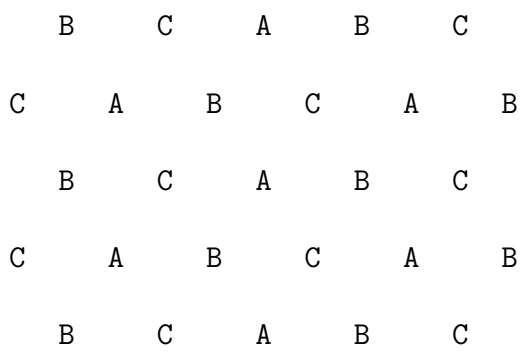
Condensed Matter Physics I
Final written test
academic year 2012/13
February 15, 2013

(*Time: 3 hours*)

Exercise 1: *Crystal lattices and structure factors*

Consider the 2D lattice here sketched, made of three different atomic species A, B, C, placed on the sites of a triangular lattice. For the sake of definiteness, consider the origin in one of the atoms A; let d be the AB distance.

1. Choose and write a set of primitive vectors and a basis, sketch the unit cell and calculate its area.
2. Write and sketch the basis of the reciprocal lattice.
3. Calculate the structure factor S($\mathbf{k}$) in the general case, when the atomic form factors f_A, f_B and f_C are different.
4. Calculate the structure factor in case of $f_A = f_B = f_C$. Specify on which reciprocal lattice vectors it is not vanishing. Comment the result.



Exercise 2: *Band structures for free electrons*

Consider the BCC lattice.

1. Plot the contours of the First Brillouin Zone in the (k_x, k_y) plane, specifying the coordinates of the relevant points.
2. Write explicitly the expression of the first three energy bands $\mathcal{E}_n(\mathbf{k})$ for $n = 1, 2, 3$ for free electrons from Γ to N=$\frac{2\pi}{a}$(1/2, 1/2, 0) as a function of the modulus of $\mathbf{k}$ and make a plot in the reduced zone scheme.

Exercise 2: *Semiclassical model of electron dynamics*

Consider the electron orbits in a 3D solid with band dispersion

$$E(\mathbf{k}) = E_0 - 2t[cos(k_x a) + cos(k_y a) + cos(k_z a)]$$

with lattice parameter a, under a uniform static magnetic field $\mathbf{H} = H\hat{y}$; $t > 0$.

1. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0 - 4t$.
2. Write the Bloch electron velocity in direct space corresponding to the orbit in (1). Describe the orbit in real space.
3. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0$.
4. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0 + 4t - t\delta^2$ and $\delta \ll 1$.
5. Write explicitely the period of that orbit. What should be t in order to have the same period as for free electrons?
6. Indicate the direction of the motion along the orbit.

NOTE:

- Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.