COMPUTER ENGINEERING

Diversity - Multiplexing

Antenna diversity in transmission

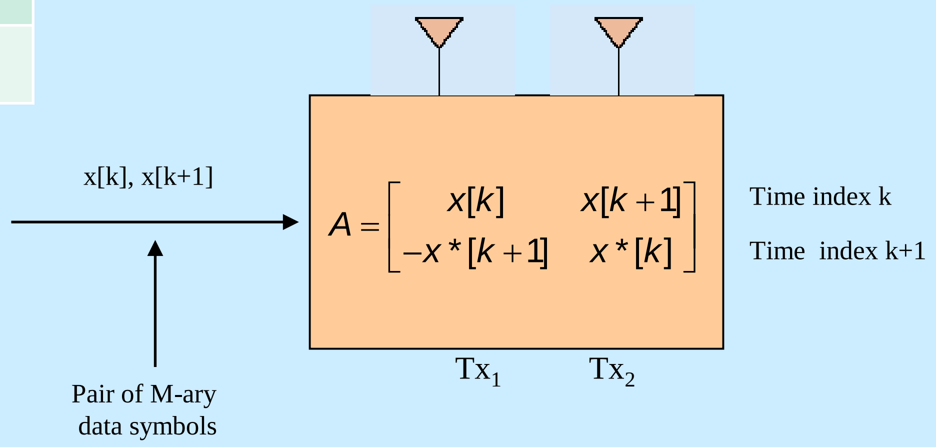
- Consider the transmission from a cellular base station to a mobile device, and consider antenna diversity.
- It may be difficult to have a multiple antenna system at the mobile device. It is worth considering whether diversity benefits can be achieved by using a multiple antenna system at the transmitter.
- The simplest scheme was developed by Alamouti, and uses N_T=2 antennas at the transmitter and N_R =1 antennas at the receiver.

S. Alamouti, " A simple transmit diversity technique for wireless communications", IEEE-JSAC, Vol. 16, N.8, 1998, pp. 1451-1458.

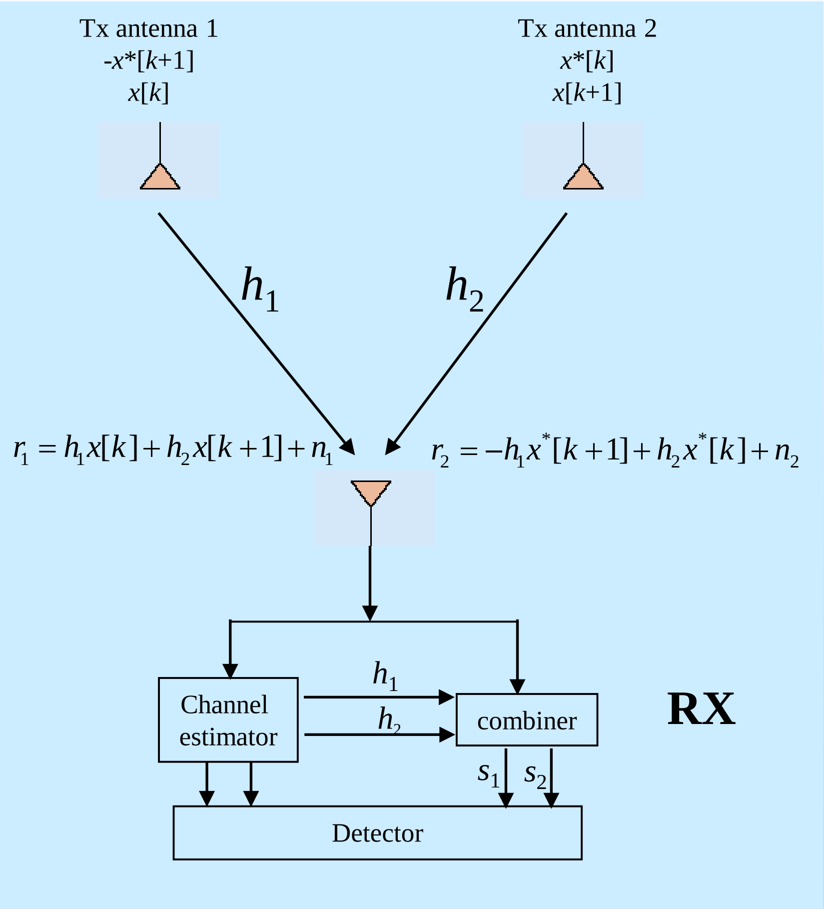
Transmission scheme

- Assume that the fading remains constant at least for the time needed to transmit two symbols, $x[k]$, $x[k+1]$.
- Transmission scheme (Space Time Coding). The two transmitting antennas according to the following scheme:

	Antenna 1	Antenna 2
Interval 1	$x[k]$	$x[k+1]$
Interval 2	$-x^*[k+1]$	$x^*[k]$



Reception scheme



Received signals

$$\underbrace{\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix}}_{\mathbf{R}} = \underbrace{\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x[k] \\ x[k+1] \end{bmatrix}}_{\mathbf{X}} + \underbrace{\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}}_{\mathbf{N}}$$

Combiner

$$\mathbf{H}^H \mathbf{R} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \mathbf{X} + \mathbf{H}^H \mathbf{N}$$

$$s_1 = (|h_1|^2 + |h_2|^2) x[k] + h_1^* n_1 + h_2 n_2^*$$

$$s_2 = (|h_1|^2 + |h_2|^2) x[k+1] + h_2^* n_1 + h_1 n_2^*$$

Diversity Gain

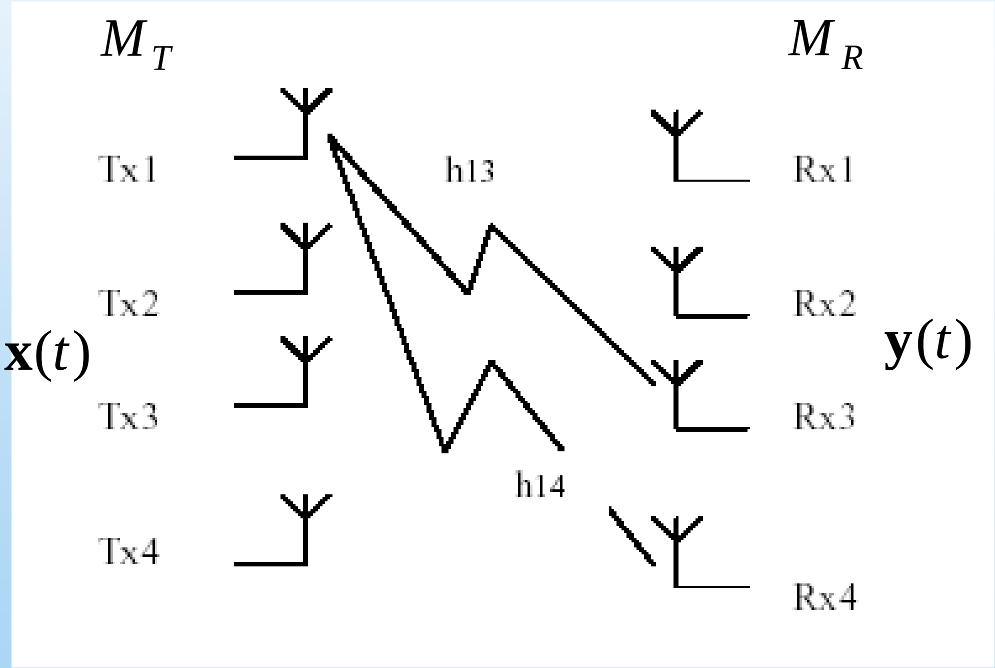
- The Alamouti scheme achieves the same diversity gain (asymptotic slope of the curve expressing the error rate as a function of the signal/noise ratio) γ), ζ=2 of the classic scheme with N_T=1 antenna at the transmitter and N_R =2 antennas at the receiver.

$$\varsigma_L = -\lim_{\gamma \to \infty} \frac{\log(P_e(E[\gamma]))}{\log(E[\gamma])}$$

- To obtain the same performance as the classic scheme (N_T =1, N_R =2) it is necessary to use double transmission power.
- In general, if you have N_T antennas at the transmitter and N_R antennas at the receiver (Multiple Input Multiple Output - MIMO - general scheme) the achievable diversity gain is equal to $N_T\, N_R$.
- More in general, we may transmit $M \leq \min(N_T, N_R)$ distinct flows. The residual degree of diversity is equal to ζ=(Nt–M+1) (Nr–M+1).

MIMO Example: Spatial multiplexing

- Multi-antenna system that uses channels (in the spatial domain) due to the multipath phenomenon.
- Low spatial correlation environments between replicas.



$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t), \quad \mathbf{H} = [h_{ij}]$$

- Sending multiple data streams to multiple antennas.
- Low probability that all paths experience deep fade at the same time.

Vertical Bell Labs Layered Space-Time (V-BLAST)

- MIMO multiplexing scheme, with interference cancellation.
- It uses $N_T > 1$ antennas at the transmitter and $N_R \geq N_T$ antennas at the receiver.
- It is capable of multiplexing N_T distinct flows.
- There is a trade-off between diversity and multiplexing, as a scheme with these characteristics has a maximum diversity gain equal to $N_T N_R$.
- If $M \leq N_T$ is the number of distinct flows transmitted, the residual diversity degree is equal to $\zeta = (N_T - M + 1)(N_R - M + 1)$.

V-BLAST

- **a**: transmitted symbols vector N_{T} x 1
- **H**: channel response matrix N_{R} x N_{T}.
- **r**=**Ha**+**ν** : received symbols vector (**ν** being the noise vector)
- Let $\mathbf{H}^{+}$ (N_t x N_r.) be the pseudo-inverse of Moore-Penrose.

$$\mathbf{H}\mathbf{H}^{+}\mathbf{H} = \mathbf{H}$$

$$\mathbf{H}^{+}\mathbf{H}\mathbf{H}^{+} = \mathbf{H}^{+}$$

$$\left(\mathbf{H}\mathbf{H}^{+}\right)^{*} = \mathbf{H}\mathbf{H}^{+}$$

$$\left(\mathbf{H}^{+}\mathbf{H}\right)^{*} = \mathbf{H}^{+}\mathbf{H}$$

V-BLAST algorithm

- Initialization

$$i = 1$$

$$\mathbf{r}_1 = \mathbf{r}$$

$$\mathbf{H}_1 = \mathbf{H}$$

$\mathbf{r}$: vector N_T x 1

$\mathbf{H}_i$: matrix N_R x N_T

$\mathbf{G}_i$: matrix N_T x N_R

$(\mathbf{G}_i)_j$ the j-th row di $\mathbf{G}_i$

$\hat{a}_{k_i}$ decided value

$(\mathbf{H}_i)_{k_i}$ the k_i-th column of $\mathbf{H}_i$

$\left(\mathbf{H}_i\right)_{k_i}^{-}$ matrix $\mathbf{H}_i$ with the column k_i set to zero (interference cancellation)

Recursion

$$\mathbf{G}_i = \mathbf{H}_i^{+}$$

$$k_i = \arg\min_j \left\| \left(\mathbf{G}_i\right)_j \right\|^2$$

$$\mathbf{w}_{k_i} = \left(\mathbf{G}_i\right)_{k_i}$$

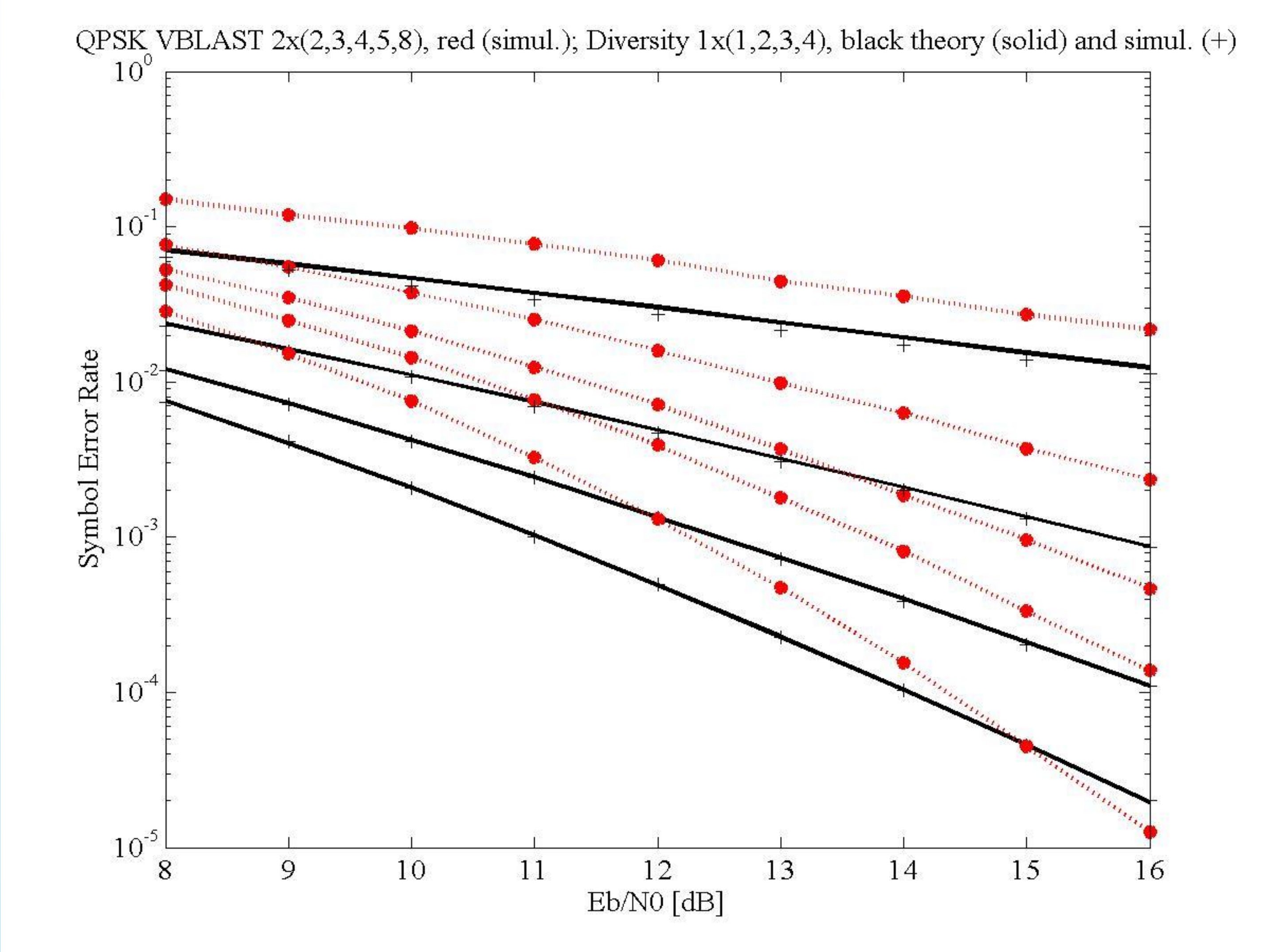
$$y_{k_i} = \mathbf{w}_{k_i}\mathbf{r}_i$$

$$\hat{a}_{k_i} = Q\left(y_{k_i}\right)$$

$$\mathbf{r}_{i+1} = \mathbf{r}_i - \hat{a}_{k_i}\left(\mathbf{H}_i\right)_{k_i}$$

$$\mathbf{H}_{i+1} = \left(\mathbf{H}_i\right)_{k_i}^{-}$$

V-BLAST: performance (Rayleigh fading)



V-BLAST: performance (Rice fading)