Problem 14.10. Forest fire model

a. Consider the following model of the spread of a forest fire. Suppose that at $t = 0$ the $L \times L$ sites of a square lattice either have a tree or are empty with probability p and $1 - p$ respectively. The sites that have a tree are on fire with probability f. At each iteration an empty site grows a tree with probability g, a tree that has a nearest neighbor site on fire catches fire, and a site that is already on fire dies and becomes empty. This model is an example of a probabilistic cellular automaton. Write a program to simulate this model and color code the three types of sites. Use periodic boundary conditions.

b. Choose $L \geq 30$ and determine the values of g for which the forest maintains fires indefinitely. Note that as long as $g > 0$, new trees will always grow.

c. Use the value of g that you found in part (b) and compute the distribution of the number of sites s_f on fire. If the distribution is critical, determine the exponent α that characterizes this distribution. Also compute the distribution for the number of trees, s_t. Is there any relation between these two distributions?

d.* To obtain reliable results it is frequently necessary to average over many initial configurations. However, the behavior of many systems is independent of the initial configuration and averaging over many initial configurations is unnecessary. This latter possibility is called *self-averaging*. Repeat parts (b) and (c), but average your results over ten initial configurations. Is this forest fire model self-averaging?