**Condensed Matter Physics I**
**final written test**
**academic year 2009/2010**
**January 19, 2011**

(*Time: 3 hours*)

*NOTE: Give all the steps necessary to understand in detail the solution procedure. Answers with the final result only or with insufficient details will not be considered valid.*

1. Consider free electrons ("empty lattice") in a 2D square lattice (lattice parameter $a$). Write the expression and plot the *first* energy band along a path from $\Gamma$ to the midpoint X of a side of the first Brillouin zone, to the corner (L) and back to $\Gamma$. Give in particular the value of the band at the high symmetry points listed above.
2. As before, but for the *second* energy band (draw the two bands in the same plot).
3. Consider the lattice made of atoms with valence Z. Give the expression of the Fermi energy $E_F$ as a function of Z.
4. Discuss the occupation of the bands for Z=1 and Z=2.
5. Give the Fermi temperature for a square lattice of sodium atoms with lattice parameter of 4.23 Å.
6. On the basis of electron counting only, is it possible in principle for a *honeycomb lattice* made of sodium atoms to show an insulating behavior? Justify your answer.

**Exercise 2**: *Semiclassical model of electron dynamics*

Consider the electron orbits in a 3D solid with band dispersion

$$E(\mathbf{k}) = E_0 - 2t[cos(k_x a) + cos(k_y a) + cos(k_z a)]$$

with lattice parameter $a$, under a uniform static magnetic field $\mathbf{H} = H\hat{y}$; $t > 0$.

1. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0 - 4t$.
2. Write the Bloch electron velocity in direct space corresponding to the orbit in (1). Describe the orbit in real space.
3. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0$.
4. Describe and write the equation of an orbit in $\mathbf{k}$ space for $k_y = \dfrac{\pi}{2a}$ and energy $E(\mathbf{k}) = constant = E_0 + 4t - t\delta^2$ and $\delta \ll 1$.
5. Write explicitely the period of that orbit. What should be $t$ in order to have the same period as for free electrons?
6. Indicate the direction of the motion along the orbit.