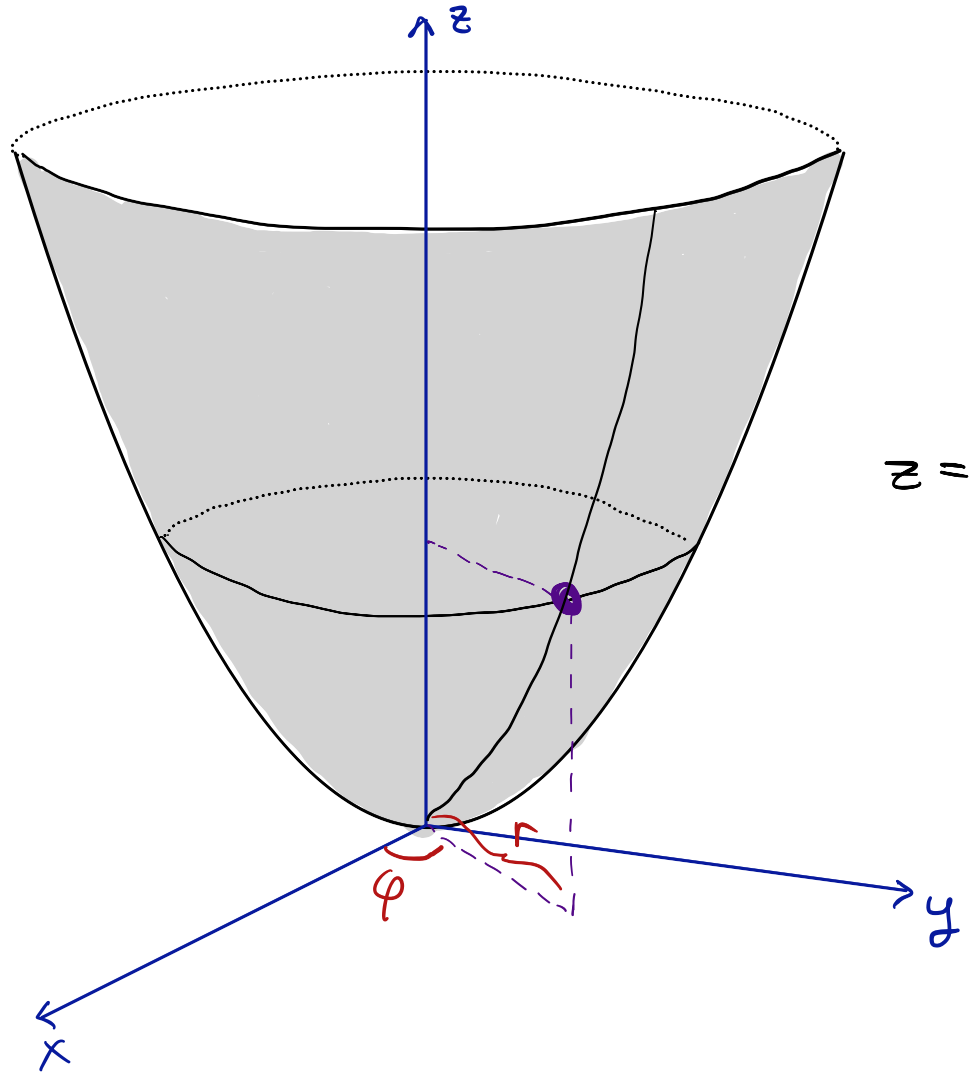
Es 2



$\downarrow \bar{g}$

$$z = \frac{x^2+y^2}{2a}$$

$$x = r\cos\varphi \qquad \dot{x} = \dot{r}\cos\varphi - r\dot{r}\,\mathrm{sen}\,\varphi$$

$$y = r\,\mathrm{sen}\,\varphi \qquad \dot{y} = \dot{r}\,\mathrm{sen}\,\varphi + r\dot{r}\cos\varphi$$

$$z = \frac{r^2}{2a} \qquad \dot{z} = \frac{r}{a}\dot{r}$$

1) $$T = \frac{m}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right) \qquad V = mgz$$

$$L = \frac{m}{2}\left(\dot{r}^2\left(1+\frac{r^2}{a^2}\right) + r^2\dot{\varphi}^2\right) - \frac{mg}{2a}r^2$$

$$a = m\begin{pmatrix} 1+r^2/a^2 & \\ & r^2 \end{pmatrix}$$

2) $$\frac{d}{dt}\frac{\partial L}{\partial \dot r} - \frac{\partial L}{\partial r} = \frac{d}{dt}\left(m\left(1+\frac{r^2}{a^2}\right)\dot r \right) = m\ddot r\left(1+\frac{r^2}{a^2}\right) + \frac{\cancel{2} m r \dot r^2}{a^2} -$$
$$- \left(\cancel{\frac{m \dot r^2}{a^2}} r + m r \dot\varphi^2 - \frac{mg}{a} r \right) = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} - \cancel{\frac{\partial L}{\partial \varphi}} = \frac{d}{dt}\left(m r^2 \dot\varphi\right) = m r^2 \ddot\varphi + 2 m r \dot r \dot\varphi = 0$$

3) ROTAZ. ATTORNO ASSE z $\rightarrow$ M_z si conserva

M è mom. ang.

4) $$\frac{\partial L}{\partial \dot\varphi} = m r^2 \dot\varphi = \ell \Rightarrow \dot\varphi = \frac{\ell}{m r^2}$$

$$L_{eff} = L - \ell\dot\varphi \Big|_{\dot\varphi = \frac{\ell}{mr^2}} =$$
$$= \frac{m}{2}\left(1+\frac{r^2}{a^2}\right)\dot r^2 - \frac{1}{2}\left(\frac{\ell^2}{mr^2} + \frac{mg}{a} r^2 \right)$$

5) $$V_{eff} = \frac{\ell^2}{2mr^2} + \frac{mg}{2a} r^2$$

$$V_{eff}' = -\frac{\ell^2}{mr^3} + \frac{mg}{a} r = 0 \rightarrow r^4 = \frac{a\ell^2}{m^2 g}$$

$$r_0 = \sqrt[4]{\frac{a\ell^2}{m^2 g}}$$

$$V_{eff}'' = \frac{3l^2}{mr^4} + \frac{mg}{a} > 0 \quad \rightarrow \quad \text{PTO eq. STAB.}$$

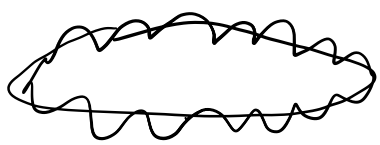
6) $B - \lambda A = 0$

$$B = \frac{3\cancel{l^2}}{\cancel{m}} \frac{m^2 g}{a\cancel{l^2}} + \frac{mg}{a} = 4\frac{mg}{a}$$

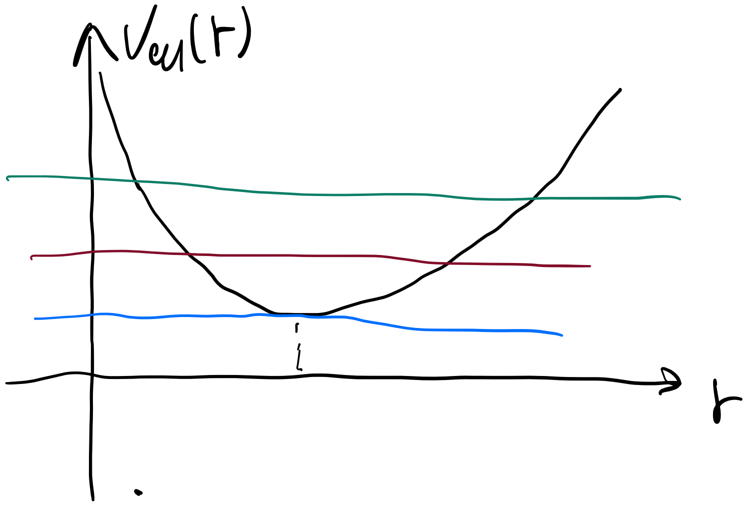
$$A = m\left(1 + \frac{1}{a^2}\frac{a^{1/2}}{g^{1/2}}\frac{l}{m}\right) = m\left(1 + \frac{l}{g^{1/2}a^{3/2}m}\right)$$

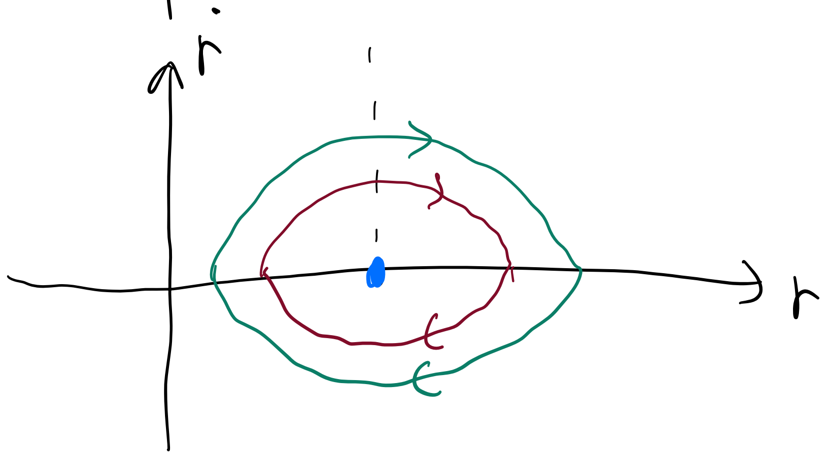
$$\omega^2 = \lambda = \frac{B}{A} = \frac{4}{1 + \frac{l}{g^{1/2}a^{3/2}m}}\frac{g}{a}$$

7) Pto stab. è traiettoria circolare. Piccole oscillaz. è moto che oscilla attorno alla circonf.



8)





9) $\vec F = \begin{pmatrix} \mu y + \rho/x \\ \mu x \\ 0 \end{pmatrix} = - \begin{pmatrix} \partial_x V \\ \partial_y V \\ \partial_z V \end{pmatrix} \leftarrow$ V indip. da z.

Seconda eq.: $V(x,y) = -\mu x y - f(x)$

Prima eq.: $\cancel{\mu y} + \frac{\rho}{x} = -\cancel{\mu y} - f'(x)$ $\quad f(x) = \rho \ln x + \cancel{\text{cost.}}$

se $x<0$ $\quad f(x) = \rho \ln(-x)$

$\hookrightarrow$ se $x>0$ $\quad f(x) = -\rho \ln(x) + \cancel{\text{cost}}$

se $x<0$ $\quad f(x) = \rho \ln(-x) + \cancel{\text{cost}}$

$$\dot z = \frac{x}{a}\dot x + \frac{y}{a}\dot y$$

$$L = \frac{m}{2}\left(\dot x^2\left(1+\frac{x^2}{a^2}\right) + \dot y^2\left(1+\frac{y^2}{a^2}\right) + \frac{2xy}{a^2}\dot x\dot y\right) - \frac{mg}{2a}\left(x^2+y^2\right) + \mu xy + \begin{cases} \rho \ln x & x>0 \\ \rho \ln(-x) & x<0 \end{cases}$$

1a) $$V(x,y) = \frac{mg}{2a}\left(x^2+y^2\right) - \mu xy + \begin{cases} -\rho \ln x \\ -\rho \ln(-x) \end{cases}$$

$$\partial_x V = \frac{mg}{a}x - \mu y - \frac{\rho}{x} = 0$$

$$\partial_y V = \frac{mg}{a}y - \mu x = 0 \rightarrow y = \frac{\mu a}{mg}x$$

$$\left(\frac{mg}{a} - \frac{\mu^2 a}{mg}\right)x - \frac{\rho}{x} = 0 \qquad x^2 = \frac{mga\rho}{m^2g^2-\mu^2a^2}$$

$$x = \pm\sqrt{\frac{mga\rho}{m^2g^2-\mu^2a^2}}$$

esistono se

$$(mg-\mu a)\rho > 0$$

$$mg > \mu a$$

$$\downarrow$$

$$y = \pm\frac{\mu a^{3/2}}{\sqrt{mg}}\sqrt{\frac{\rho}{m^2g^2-\mu^2a^2}}$$

$$\frac{1}{x^2} = \frac{mg}{a\rho} - \frac{\mu^2 a}{mg\rho}$$

$$\partial^2 V = \begin{pmatrix} \frac{mg}{a} + \frac{\rho}{x^2} & -\mu \\ -\mu & \frac{mg}{a} \end{pmatrix}$$

$$\partial^2 V\Big|_{\text{pto eq.}} = \begin{pmatrix} \frac{2mg}{a} - \frac{a\mu^2}{mg} & -\mu \\ -\mu & \frac{mg}{a} \end{pmatrix} \rightarrow \det = \frac{2m^2g^2}{a^2} - 2\mu^2$$

$$= \frac{2}{a^2}(mg - \mu a)(mg + \mu a)$$

stab. punto esiste