Testi del Syllabus

Resp. Did.	**CUCCAGNA SCIPIO**	**Matricola: 015277**
Docente	**CUCCAGNA SCIPIO, 6 CFU**	

Anno offerta:	**2024/2025**
Insegnamento:	**529SM-1 - ADVANCED ANALYSIS - mod. A**
Corso di studio:	**SM28 - MATHEMATICS**
Anno regolamento:	**2024**
CFU:	**6**
Settore:	**MAT/05**
Tipo Attività:	**B - Caratterizzante**
Anno corso:	**1**
Periodo:	**Annualità Singola**
Sede:	**TRIESTE**

Testi in italiano

Lingua insegnamento	English
Contenuti (Dipl.Sup.)	a) Topological vector spaces. Locally convex spaces. Banach and Hilbert spaces. b) Continuous linear operators. Spectrum of a continuous linear operator. Projections. b) Examples of Banach spaces : continuous functions, Lebesgue spaces (L^p), Sobolev spaces on Tori. c) Weak topologies. d) Compact operators and their spectrum. The Fredholm alternative. The Lax Milgram Theorem.
Testi di riferimento	- H. Brezis, Functional Analysis, Springer - K. Yosida, Functional analysis, Springer. - M. Reed, B. Simon, Functional analysis, Academic Press. - W. Rudin, Analisi reale e complessa, Boringhieri. The instructor will provide before the course his printed notes.
Obiettivi formativi	By the end of the course the student will be able to manage the fundamental tools of Functional Analysis and to approach the further steps of this area of Mathematics, like the theory of Distributions and of Sobolev spaces, in order to face the first arguments in Partial Differential Equations and Calculus of Variations. The student will be able to read autonomously advanced monographs of Functional Analysis, to understand proofs and applications of theorems of any level, and to apply them to various fields of Mathematics. He/she will be able to solve problems of basic level.
Prerequisiti	Calculus I and II and Complex Analysis. Basic notions in measure theory. Basic notions in general topology.

Metodi didattici	Lectures and solutions of problems.
Modalità di verifica dell'apprendimento	The final exam is in two parts. The written part (2 hours) consists in the solution of some problems, concerning the contents of the course. The oral part is devoted to ascertain the comprehension and the managing of the topics reached by the candidate.
Programma esteso	0. Metric and normed spaces. Converging and Cauchy sequences; completeness, compactness, precompactness and relative compactness; density and separablity. Topological vector spaces. Locally convex spaces. Continuous linear operators between topological vector spaces. Frechet spaces. Dual spaces. Norm of a continuous linear operator between two Banach spaces. Convergence of sequences of operators: uniform convergence and strong convergence. Some examples. Bounded operators form a Banach space into itself and their spectrum. Projections. Spectral projections. Some examples of functions of operators, defined using power sequence: the exponential; the Neumann series. 1. Analytic form of Hahn-Banach theorem, gauge of a convex set, first and second geometric form of Hahn-Banach theorem. Applications taken from the theory of harmonic functions. Baire theorem, Banach-Steinhaus theorem, open mapping theorem. Some application to the theory of Fourier Series. Inverse mapping theorem, closed graph theorem. Diagonal procedure. Convex sets and convex functions. Examples of spaces: space of continuos functions on a real interval and L^p spaces. 2. Inner product spaces, orthogonality and orthonormality, orthonormal systems. Pitagorean theorem, Bessel inequality, Schwarz inequality, polarization identity. Hilbert spaces, orthogonal complement, projection operators, direct sum. Riesz theorem. Hilbert bases, countablity of the basis and separability, Parseval identity. Lax-Milgram theorem. Selfadjoint operators. 3. Space of continuous functions on a compact metric space. Completeness and separability. Partition of unity. Equicontinuity and Ascoli-Arzelà theorem. 4. Topology generated by a family of functions; weak topology in a Banach space, basis of a weak topology; properties of weakly converging sequences, weak and strong closure, Mazur lemma. Bidual space and reflexive spaces; strong and weak continuity of linear operators. Weak* topology, properties of weak* converging sequences, Banach-Alaoglu theorem. Helly lemma, Kakutani theorem. Properties of reflexive (and separable) Banach spaces. Sequential relative compactness theorem in a reflexive Banach space. Weiertrass theorem on the minimum of a sequential weakly lower semicontinuous functional. Uniform convexity and Millman theorem, weak-strong convergence in a uniform convex space. 5. L^p spaces. Definition, Holder inequality, Minkowsky inequality, separability, interpolation, Clarkson inequality and uniform convexity, reflexivity. Duality and Riesz theorem. Convolution, Young inequality, function with compact support, mollifiers, strong compactness criterion. Weak convergence. 6. Compact operators. Freholm alternative.

Obiettivi per lo sviluppo sostenibile

Codice	Descrizione

Testi in inglese

	English
	a) Topological vector spaces. Locally convex spaces. Banach and Hilbert spaces. b) Continuous linear operators. Spectrum of a continuous linear operator. Projections. b) Examples of Banach spaces : continuous functions, Lebesgue spaces (L^p), Sobolev spaces on Tori. c) Weak topologies. d) Compact operators and their spectrum. The Fredholm alternative. The Lax Milgram Theorem.ologies.
	- H. Brezis, Functional Analysis, Springer - K. Yosida, Functional analysis, Springer. - M. Reed, B. Simon, Functional analysis, Academic Press. - W. Rudin, Analisi reale e complessa, Boringhieri. The instructor will provide before the course his printed notes.
	By the end of the course the student will be able to manage the fundamental tools of Functional Analysis and to approach the further steps of this area of Mathematics, like the theory of Distributions and of Sobolev spaces, in order to face the first arguments in Partial Differential Equations and Calculus of Variations. The student will be able to read autonomously advanced monographs of Functional Analysis, to understand proofs and applications of theorems of any level, and to apply them to various fields of Mathematics. He/she will be able to solve problems of basic level.
	Calculus I and IInd Complex Analysis . Basic notions in measure theory. Basic notions in general topology.
	Lectures and solutions of problems.
	The final exam is in two parts. The written part (2 hours) consists in the solution of some problems, concerning the contents of the course. The oral part is devoted to ascertain the comprehension and the managing of the topics reached by the candidate.
	0. Metric and normed spaces. Converging and Cauchy sequences; completeness, compactness, precompactness and relative compactness; density and separablity. Topological vector spaces. Locally convex spaces. Continuous linear operators between topological vector spaces. Frechet spaces. Dual spaces. Norm of a continuous linear operator between two Banach spaces. Convergence of sequences of operators: uniform convergence and strong convergence. Some examples. Bounded operators form a Banach space into itself and their spectrum. Projections. Spectral projections. Some examples of functions of operators, defined using power sequence: the exponential; the Neumann series. 1. Analytic form of Hahn-Banach theorem, gauge of a convex set, first and second geometric form of Hahn-Banach theorem. Applications taken from the theory of harmonic functions. Baire theorem, Banach-Steinhaus theorem, open mapping theorem. Some application to the theory of Fourier Series. Inverse mapping theorem, closed graph theorem. Diagonal procedure. Convex sets and convex functions. Examples of spaces: space of continuos functions on a real interval and L^p spaces. 2. Inner product spaces, orthogonality and orthonormality, orthonormal

systems. Pitagorean theorem, Bessel inequality, Schwarz inequality, polarization identity. Hilbert spaces, orthogonal complement, projection operators,
direct sum. Riesz theorem. Hilbert bases, countablity of the basis and separability,
Parseval identity. Lax-Milgram theorem. Selfadjoint
operators.
3. Space of continuous functions on a compact metric space. Completeness and separability. Partition of unity. Equicontinuity and Ascoli-Arzelà theorem.
4. Topology generated by a family of functions; weak topology in a Banach space, basis of a weak topology; properties of weakly converging sequences, weak and strong closure, Mazur lemma. Bidual space and reflexive spaces; strong and weak continuity of linear operators. Weak* topology, properties of weak* converging sequences, Banach-Alaoglu theorem. Helly lemma, Kakutani theorem. Properties of reflexive (and separable) Banach spaces. Sequential relative compactness theorem in a reflexive Banach space. Weiertrass theorem on the minimum of a sequential weakly lower semicontinuous functional. Uniform convexity and Millman theorem, weak-strong convergence in a uniform convex space.
5. L^p spaces. Definition, Holder inequality, Minkowsky
inequality, separability, interpolation, Clarkson inequality and uniform convexity, reflexivity. Duality and Riesz theorem. Convolution, Young inequality, function with compact support, mollifiers, strong compactness criterion. Weak convergence.
6. Compact operators. Freholm alternative.

Obiettivi per lo sviluppo sostenibile

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