

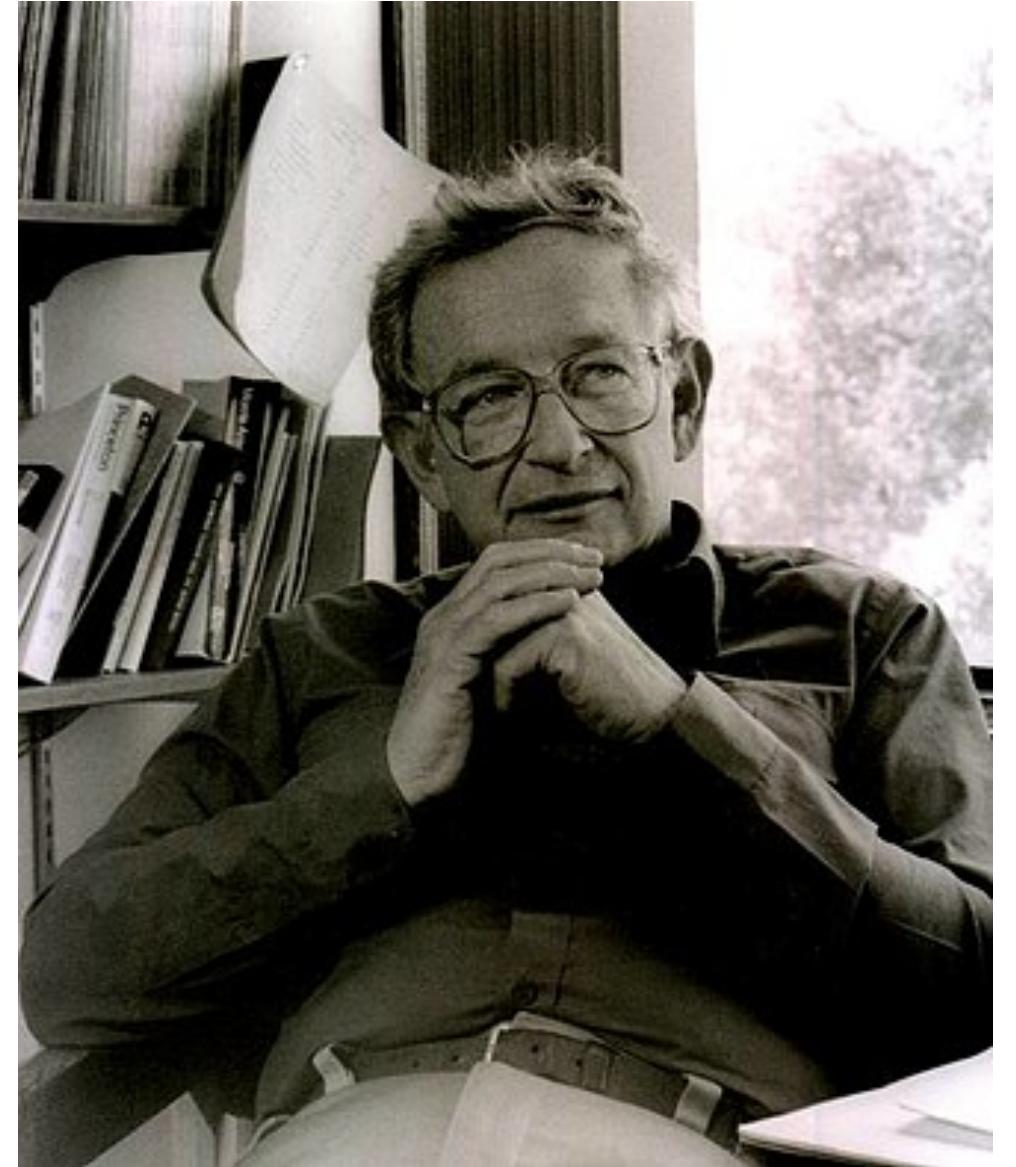
## More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

1972

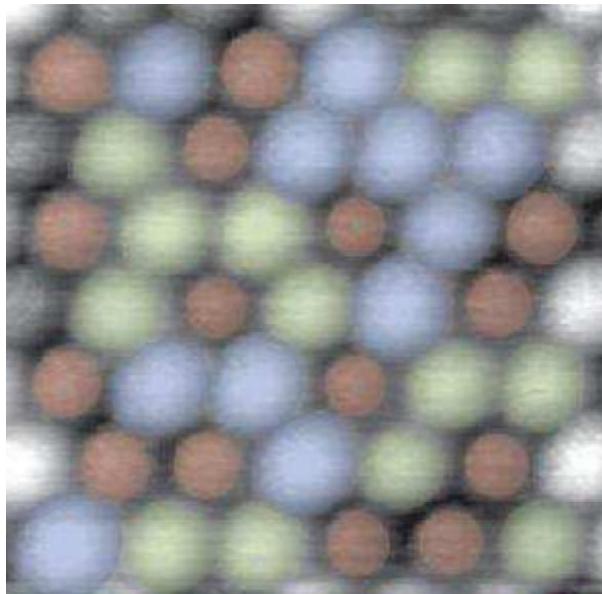
P. W. Anderson

I do not mean to give the impression that all is settled. For instance, I think there are still fascinating questions of principle about glasses and other amorphous phases, which may reveal even more complex types of behavior. Nevertheless, the role of this type of broken symmetry in the properties of inert but macroscopic material bodies is now understood, at least in principle. In this case we can see how the whole becomes not only more than but very different from the sum of its parts.

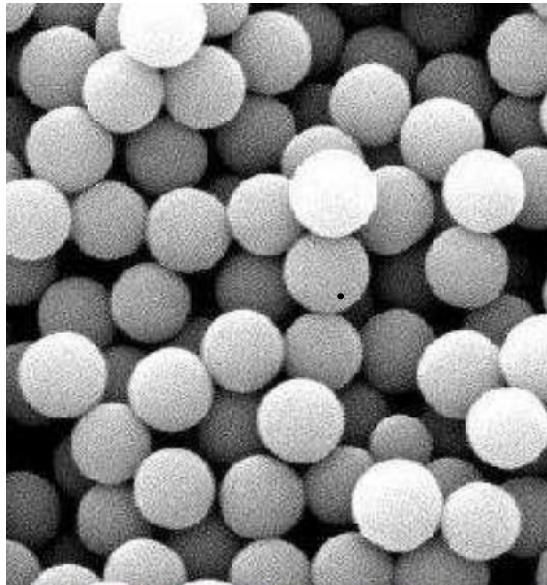


Nobel prize 1977

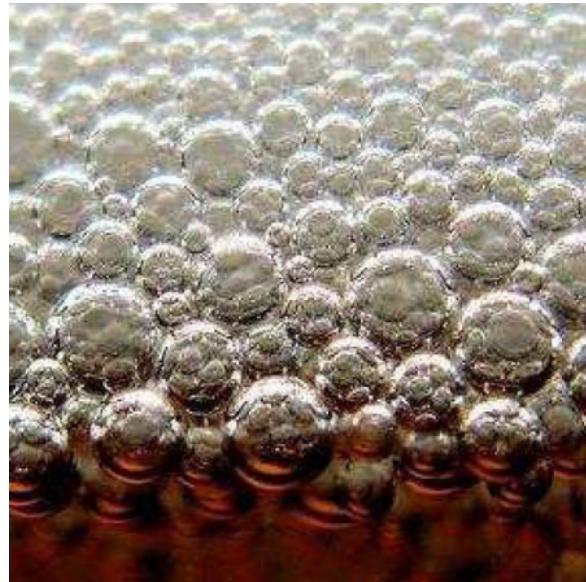
lega metallica



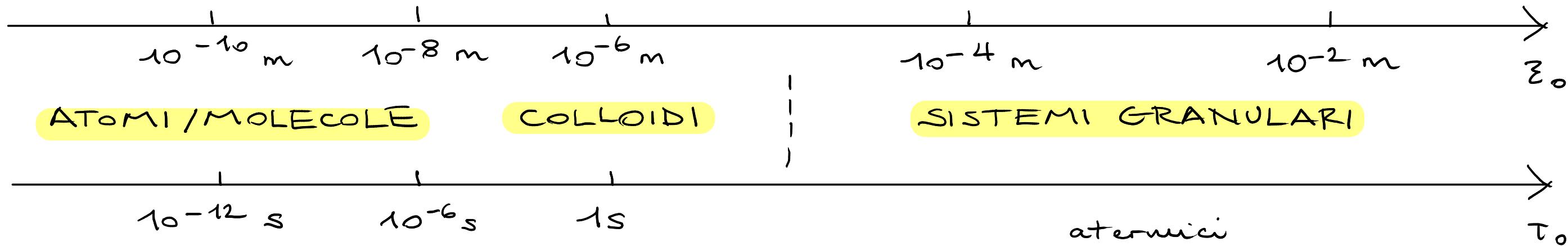
PMMA

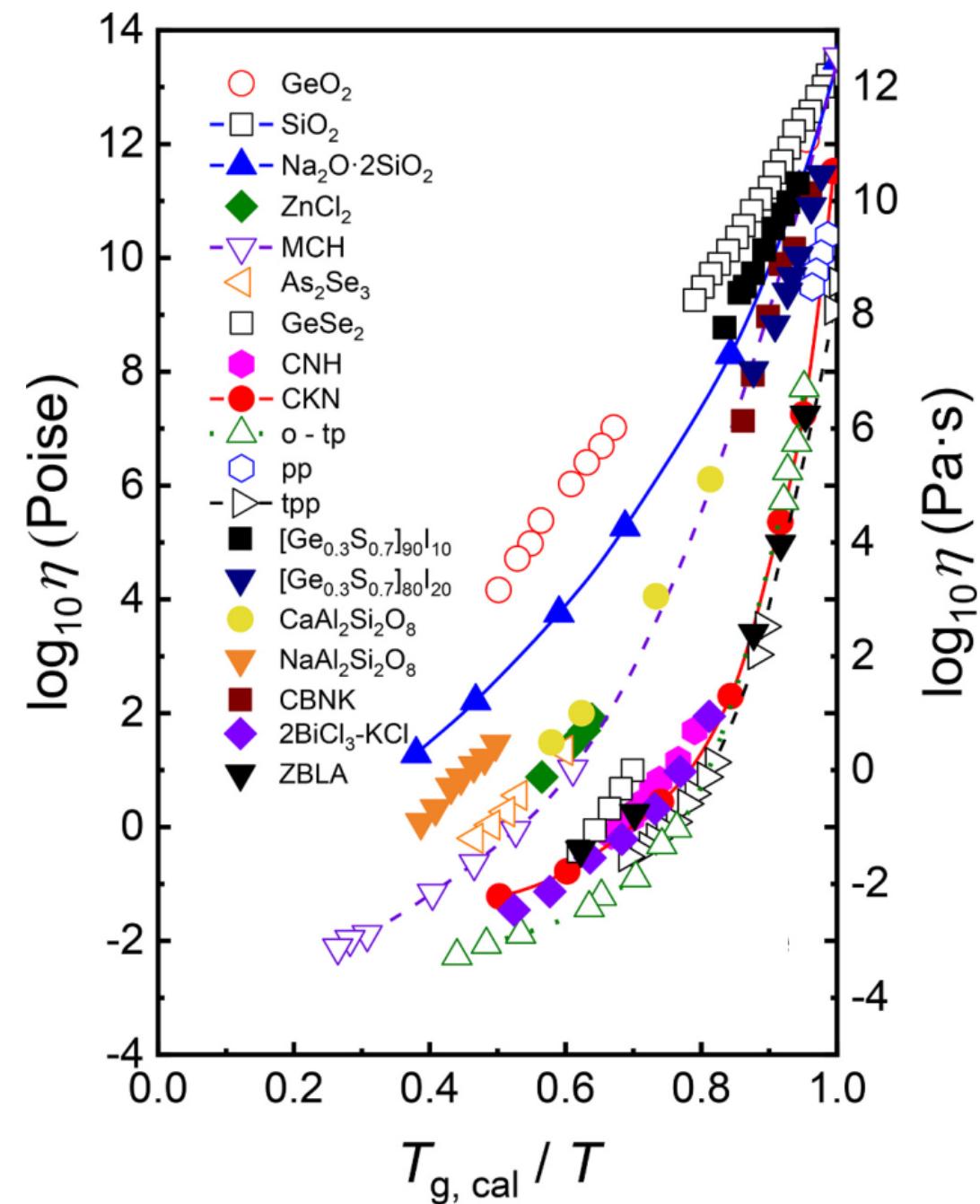
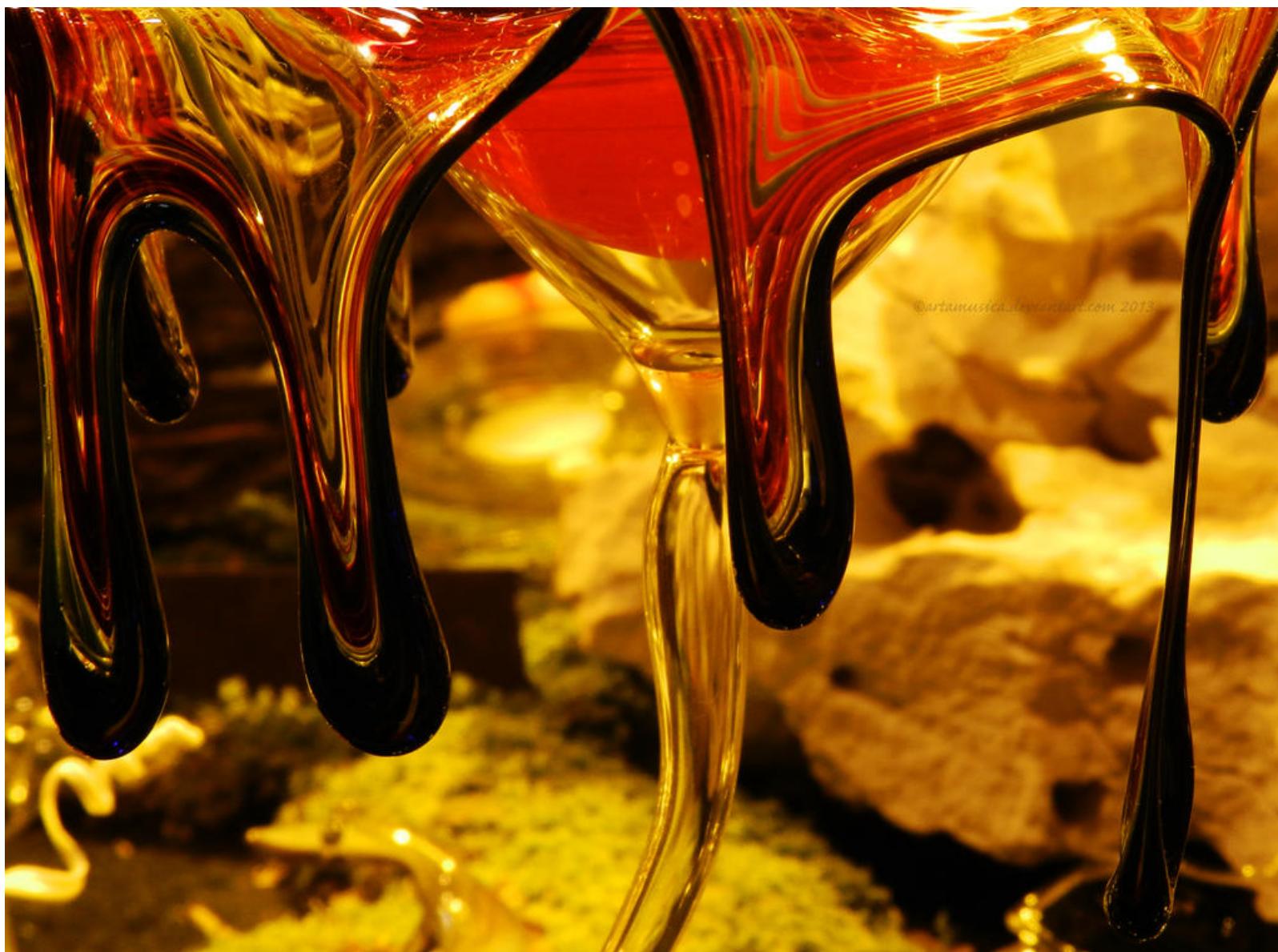


schiuma



fertilizzante





# INTRODUZIONE

Disordinato: assenza di ordine a lungo raggio

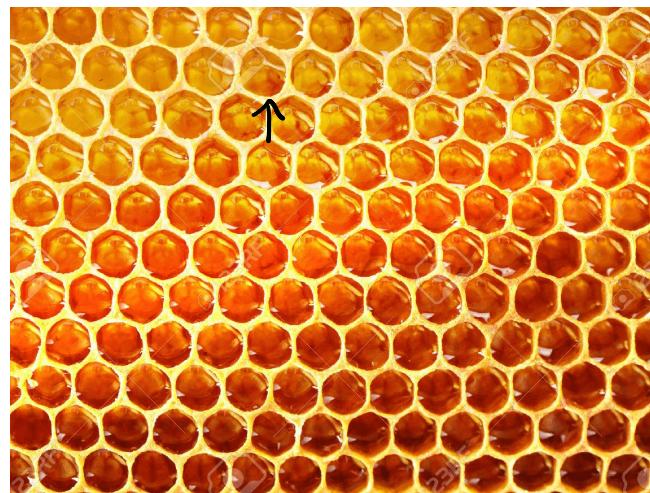
1) Dots posizionali  $\{\vec{r}_i\}$

$$H = K + U(\{\vec{r}_i\})$$

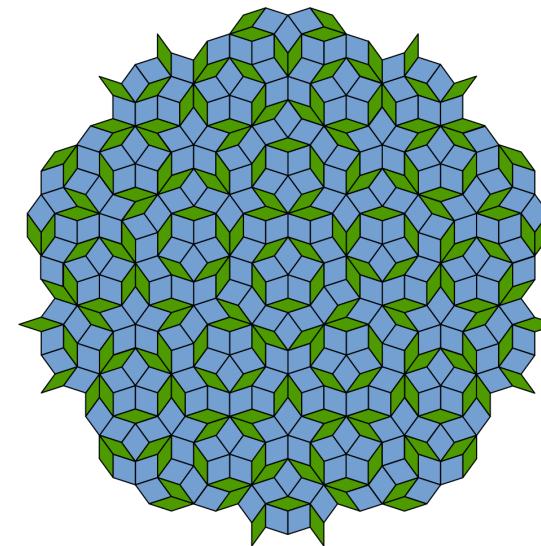
/ cristalli periodici

\ quasi cristalli (190)

cristallo: spettro di diffrazione discreto



honey comb 2d



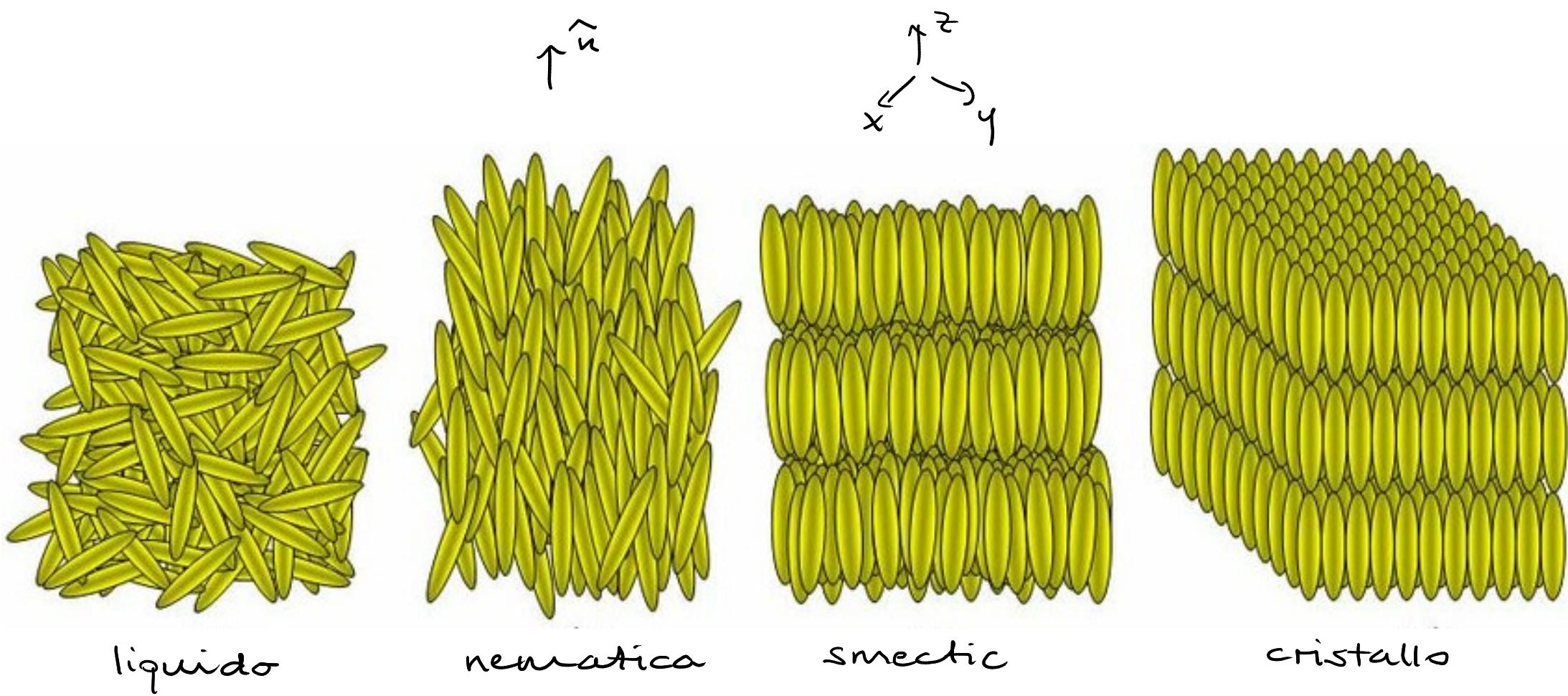
Penrose tiling

2) Dots orientazionali  $\{\vec{\sigma}_i\}$

$$H = H(\{\vec{\sigma}_i\}) \quad \uparrow \downarrow \uparrow \dots \uparrow \downarrow \uparrow \quad \text{es. magnetismo}$$

3) Dots misti  $\{\vec{r}_i, \vec{\sigma}_i\}$

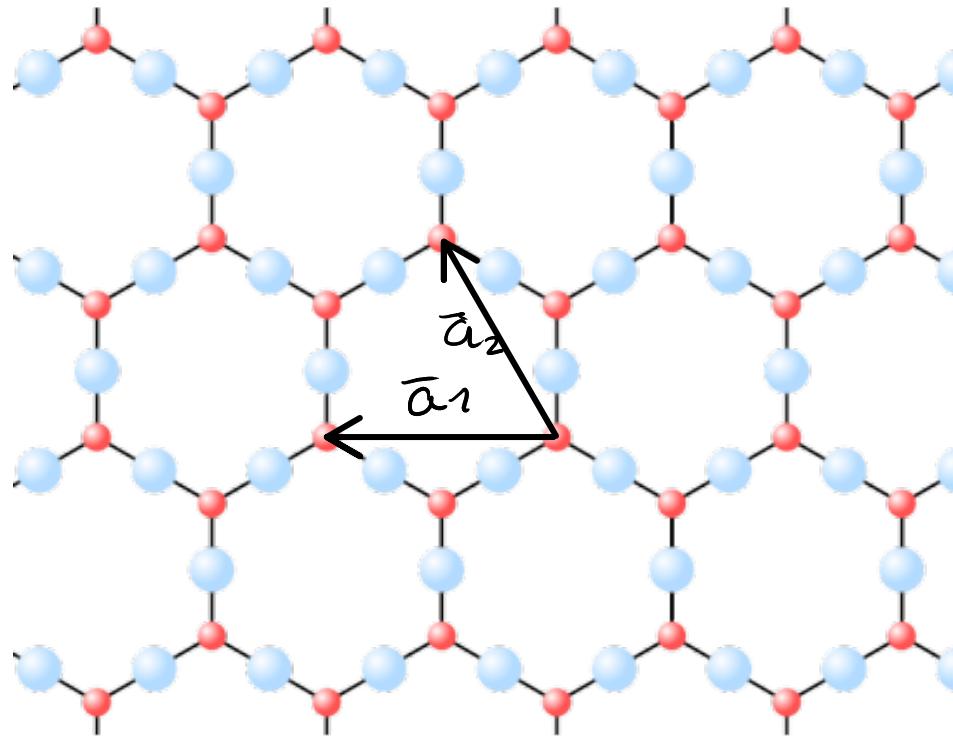
es.: cristalli liquidi, sistemi 2d



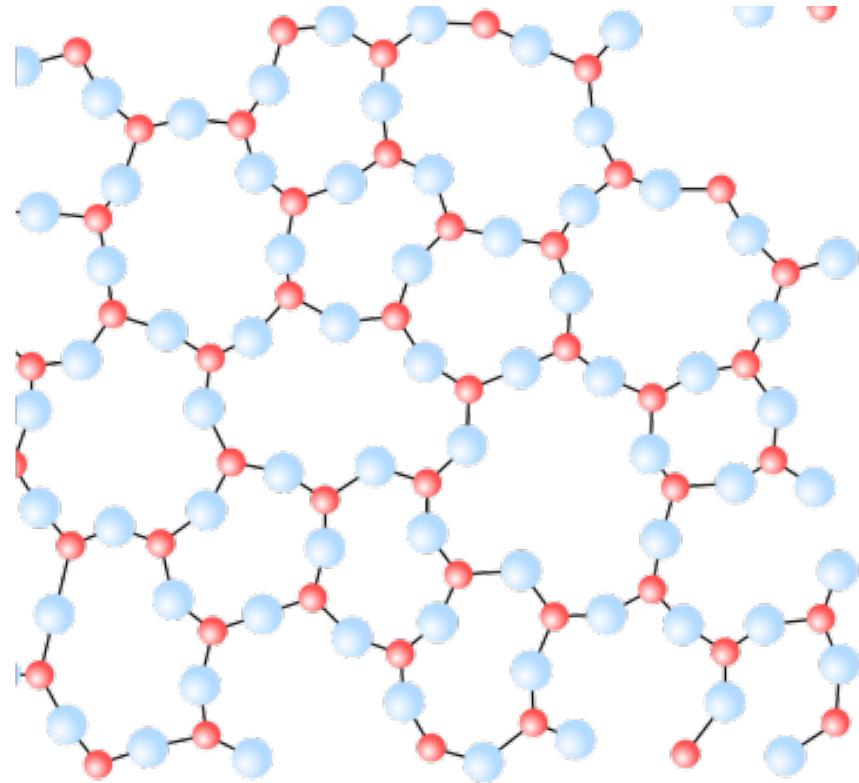
# SIMMETRIE E FASI DELLA MATERIA

1) Configurazione

$$T: \vec{r} \rightarrow \vec{r} + \vec{R}_\ell \quad \vec{R}_\ell = l_1 \vec{a}_1 + l_2 \vec{a}_2 \quad l_1, l_2 \in \mathbb{Z} \quad \text{traslazione discreta}$$



cristallo periodico



amorf

## 2) Sistema

$F(\{\bar{r}_i\})$  invariante rispetto a  $T$  (o per  $T$ ) se  $F(\{\bar{r}_i\}) = F(T(\{\bar{r}_i\}))$

Sistema possiede simmetria  $T$  se  $H(\{\bar{r}_i\}) = H(T(\{\bar{r}_i\}))$

Traslazione continua:  $T: \vec{r} \rightarrow \vec{r} + \vec{R} \quad \vec{R} \in \mathbb{R}^3$

Es.:  $H = K + U(\bar{r}_i - \bar{r}_j)$  sistema invariante per traslazione continua

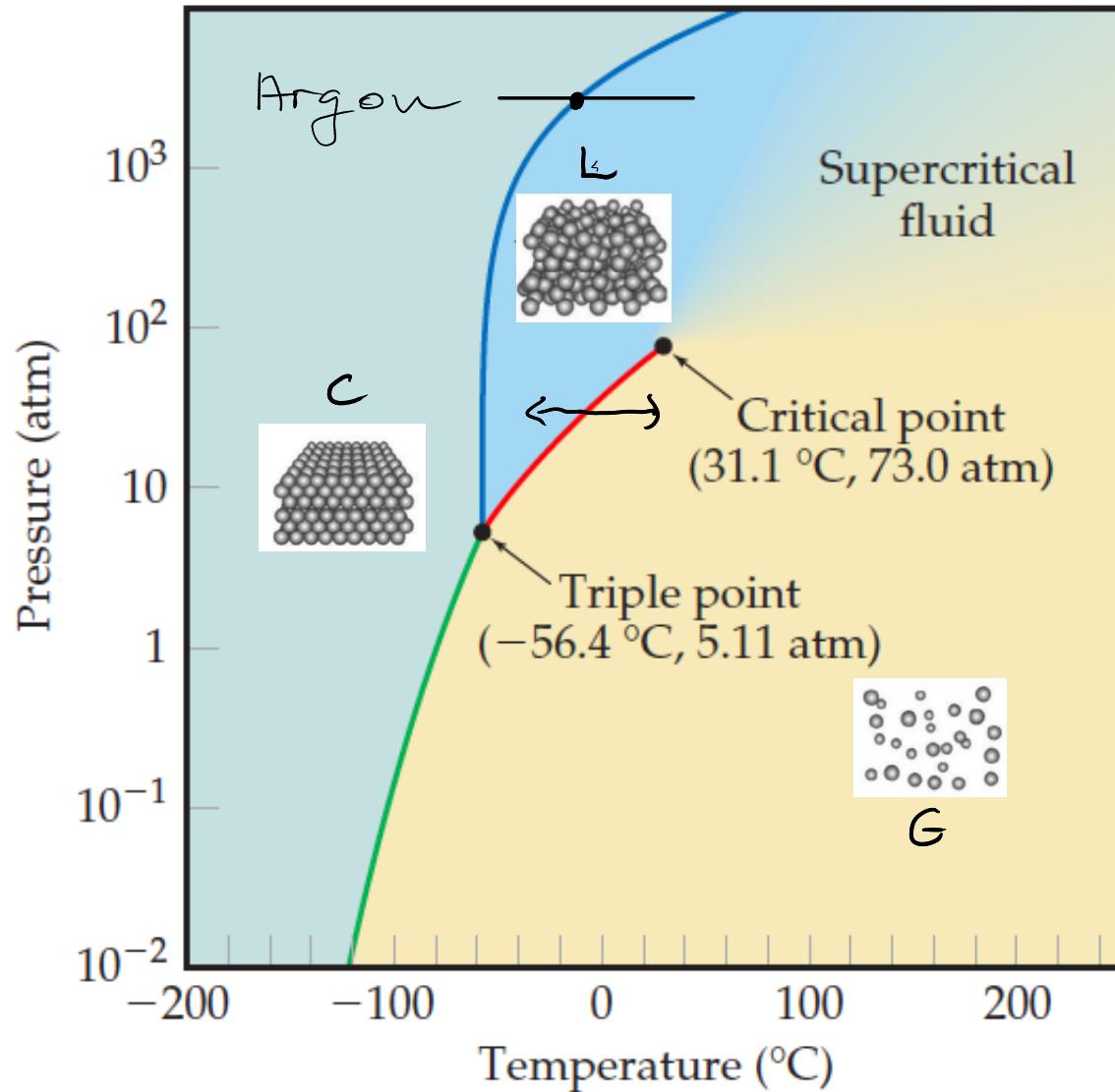
## 3) Fase

Regione di analiticità dell'energia libera ( $N \rightarrow \infty$ ,  $N/V = \text{cost}$ )

Fase: insieme di microstati

Diagramma di fase : gas rari

$\alpha = C, L, G$



$$F(\{\vec{r}_i\}, \vec{r}) = \hat{F}(\vec{r})$$

$$\langle \hat{F}(\vec{r}) \rangle_\alpha = F(\vec{r}) = \frac{\text{Tr}_\alpha [e^{-\beta H} \hat{F}(\vec{r})]}{\text{Tr}_\alpha [e^{-\beta H}]}$$

Es. :  $\hat{g}(\vec{r}) = \sum_{i=1}^N \delta(\vec{r} - \vec{r}_i)$

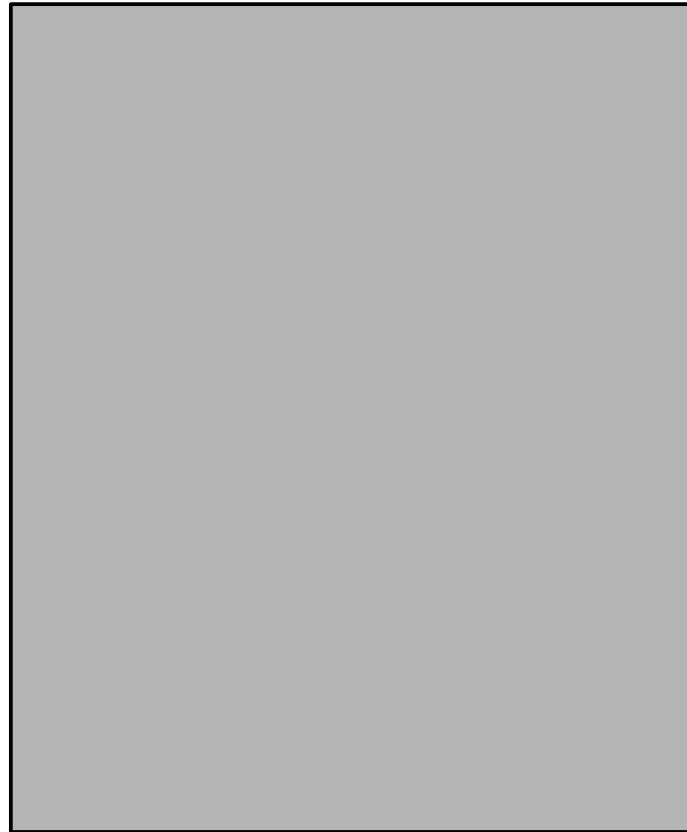
densità micro

$$g(\vec{r}) = \langle \hat{g}(\vec{r}) \rangle_\alpha$$

densità locale

traslazione discreta:  $\vec{T} : \vec{r} \rightarrow \vec{r} + \vec{R}_e$   
traslazione continua:  $T : \vec{r} \rightarrow \vec{r} + \vec{R}$

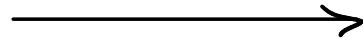
$g(\vec{r})$  profilo della densità locale



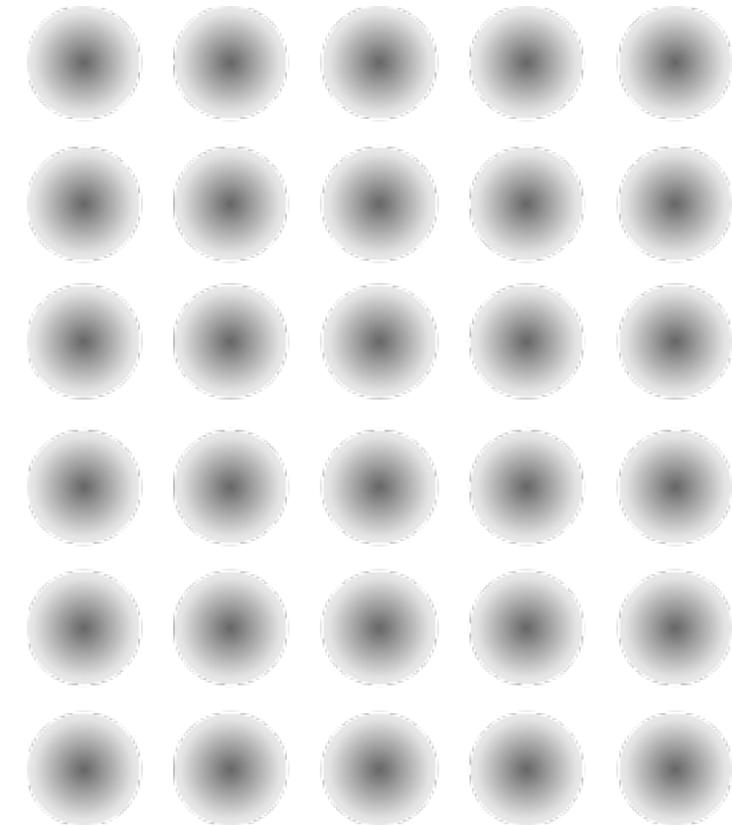
LIQUIDO

$$g(\vec{r}) = \text{cost}$$

ordine a  
lungo raggio



ROTTURA  
SPONTANEA DI  
SIMMETRIA

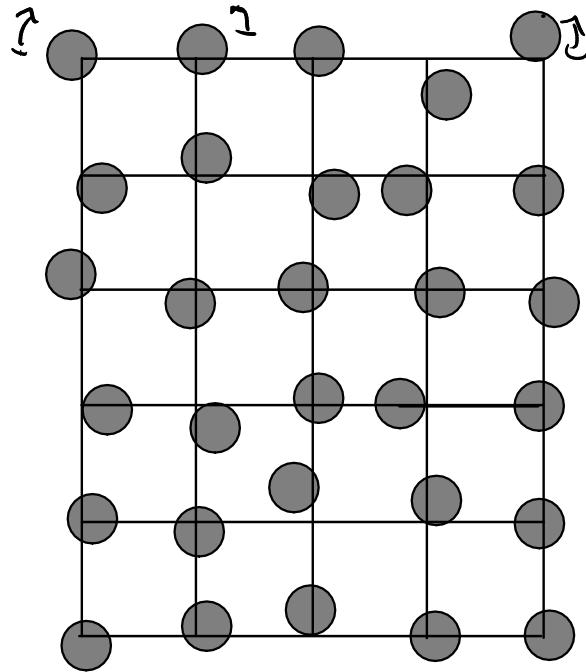


CRISTALLO

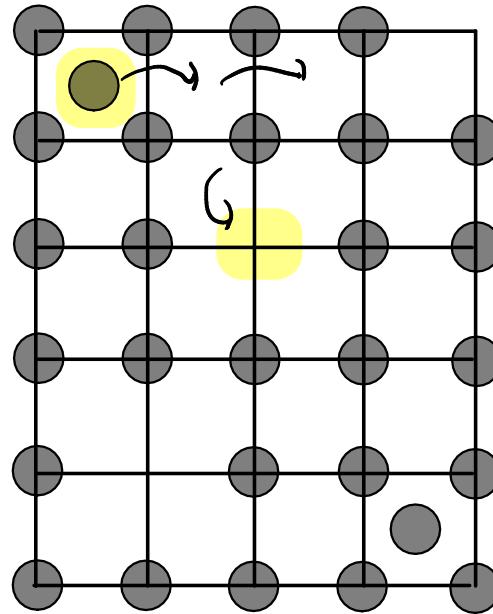
# TIPOLOGIE DI DISORDINE

Casuale / aleatorio  $\sim$  disordine  $\Delta$   $\Rightarrow$  ENTROPIA

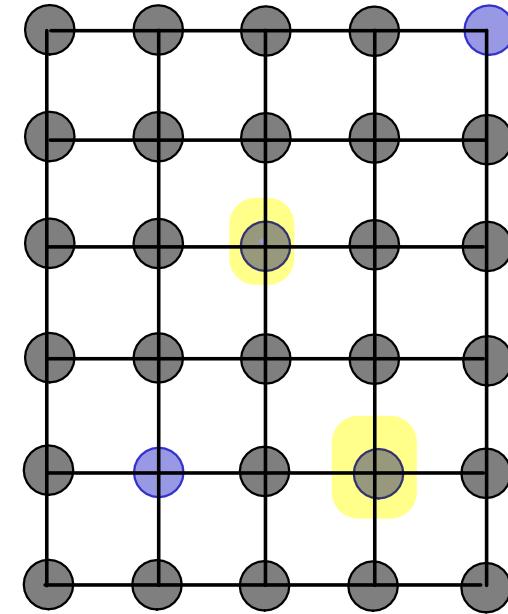
## 1) Disordine sostituzionale



moto  
vibrazionale



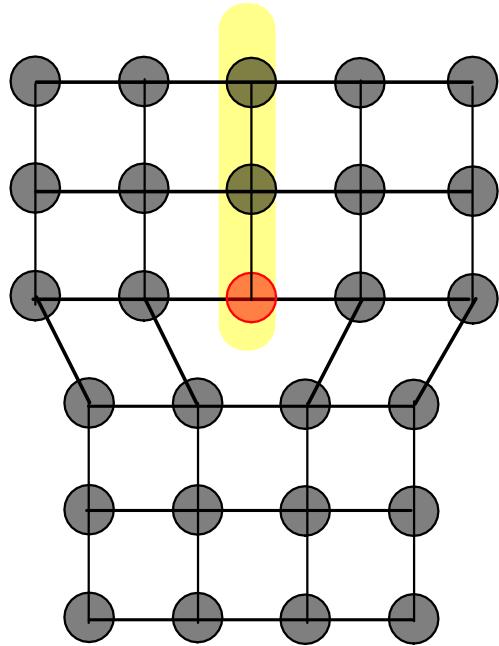
difetti:  
interstiziali  
vacanze



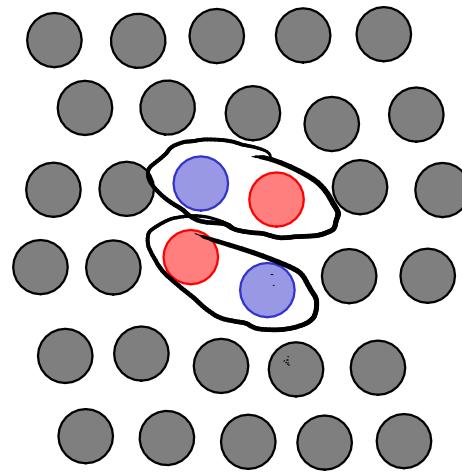
miscela AB

2) **Disordine topologico**

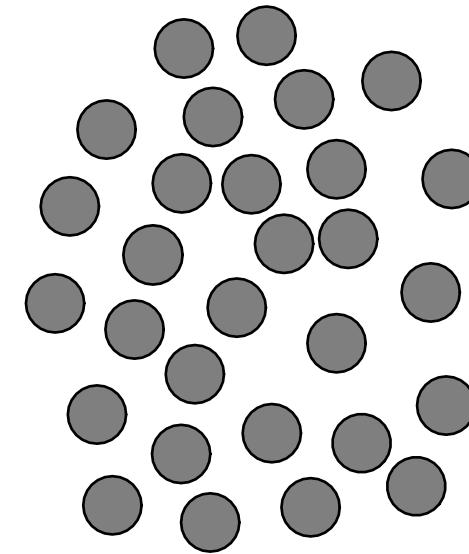
difetti  $\rightarrow$  invarianti topologici



dislocazioni



dislocazioni



amorfo

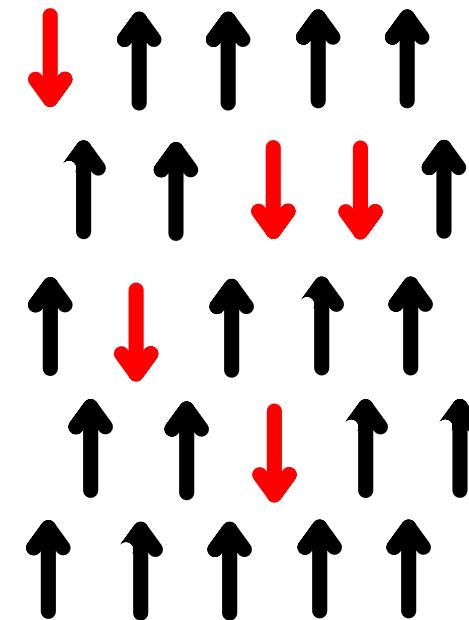
### 3) Disordine gelato

fluidi in un mezzo poroso



$$H = H(\{\bar{F}_i\}, \{\bar{r}_m\})$$

vetri di spin  $\sigma_i = \pm 1$



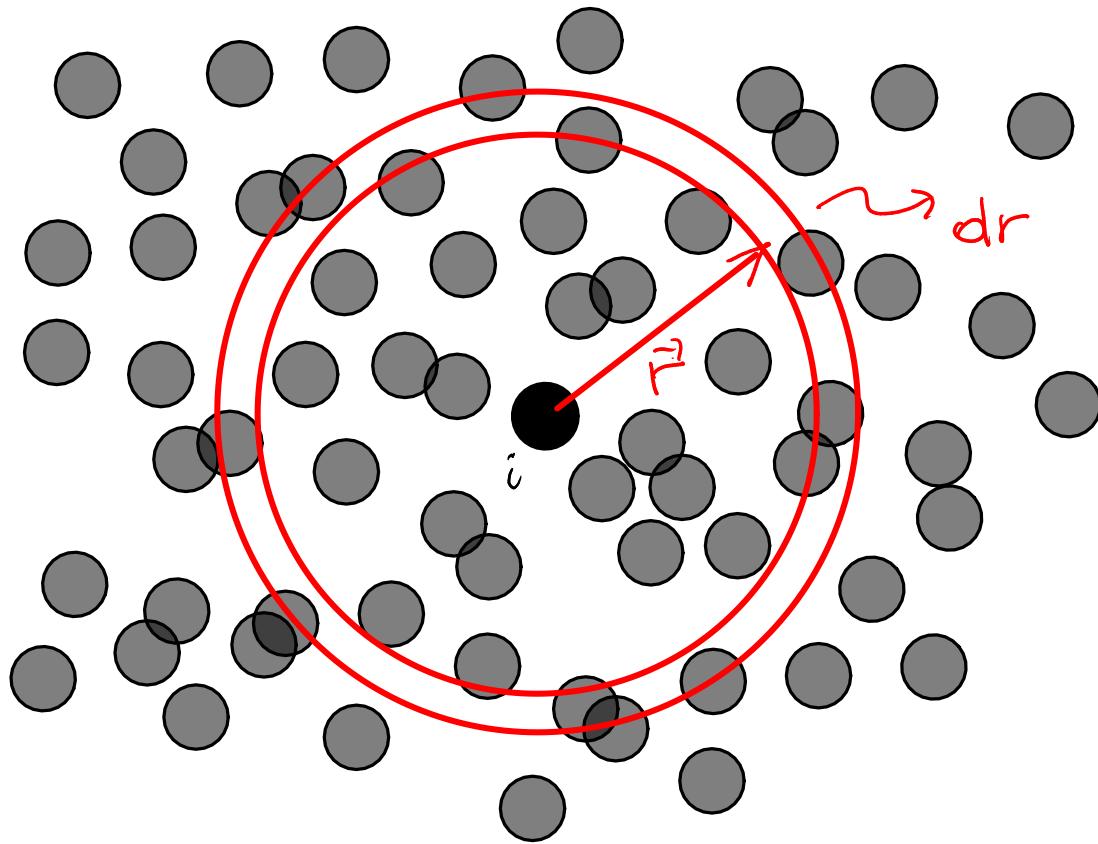
Ising :  $H = - \sum_{\langle ij \rangle} J \sigma_i \sigma_j$

Edwards Anderson :  $H = - \sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$

$P(J_{ij})$  gaussiana

# ORDINE E CORRELAZIONI

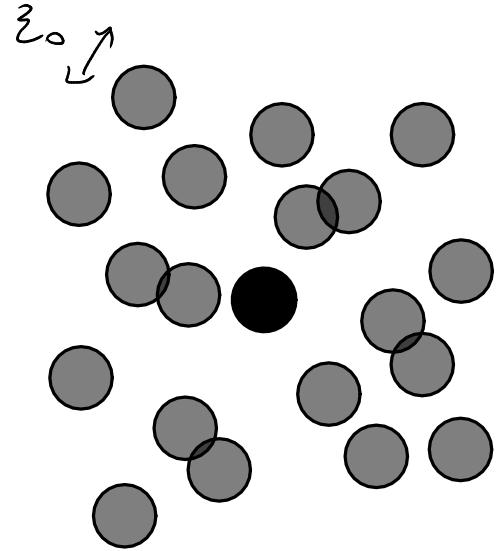
$$\rho = \frac{N}{V}$$



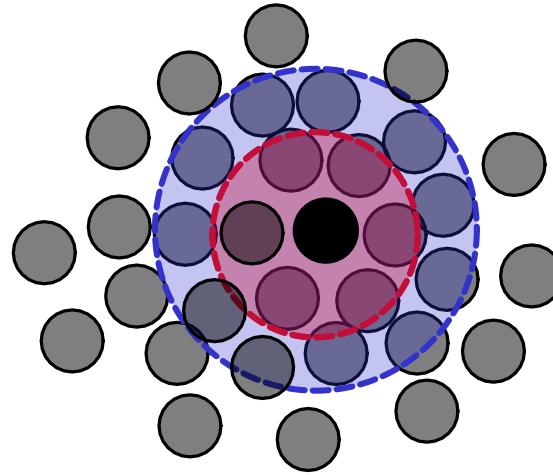
Funzione di distribuzione radiale

$$g(r) = \frac{\left\langle \frac{1}{N} \sum_{i=1}^N n_i(r) \right\rangle}{\rho \cdot 4\pi r^2 dr}$$

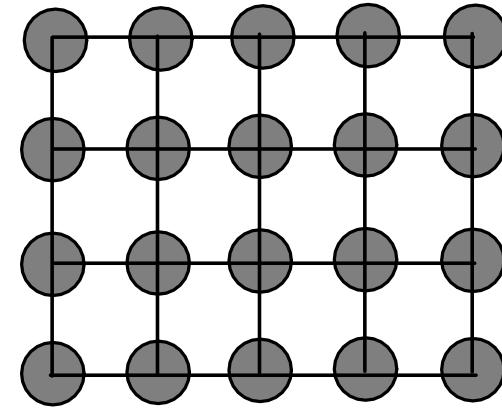
gas  
corto raggio



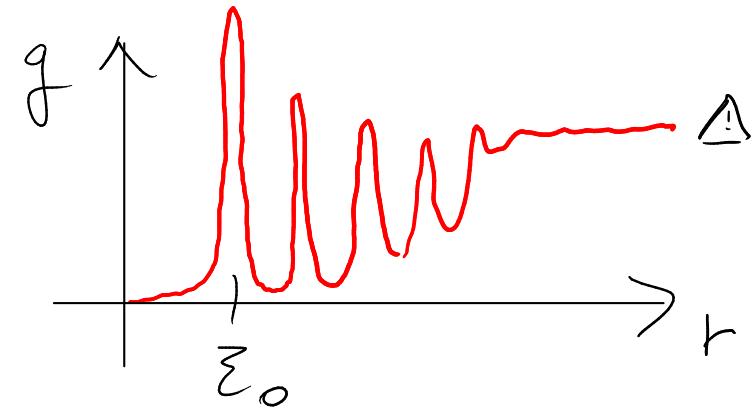
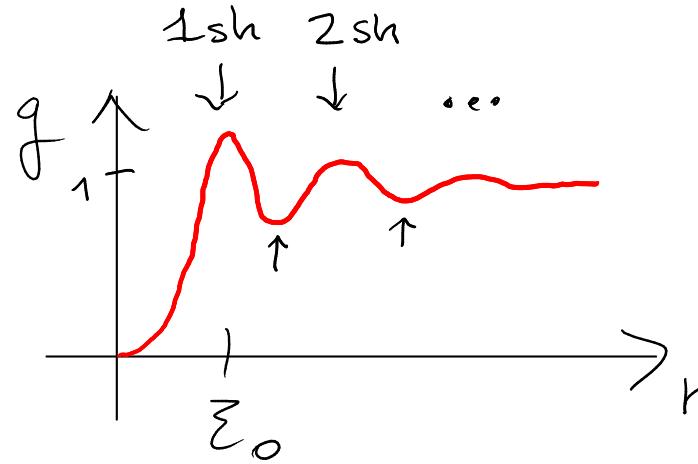
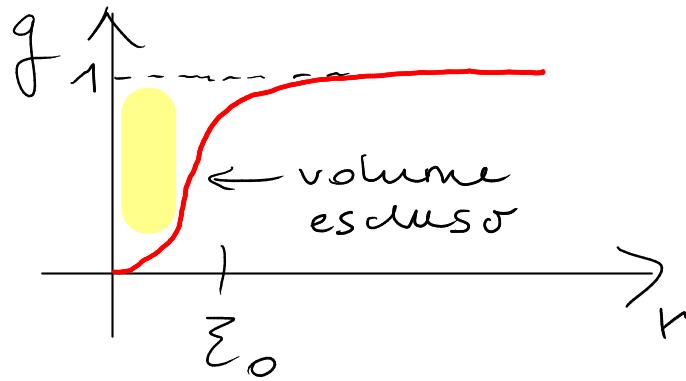
liquido  
medio raggio

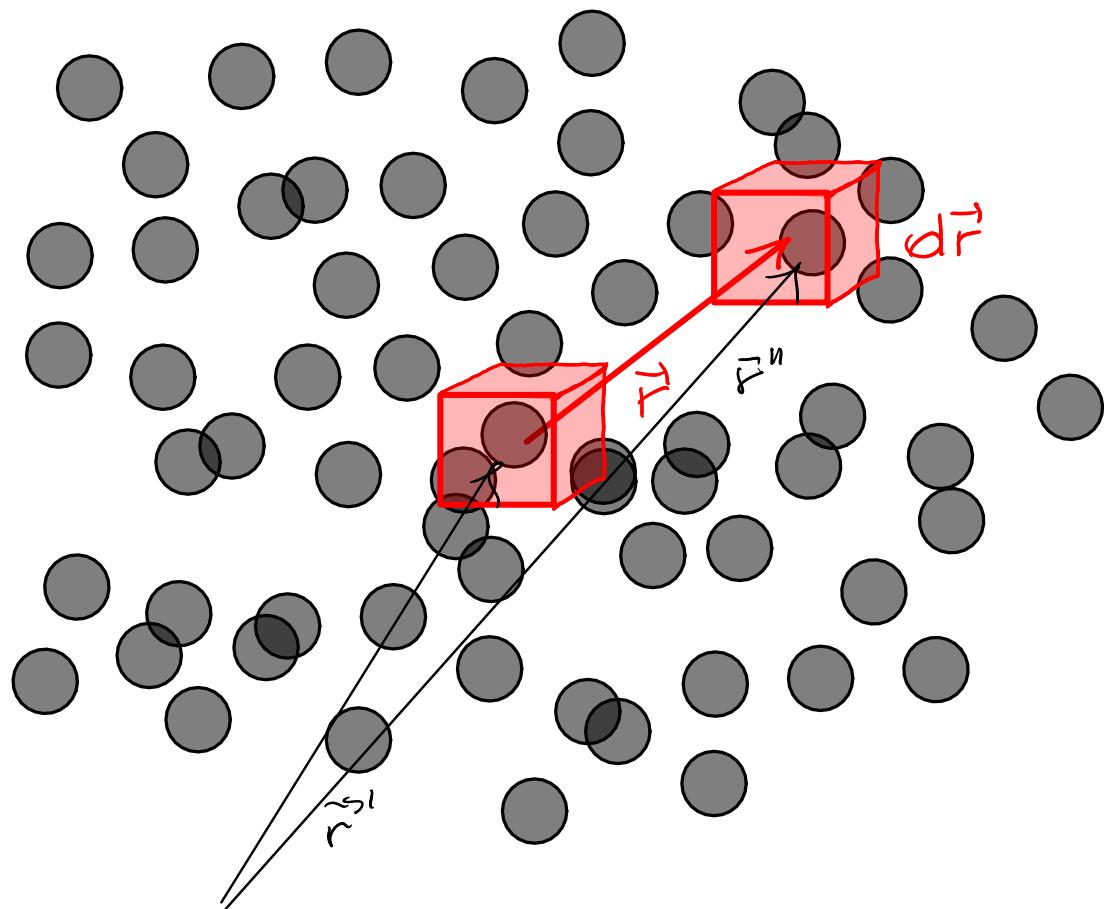


cristallo  
lungo raggio



g.p.:  
 $g(r) = 1$





Funzione di correlazione della densità microscopica

$$\hat{\rho}(\vec{r}) = \sum_{i=1}^N \delta(\vec{r} - \vec{r}_i)$$

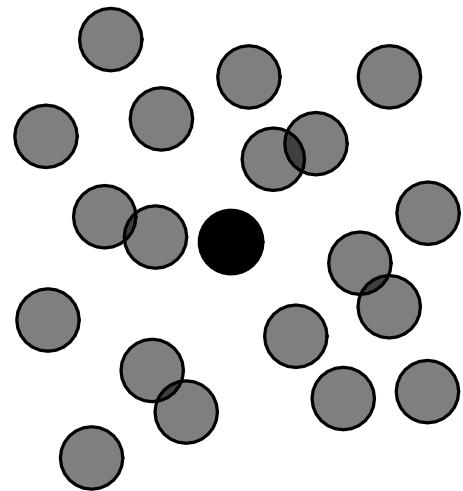
$$G(\vec{r}_1, \vec{r}_2) = \langle (\hat{\rho}(\vec{r}_1) - \rho) (\hat{\rho}(\vec{r}_2) - \rho) \rangle$$

↓

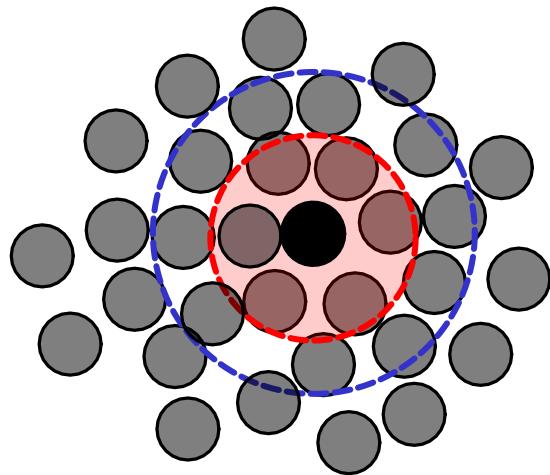
$$G(\vec{r}) \quad \vec{r} = \vec{r}_2 - \vec{r}_1 \text{ omogeneo}$$

$$G(\vec{r}) = \rho (g(\vec{r}) - 1) + \delta(\vec{r})$$

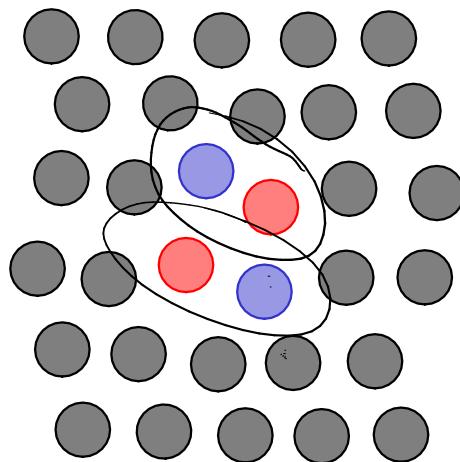
gas  
corto



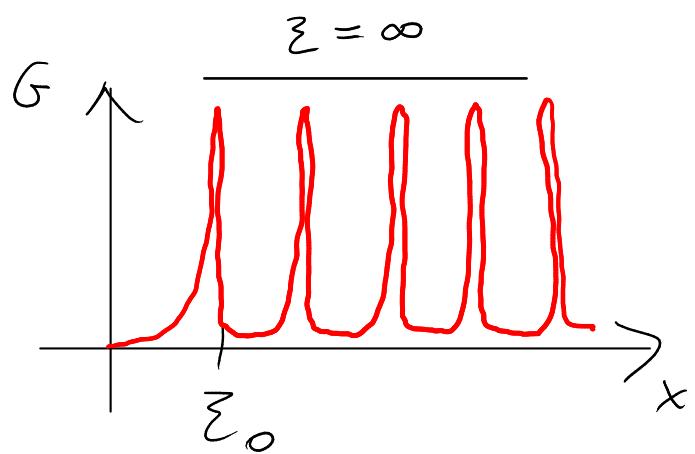
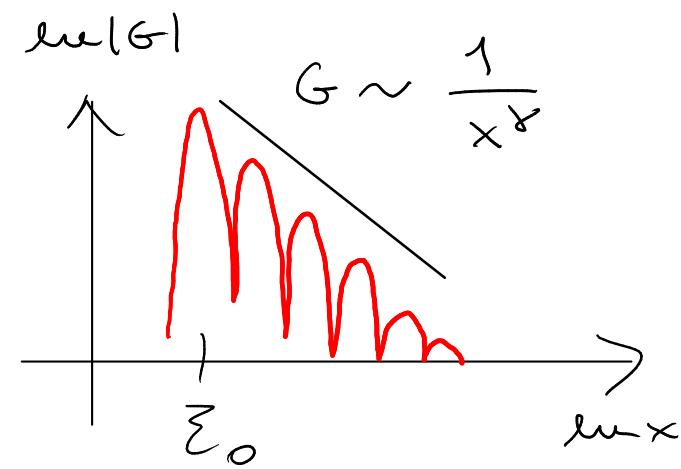
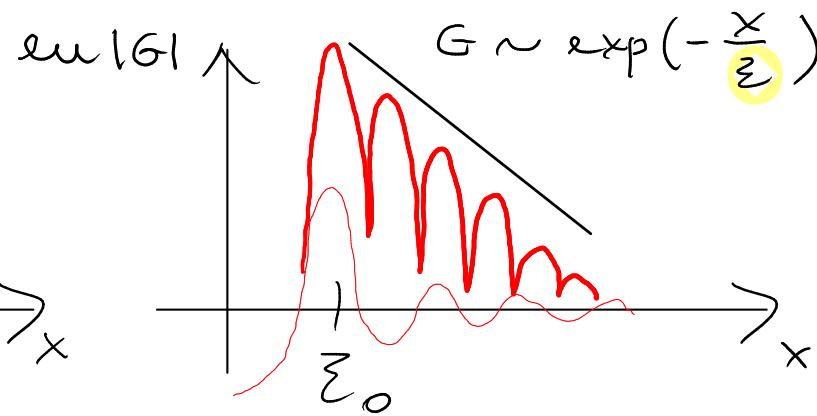
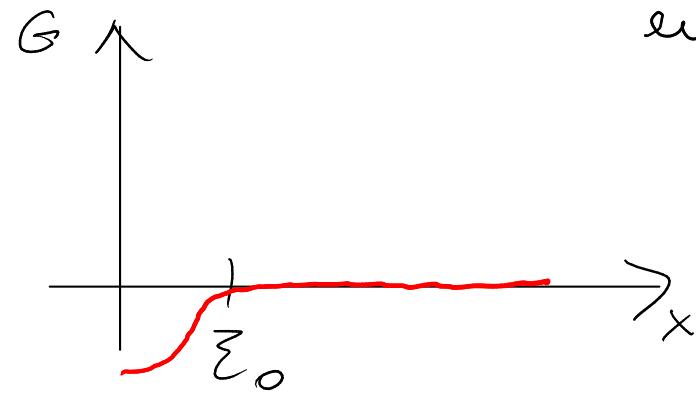
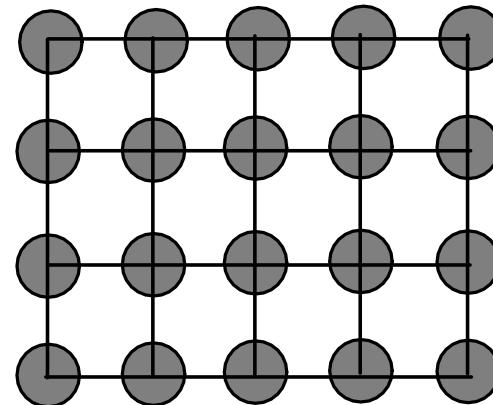
liquido  
medio  $\xi \approx \xi_0$



solidi 2d  
quasi-lungo



cristallo  
lungo



ORDINE E DIMENSIONALITÀ

130-140: Peierls, Landau  
 1d no ordine a lungo raggio  
 lowest critical dimension  
 simmetria discreta / continua

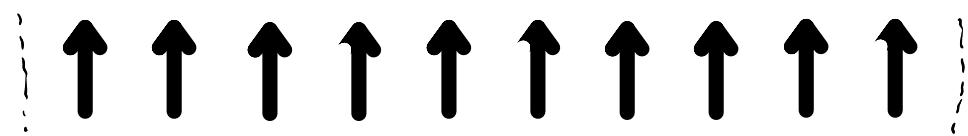


Peierls



Landau

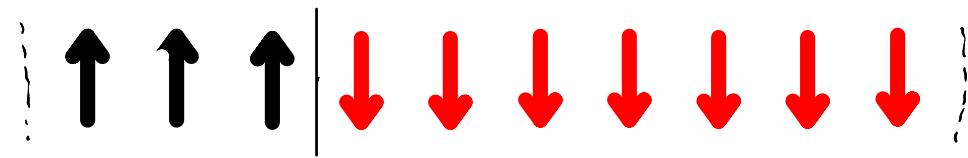
1) Spins 1d, N,  $\sigma_i = \pm 1$ , PBC, senza campo esterno



$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j \quad J > 0$$

$$F = E - TS = NJ \text{ cristallo}$$

interfaccia



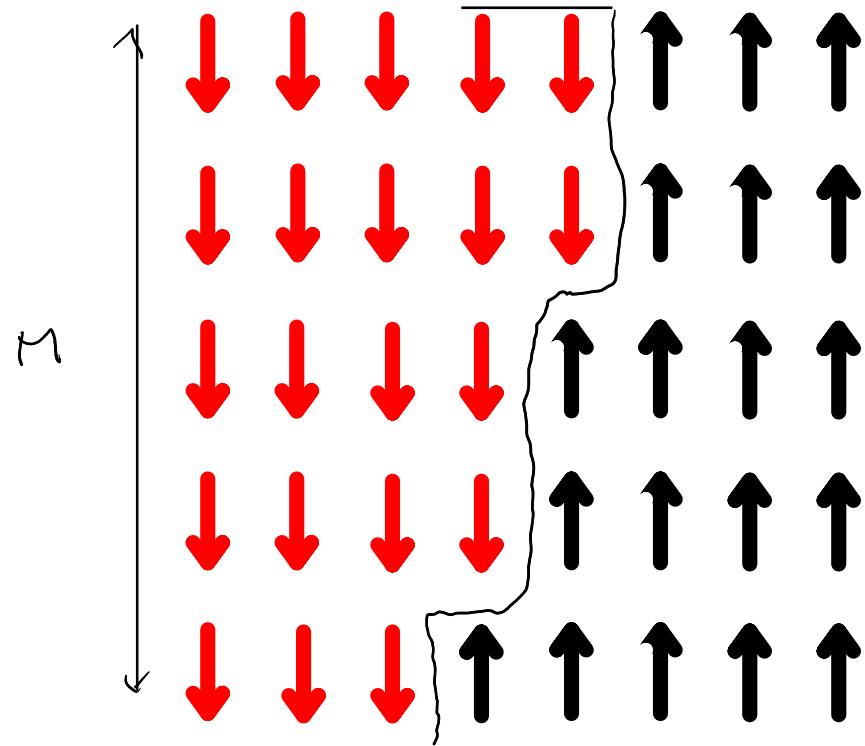
$$\Delta E = 2J$$

$$\Delta S = k_B \ln N$$

$$\Delta F = 2J - k_B T \ln N$$

$\forall T \exists N$  t.c.  $\Delta F < 0$ : interfaccia "destabilizza" XTAL  $\Rightarrow$  no ordine a lungo raggio

2) Spins  $2d$ ,  $N$ ,  $\sigma_i = \pm 1$ , PBC, senza campo esterno,  $z = n$ -di coordinazione



$N \rightarrow \infty, n \rightarrow \infty$

$n$  coppie di spins "a cavallo" dell'interfaccia

$$\Delta E = nJ + nJ \approx 2nJ$$

$$\Delta S \approx k_B \ln [(z-1)^n] = n k_B \ln (z-1)$$

$$\Delta F = \underbrace{[2J - k_B T \ln (z-1)]}_{} n$$

$$T_c = \frac{2J}{k_B \ln (z-1)} \xrightarrow{z=4} T_c = 1.82 \frac{J}{k_B}$$

$T > T_c$ :  $\Delta F \rightarrow -\infty$  se  $n \rightarrow \infty \Rightarrow$  disordinato

$T < T_c$ :  $\Delta F$  è minimo per  $n=0$

$\Rightarrow$  ordine a lungo raggio

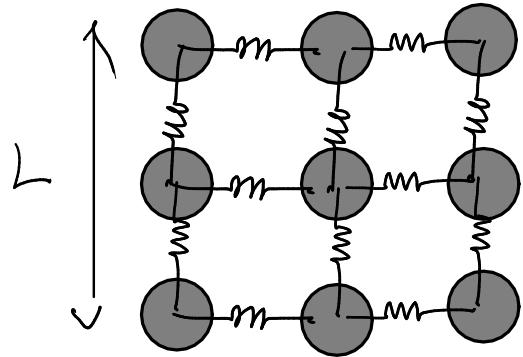
lowest critical dim = 1

1941: Kramers-Wannier duality  $T_c = 2.269 J/k_B$

1943: Onsager prima soluzione esatta del modello di Ising 2d

1953: Kac & Ward metodo combinatorio

3) **particelle**,  $N$ , vibrazioni armoniche,  $\mathbb{H}$  simmetria per traslazione, PBC  
 $d = \text{dimensionalità}$



$\Delta \vec{r}_i$   $\langle |\Delta \vec{F}|^2 \rangle = \langle \frac{1}{N} \sum_{i=1}^N |\Delta \vec{r}_i|^2 \rangle$  spostamento quadratico medio

modi normali  $\rightarrow$  densità degli stati  $g(\omega)$

$$\langle |\Delta \vec{F}|^2 \rangle = \frac{d \cdot k_B T}{m} \int_{\omega_{\min}}^{\infty} d\omega \frac{g(\omega)}{\omega^2}$$

$$\omega_{\min} = \frac{2\pi}{L}$$

$$\omega_{\min} \approx c \frac{2\pi}{L}$$

$\uparrow$   
piccoli  $\omega$

Debye:  $g(\omega) \sim \omega^{d-1}$

$$\langle |\Delta \vec{F}|^2 \rangle \sim \int_{\frac{c2\pi}{L}}^{\infty} d\omega \omega^{d-3}$$

$d \geq 3$ :  $\langle |\Delta \vec{F}|^2 \rangle \rightarrow \text{cost}$

$d = 2$ :  $\langle |\Delta \vec{F}|^2 \rangle \sim \ln L$

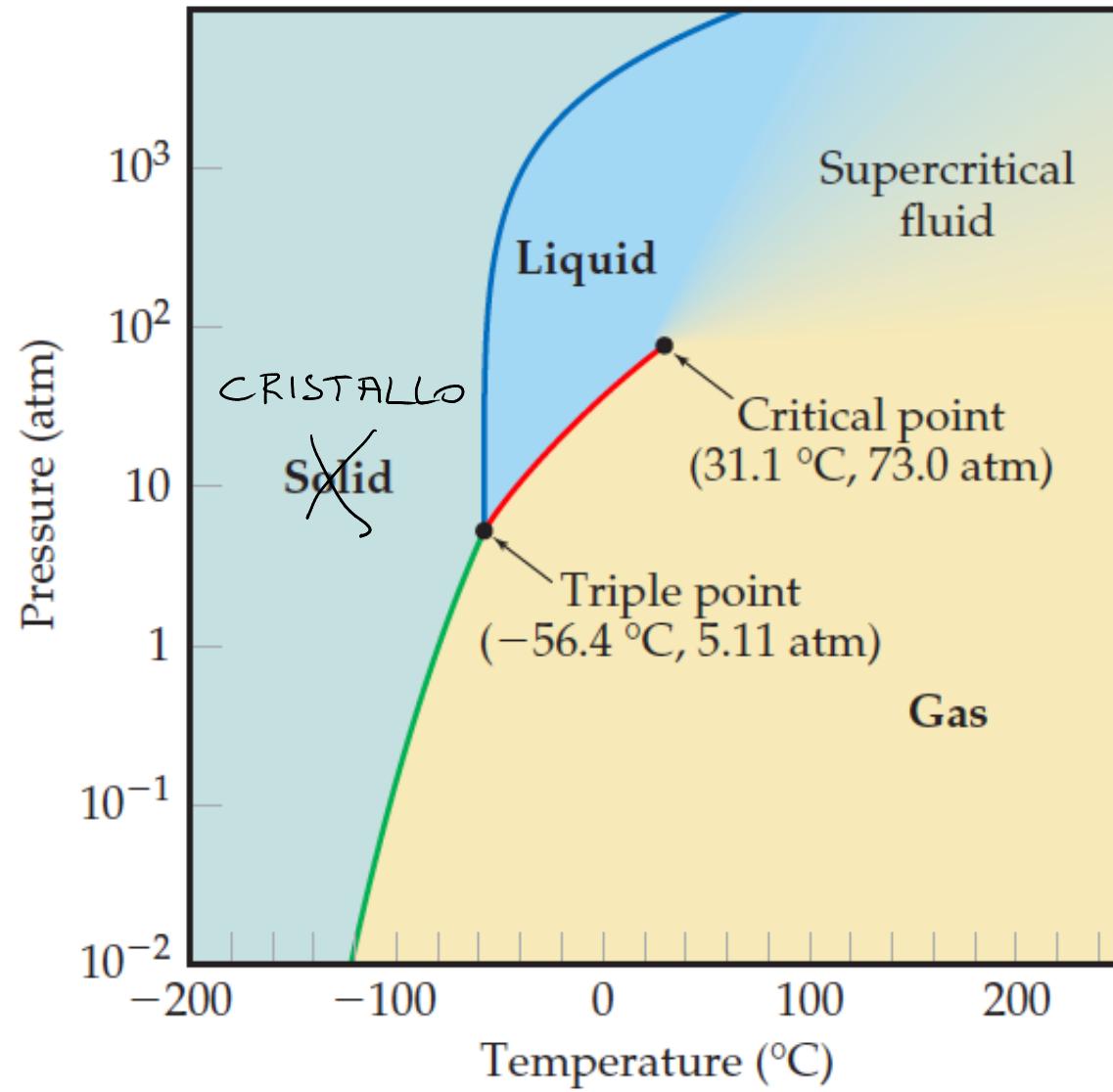
$d = 1$ :  $\langle |\Delta \vec{F}|^2 \rangle \sim L$

}  $\langle |\Delta \vec{F}|^2 \rangle \rightarrow \infty$  se  $L \rightarrow \infty$

160. Hohenberg, Mermin, Wagner

Sotto ipotesi abbastanza generali sulla natura delle interazioni e dei costituenti elementari, sistemi con simmetria continua non possiedono ordine a lungo raggio in 1d e 2d

Lowest critical dimension: 2



meccanica



SOLIDO

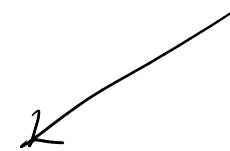
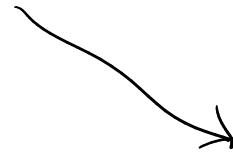
elasticità

reologia



FLUIDO

viscosa



visco-elasticità

LIQUIDI

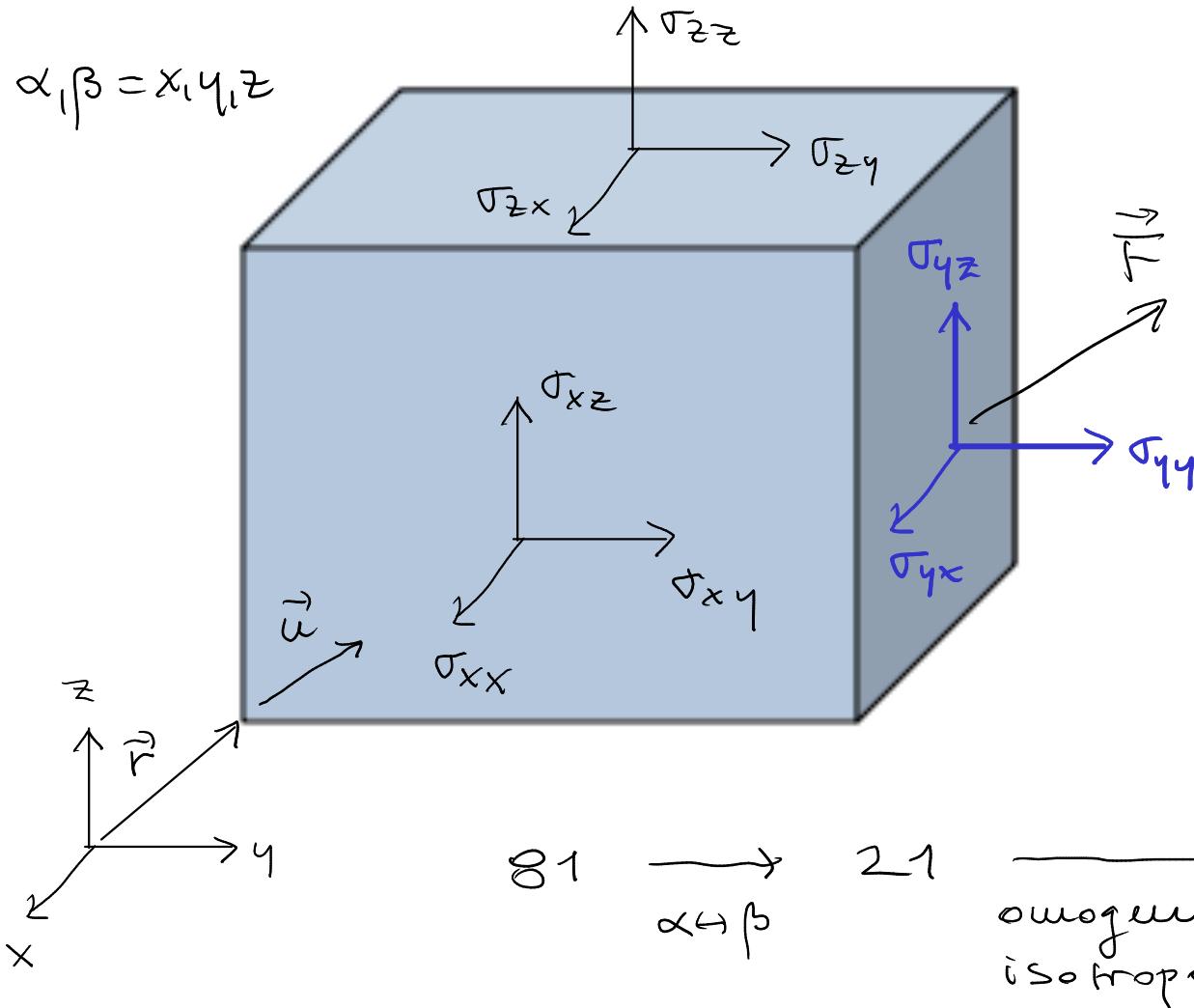
SOTTORAFFREDDATI

MATERIA SOFFICE

# ELASTICITÀ

Sforzo = modulo  $\times$  deformazione  
 $F/A$  SI: Pa relativa

Hooke:  $F = -k \Delta x$   
 $\rightarrow$  solido hookiano



Sforzo:  $\sigma_{\alpha\beta}$   $\alpha \leftrightarrow \beta$

Spostamento:  $\vec{u}$

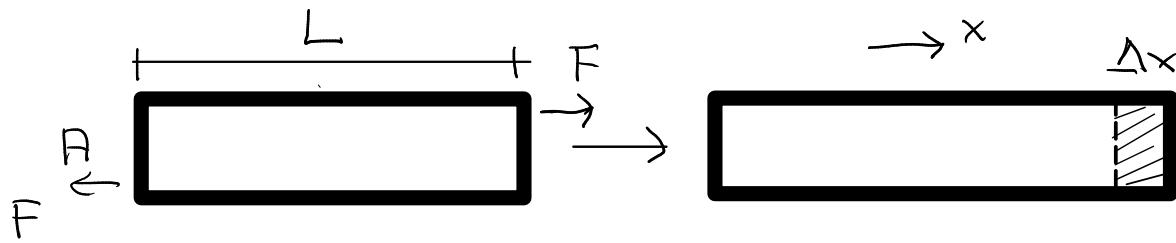
Deformazione:  $\gamma_{\alpha\beta} = \frac{\partial u_\beta}{\partial r_\alpha}$

$$\rightarrow \frac{1}{2} \left( \frac{\partial u_\alpha}{\partial r_\beta} + \frac{\partial u_\beta}{\partial r_\alpha} \right)$$

$$\sigma_{\alpha\beta} = \frac{1}{\delta\theta} C_{\alpha\beta\delta\theta} \gamma_{\delta\theta}$$

$\uparrow$   
**modulo** (costanti elastiche)

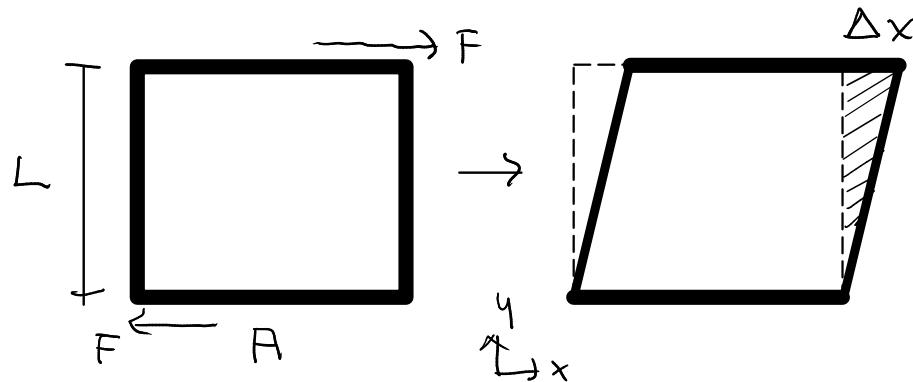
## 1) Sforzo di trazione



$$\left. \begin{aligned} \sigma_{xx} &= \sigma = \frac{F}{A} \\ \gamma_{xx} &= \gamma = \frac{\Delta x}{L} \end{aligned} \right\} \sigma = \gamma \text{ } \underline{\text{modulus di Young}}$$

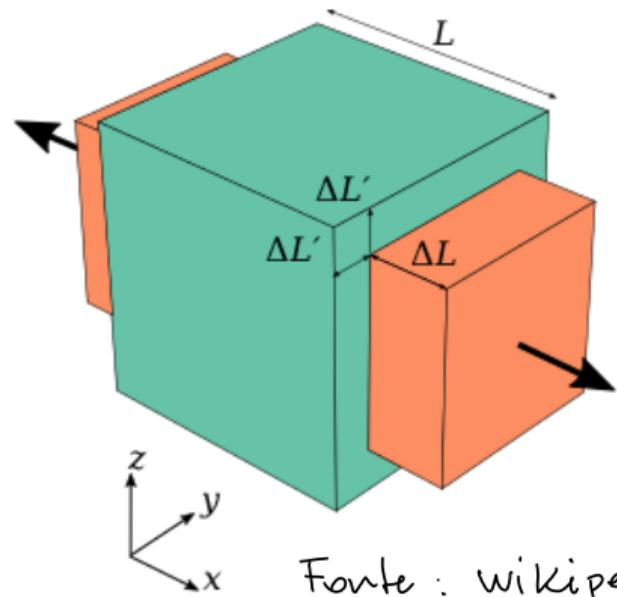
Es: gomma  $\gamma \simeq 0.1 - 0.01 \text{ GPa}$ , diamante  $\gamma \simeq 10^3 \text{ GPa}$  @ Ta

## 2) Sforzo di taglio



$$\left. \begin{aligned} \sigma_{yx} &= \sigma = \frac{F}{A} \\ \gamma_{yx} &= \gamma = \frac{\Delta x}{L} \end{aligned} \right\} \sigma = \gamma \text{ } \underline{\text{modulus di taglio}}$$

Es.: gomma  $6 \times 10^{-4} \text{ GPa}$  diamante  $500 \text{ GPa}$  @ Ta



Fonte: wikipedia  
"Poisson's ratio"

Isotropo, omogeneo:  $\gamma = 2G(1 + \nu)$

$$\nu = - \frac{\Delta L'}{\Delta L}$$

← trasversale  
← longitudinale

↑  
coefficiente di  
Poisson

$$0 \leq \nu < 1/2$$

↑

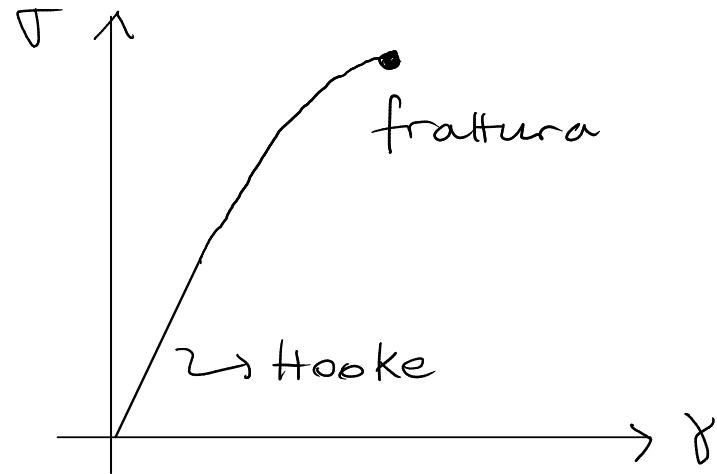
es. sughero

↑

incomprimibile es. gomma

auxetici:  $\nu < 0$

fragile



duttile



**Brittle Fracture**

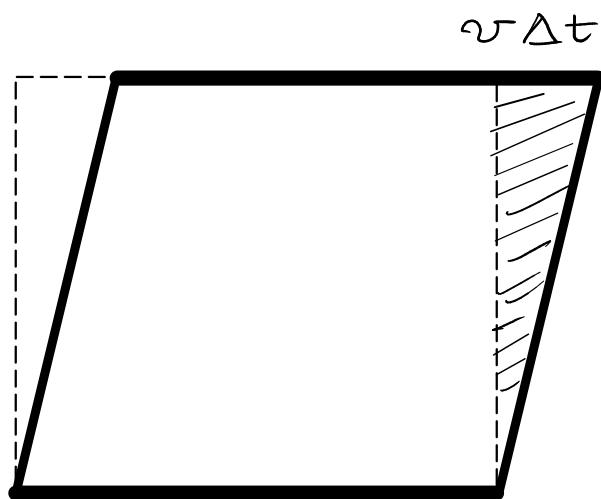
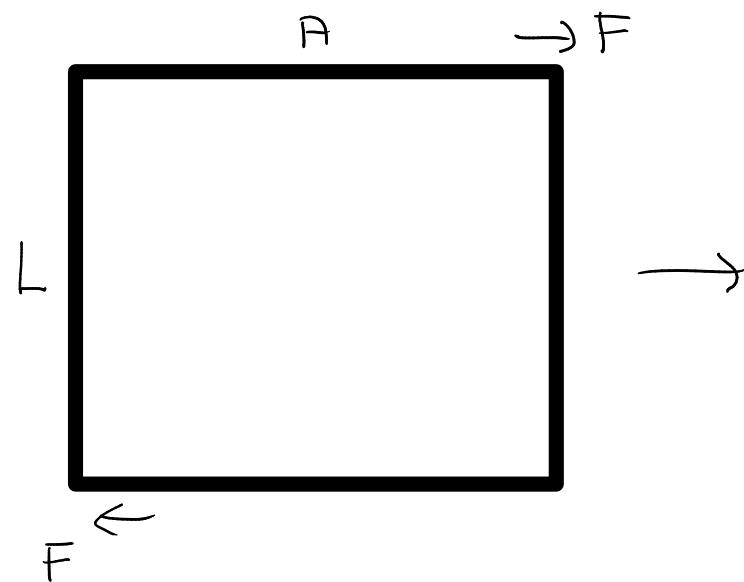


**Ductile Fracture**



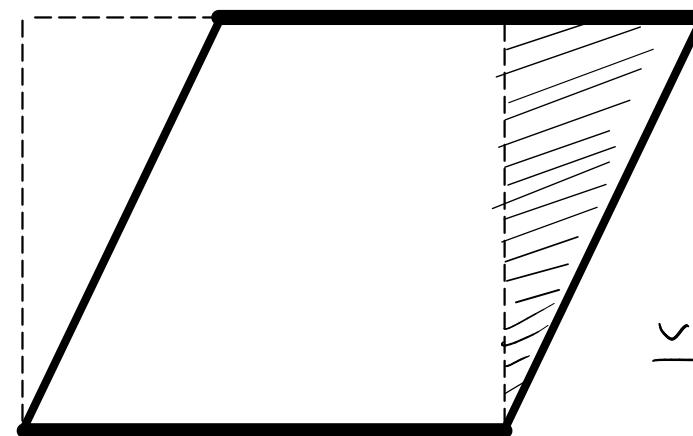
# VISCOSITÀ

Sforzo  $\sim$  deformazione  
Hooke equilibrio



$$\gamma = \frac{v\Delta t}{L}$$

Sforzo  $\sim$  tasso di deformazione  
Newton **stazionario**



$$\dot{\gamma} = \frac{v}{L}$$

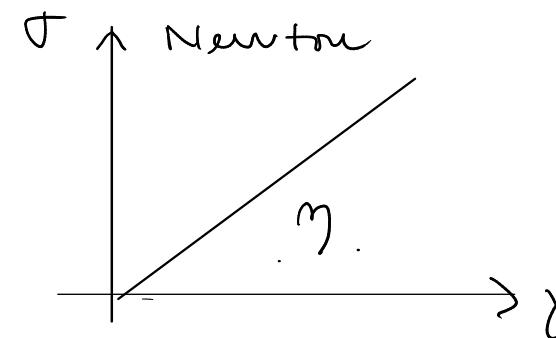
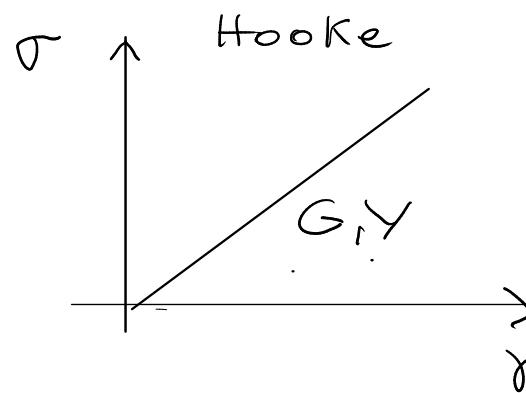
$$\Rightarrow \sigma = \eta \dot{\gamma}$$

viscosità  
↓

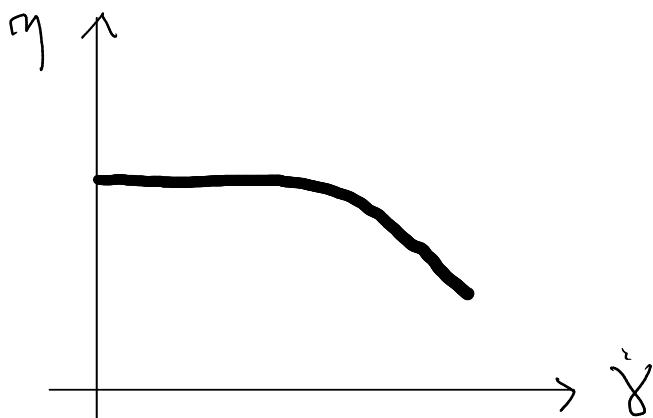
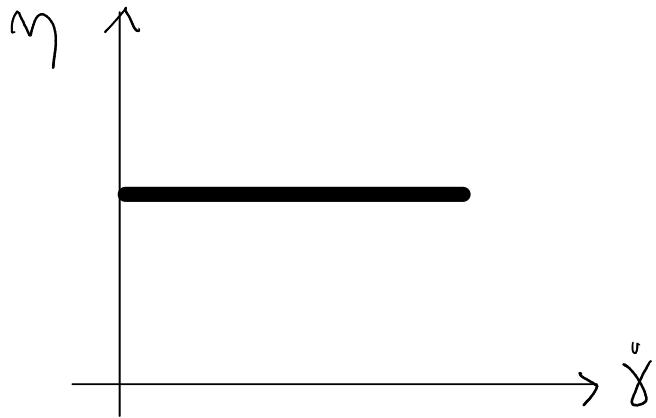
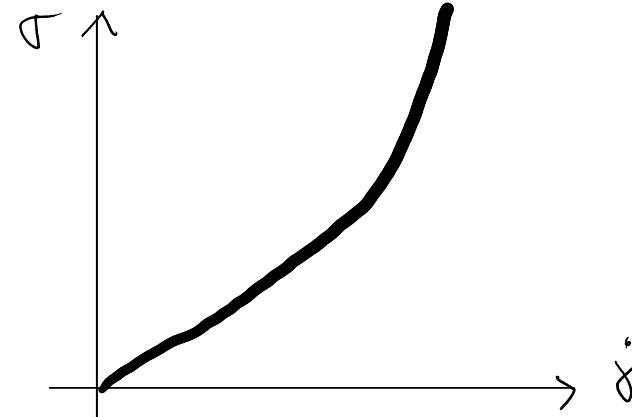
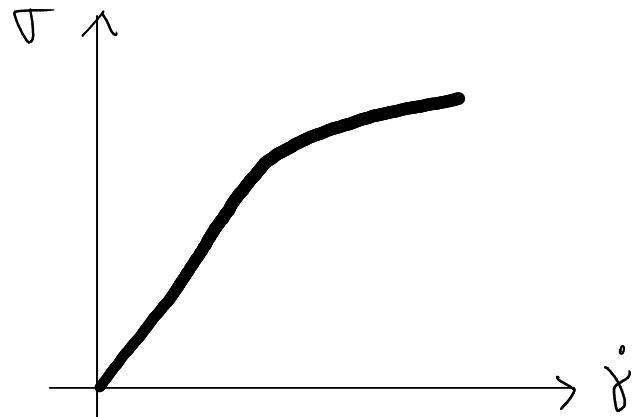
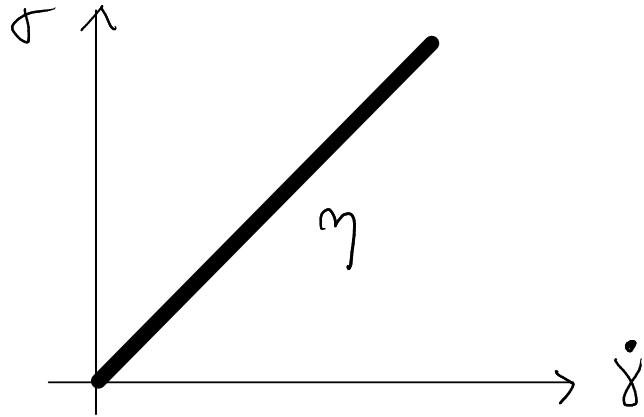
SI: Pa·s

ES.:  $H_2O @ T_{amb}$   
 $\eta \sim 10^{-3} \text{ Pa}\cdot\text{s}$   
Miele  
 $\eta \sim 1 \text{ Pa}\cdot\text{s}$

$T \approx 90^\circ\text{C}$   
 $\eta \sim 10^{-4} \text{ Pa}\cdot\text{s}$   
Liquido @  $T_g$   
 $\eta \sim 10^{12} \text{ Pa}\cdot\text{s}$



$$\sigma = \eta(\dot{\gamma}) \dot{\gamma}$$



fluids Newtoniani

assottigliamento al taglio

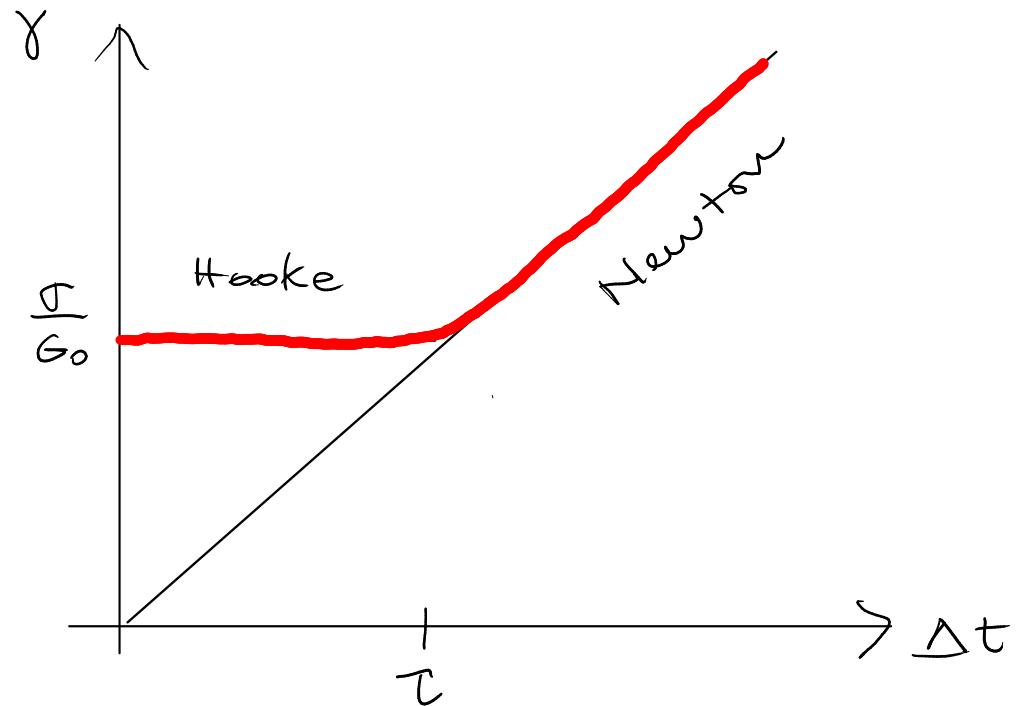
shear thinning

ispessimento al taglio

shear thickening

# VISCO-ELASTICITA'

Dipendenza dall'intervallo di tempo  $\Delta t$  durante il quale perturbo il sistema



$$\sigma = G_0 \gamma \quad G_0 = \text{modulo di taglio istantaneo}$$

$$\sigma = \eta \dot{\gamma}$$

$\tau$  = tempo di rilassamento

$$\begin{cases} \dot{\gamma} = \frac{\sigma}{G_0 \tau} \\ \dot{\gamma} = \frac{\sigma}{\eta} \end{cases} \Rightarrow \text{relazione di Maxwell} \quad \eta = G_0 \tau$$

Es.:  $G_0 \sim 10^9 \text{ Pa}$

$\tau \sim 10^{-12} \text{ s}$

$\eta \approx 10^9 \text{ Pa} \times 10^{-12} \text{ s} \approx 10^{-3} \text{ Pa}\cdot\text{s}$

## Interpretazione micro

$$[G] = [Y] = \frac{E}{V} \quad \text{energia elastica / volume}$$

viscosità  $\rightarrow$  tasso di dissipazione di energia / volume

$$\frac{dW}{dt} = F \cdot v = (\sigma A) \cdot (\dot{\gamma} L) = \sigma \dot{\gamma} V$$

$$\dot{w} = \sigma \dot{\gamma} = \eta \dot{\gamma}^2 \leftarrow \text{fluido Newtoniano} \quad \dot{w} \sim \dot{\gamma}^2 \sim \eta$$

$\uparrow$   
tasso per unità di volume

$$\sigma = \eta \dot{\gamma}$$

