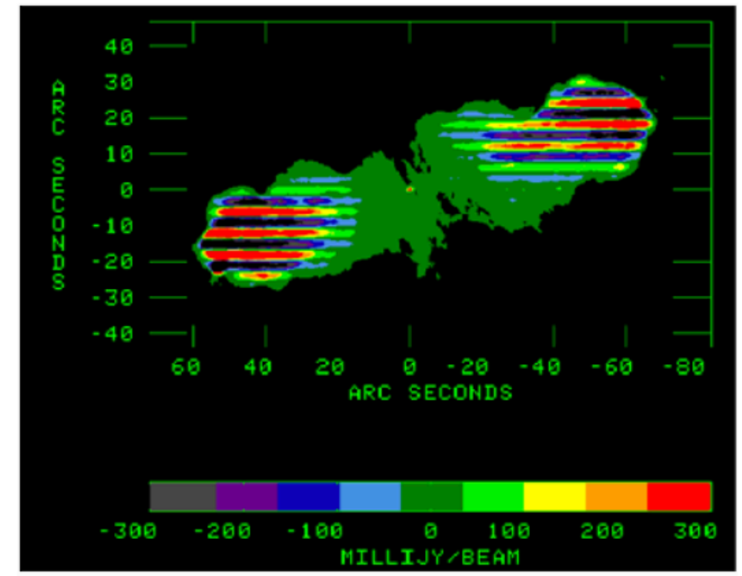

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Actual brightness
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How a 5-km baseline interferometer "sees" the source

Example of fringes: Cygnus A seen by the VLA (uniform response)

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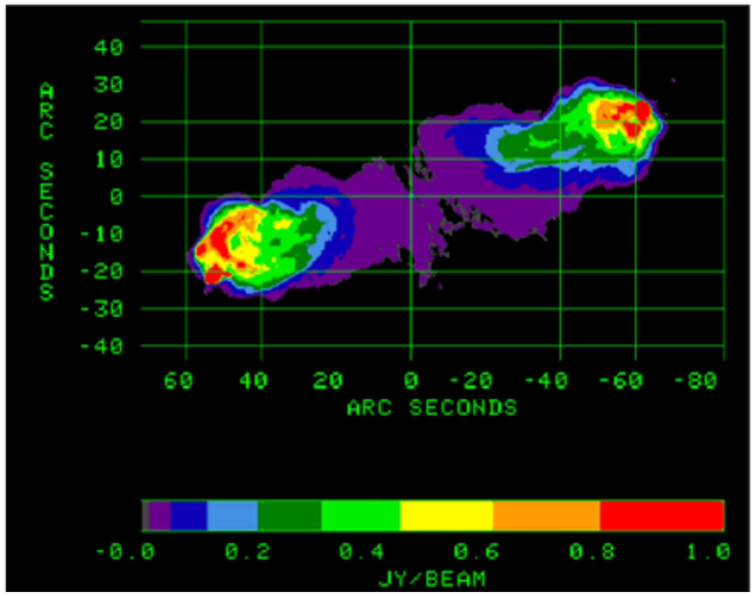

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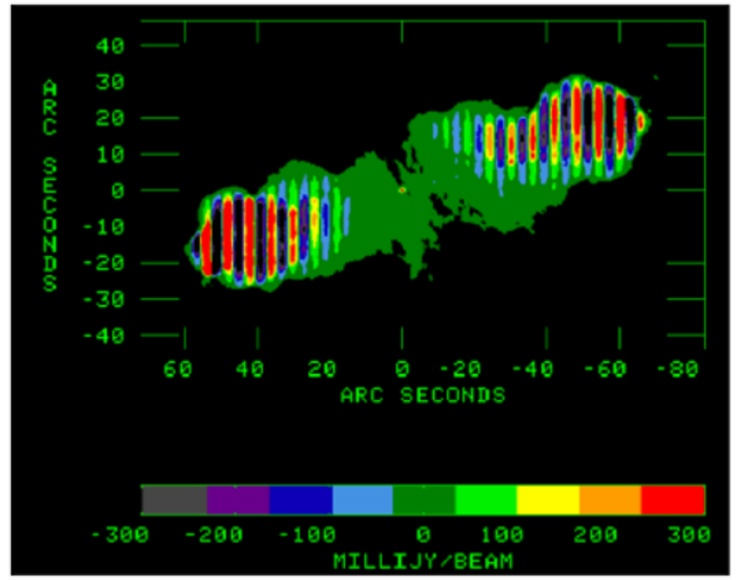

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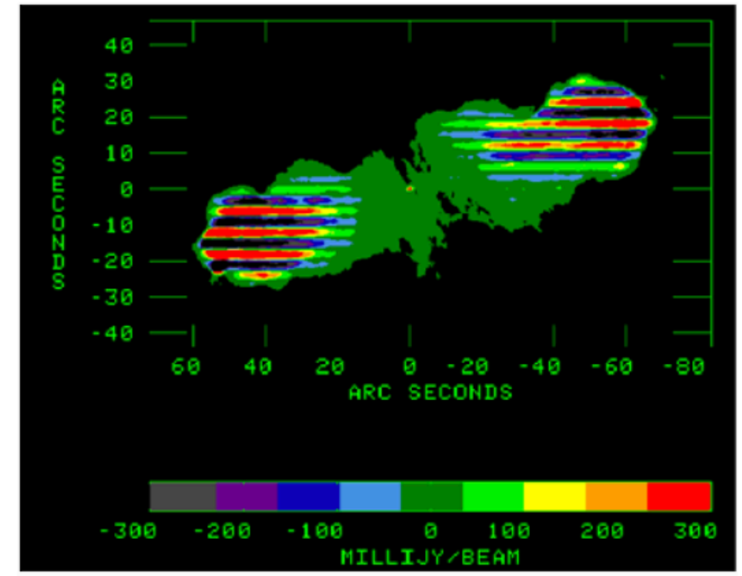


5 km NS spacing

Sum = -16 Jy

Actual brightness of the source (single dish)

How a 5-km baseline interferometer “sees” the source

Basic considerations

- For a point source (by definition << then the fringe spacing), the interferometer response is the same for every baseline.
- The interferometer response to a real source can be negative
- As the baseline gets longer, the response goes to zero (the source is resolved out)
- As the baseline gets shorter, the response goes to the total source flux (zero spacing)

Response for an extended source: the complex correlator

The correlator response

$$\boxed{R_c = \int_{\Omega\text{source}} I_\nu(\hat{s}) cos(2\pi\nu \overrightarrow{b} \cdot \hat{s}/c) d\Omega}$$

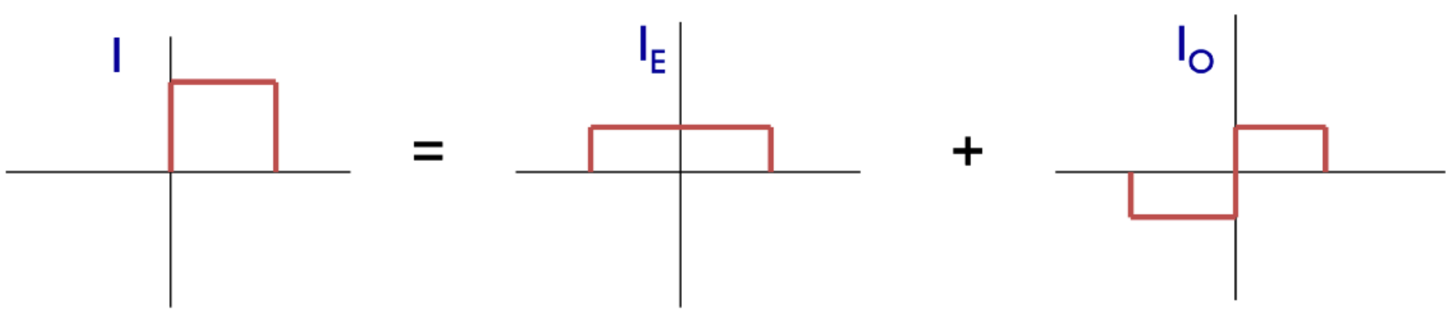
is not enough to recover the actual brightness…why?

Let’s recall that any real function can be written as the sum of an even and an odd part

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

An even part: $$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

An odd part: $$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



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is not enough to recover the actual brightness…why?

Suppose that the source has a component with odd symmetry, for which $I_{\nu,\mathrm{O}}(\hat{s}) = -I_{\nu,\mathrm{O}}(-\hat{s})$. We have that

$$R_c = \int_{\Omega\text{source}} \underbrace{I_{\nu,O}(\hat{s})}_{\text{odd}} \underbrace{cos(2\pi\nu \overrightarrow{b} \cdot \hat{s}/c)}_{\text{even}} d\Omega = 0$$

To detect $I_{\nu,\mathrm{O}}$ we need a sinusoidal correlator, whose output is odd

$$\boxed{R_s = \int_{\Omega\text{source}} I_\nu(\hat{s}) sin(2\pi\nu \overrightarrow{b} \cdot \hat{s}/c) d\Omega}$$

and that can be implemented as a second correlator that follows a $\pi/2$ delay inserted into the ouput of the antenna, as $sin(\omega\tau_g) = cos(\omega\tau_g - \pi/2)$

The combination of cosine and sine correlators is called a complex correlator, because it is mathematically convenient to treat cos and sin as complex exponentials using Euler's formula

$$e^{i\phi} = cos\phi + isin\phi$$

Response for an extended source: the complex visibility

The correlator response

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$$\boxed{R_s = \int_{\Omega \text{source}} I_\nu(\hat{s}) sin(2\pi\nu \overrightarrow{b} \cdot \hat{s}/c) d\Omega}$$

We define the **complex visibility** V from the two independent (real) correlator outputs R_c and R_s as:

$$\boxed{V = R_c - iR_s = Ae^{-i\phi}}$$

where $A = \sqrt{R_c^2 + R_s^2}$ is the visibility amplitude

$\phi = tan^{-1}(R_s/R_c)$ is the visibility phase

Response for an extended source: the complex visibility

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where $A = \sqrt{R_c^2 + R_s^2}$ is the visibility amplitude

$\phi = tan^{-1}(R_s/R_c)$ is the visibility phase

This gives us the relation between the source brightness and the response of an interferometer:

$$\boxed{V = \int_{\Omega\mathrm{source}} I_\nu(\hat{s}) e^{-2\pi i\nu \vec{b}\cdot\hat{s}/c} d\Omega}$$

which (under some circumstances) is a 2D Fourier transform, giving us a well established way to recover $I_\nu(\hat{s})$ from *V*.

As $I_\nu(\hat{s})$ is a real function, *V* is a complex function and is hermitian: $V^*(\phi) = V(-\phi)$

Response for an extended source: the complex visibility

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which (under some circumstances) is a 2D Fourier transform, giving us a well established way to recover $I_\nu(\hat{s})$ from *V*.

The power measured by the correlator is then

$$\boxed{P = < V_1 V_2^* > = A^2 e^{-i\omega \overrightarrow{b} \cdot \hat{s}/c}}$$

where

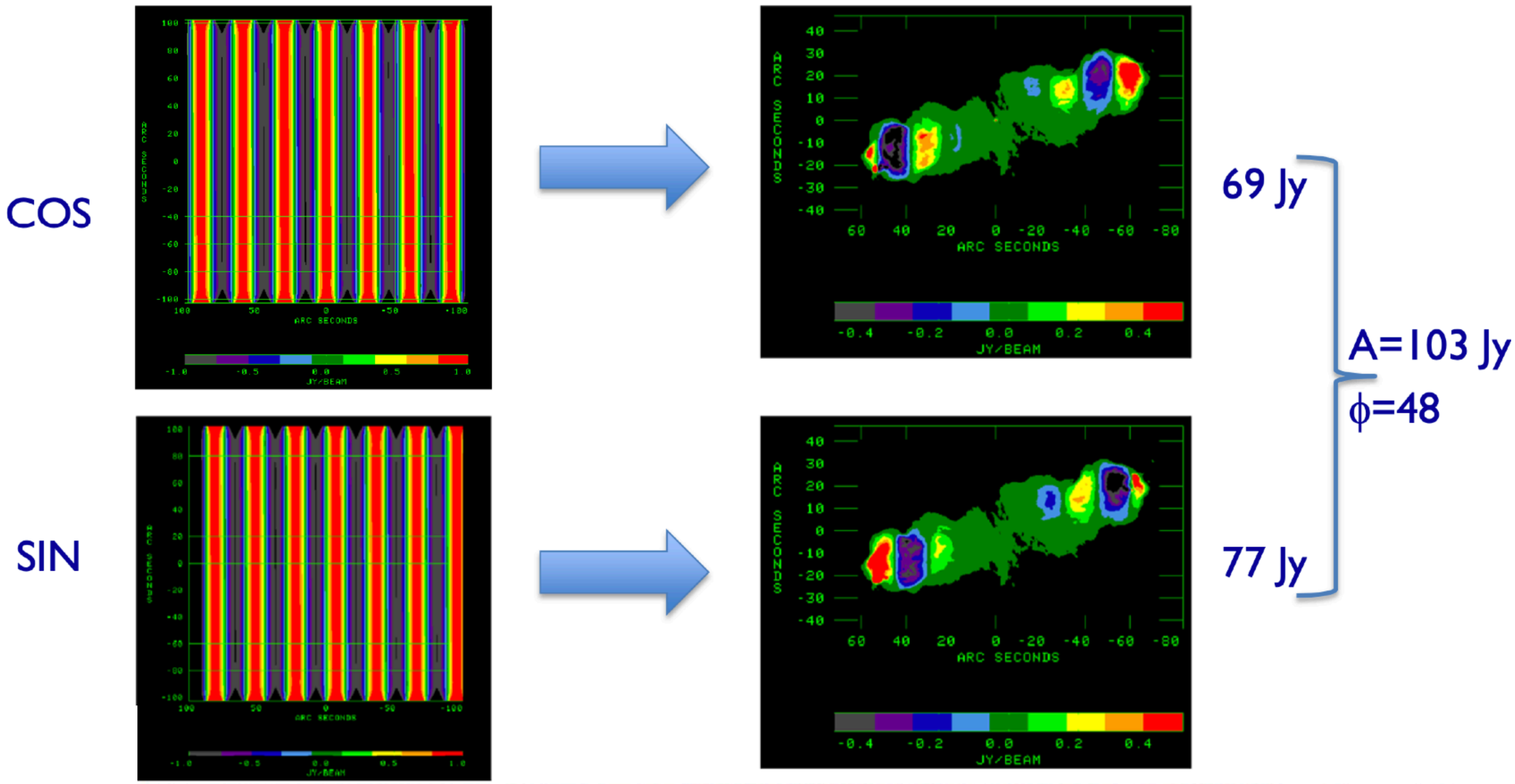
$$V_1 = Acos(\omega t) = \mathrm{Re}(A^2 e^{-i\omega t})$$

$$V_2 = Acos[\omega(t - \overrightarrow{b} \cdot \hat{s}/c)] = \mathrm{Re}(A^2 e^{-i\omega \overrightarrow{b} \cdot \hat{s}/c})$$

$$\omega = 2\pi\nu$$

Example of fringes: Cygnus A seen by the VLA (uniform response)

We now have two real correlators, whose patterns are phase shifted by 90 deg on the sky:



Basic considerations

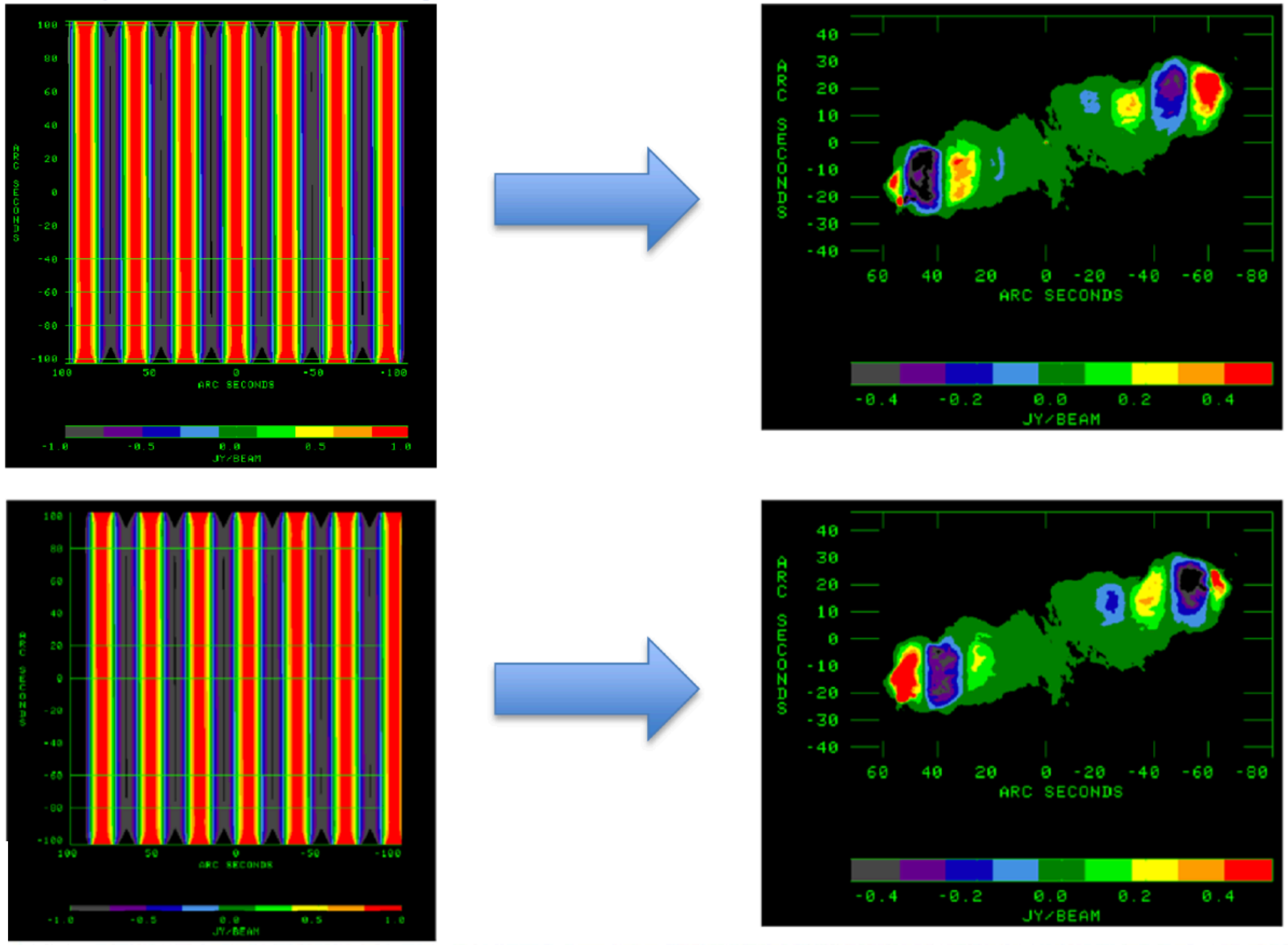
- The complex visibility amplitude is independent of the source location, and is linearly related to the source flux density
- The complex visibility phase is a function of source location, and independent of source flux density
- The visibility is a unique function of the source brightness

Example of fringes: Cygnus A seen by the VLA (uniform response)

We now have two real correlators, whose patterns are phase shifted by 90 deg on the sky:

COS

SIN



69 Jy

A=103 Jy

ϕ=48

77 Jy

Basic considerations

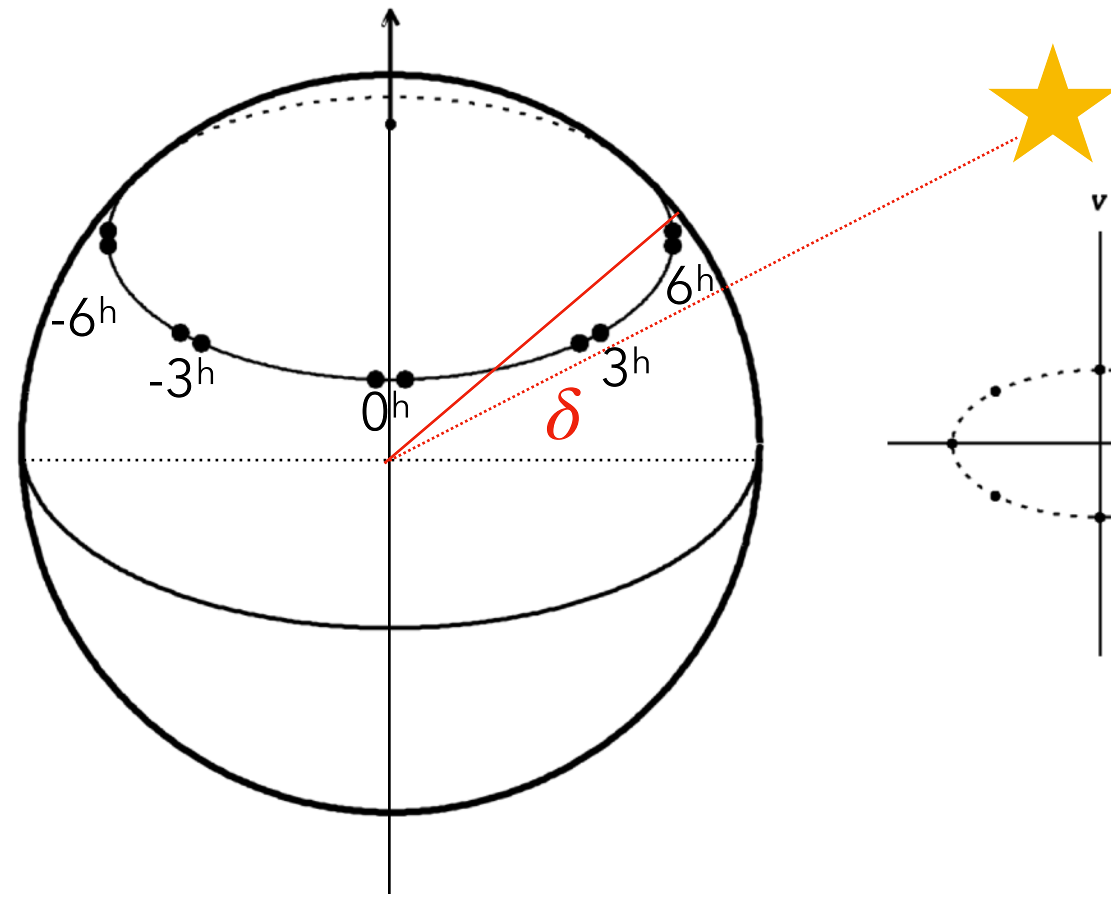
- The two functions are related through a Fourier transform $V(u,v) = I(l,m)$ where (u,v) are baseline coordinates and (l,m) are source coordinates
- An interferometer, at a given time, makes one measurement of the visibility, at baseline coordinate (u,v).
- Sufficient knowledge of the visibility function (as derived from an interferometer) will provide us a “reasonable estimate” of the source brightness

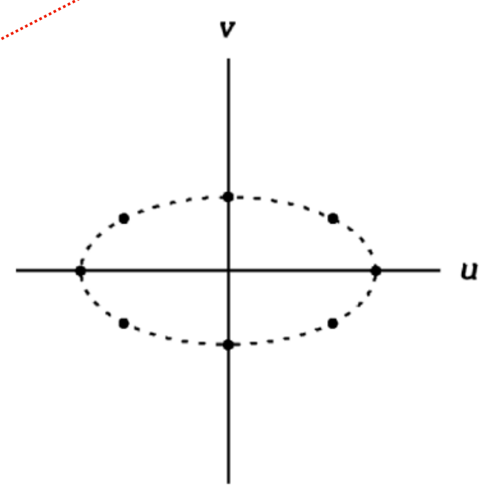
Earth rotation aperture synthesis

Most astronomical sources are stationary , that is their brightness does not change on the timescale of the observation. Earth rotation and moving antennas can be exploited to increase the number of effective "antenna pairs".

Consider an east-west two-element interferometer at latitude +40° as seen by a source at declination $\delta = +30°$.

The projected east-west component (in wavelength units) is *u* and the north-south projected component is *v*.





During a 12-hour period, the interferometer traces out a complete ellipse in the (u,v) plane. The maximum value of u equals the actual antenna separation in wavelengths, and the maximum value of v is smaller by the projection factor $sin\delta$, where δ is the source declination

If the interferometer has N>2 antennas, or if the spacing of the two antennas is changed daily, the (u,v) coverage will become a number of concentric ellipses having the same shape. Thus the synthesized beam obtained can approach an elliptical Gaussian.

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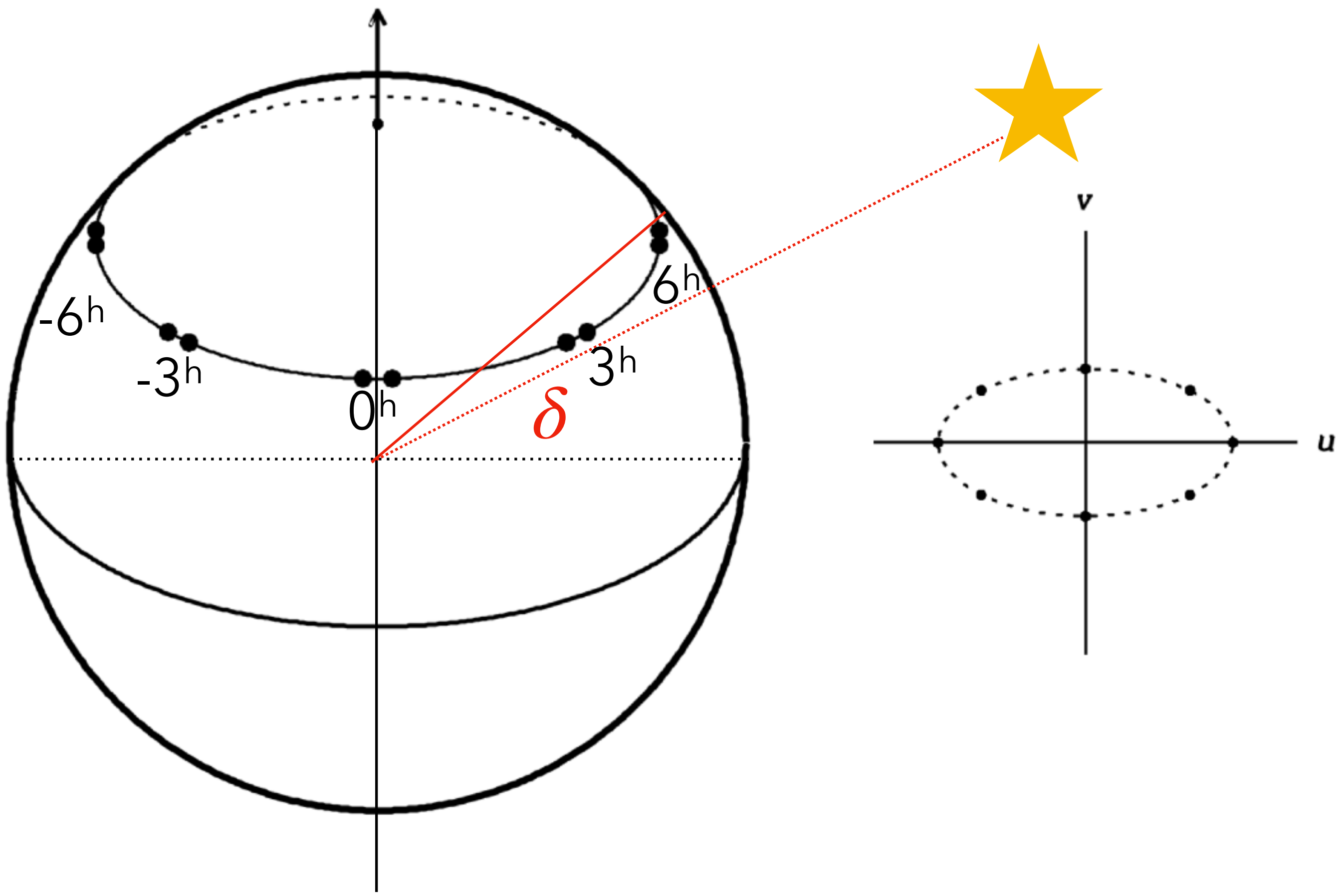
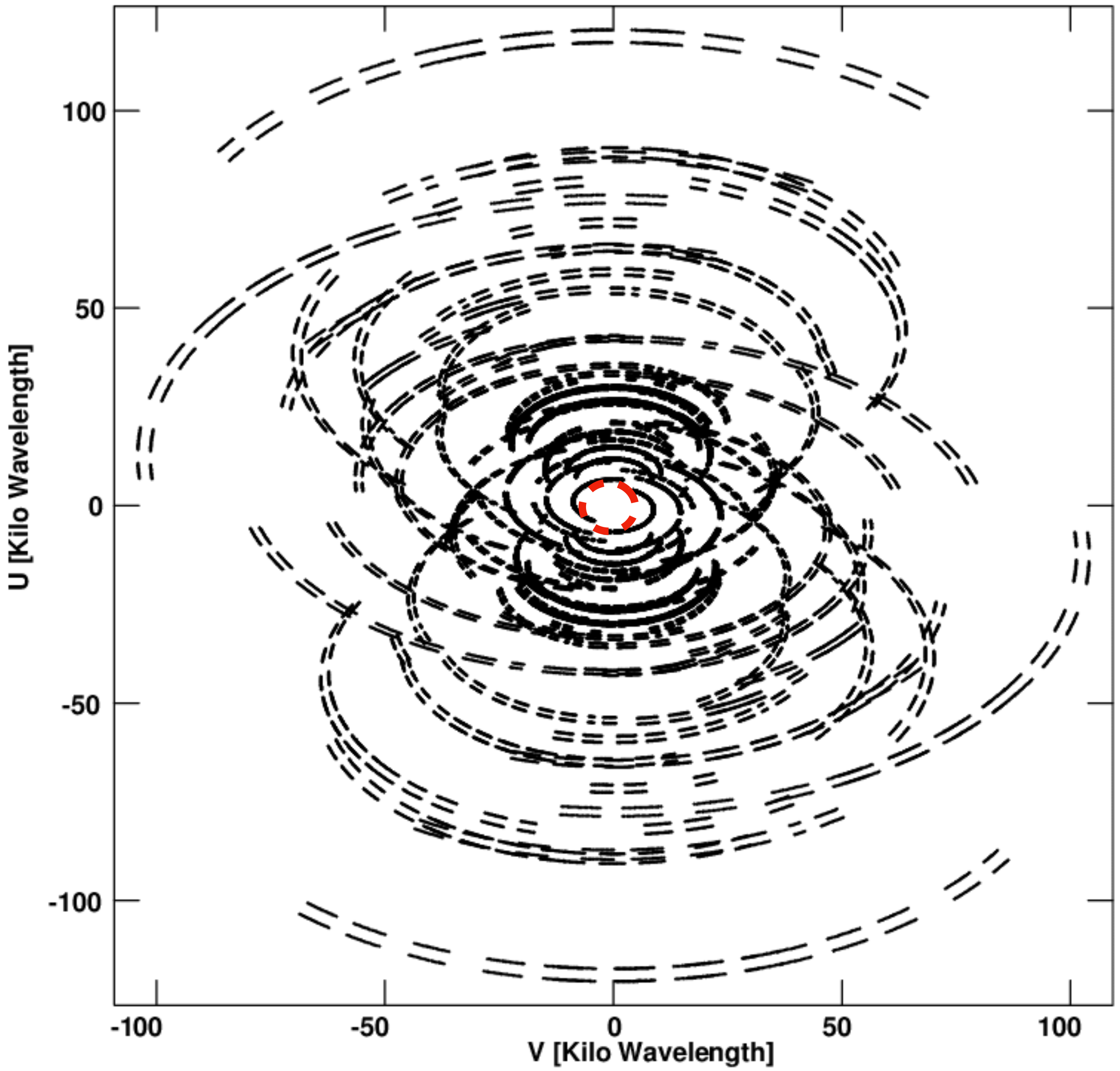
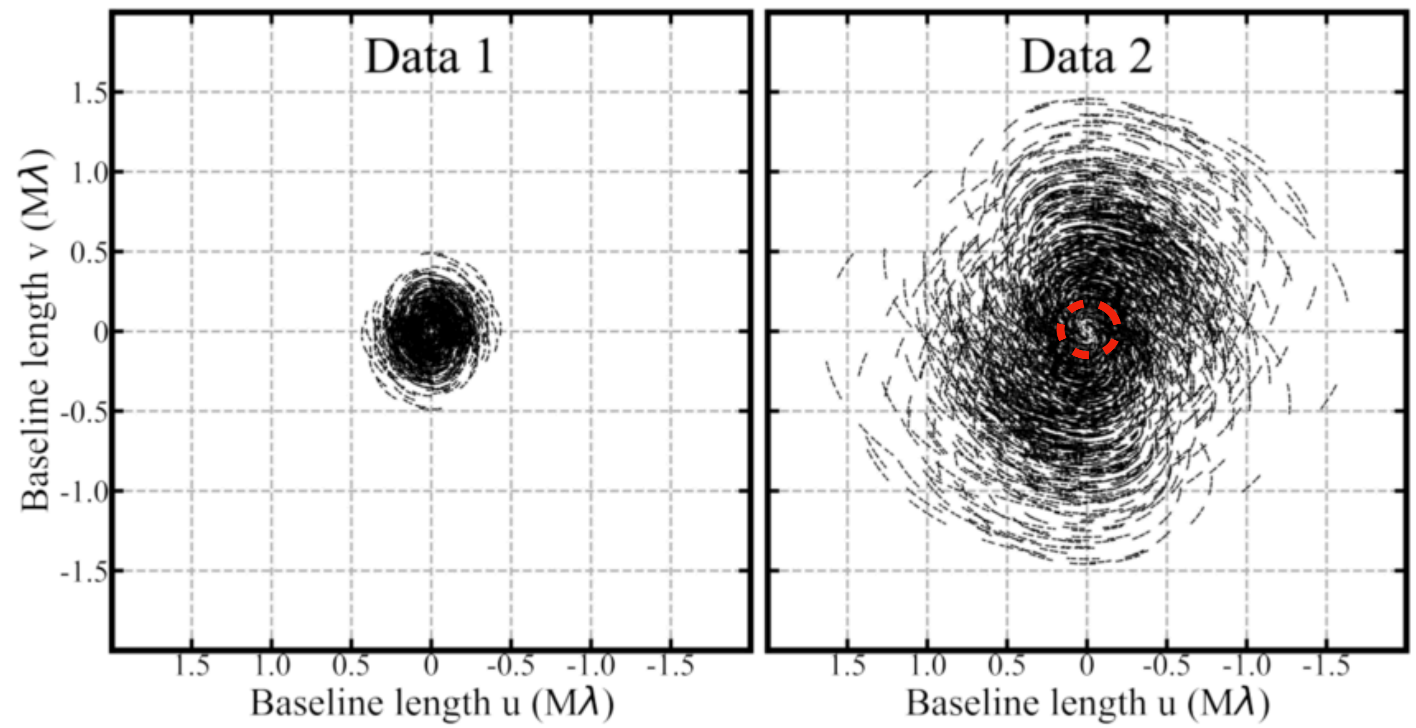


The synthesized beamwidth is ~ u^{-1} rad east–west and ~$u^{-1}/sin\delta$ rad in the north–south direction.

The synthesized beam is almost circular for a source near the celestial pole, but the north-south beamwidth is very large for a source near the celestial equator.

Earth rotation aperture synthesis





IRAM PdBI (now NOEMA)
4 to 6 antennas in 3 configurations
On source time ~27 hours

De Breuck et al. 2003

Two ALMA configurations
compact (left) and extended (right)
On source time ~1 (left) and ~2(right) hours

Yamaguchi et al. 2020

Solutions