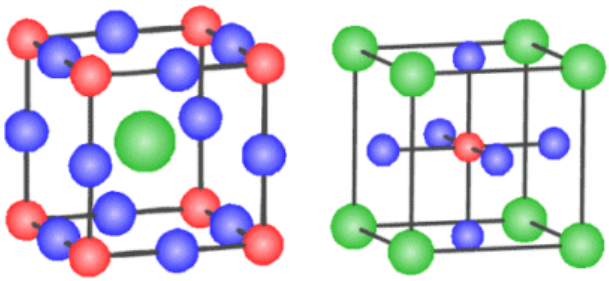
Condensed Matter Physics I
Final written test - 8 February 2016
(3 hours)

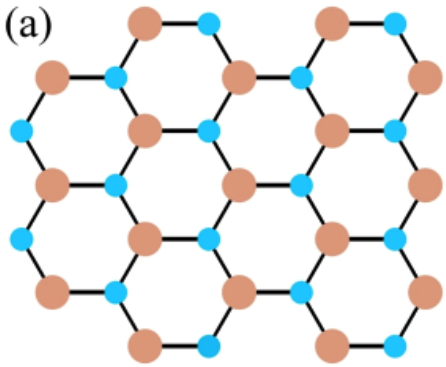
- Solve all the exercises.
- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Crystalline structures*

1. What is $(10\bar{3})$? Sketch it in a cubic unit cell.
2. What is $[10\bar{3}]$? Sketch it in a cubic unit cell.
3. In the Figure below, two representations of the perovskite structure are shown. Given red=Ca, green=Ti, blue=O, what is the formula for the chemical composition of the solid corresponding to each representation? Are the two representations equivalent? (Justify your answer)



4. Calculate the structure factor of BN (Boron Nitride), which has the honeycomb arrangement (see Figure below).
5. Assuming that B and N have the same atomic form factor, for which **K** there is the maximum constructive interference?



Exercise 2: *Electron orbits in empty triangular lattice*

A two-dimensional free-electron-like solid has a triangular lattice structure and 2 electrons per unit cell.

1. Show that the Fermi wave vector k_F is $(2\sqrt{3}/\pi)^{1/2}$ times larger than the distance of the Bragg planes from the Γ point.

2. Show that this implies that the solid is a metal with two partially filled bands. Sketch the Fermi surface in the first and in the second Brillouin zones.

3. Consider a uniform magnetic field H applied perpendicular to the plane of the metal, pointing out of the page. Show the orbits of one electron lying on the Fermi surface in the lowest band and in the second band. Are the two orbits open or closed? Are they electron-like or hole-like (are they covered in a clockwise or anticlockwise direction)?

4. Which is the perimeter of the orbit at the Fermi surface in the second band (in terms of k_F)?
   (Hint: consider the angle subtended by the arc of the Fermi circle outside the first Brillouin zone...)

5. Find the orbital period T_{free} for an electron at the Fermi surface in absence of the lattice (a totally free electron).

6. In the case of the empty triangular lattice, find the ratio between the orbital period T for an electron at the Fermi surface of the second band and T_{free}.
   (Hint: The time taken to traverse a portion of the orbit between two points $\mathbf{k}_1$ *and* $\mathbf{k}_2$ *is given by* $T = \int_{\mathbf{k_1}}^{\mathbf{k_2}} dk/|\dot{\mathbf{k}}| \ldots$*; here* $\dot{\mathbf{k}}$ *is constant independent on* $\mathbf{k}$ *(why?); consider the perimeter of the second-band and that of the free electron Fermi circle...; at the end you should arrive to a number)*