COMPUTER & ELECTRONIC ENGINEERING

Transmission on the radio channel
Narrow band signals

Fading

- Signal bandwidth smaller than coherence bandwidth.
- Non-selective frequency fading.
- Received signal:

$$r(t) = \alpha e^{j\phi} u_m(t) + z(t), 0 \leq \mathrm{t} \leq \mathrm{T}$$

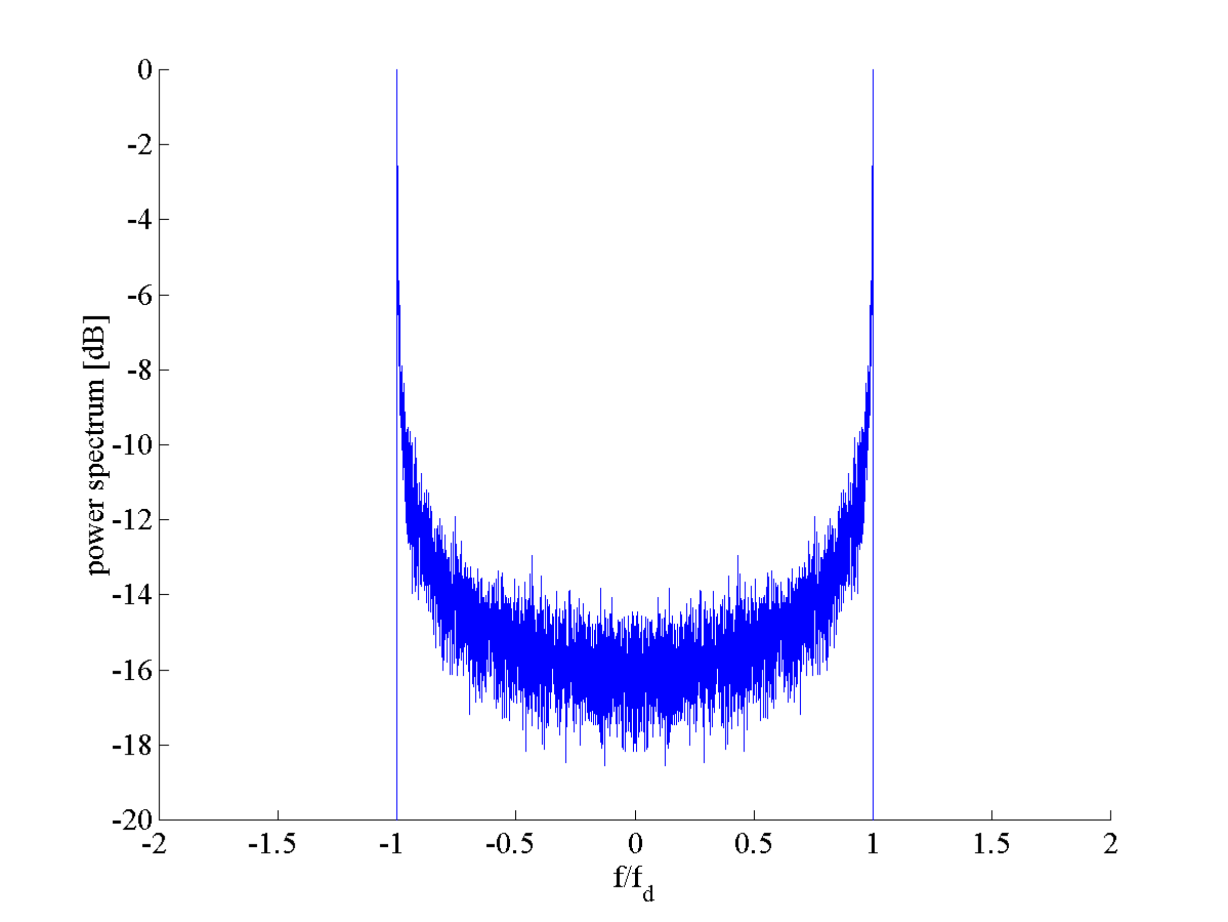
where α and ϕ are random variables, and $z(t)$ represents the Gaussian noise process.

Fading: Clarke's model

- Assumptions:
  - Fixed transmitter equipped with a vertically polarized antenna.
  - Mobile receiver equipped with an omnidirectional antenna on the horizontal plane.
  - Incident field obtained as the resultant of many components coming from all directions.
- Resulting model:
  - The real and imaginary components of the field are Gaussian random variables (with zero mean if the transmitter and receiver are not in visibility - Rayleigh fading, non-zero otherwise - Rice fading), uncorrelated.
  - The autocorrelation coefficient (normalized autocovariance) of each of the two components is expressed by: $\rho(\tau) = J_0(2\pi f_d \tau)$ (J_0 is the Bessel function of the first kind and of order 0, and f_d is the (maximum) Doppler frequency (Doppler band)).

Power spectrum of phase and quadrature components

- Spectrum of components (average value excluded): $S(f) = \frac{1}{2\pi f_d} \frac{1}{\sqrt{1-(f/f_d)^2}}, \quad 0 \le |f| \le f_d$



Overall spectrum (Rayleigh fading)

- The autocorrelation coefficient (normalized autocovariance) of the envelope and power can both be described by the following expression (for the envelope it is an approximate description, while for the power it is an exact expression):

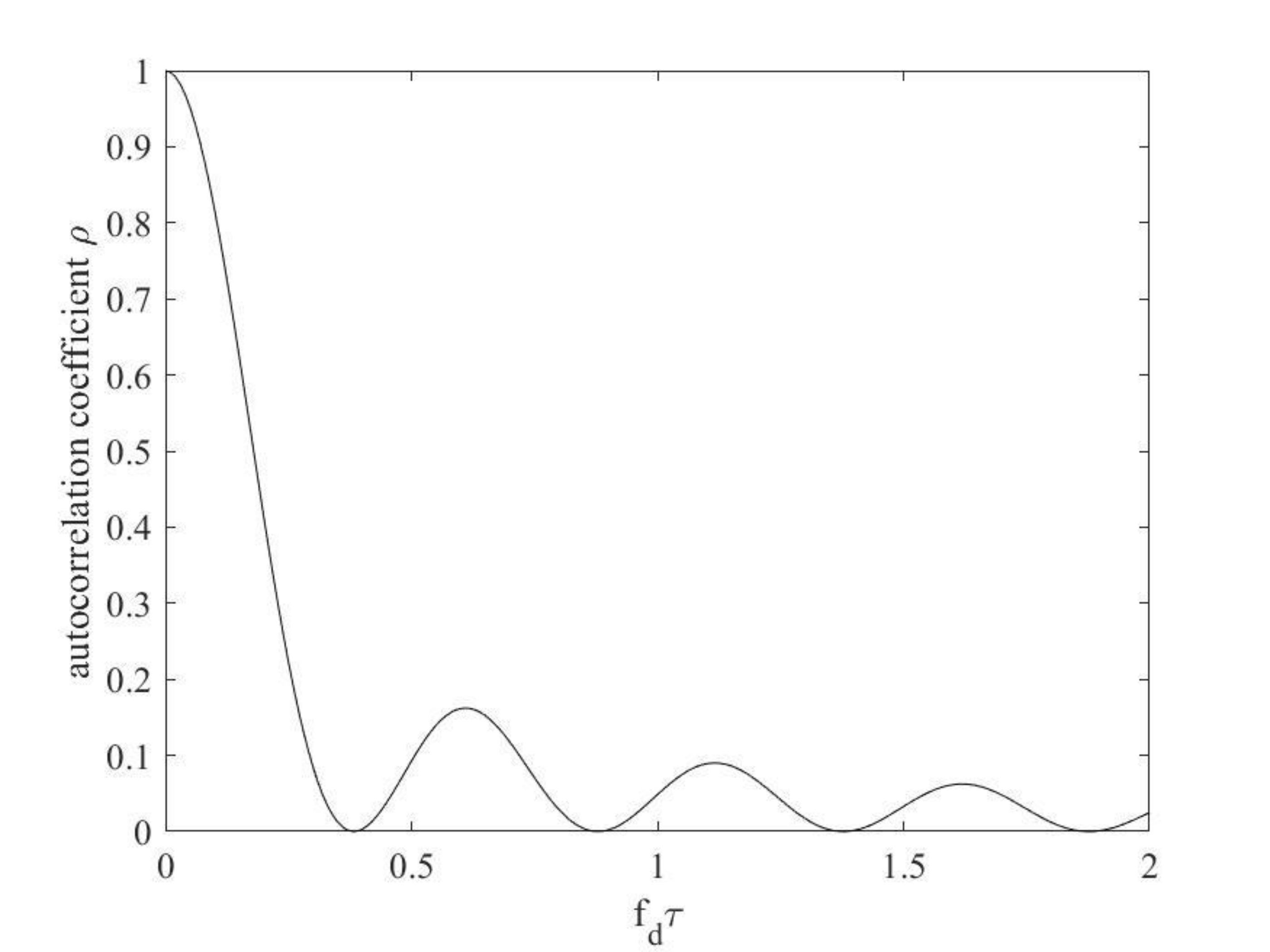
$$\rho(\tau) = J_0^2(2\pi f_d \tau)$$

- The power spectrum is expressed by the :

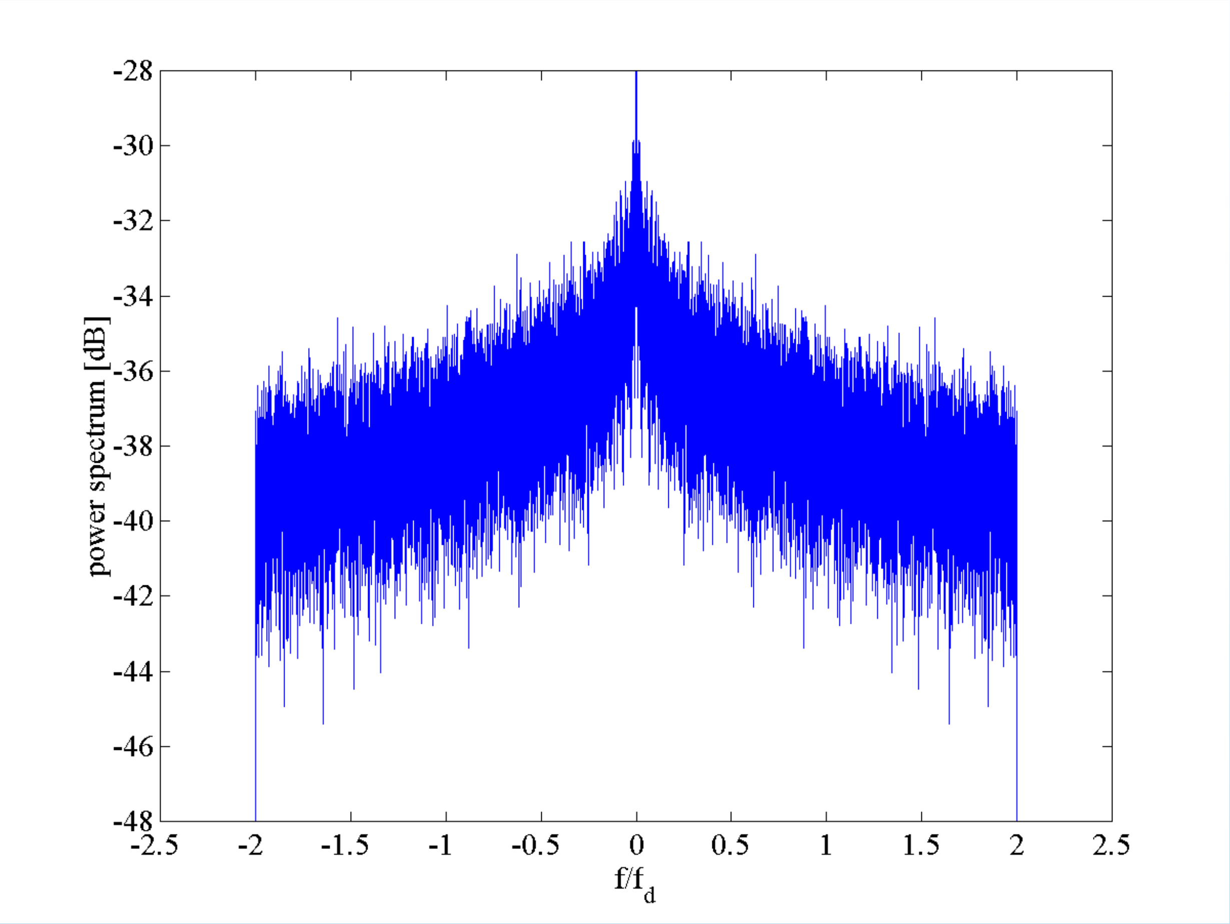
$$S(f) = \frac{P}{64\pi} \frac{1}{f_d} \mathrm{K}\left(\sqrt{1 - \left(\frac{f}{2 f_d} \right)^2} \right), \quad 0 \le |f| \le 2 f_d$$

where K(x) is the complete elliptic integral of the first kind.

Overall spectrum (Rayleigh fading)



Overall spectrum (Rayleigh) (measurement)



Fast-slow fading

- In the hypothesis of a non-selective channel (signal bandwidth smaller than the coherence bandwidth), the signal undergoes a complex attenuation, $\alpha(t)$, that varies over time (fading).
- Observing the autocorrelation coefficient, we note that interval fading values of $t \ll 0.1/f_d$ are strongly correlated, while interval values of $t \gg 0.1/f_d$ are uncorrelated.
- Called T the time interval of interest (for example duration of a symbol or a packet), the fading is said to be slow if $T \ll 0.1/f_d$ (consecutive symbols undergo the same attenuation) and fast if $T \gg 0.1/f_d$ (consecutive symbols undergo independent attenuations).
- Example: v=100 km/h, f_c =1.0 GHz: results in f_d =93 Hz; Slow fading: $1/T \gg 930$ symbols/s; Fast fading: $1/T \ll 930$ symbols/s.

Fading: A model for amplitude

- The envelope of the received signal, ρ, generally follows the Rice distribution, whose probability density function is given by:

$$f_\rho(\rho|P,K) = (1+K)e^{-K}\frac{\rho}{P}e^{-\frac{1+K}{2P}\rho^2} I_0\left(\rho\sqrt{\frac{2K(1+K)}{P}}\right)$$

where K is the Rice factor, (ratio of the direct path power to the power received through reflections), P is the average received power, and I_0 is the modified Bessel function of the first kind and of order 0.

Rice fading

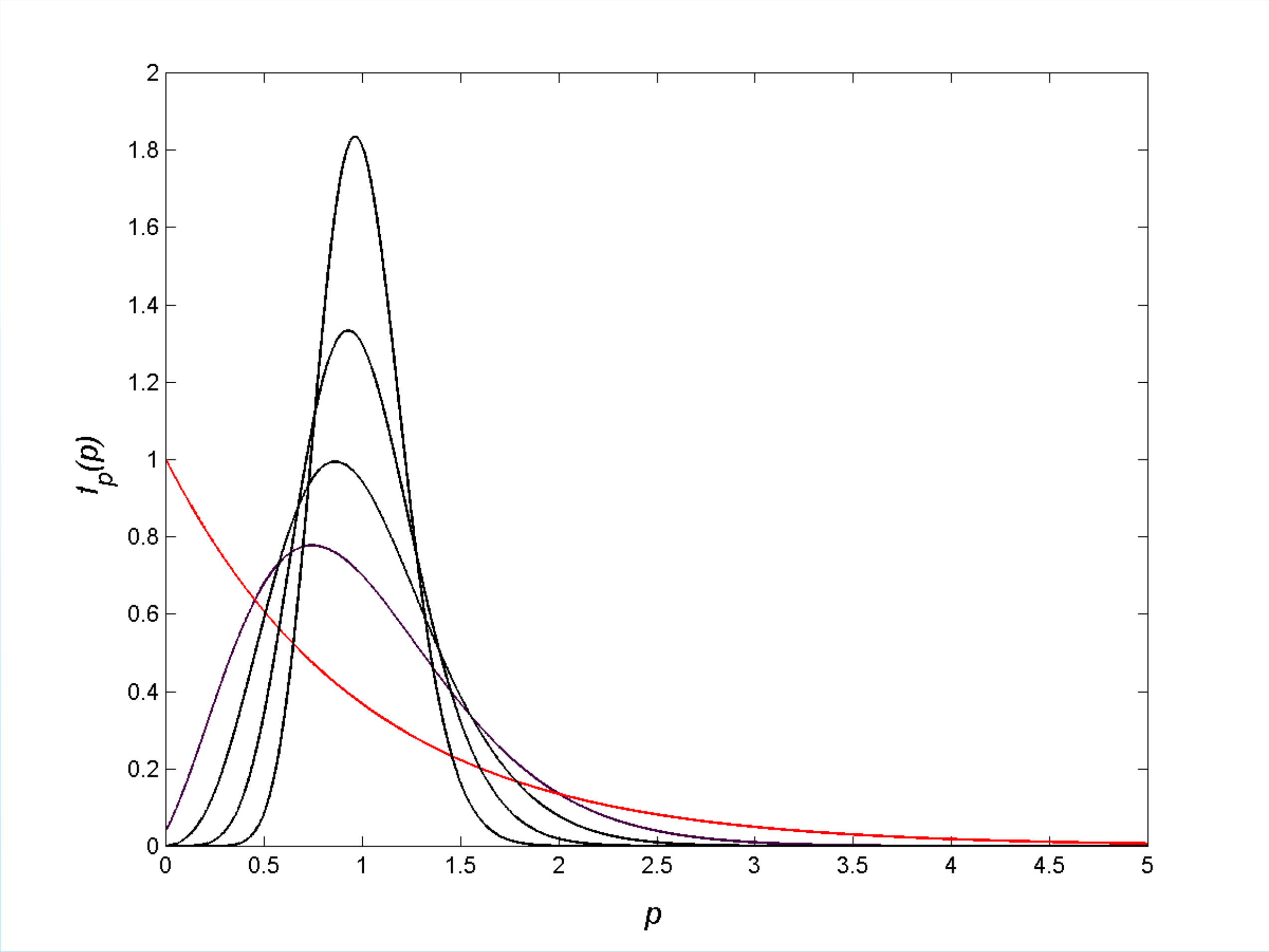
- Similarly the power, $p=\rho^2/2$, follows, in general, the distribution non central χ^2 distribution with 2 degrees of freedom, whose probability density function is given by :

$$f_p(p|P,K) = (1+K)\frac{e^{-K}}{P}e^{-\frac{1+K}{P}p} I_0\left(\sqrt{4K(1+K)\frac{p}{P}}\right) \qquad \sigma^2 = P^2\frac{1+2K}{(K+1)^2}$$

- For $K=0$ we obtain the Rayleigh fading for which it is (envelope and power):

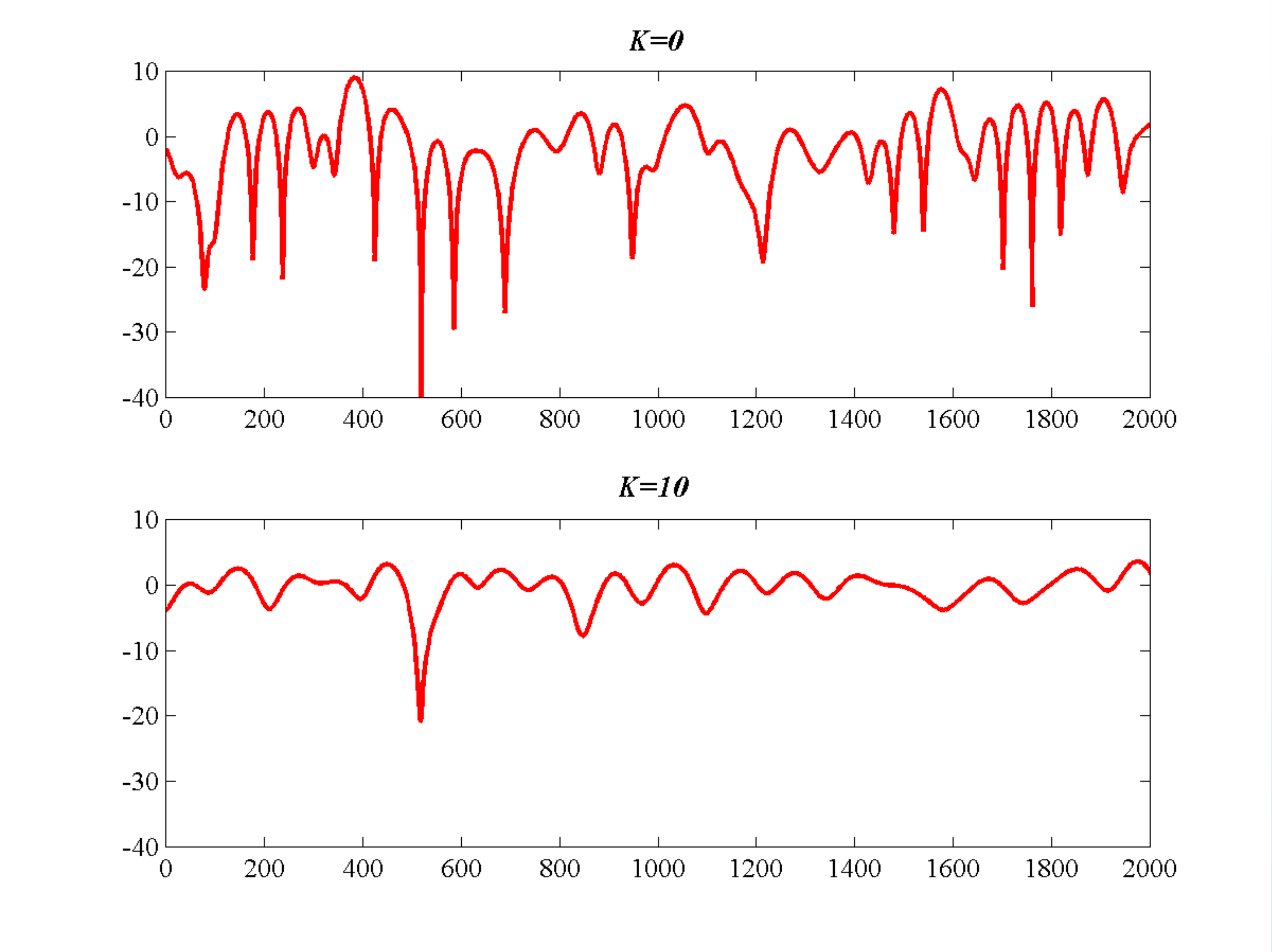
$$f_\rho(\rho|P) = \frac{\rho}{P}e^{-\frac{\rho^2}{2P}} \qquad f_p(p|P) = \frac{1}{P}e^{-\frac{p}{P}}$$

Rice fading



- Probability density of power p for $E[p]=P=1$ and $K=0,5,10,20,40$

Fading: examples of behaviors



Coherent demodulation performance

- Assume that phase shift, ϕ, introduced by the channel is known.
- Matched filter or correlator demodulation.
- Let γ_b be the instantaneous S/N ratio. Once the modulation is chosen, the error probability is a known function of γ_b.
- The average error probability is obtained by integrating the error probability with respect to the probability density of γ_b.

$$\overline{P}_e = \int_0^\infty P_e(\gamma_b) f_{\gamma_b}(\gamma_b) d\gamma_b$$

Coherent demodulation performance

- With Rayleigh fading, the error probability can be determined in closed form, and is given by:

$$\overline{P}_e = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_b}{1+\overline{\gamma}_b}}\right) \underset{\overline{\gamma}_b \to \infty}{\longrightarrow} \frac{1}{4\overline{\gamma}_b}$$

for binary antipodal modulations (ASK, PSK)

$$\overline{P}_e = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_b}{2+\overline{\gamma}_b}}\right) \underset{\overline{\gamma}_b \to \infty}{\longrightarrow} \frac{1}{2\overline{\gamma}_b}$$

for orthogonal modulations (FSK)

- The dependence of the average error rate on the average signal-to-noise ratio is not exponential, so the decrease in the error rate as the S/N ratio increases is much slower.

Non Coherent demodulation performance

- Even in this case, with Rayleigh fading, the error probability can be determined in closed form, and is given by:

$$\overline{P}_e = \frac{1}{2\left(1+\overline{\gamma}_b\right)} \xrightarrow[\overline{\gamma}_b \to \infty]{} \frac{1}{2\overline{\gamma}_b}$$

DPSK (asymptotically coincident with coherent FSK)

$$\overline{P}_e = \frac{1}{\left(2+\overline{\gamma}_b\right)} \xrightarrow[\overline{\gamma}_b \to \infty]{} \frac{1}{\overline{\gamma}_b}$$

FSK with envelope demodulation

- Note that phase estimation is not necessary. Such estimation is not necessary even when considering DPSK, in which case it is sufficient that the phase is stable in a double symbol period.

Diversity

- Availability of L independent channels (branches).
  - Space: antennas (in reception) appropriately spaced.
  - Polarization (L=2).
  - Direction (angle): use of directive antennas.
  - Frequency: transmission on bands separated by a Δ_f bigger than the coherence band.
  - Time: transmission in time intervals separated by a Δ_t bigger than the coherence time.

Diversity (considerations)

- Comparison of techniques.
  - Space, polarization, and direction diversity require more complex antenna systems;
  - Frequency diversity requires a larger bandwidth;
  - Time diversity, which implies a less efficient use of time (and therefore bandwidth), cannot be used in low mobility systems.

- Use modes.
  - Selection: the channel with the highest S/N ratio or, more simply, with the highest received power is used;
  - MRC (Maximal Ratio Combining): the signals are re-phased and combined linearly (coefficients proportional to the respective S/N).
  - ECG (Equal Gain Combining) the signals are re-phased and added.

Selection diversity performance

- Γ: average S/N ratio on the generic channel. Rayleigh fading. The distribution function of the S/N ratio on the single channel is:

$$F_{\gamma_i}(\gamma_i) = \Pr[\gamma_i \le \gamma] = 1 - \exp(-\gamma/\Gamma)$$

- S/N ratio distribution function on the selected channel :

$$F_{\gamma_S}(\gamma_s) = \Pr[\gamma_1, \gamma_2, \ldots, \gamma_L \le \gamma_s] = (1 - \exp(-\gamma_s/\Gamma))^L$$

- Probability density of the S/N ratio on the selected channel :

$$f_{\gamma_S}(\gamma_s) = \frac{dF_{\gamma_s}}{d\gamma_s} = \frac{L}{\Gamma}(1 - \exp(-\gamma_s/\Gamma))^{L-1} \exp(-\gamma_s/\Gamma)$$

- Resulting average signal-to-noise ratio :

$$E[\gamma_S] = \Gamma \sum_{l=1}^{L} \frac{1}{k}$$

MRC performance

- Be L=2. Suppose we transmit the symbol $x_i = \pm\sqrt{E_s}$. Be $h_{11}=\alpha_1\exp(j\varphi_1)$ and $h_{12}=\alpha_2\exp(j\varphi_2)$ the (complex) responses of the two independent channels. Indicated with z_1 e z_2 the noise present on the channels with variance $N_0/2$, the received signals are:

$$r_1 = x_i h_{11} + z_1,$$
$$r_2 = x_i h_{12} + z_2.$$

- MRC: build the signal $r=r_1 h_{11}^* + r_2 h_{12}^* = x_i(\alpha_1^2+\alpha_2^2) + z_1 h_{11}^* + z_2 h_{12}^*$ with energy $E_s(\alpha_1^2+\alpha_2^2)^2$ and noise variance $(\alpha_1^2+\alpha_2^2)\, N_0/2$.

- Therefore the resulting signal-to-noise ratio, γ_t, is given by:

$$\gamma_t = \frac{E_s\left(\alpha_1^2+\alpha_2^2\right)^2}{\left(N_0/2\right)\left(\alpha_1^2+\alpha_2^2\right)} = \frac{E_s}{\left(N_0/2\right)}\left(\alpha_1^2+\alpha_2^2\right) = \gamma_1+\gamma_2.$$

- In general, in the presence of L independent replicas $\gamma_t = \sum_{l=1}^{L} \gamma_l$

MRC performance

- Moment generating function (definitions and properties):

$$\Phi_1(s) = E[\exp(s\gamma)] = \int f_\gamma(\gamma)\exp(s\gamma)d\gamma \qquad E[\gamma^n] = \left.\frac{d^n\Phi_1(s)}{ds^n}\right|_{s=0}$$

- Rayleigh fading: $\Phi_1(s) = \dfrac{1}{1-s\Gamma}$

- MRC: $\Phi_L(s) = (\Phi_1(s))^L$

- Rayleigh fading $f_L(\gamma,\Gamma) = \dfrac{1}{(L-1)!}\left(\dfrac{\gamma}{\Gamma}\right)^{L-1}\dfrac{1}{\Gamma}\exp\left(-\dfrac{\gamma}{\Gamma}\right) \qquad E[\gamma] = L\Gamma$

$$F_L(\gamma,L) = 1 - \left(\sum_{k=0}^{L-1}\frac{(\gamma/\Gamma)^k}{k!}\right)\exp\left(-\frac{\gamma}{\Gamma}\right)$$

MRC Asymptotic Performance

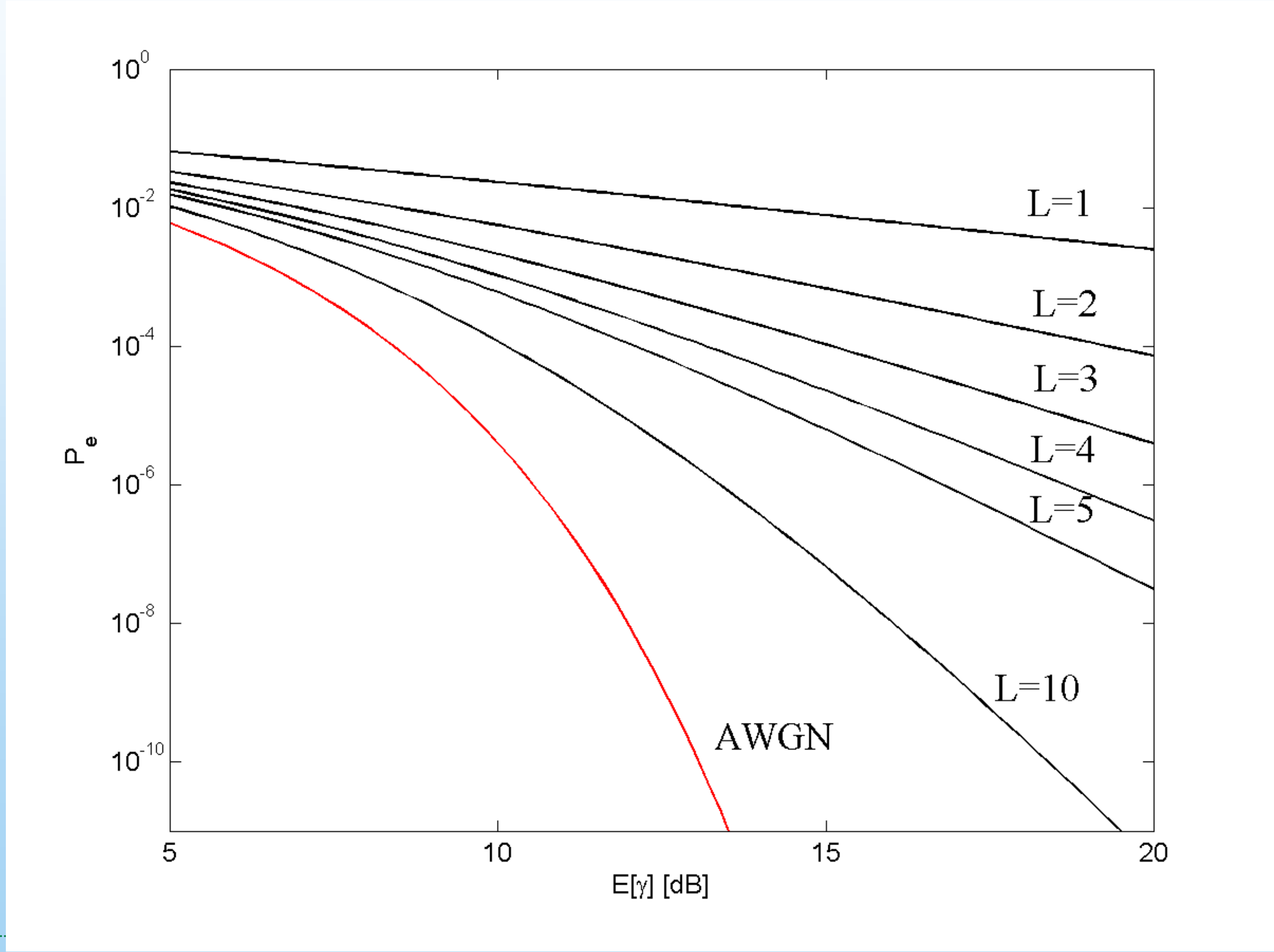
- Rayleigh Fading. The average error probability, in many cases of interest, is expressed by the relation:

$$P_e = \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k$$

Modulation	Parameter μ	Asymptotic Performance
Antipodal Coherent	$\mu = \sqrt{\frac{E[\gamma]}{1+E[\gamma]}}$	$P_e = \left(\frac{1}{4E[\gamma]}\right)^L \binom{2L-1}{L}$
Orthogonal Coherent	$\mu = \sqrt{\frac{E[\gamma]}{2+E[\gamma]}}$	$P_e = \left(\frac{1}{2E[\gamma]}\right)^L \binom{2L-1}{L}$
DPSK	$\mu = \frac{E[\gamma]}{1+E[\gamma]}$	$P_e = \left(\frac{1}{2E[\gamma]}\right)^L \binom{2L-1}{L}$
FSK non coherent	$\mu = \frac{E[\gamma]}{2+E[\gamma]}$	$P_e = \left(\frac{1}{E[\gamma]}\right)^L \binom{2L-1}{L}$

BPSK performance

- Hypothesis: E[γ]=Γ (independent of *L*). Each independent path uses an *L*-th fraction of the power.



Nakagami fading

- For power, the gamma probability density is used..

$$f_p\left(p|P,m\right)=\left(\frac{m}{P}\right)^m \frac{p^{(m-1)}}{\mathrm{Gamma}(m)}\exp\left(-\frac{mp}{P}\right) \qquad \Phi_p\left(s|P,m\right)=\left(1-\frac{sP}{m}\right)^{-m}$$

for integer values of *m*, Gamma(*m*)=(*m*-1)!

$$\sigma^2=\frac{P^2}{m}$$

- It is possible to pass, with an acceptable approximation, from Rice's description to Nakagami's description (and vice versa) with the substitutions :

$$m=\frac{(K+1)^2}{(2K+1)}; \qquad K=\frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}, \quad m>1$$

- For DPSK, the average error probability is given by:

$$\overline{P}_e=\frac{1}{2(1+\Gamma/m)^m}$$

Shadowing

- The attenuation due to obstacles, foliage follow a log-normal type statistic (shadowing).

- Given a random variable $V \geq 0$, it follows a log-normal statistic if the variable $U=\ln(V)$ (*or* $U=10\log(V)$, or $U=20\log(V)$ is a Gaussian variable.

- Average received power in dBm

$$U = 10\log_{10} P_R \qquad U_m = E[U] \qquad \sigma^2_{\log n} = E\left[(U-U_m)^2\right]$$

$$f_U(U) = \frac{1}{\sqrt{2\pi\sigma^2_{\log n}}} e^{-\frac{(U-U_m)^2}{2\sigma^2_{\log n}}} \Rightarrow f_{P_R}(P_R) = \frac{C}{\sqrt{2\pi\sigma^2_{\log n}}} \frac{1}{P_R} e^{-\frac{(10\log_{10} P_R - U_m)^2}{2\sigma^2_{\log n}}}$$

with $C = 10\log_{10} e$

defined $\mu = U_m / C$ and $\sigma = \sigma_{\log n}/C$, we have $f_{P_R}(P_R) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{P_R} e^{-\frac{(\log P_R - \mu)^2}{2\sigma^2}}$

from which $E[P_R] = \exp(\mu + \sigma^2/2)$ and $\mathrm{var}[P_R] = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$

Generating uncorrelated variables

- Rice fading generation of *M* sequences with *N* values: parameters *P* and *K*

```
function p=RicerandWN(M,N,P,K)
%function p=RicerandWN(M,N,P,K)
    scatter=P/(1.0+K); %mean value of scattered component
    dom=P-scatter;     %mean value of dominant component
    p=(scatter*randn(M,N).^2+(scatter^.5*randn(M,N)+(2*dom)^.5).^2)/2;
    %mean value P; variance P^2*(1+2*K)/(1+K)^2
```

- Shadowing generation of *m* sequences with *n* values : parameters m_{dB} and σ_{dB}, being $m = 10^{m_{\mathrm{dB}}/10}$, $\mu = U_m / C = \log(m)$ and $\sigma_{\log n} = \sigma_{dB}$

```
function p=lnorrandWN(M,N,mdB,sdB)
%function p=lnorrandWN(M,N,mdB,sdB)
    C=10*log10(exp(1));
    s=sdB/C;
    m=10^(mdB/10);
    p=m*exp(randn(M,N)*s);
    %mean value exp(log(m)+s^2/2);
    %variance exp(2*log(m)+s^2)*(exp(s^2)-1);
```

Generating correlated Rice variables

- Use function obtained by the algorithm (Sum of Sinusoids) in
  Y. Li, X. Huang, "The simulation of independent Rayleigh faders", IEEE Transactions on Communic, Vol. 50, N 9, pp 1503-1514.

```
function [rt,zt,rt1,rt2]=sosfadeli(nc,fd,T,K,N,M)

rt  fading (complex value: rt=rt1+jrt2)
zt  power (in dB)
rt1 in phase component
rt2 in quadrature component
nc samples
the results are M x nc matrixes
fd maximum Doppler frequency (Hz)   fd*T<=1
T sampling time (s)
K Rice factor (dB); K=-inf for Rayleigh
N oscillators per sequence
M sequences
```