Starting from eq. 6.50 of the Karttunen book after the derivation of the virial theorem, which can be applied to a system of mass-points, e.g. stars (globular cluster, elliptical galaxy) or galaxies (e.g., galaxy cluster).

In the case **the system is stationary**, i.e. it has already reached the virial dynamical equilibrium, we can avoid to consider the time averages and write:

$$2T + U = 0, \tag{1}$$

where

$$T = \frac{1}{2} M\sigma_v^2 \quad \text{with} \quad \sigma_v^2 = \frac{\sum_i m_i(\vec{v_i} - <\vec{v_i}>)^2}{\sum_i m_i} \tag{2}$$

and

$$U = -\frac{GM^2}{R_V} \quad \text{with} \quad R_V = \frac{(\sum_i m_i)^2}{\sum_{i>j} m_i m_j / r_{ij}}. \tag{3}$$

Thus, in the estimate of virial mass from the virial system, the two important quantities are the velocity dispersion of the mass-points and the "virial radius" defined as above. Note that the harmonic radiu is $\sim 2R_V$ for a large number of objects, in fact:

$$R_H = \frac{(\sum_{i>j} m_i m_j)}{\sum_{i>j} m_i m_j / r_{ij}}, \tag{4}$$

or in no-weighted quantities:

$$R_H = \frac{N(N-1)/2}{\sum_{i>j} 1/r_{ij}}, \tag{5}$$

$$R_V = \frac{N^2}{\sum_{i>j} 1/r_{ij}}, \tag{6}$$

However, the **real observables quantities** are the line-of-sight velocity (or radial velocity, v_r) and the distance between the two mass-points is projected onto the sky (r'_{ij}).

One can shows that in a **spherical system**, $\sigma_v^2 = 3 \times \sigma_{v,r}^2$ and $R_V = \pi/2 \times R_V$.

Moreover, instead of the mass m_i, one has to use the luminosity l_i, which is the real observable, and to assume $m_i \propto l_i$. Unfortunately, this is not streactly true for all the astronomical objects. no luminosity

In the cases where there is no luminosity segregation in the velocity space, i.e. there is a status of velocity equipartition, e.g. **in the case of galaxy clusters**, it is more reliable to avoid of mass weighting, i.e.

$$M = \frac{\sigma_v^2 R_V}{G} = 3\pi/2 \times \frac{\sigma_{v,r}^2 R'_V}{G}. \tag{7}$$

For galaxy clusters, where size ~ 1 Mpc and $\sigma_{v,r} \sim 1000$ km/s, the above equation gives:

$$M/M_\odot = 7 \times 10^{14} [\sigma_{v,r}/(1000\text{kms}^{-1})]^2 [R'_v/Mpc] \tag{8}$$