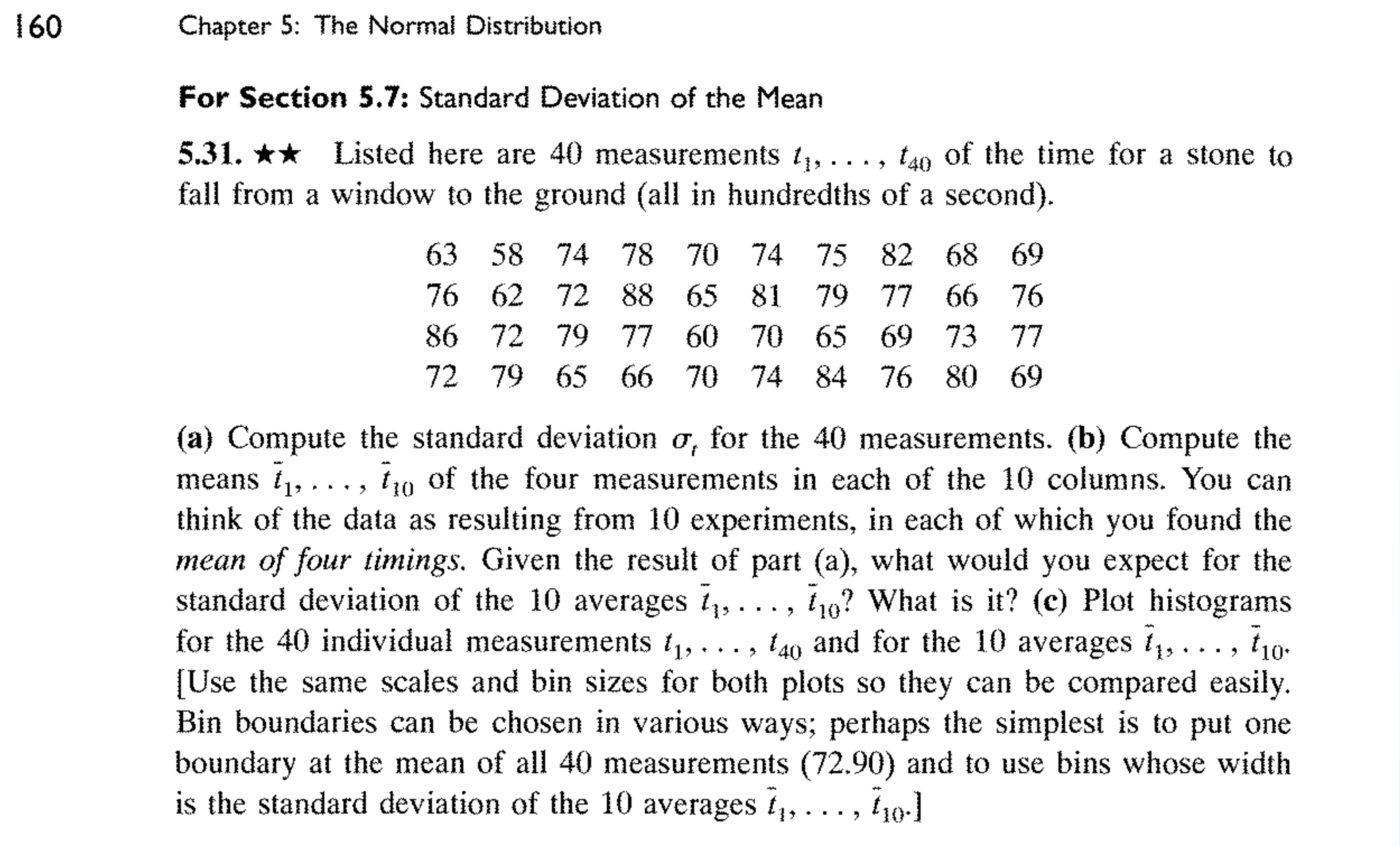
Es. 5.12 (5.31)



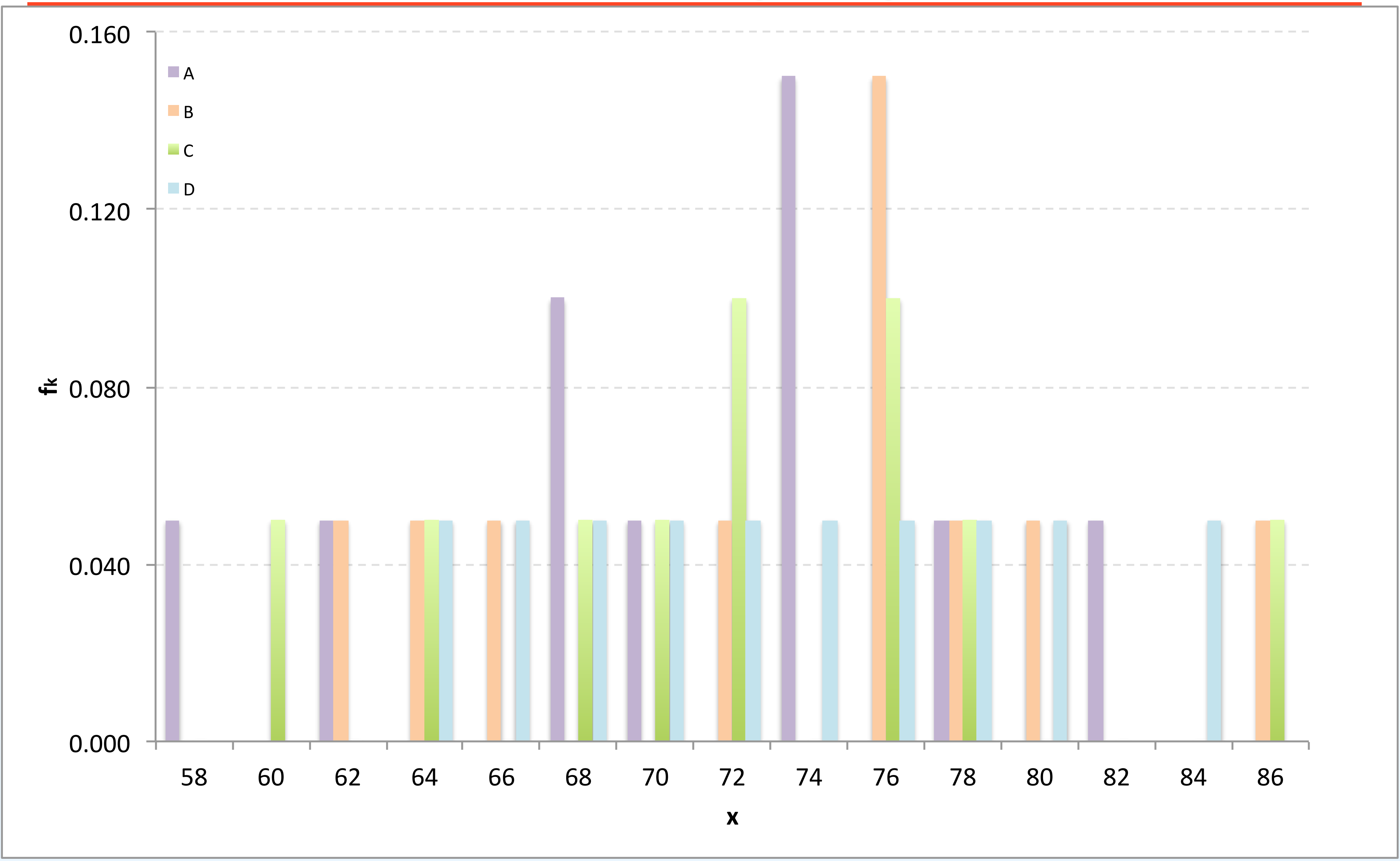
For Section 5.7: Standard Deviation of the Mean

5.31. ★★ Listed here are 40 measurements $t_1, \ldots, t_{40}$ of the time for a stone to fall from a window to the ground (all in hundredths of a second).

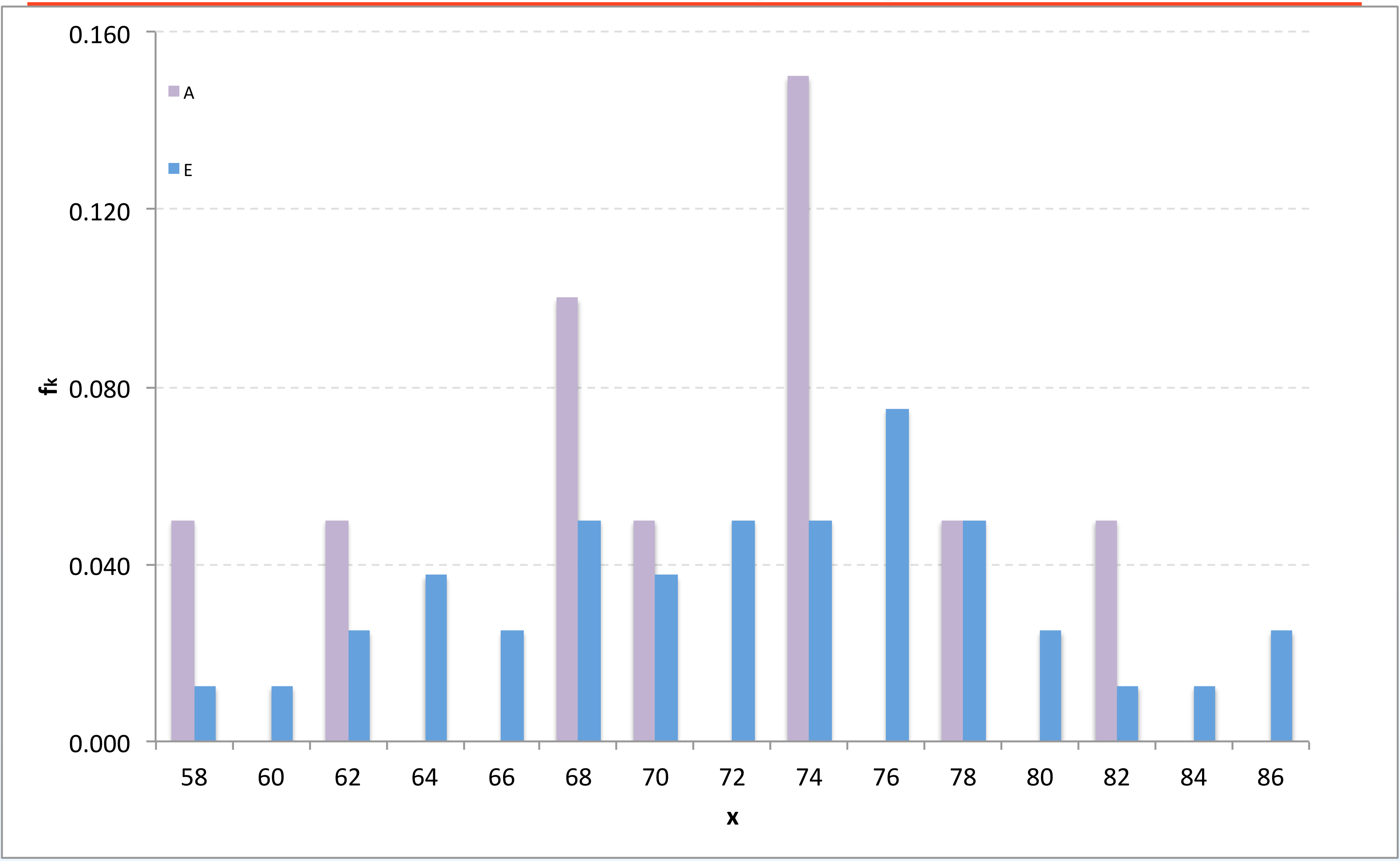
63	58	74	78	70	74	75	82	68	69
76	62	72	88	65	81	79	77	66	76
86	72	79	77	60	70	65	69	73	77
72	79	65	66	70	74	84	76	80	69

(a) Compute the standard deviation σ_t for the 40 measurements. **(b)** Compute the means $\bar{t}_1, \ldots, \bar{t}_{10}$ of the four measurements in each of the 10 columns. You can think of the data as resulting from 10 experiments, in each of which you found the *mean of four timings*. Given the result of part (a), what would you expect for the standard deviation of the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$? What is it? **(c)** Plot histograms for the 40 individual measurements $t_1, \ldots, t_{40}$ and for the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$. [Use the same scales and bin sizes for both plots so they can be compared easily. Bin boundaries can be chosen in various ways; perhaps the simplest is to put one boundary at the mean of all 40 measurements (72.90) and to use bins whose width is the standard deviation of the 10 averages $\bar{t}_1, \ldots, \bar{t}_{10}$.]

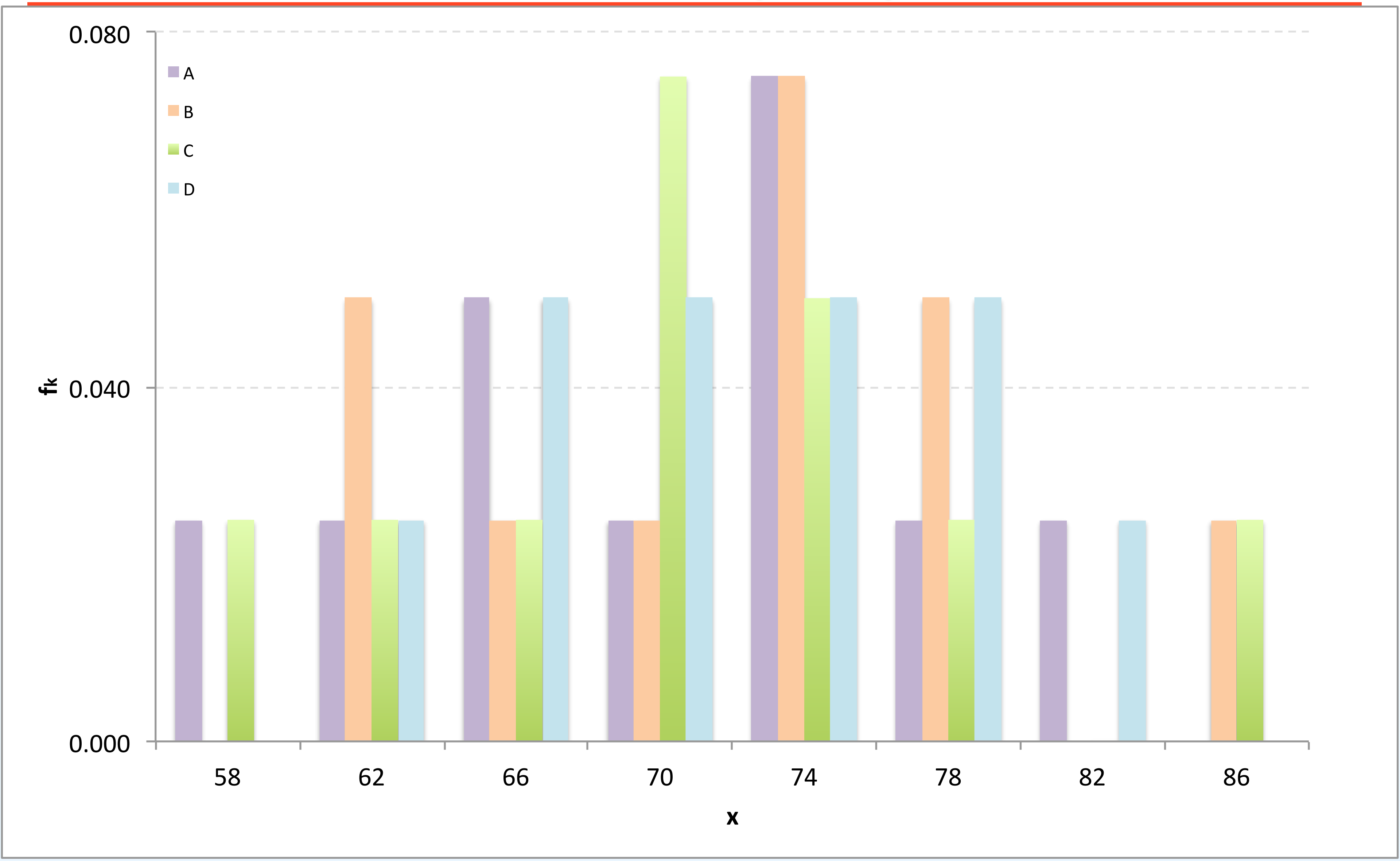
Es. 5.12 (5.31)


0.160
0.120
0.080
0.040
0.000
f_k
A
B
C
D
58
60
62
64
66
68
70
72
74
76
78
80
82
84
86
x

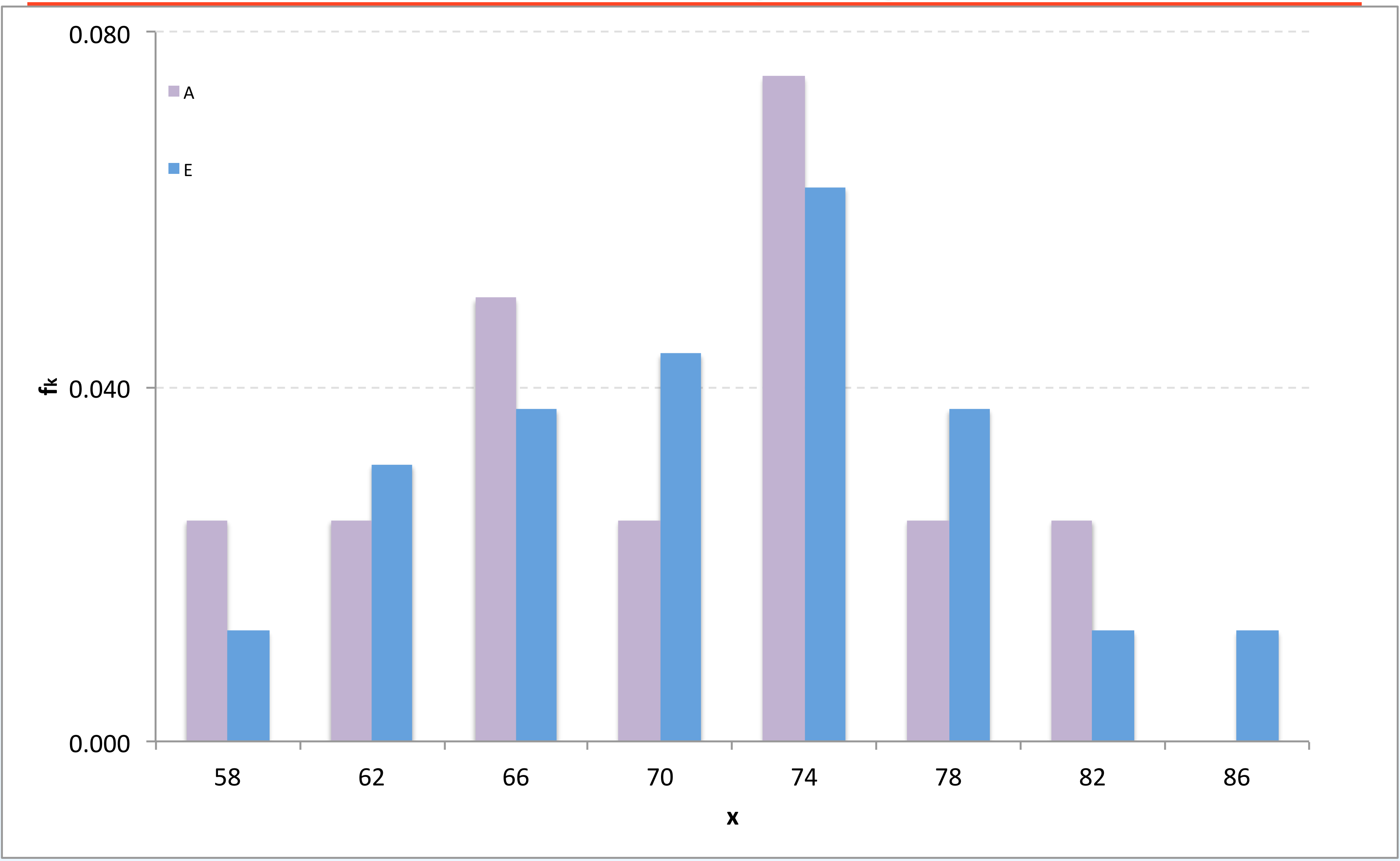
Es. 5.12 (5.31)


0.160
0.120
0.080
0.040
0.000
fk
A
E
58
60
62
64
66
68
70
72
74
76
78
80
82
84
86
x

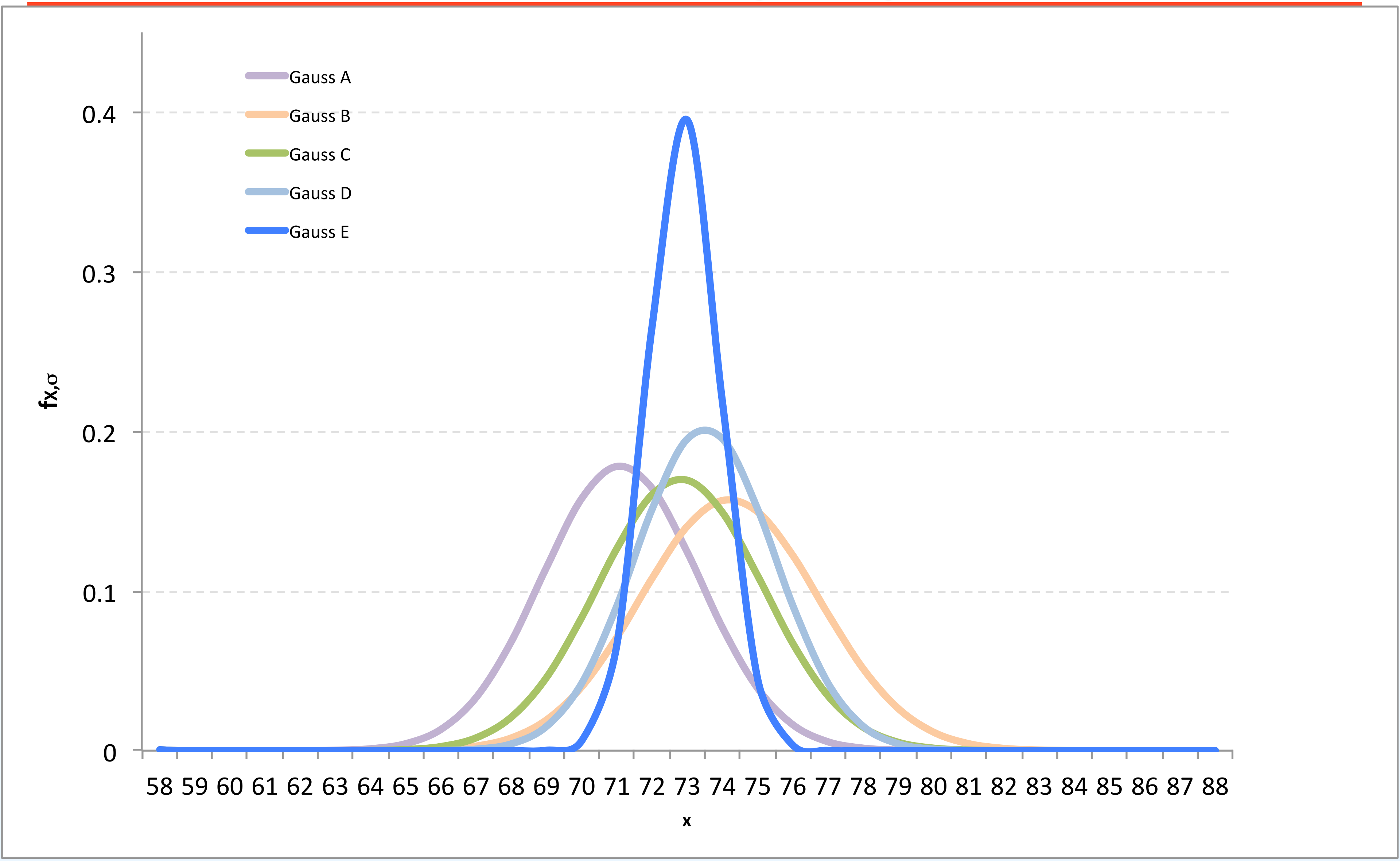
Es. 5.12 (5.31)



Es. 5.12 (5.31)



Es. 5.12 (5.31)


Gauss A
Gauss B
Gauss C
Gauss D
Gauss E
0.4
0.3
0.2
0.1
0
$f_{X,\sigma}$
58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88
x

Link 05/05/2020

- Link alle due lezioni registrate:
  - https://drive.google.com/open?id=1DPeN0xGPuF4tmY_ooZWIRTSIUhIZdxN-

05/05/2020

①

x . mis X valore vero

x, y, z ..... X, Y, Z .....

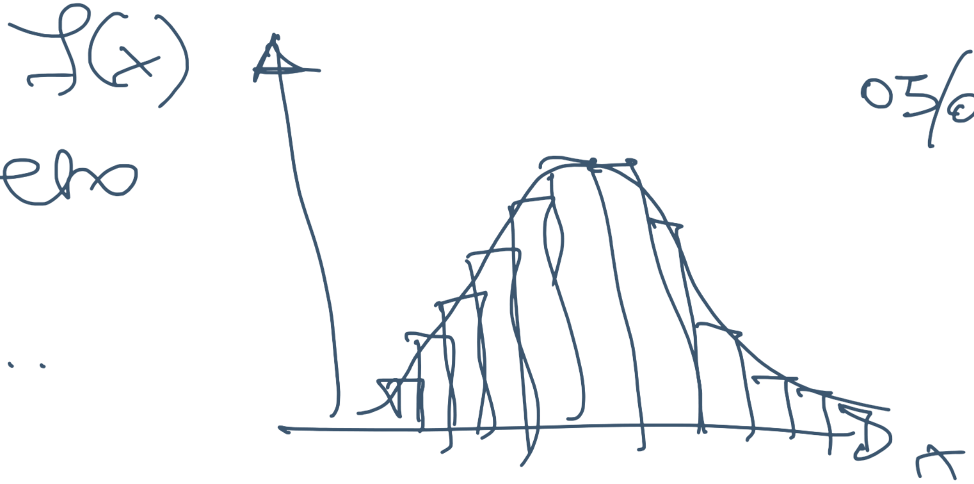
$$f(x) = e^{-\frac{x^2}{2\boxed{\sigma^2}}}$$

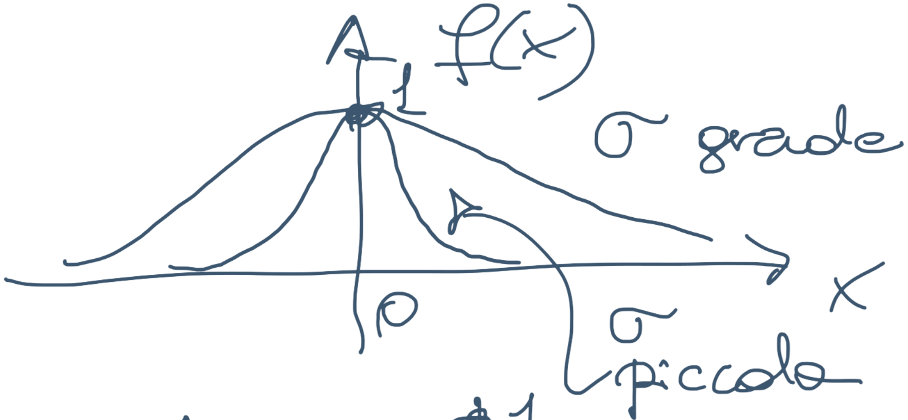
$$f(0) = 1$$

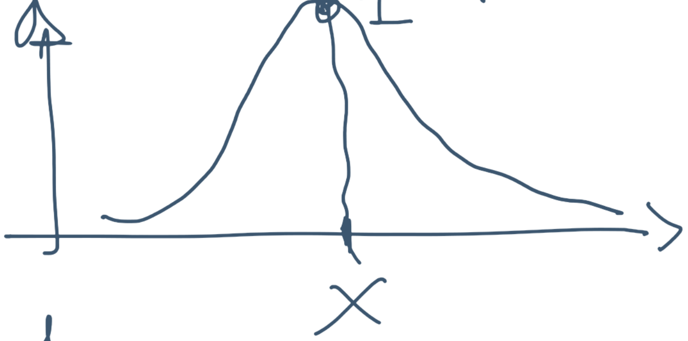
$$f(x) = e^{-\frac{(x-X)^2}{2\sigma^2}}$$

$$f(x) = \boxed{A} e^{-\frac{(x-X)^2}{2\sigma^2}}$$

$$\int_{-\infty}^{+\infty} f(x) = \int A e^{-\frac{(x-X)^2}{2\sigma^2}} dx = 1$$







$$\int_{-\infty}^{+\infty} A e^{-\frac{(x-X)^2}{2\sigma^2}} dx = 1 \qquad y = x - X \qquad dy = dx \qquad (2$$

$$z = \frac{y}{\sigma}$$

$$1 = A \int_{-\infty}^{+\infty} e^{-\frac{y^2}{2\sigma^2}} dy =$$

$$dz = \frac{dy}{\sigma}$$

$$dy = \sigma \, dz$$

$$= A \int_{-\infty}^{+\infty} e^{-z^2/2} dz \cdot \sigma =$$

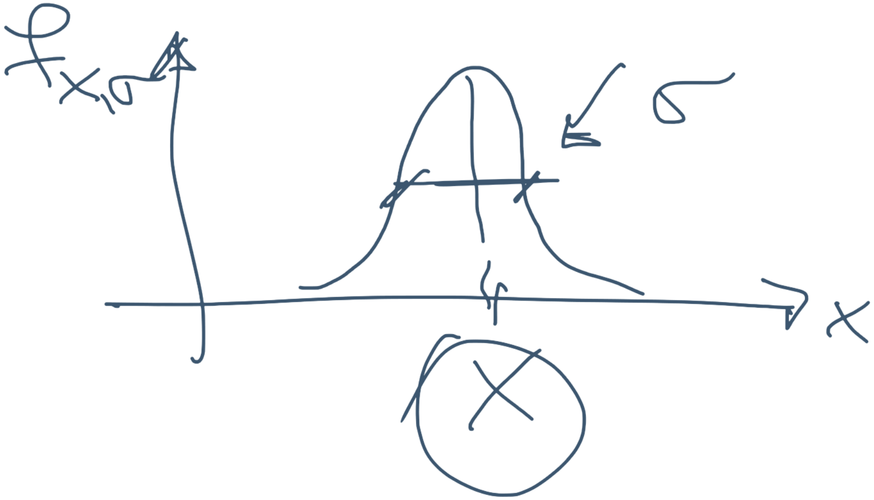
$$= A\sigma \cdot \boxed{\int_{-\infty}^{+\infty} e^{-z^2/2} dz} = \sqrt{2\pi}$$

$$= A \cdot \sigma \sqrt{2\pi} \qquad A = \frac{1}{\sigma \sqrt{2\pi}}$$

$$f(x) = \frac{1}{\boxed{\sigma}\sqrt{2\pi}} e^{-\frac{(x-\boxed{X})^2}{2\boxed{\sigma}^2}} = f_{X,\sigma}(x)$$

③

$$f_{X,\sigma}(x) = \boxed{\frac{1}{\sigma\sqrt{2\pi}}}\, e^{-\frac{(x-X)^2}{2\sigma^2}}$$



$$\bar{x} = \int_{-\infty}^{+\infty} x\, f_{X,\sigma}(x)\,dx =$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} x\, e^{-\frac{(x-X)^2}{2\sigma^2}}\,dx$$

$y = x - X \quad dy = dx$

$x = y + X$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} (y+X)\, e^{-\frac{y^2}{2\sigma^2}}\,dy =$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \left[\overset{0}{\cancel{\boxed{\int_{-\infty}^{+\infty} y e^{-\frac{y^2}{2\sigma^2}}\,dy}}} + \int_{-\infty}^{+\infty} X e^{-\frac{y^2}{2\sigma^2}}\,dy \right] =$$

$z = \frac{y}{\sigma} \quad dz = \frac{dy}{\sigma}$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \cdot X \cdot \int_{-\infty}^{+\infty} e^{-\frac{z^2}{2}}\,dz \cdot \cancel{\sigma} = \frac{1}{\cancel{\sqrt{2\pi}}} \cdot X \cdot \cancel{\sqrt{2\pi}}$$

$$= X$$

$$\boxed{\sigma_x^2} = \int_{-\infty}^{+\infty} (x-\overline{X})^2 f(x)\,dx \qquad f(x)=f_{X,\sigma}(x) \qquad (4$$

$$= \int_{-\infty}^{+\infty} (x-X)^2 \frac{e^{-\frac{(x-X)^2}{2\sigma^2}}}{\boxed{\sigma\sqrt{2\pi}}}\,dx \qquad y = x - X$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} y^2 e^{-\frac{y^2}{2\sigma^2}}\,dy \qquad z = y/\sigma \qquad dz = \frac{dy}{\sigma}$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{+\infty} z^2 \boxed{\sigma^2} e^{-z^2/2}\,dz\cdot\cancel{\sigma} = \frac{\sigma^2}{\sqrt{2\pi}} \boxed{\int_{-\infty}^{+\infty} z^2 e^{-z^2/2}\,dz} = \frac{\sigma^2}{\cancel{\sqrt{2\pi}}}\cdot\cancel{\sqrt{2\pi}}$$

$$u = z \qquad v = e^{-z^2/2} \qquad dv = -\frac{\cancel{2}z}{\cancel{2}} e^{-z^2/2}\,dz$$

$$-u\cdot dv = -z\cdot(-z)e^{-z^2/2}\,dz = z^2 e^{-z^2/2}\,dz$$

$$\int u\,dv = u\cdot v - \int du\,v \qquad -\int u\,dv = -uv + \int du\,v$$

$$\int_{-\infty}^{+\infty} z^2 e^{-z^2/2}\,dz = \cancel{-z\cdot e^{-z^2/2}\Big|_{-\infty}^{+\infty}} + \boxed{\int dz\cdot e^{-z^2/2}} = \sqrt{2\pi}$$

$\boxed{\bar{X}, \sigma}$

$\bar{x} = \boxed{X}$

σ_1 piccolo

σ_2 largo