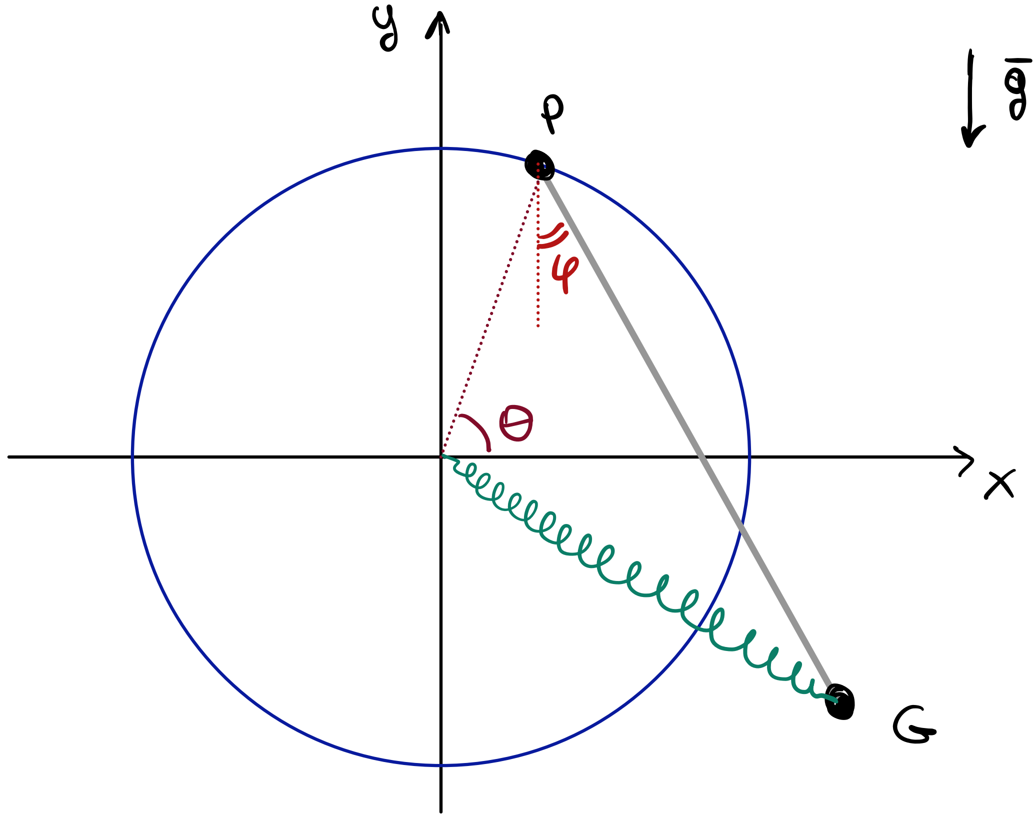

$$x_p = R\cos\theta \qquad x_Q = R\cos\theta + 2R\,\text{sen}\,\varphi$$

$$y_p = R\,\text{sen}\,\theta \qquad y_Q = R\,\text{sen}\,\theta - 2R\cos\varphi$$

$$T = \frac{m}{2}\left(\dot{x}_p^2 + \dot{y}_p^2 + \dot{x}_Q^2 + \dot{y}_Q^2\right) =$$

$$= \frac{1}{\cancel{2}} m \left(2R^2\dot{\theta}^2 + {}^2\cancel{4}R^2\dot{\varphi}^2 - {}^2\cancel{4}R^2\dot{\theta}\dot{\varphi}\,\text{sen}\,(\theta-\varphi)\right)$$

$$a = m\begin{pmatrix} 2R^2 & -2R^2\,\text{sen}(\theta-\varphi) \\ -2R^2\,\text{sen}(\theta-\varphi) & 4R^2 \end{pmatrix}$$

$$V = \frac{k}{2}\left[x_Q^2 + y_Q^2\right] + mg\left(y_p + y_Q\right)$$

$$= 2mgR\left[\text{sen}\,\theta - \cos\varphi + \frac{kR}{4mg}\left(5 - 4\,\text{sen}(\theta-\varphi)\right)\right]$$

$$\cong \quad 2mgR\left[\operatorname{sen}\theta - \cos\varphi - \frac{KR}{mg}\operatorname{sen}(\theta-\varphi)\right]$$

1) $$L = mR^2\left(\dot\theta^2 + 2\dot\varphi^2 - 2\dot\theta\dot\varphi\operatorname{sen}(\theta-\varphi)\right)$$

$$- 2mgR\left[\operatorname{sen}\theta - \cos\varphi - \frac{KR}{mg}\operatorname{sen}(\theta-\varphi)\right]$$

2) $$\frac{d}{dt}\frac{\partial L}{\partial\dot\theta} = \frac{d}{dt}\left(2mR^2\dot\theta - 2mR^2\dot\varphi\operatorname{sen}(\theta-\varphi)\right) =$$

$$= 2mR^2\left[\ddot\theta - \ddot\varphi\operatorname{sen}(\theta-\varphi) - \dot\varphi(\dot\theta-\dot\varphi)\cos(\theta-\varphi)\right]$$

$$\frac{\partial L}{\partial\theta} = -mR^2\cos(\theta-\varphi)\,\dot\theta\dot\varphi - 2mgR\cos\theta + 2KR^2\cos(\theta-\varphi)$$

$$\ddot\theta - \ddot\varphi\operatorname{sen}(\theta-\varphi) + \dot\varphi^2\cos(\theta-\varphi) - \frac{\dot\theta\dot\varphi}{2}\cos(\theta-\varphi)$$
$$+ \frac{g}{R}\cos\theta - \frac{K}{m}\cos(\theta-\varphi) = 0$$

3) $g=0 \longrightarrow$ $$\left.\begin{aligned}\varphi &\to \varphi + a\\ \theta &\to \theta + a\end{aligned}\right\}$$ rotaz. attorno asse $z \perp (xy)$.

4)

$$V = 2mgR\left[\operatorname{sen}\theta - \cos\varphi - \frac{KR}{mg}\operatorname{sen}(\theta-\varphi)\right.$$

$$\partial_\theta V = 2mgR\left[\cos\theta - \frac{KR}{mg}\cos(\theta-\varphi)\right]$$

$$\partial_\varphi V = 2mg \qquad + \frac{KR}{mg}\cos(\theta-\varphi)\Big]$$

$$dV=0 \rightarrow \operatorname{sen}(-\varphi) = \cos\theta \longrightarrow \theta = \frac{\pi}{2}+\varphi \nearrow \theta-\varphi = \frac{\pi}{2}$$

o

$$\cos(\theta-\varphi) = -\frac{mg}{KR}\operatorname{sen}\varphi \qquad -\theta = \frac{\pi}{2}+\varphi \nearrow \varphi-\theta = 2\varphi\frac{\pi}{2}$$

$$\theta = \varphi+\frac{\pi}{2} \rightarrow \operatorname{sen}\varphi = 0 \rightarrow \varphi = 0,\pi \rightarrow (\theta,\varphi) = \begin{cases}(\pi/2, 0)\\ (3\pi/2, \pi)\end{cases}$$

$$\theta = -\varphi-\frac{\pi}{2} \rightarrow \operatorname{sen}\varphi\left(\cos\varphi - \frac{mg}{2KR}\right) = 0 \rightarrow$$

$$\rightarrow \varphi = 0, \pi, \underbrace{\pm\arccos\frac{mg}{2KR}}_{}\overset{\varphi_\pm}{} \rightarrow (\theta,\varphi) = \begin{cases}(3\pi/2, 0)\\ (\frac{\pi}{2}, \pi)\\ (-\varphi_+ -\frac{\pi}{2}, \varphi_+)\\ (-\varphi_- -\frac{\pi}{2}, \varphi_-)\end{cases}$$

$$\exists \text{ se } \frac{mg}{2KR} \leq 1$$

$$\partial^2 V = 2mgR \begin{pmatrix} -\operatorname{sen}\theta + \frac{KR}{mg}\operatorname{sen}(\theta-\varphi) & -\frac{KR}{mg}\operatorname{sen}(\theta-\varphi) \\ -\frac{KR}{mg}\operatorname{sen}(\theta-\varphi) & \cos\varphi + \frac{KR}{mg}\operatorname{sen}(\theta-\varphi) \end{pmatrix}$$

$\theta = \varphi + \frac{\pi}{2}$

$$\partial^2 V = \begin{pmatrix} \frac{KR}{mg} - \cos\varphi & -\frac{KR}{mg} \\ -\frac{KR}{mg} & \frac{KR}{mg} + \cos\varphi \end{pmatrix} 2mgR$$

$$\hookrightarrow \frac{\det}{(2mgR)^2} = \left(\frac{KR}{mg}\right)^2 - \cos^2\varphi - \left(-\frac{KR}{mg}\right)^2 = -\cos^2\varphi < 0$$

$\neq 0$ per $\varphi = 0, \pi$

$\longrightarrow$ instabili

$\left(\frac{\pi}{2}, 0\right) \quad \left(\frac{3\pi}{2}, \pi\right)$

$$\theta = -\varphi - \frac{\pi}{2}$$

$$\frac{\partial^2 V}{2mgR} = \begin{pmatrix} \cos\varphi - \frac{kR}{mg}\cos 2\varphi & \frac{kR}{mg}\cos\varphi \\ \frac{kR}{mg}\cos\varphi & \cos\varphi - \frac{kR}{mg}\cos 2\varphi \end{pmatrix}$$

$$\varphi = 0 \to \begin{pmatrix} 1 - \frac{kR}{mg} & \frac{kR}{mg} \\ \frac{kR}{mg} & 1 - \frac{kR}{mg} \end{pmatrix}$$

$\det = 1 - \frac{2kR}{mg} > 0$

quando $\frac{mg}{2kR} > 1$

$\Rightarrow \mathrm{tr} > 0 \to$ STAB $\left(\frac{3\pi}{2}, 0\right)$

se $\frac{mg}{2kR} > 1$

$$\varphi = \pi \to \begin{pmatrix} -1 - \frac{kR}{mg} & \frac{kR}{mg} \\ \frac{kR}{mg} & -1 - \frac{kR}{mg} \end{pmatrix}$$

$\det = 1 + 2\frac{kR}{mg} > 0$

$\mathrm{tr} < 0 \to \left(\frac{\pi}{2}, \pi\right)$ INSTAB.

$$\varphi = \varphi_\pm \to \begin{pmatrix} \frac{kR}{mg} & 1 - \frac{kR}{mg} \\ 1 - \frac{kR}{mg} & -1 + \frac{mg}{2kR} + \frac{kR}{mg} \end{pmatrix}$$

$\det = -\frac{1}{2} + \frac{kR}{mg} =$

$\frac{1}{2}\left(\frac{2kR}{mg} - 1\right) > 0$ quando esistono

$\left(-\varphi_\pm - \frac{\pi}{2}, \varphi_\pm\right)$ STAB

quando esistono

5) $\frac{mg}{kR} = \frac{25}{11}$ $\qquad$ $kR = \frac{11}{25} mg$

$$B = \partial^2 V\left(-\frac{\pi}{2}, 0\right) = \begin{pmatrix} 2R\,(mg - kR) & 2(kR)R \\ 2(kR)R & 2R(mg - kR) \end{pmatrix} =$$

$$\underset{kR = \frac{11}{25} mg}{\overset{\uparrow}{=}} \begin{pmatrix} \frac{28}{25} mgR & \frac{22}{25} mgR \\ \frac{22}{25} mgR & \frac{28}{25} mgR \end{pmatrix} = \frac{2mgR}{25} \begin{pmatrix} 14 & 11 \\ 11 & 14 \end{pmatrix}$$

$$A = \begin{pmatrix} 2mR^2 & 2mR^2 \\ 2mR^2 & 4mR^2 \end{pmatrix} = 2mR^2 \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\det(B - \lambda A) = \det\left(2mR^2 \left[\begin{pmatrix} \frac{14}{25} g/R & \frac{11}{25} g/R \\ \frac{11}{25} g/R & \frac{14}{25} g/R \end{pmatrix} - \begin{pmatrix} \lambda & \lambda \\ \lambda & -2\lambda \end{pmatrix} \right] \right) =$$

$$(2mR^2)^2 \left[\left(\frac{14}{25} \frac{g}{R} - \lambda \right)\left(\frac{14}{25} \frac{g}{R} - 2\lambda \right) - \left(\frac{11}{25} \frac{g}{R} - \lambda \right)^2 \right] =$$

$$= \frac{4}{25} m^2 R^4 \left(\frac{g}{R} - 5\lambda \right)\left(\frac{3g}{R} - 5\lambda \right)$$

$$\omega^2 = \lambda = \begin{cases} \frac{g}{5R} \\ \frac{3g}{5R} \end{cases}$$

$$\bar{q} = \begin{pmatrix} \theta + \pi/2 \\ \varphi \end{pmatrix}$$

$$L_{lin} = \frac{1}{2} \dot{\bar{q}} \cdot A \dot{\bar{q}} - \frac{1}{2} \bar{q} \cdot B \bar{q}$$

6) $\theta(t) = -\frac{\pi}{2} + A_\theta \cos(\omega t + \theta_0)$

$\varphi(t) = A_\varphi \cos(\omega t + \varphi_0)$

7) $x_P = R\cos(\theta)\cos(\omega t)$

$z_P = R\cos(\theta)\,\mathrm{sen}(\omega t)$

$y_P = R\,\mathrm{sen}\,\theta$

$x_Q = (R\cos\theta + 2R\,\mathrm{sen}\,\varphi)\cos\omega t$

$z_Q = (R\cos\theta + 2R\,\mathrm{sen}\,\varphi)\,\mathrm{sen}\,\omega t$

$y_Q = R\,\mathrm{sen}\,\theta - 2R\cos\varphi$

$T' = \frac{m}{2}(\dot{x}_P^2 + \dot{y}_P^2 + \dot{x}_Q^2 + \dot{y}_Q^2 + \dot{z}_P^2 + \dot{z}_Q^2) - V$

$= L_{\text{di pendolo}} + \frac{m\omega^2 R^2}{2}(3 + \cos(2\theta) - 2\cos(2\varphi) + 4\cos\theta\,\mathrm{sen}\,\varphi)$