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Minority games and stylized facts

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Abstract

The minority game is a generic model of competing adaptive agents, which is often believed to be a model of financial markets. We discuss to which extent this is a reasonable statement, and present minimal modifications that make this model reproduce stylized facts. The resulting model shows that without speculators, prices follow random walks, and that stylized facts disappear if enough speculators take into account their market impact. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The minority game (MG) [1] was introduced a few years ago as a physicists' simplification of the famous Arthur's El Farol's Bar problem [2]. Since then it has attracted much attention from the econophysics community (see Ref. [3]). The MG was not initially thought of as a model of financial market, but rather as a generic model of competing adaptive agents in the economy. Nevertheless, a majority of papers on the MG are motivated by the study of financial markets. It is therefore worth investigating why and to what extent this is justified.

In this game, N agents have to select one choice between two at each time step, and those who are in minority are rewarded. They do not act randomly, but rather inductively. This is achieved by giving to all agents their own set of S strategies, or theories of the world, which predict a winning action for all P possible states of the world. The agents are inductive in the sense that they assign a score to each of their strategies which reflect the perceived performance of the latter and use at a given time,

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the strategy with the highest score. The strategy sets of all agents are fixed before the beginning of the game, hence, play the role of a quenched disorder. If agents have no memory ($P = 1$), there is no frustration in the physical sense, and an exact solution is straightforward [4]. If $P > 1$, frustration arises because agents cannot optimize their behavior simultaneously for all states of the world. In that case, the spin glass nature of the MG is revealed by a fruitful mathematical formalism [5–7]; spin-glass techniques solve the model [6–9].

The minority mechanism has three fundamental consequences:

(1) Competition for limited resources: not all agents can win at the same time. [1]

(2) There is no good behavior: a behavior is good only with respect to others' behavior.

(3) A good behavior may become bad when others' behavior changes.

Agents' inductive behavior complete the definition of the game:

(4) Adaptive agents try to predict next winning choice, which is determined only by their own choices.

2. MG and financial markets

In a metaphorical way, this sounds like a financial market. [2] At this point, there are, however, several characteristics that the MG does not share with financial markets.

2.1. Producers

The first problem is that the MG is a negative sum game, hence, it is unclear why speculators would be willing to play such games. Indeed, in the basic MG agents are forced to play at each time step; one expects that no agent would remain in the game if they were not allowed to play. This amounts to asking why speculators are interested in real markets.

The money does not come from the speculators themselves; indeed there are other types of agents, in particular agents called producers in Ref. [11] who are not interested in making money inside markets, but who use markets for exchanging goods. They introduce predictable patterns in the market, which speculators exploit for their own profit. The producers are much less adaptive than the speculators; in the MG context, this is reflected by giving significantly less strategies to the producers than to the speculators; as in Ref. [8], we consider N_p producers (non-adaptive agents) [3] and N_s speculators (standard inductive agents). The resulting model already allows one to study

[1] Note that competition only causes a *psychological* frustration, but no physical frustration: the latter is due to the memory of agents, as stated above.

[2] The question why financial markets can be modeled by a minority mechanism is discussed in Ref. [10].

[3] I.e., with one strategy only. This is in contrast with Ref. [12] where speculators have one strategy and producers behave periodically.

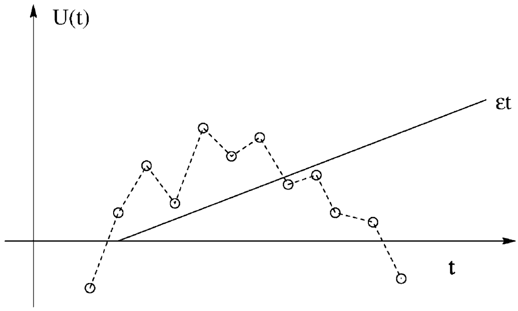

Fig. 1. Graphic illustration of the proposed grand canonical mechanism. $U(t)$ is the score of the best strategy of the considered agent.

the interplay between the information content H left by producers' behavior, and the gain of the speculators. In addition, it is also exactly solvable by spin-glass techniques.

2.2. A realistic grand canonical mechanism

Now that agents have a good reason to enter into the market, they still need a criterion which should tell them when to enter and when to withdraw. Let us review carefully agents' behavior. For an agent of the basic MG, being inductive means *comparing* the performance of all her strategies and nothing more. In other words, only the *relative* strategies' value is considered by agents, whereas actual value of scores does not matter; consequently, an agent is not worried about her real gain. [4] But having a positive or negative real gain is of great relevance in reality. Therefore, it seems reasonable to consider the following grand canonical mechanism: at a given time, an agent plays if she has at least one strategy with a positive score [5] [13,8,14]. However, this mechanism is problematic for two reasons. First, if there are no producers, hence no incentive for the speculators to play, one ends with about 75% of speculators in the market if N_s is large enough. [6] A more subtle but even more important problem is the fact that an agent who enters into market at time t and withdraws from it at time $t+\Delta t$ is sure to suffer a loss of at least $U(t)-U(t+\Delta t)$ where $U(t+\Delta t)<0$; indeed, the increase of score—a possible gain—between $t-1$ and t is *virtual*, since the agent is not in the market, whereas the loss is real (see Fig. 1).

Consequently, there is a need for another grand canonical mechanism. [7] The idea is to give a benchmark to agents such that they only stay in the market if they perform well *in the market* and not only outside it; as a consequence, the more time they can spend in the market, the more successful they are. The benchmark we propose is

[4] This makes sense as long as she is forced to play.
[5] Mathematically, if $U_i^{\max}(t)$ be the score of her best strategy: the criterion is, play if $U_i^{\max}(t)>0$.
[6] Except if $N<N_c=P/\alpha_c$ [15].
[7] The mechanism we proposed in [16] can be found in Refs. [12,17] where the strategies' scores are kept during a small past time window, assumption which is probably more realistic, but not needed for our purpose.

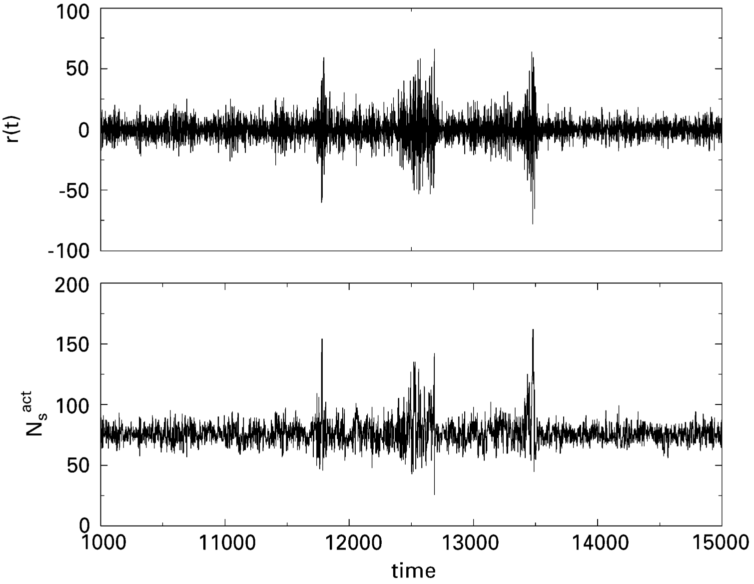

Fig. 2. Snapshot of returns and number of speculators in the market versus time. The volatility and the volume are clustered, the distribution of returns and volume have power-law tails ($P = 16$, $S = 2$, $N_s = 501$, $N_p = 1001$, $\varepsilon = 0.01$).

simply that an agent only plays if she has at least one strategy with a score higher than $\varepsilon t/P$ (t/P is the system size's independent time; see Ref. [7]). Note that it is clear from Fig. 1 that agents with this benchmark withdraw quicker from the market.

The ε parameter consists of two parts. The first one is a "common sense" factor C, which remedies the sure loss problem of the $\varepsilon = 0$ case; the second one can be interpreted as the interest rate I of a risk-free account, hence $\varepsilon = C + I$, that is, even if the interest rate is zero, one still has to consider $\varepsilon > 0$. Strikingly, as soon as $\varepsilon > 0$, there is a phase transition of first order for $n_s = N_s/P > n_s^*$ and the average number of speculators inside the market is proportional to the amount of information left by the producers. [8] In addition, in this region, a whole set of stylized facts [19] arises: [9,10] clustered volatility, power-law tails of returns' and volume's distribution can be obtained (see Fig. 2). Note also that this region is *marginally* efficient if $\varepsilon > 0$; when the number of speculators increases, the market becomes more efficient. Since producers' contribution to the price dynamics is binomial, the price follows a random walk without the speculators, and stylized facts arise only if there are enough speculators. [11]

[8] Note that here $n_s = N_s/P = 1/\alpha$, where α is the usual control parameter [18].
[9] Stylized facts have also been observed in numerous other well-known models of financial markets [21–23].
[10] The value of exponents depends on the system parameters, which can be adjusted in order to reproduce real market data.
[11] The above behavior *crucially* depends on the price-taking behavior of a large fraction of the speculators. If there are enough speculators who account for their impact, all stylized facts disappear.

Therefore, the combination of the presence of producers and the proposed grand canonical mechanism not only answers the questions of why and how should speculators participate in markets, but also reproduces some markets' characteristics. Most importantly in the MG context, this combination is still exactly solvable [20].

2.3. Evolving capital—re-investment

This extension has been considered in Refs. [12,17,24,25]. Even if evolving capitals are not needed in order to reproduce stylized facts in MG-like models, it has a particular economic relevance, so that it makes sense to complete the above models with this feature. In this case, the presence of producers is needed, else all speculators are eventually ruined; even if agents are forced to play, near the critical point, stylized facts also arise; the resulting model is also exactly solvable [25], although much more difficult to tackle analytically.

3. Conclusions

Very minimal modifications are needed in order to obtain stylized facts in MG-like models. What we believe to be the minimal modification to the standard MG is the combination of producers and speculators playing or not according to their benchmark. A more realistic and still exactly solvable model of markets is a MG with producers, benchmark and evolving capitals, but the latter are not needed in order to produce stylized facts.

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