Condensed Matter Physics I
final written test
academic year 2013/2014
January 27, 2014

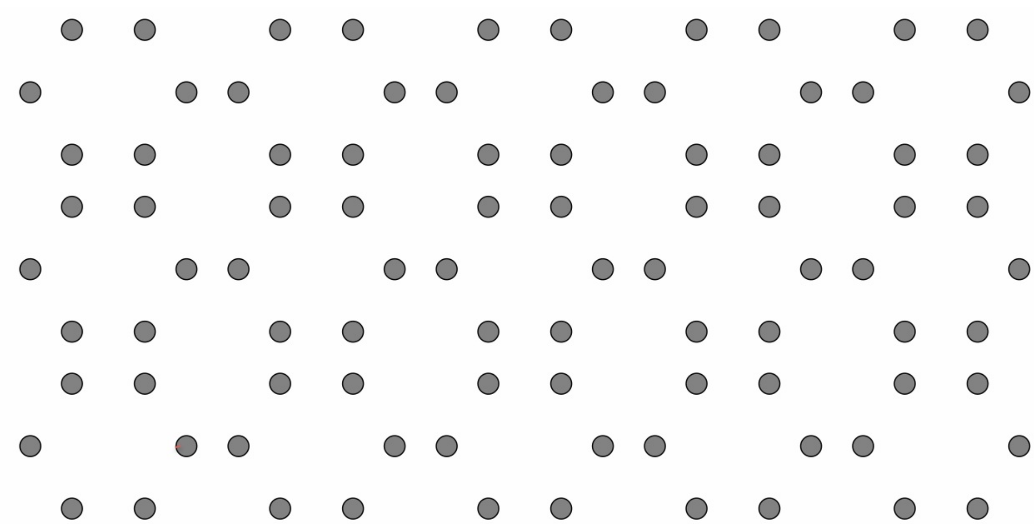
(*Time: 3 hours*)

NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.

Exercise 1: *Crystalline structures*

(You may solve this first exercise directly on this sheet of paper)
The figure below shows a periodic array.

1. Identify the type of Bravais lattice, write the primitive basis vectors and those of the basis (if any), sketch in the figure the primitive basis vectors, the basis, the primitive unit cell and the Wigner-Seitz one.



Exercise 2: *Free electrons*

1. The alkali metal sodium (BCC) has one conduction electron per atom, and an average atomic separation of 3.72 Å. Its resistivity at 273 K is 4.2 $\mu\Omega$ cm. Give an estimate of the relaxation time τ.

2. Other systems exist in nature which can be treated (at various levels of approximation) as quantum Fermi gases. One is liquid ^{3}He, which has a density of 81 $kg\ m^{-3}$. Estimate T_F, k_F and v_F in this case.

Exercise 3: *Energy bands*

1. Consider now an orthorhombic lattice with $|\mathbf{a}_1| = a$, $|\mathbf{a}_2| = b = 2a$, $|\mathbf{a}_3| = c = 4a$ ($\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are the orthogonal primitive vectors), write and sketch the reciprocal lattice vectors $\{\mathbf{b}_i\}$ and draw the first Brillouin zone.

2. Show that a s-like energy band for electrons in this orthorhombic lattice, in the tight-binding model with no overlap and only nearest neighbour interactions reads:

$$E(k_x, k_y, k_z) = E_0 + 2\gamma_1 cos(ak_x) + 2\gamma_2 cos(bk_y) + 2\gamma_3 cos(ck_z).$$

   Assume $\gamma_1 > \gamma_2 > \gamma_3 > 0$: is this assumption physically reasonable considering a, b, and c as above? Justify your answer.

3. Find the maximum of the band and calculate the effective mass in that point.

4. In the plane $k_z = 0$, draw the first Brillouin zone and plot a constant energy contour for small k. How does the contour look like?

5. In that plane, how do the shape of the constant energy contours change at slightly larger k? Make a sketch. Do all constant energy contours close within the first Brillouin zone?

6. In that plane, draw also the second Brillouin zone.

Exercise 4: *Weak potential*

Consider a 2D square lattice with a weak crystalline potential:

$$U(x, y) = -4U \cos(2\pi x/a) \cos(2\pi y/a)$$

1. Give the primitive lattice vectors of the direct lattice.

2. Give the primitive vectors of the reciprocal lattice.

3. Find the energy gap separating the two otherwise degenerate levels at $(\pi/a, \pi/a)$ point of the Brillouin zone. Which point is it?