Testi del Syllabus

Resp. Did.	**CUCCAGNA SCIPIO**	**Matricola: 015277**
Docente	**CUCCAGNA SCIPIO, 6 CFU**	

Anno offerta:	**2024/2025**
Insegnamento:	**527SM - DIFFERENTIAL EQUATIONS**
Corso di studio:	**SM34 - MATEMATICA**
Anno regolamento:	**2023**
CFU:	**6**
Settore:	**MAT/05**
Tipo Attività:	**C - Affine/Integrativa**
Anno corso:	**2**
Periodo:	**Secondo Semestre**
Sede:	**TRIESTE**

Testi in italiano

Lingua insegnamento	INGLESE
Contenuti (Dipl.Sup.)	The course considers advanced topics in partial differential equations. After an introductory part focused on harmonic analysis, the course introduces various equations, such as Navier Stokes and nonlinear Schroedinger, and looks at the initial value problem. Among the topics covered there will be the proof of Leray Theorem on the existence of weak solutions of the Navier Stokes equation, which is probably the most important result in the modern theory of PDE's. Another important topic we will discuss is the proof of scattering in energy space of for defocusing pure power Semilinear Schroedinger equations . Here the main point is the so called Morawetz inequality that is one of the most remarkable results in the theory of Hamiltonian PDE's.
Testi di riferimento	Cazenave, Semilinear Schroedinger Equations, AMS Bahouri, Chemin, Danchin, Raphaël Fourier analysis and nonlinear partial differential equations. Springer, Heidelberg, 2011.
Obiettivi formativi	The purpose of the course is to introduce the students to the rigorous analytic theory of nonlinear time dependent PDE's in Euclidean space and to show the deep differences between the notion of PDE and of ODE. PDE's are intrinsically more complex and in modern Mathematical Analysis are understood and treated through the framework of Functional Analysis. In particular, students of Mathematics will see in this course various application of ideas of Functional Analysis, like the notion of weak topology, that can be fully appreciated only when applied to concrete problems. The course assumes some background in the theory of Banach spaces, Fourier transform, distributions, tempered distributions, which are taught in the master course Advanced Analysis . Interested students who don't have such a mathematical background shouldn't be discouraged to attend the course, because to get a sense of what will be taught in the course it is not necessary to have a rigorous understanding but rather an intuitive grasp of such background that the

lecturer can provide to these students upon request. However, to fully benefit from this course, especially in view of an authonomous elaboration of the mathematical formalism in their future research, it is necessary that the students fill the background gaps. The course is aimed mainly to 5th year students of Mathematics. The emphasis is on depth of analysis rather than on width of material treated.

1) Knowledge and understanding
By seeing a thorough analysis , "forensic" in detail and slow paced, of some case studies of time dependent PDEs, the students will acquire experience and develop a clear sense of how the time dependent nonlinear PDE work. No "abstract theory" is a substitute of a detailed analysis of specific PDE's.

2)Applying knowledge and understanding
After the experience and the insights gained in this course, by proceeding by analogy, the students will be able to apply in new contexts the understanding gained in the examples discussed during the course and they will need to show this skill during the seminar at the exam.

3) Making judgements
The course aims to build insight and experience in the topic of time dependent semilinar PDEs. This will allow the students to develop basic ideas of what to expect and a critical sense of what to look for, when facing new problems

4) Communication Skills
Contingent on the communication abilities of the Instructor, the students will see how an expert of PDEs talks and thinks about them and will have a chance, during their final seminar, to demonstrate their own communications skills in the subject of time dependent semilinar PDEs.

5) Learning skills
The subject of time dependent PDEs is large and deep and requires a long training. The course will touch only few aspects of it. The course, nonetheless, has the ambition of giving an in depth introduction to the subject, so that, at the end of the course, the students will have a clear sense on how to proceed further, when facing new problems.

Prerequisiti	Functional analysis, specifically Sobolev spaces and, broadly speaking, the topics of the 1st year courses ADVANCED ANALYSIS parts A e B.
Metodi didattici	The course consists of lectures during which the Instructor discusses all the details of the topics covered, answers student's questions and tries to get them involved. The students will receive before the lectures the lecture notes of the Instructor
Modalità di verifica dell'apprendimento	The exam consists of a student seminar of about 30 minutes on a topic arranged with the Instructor, during which the student will show whether or not is able to apply the main ideas presented during the lectures by the Instructor in specific and analogous contexts. The Instructor might ask some questions on the topics covered during the course in class.
Programma esteso	Trasformata di Fourier e sua inversa. Nucleo del calore. Disuguaglianza di dispersione. Spazi di Sobolev omogenei e non omegenei in R^d basati su L^2 . Completezza degli spazi omogenei solo se il loro indice soddisfa s<d/2 e necessità di considerare s>-d/2. Proiettore di Leray. Spazi H(R^d) e V(R^d) e densità al loro interno dei campi C^∞ a supporto compatto e divergenza nulla (dimostrazione per d=3). Spazi di Lorentz L^(p,∞) (R^d) ed enunciato di una estensione della disuguaglianza di Young per convoluzioni. Dimostrazione del teorema di Hardy – Littlewood – Sobolev. Calcolo della trasformata di Fourier |x |^(-γ) per 0<γ<d in R^d. Una versione del Teorema di immersione di Sobolev. Un lemma di interpolazione. Una versione della disuguaglianza di Gagliardo e Nirenberg. La disuguaglianza di Gronwall. Una definizione di soluzioni deboli dell’equazione del calore ed enunciato dell’esistenza

ed unicità della soluzione debole, validità dell’identità dell’energia e continuità nel tempo. Equazione di Navier Stokes di densità costante, incompressibile. Pressione, identità dell’energia, prodotto tensoriale di due campi vettoriali e sua divergenza, identità dell’energia. Definizione di soluzione debole. Dimostrazione del teorema di esistenza di soluzioni deboli di Leray. Teoria di Kato delle mild solutions e dimostrazione dell'esistenza locale nello spazio di Sobolev critico. Equazione di Schrödinger lineare a coefficienti costanti e sua soluzione fondamentale. Disuguaglianze di Strichartz. Vari teoremi di buona positura locale. Conservazione di energia, massa e momenti lineari. Caso delle equazioni defocalizzanti. Disuguaglianza di Morawetz e dimostrazione dello scattering nello spazio dell'energia.

Obiettivi per lo sviluppo sostenibile

Codice	Descrizione

Testi in inglese

English

The course considers advanced topics in partial differential equations. After an introductory part focused on harmonic analysis, the course introduces various equations, such as Navier Stokes and nonlinear Schroedinger, and looks at the initial value problem. Among the topics covered there will be the proof of Leray Theorem on the existence of weak solutions of the Navier Stokes equation, which is probably the most important result in the modern theory of PDE's. Another important topic we will discuss is the proof of scattering in energy space of for defocusing pure power Semilinear Schroedinger equations . Here the main point is the so called Morawetz inequality that is one of the most remarkable results in the theory of Hamiltonian PDE's.

Cazenave, Semilinear Schroedinger Equations, AMS

Bahouri, Chemin, Danchin, Raphaël Fourier analysis and nonlinear partial differential equations. Springer, Heidelberg, 2011.

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The course assumes some background in the theory of Banach spaces, Fourier transform, distributions, tempered distributions, which are taught in the master course Advanced Analysis .
Interested students who don’t have such a mathematical background shouldn’t be discouraged to attend the course, because to get a sense of what will be taught in the course it is not necessary to have a rigorous understanding but rather an intuitive grasp of such background that the lecturer can provide to these students upon request. However, to fully benefit from this course, especially in view of an authonomous elaboration of the mathematical formalism in their future research, it is necessary that the students fill the background gaps. The course is aimed mainly to 5th year students of Mathematics. The emphasis is on depth of analysis rather than on width of material treated.

	1) Knowledge and understanding At the end of the course, after an in depth analysis of some case studies, the students will have a clear sense of how the time dependent nonlinear PDE work. 2)Applying knowledge and understanding After the course, by proceeding by analogy, the students will be able to apply in new contexts the understanding gained in the examples discussed during the course and they will need to show this skill during the exam. 3) Making judgements After the course, they will have a basic idea of what it means to understand a time dependent nonlinear PDE and will get a critical sense of what to look for when facing new problems 4) Communication Skills Contingent on the communication abilities of the Instructor, the students will see how an expert of PDE's talks and thinks about them and will have a chance, during the seminar, to demonstrate their own communications skills in the subject of PDE's. 5) Learning skills The subject of time dependent Nonlinear PDE's is large and deep and requires a long training. The course will touch only few aspects of it. The course, nontheless, has the ambition of giving an in depth introduction to the subject, so that, at the end of the course, the students will have a clear sense on how to proceed when facing new problems.
	Functional analysis, specifically Sobolev spaces and, broadly speaking, the topics of the 1st year courses ADVANCED ANALYSIS parts A e B.
	The course consists of lectures during which the Instructor discusses all the details of the topics covered, answers student's questions and tries to get them involved. The students will receive before the lectures the lecture notes of the Instructor
	The exam consists of a student seminar of about 30 minutes on a topic arranged with the Instructor, during which the student will show whether or not is able to apply the main ideas presented during the lectures by the Instructor in specific and analogous contexts. The Instructor might ask some questions on the topics covered during the course in class.

Obiettivi per lo sviluppo sostenibile

Codice	Descrizione