COMPUTER ENGINEERING

Radio channel transmission
Wideband signals

Motivations

- Low power spectral density (hard to intercept).
- Protection (encryption).
- Diversity.
- Flexible multiple access.
- Coexistence with legacy systems (radio spectrum crowding).
- Interference rejection.
- High temporal resolution.

Spread Spectrum: Expansion Modes

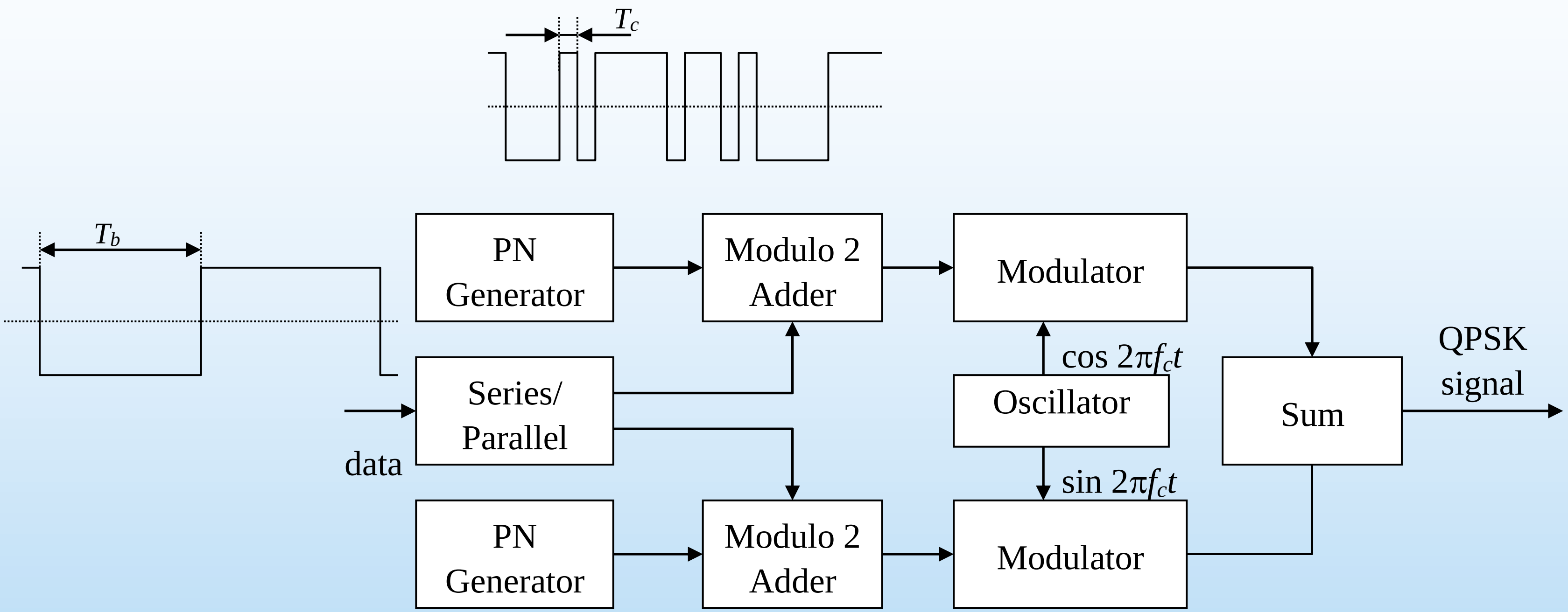
- Direct Sequence (DS) Spread Spectrum
  - The expansion is obtained by ‘combining’ the information sequence with a sequence with a higher rate of pseudo-random features (Pseudo Noise – PN – sequence), which occupies a significantly larger bandwidth than the coherence bandwidth.

- Frequency Hopping (FH)
  - The expansion is obtained by pseudo-randomly varying the carrier frequency over time, with variations significantly larger than the coherence bandwidth.

FH – Main Features

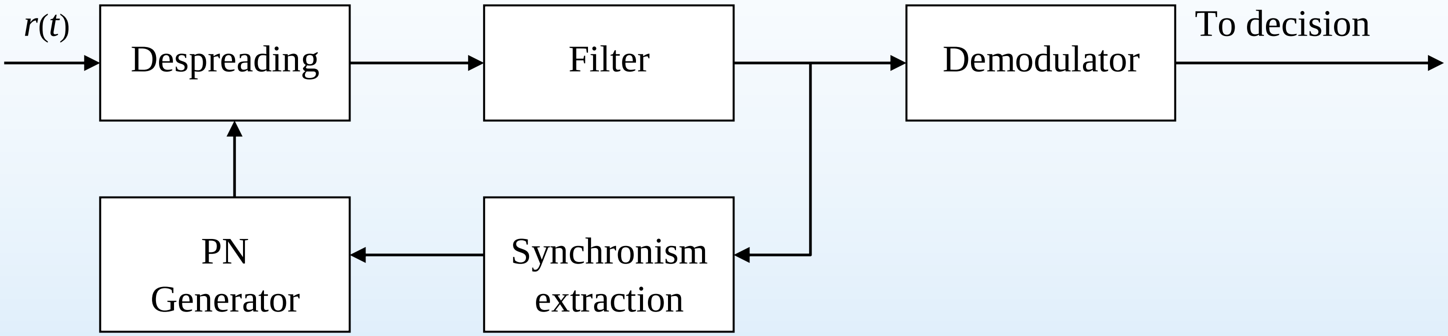
- Mode easily inserted in systems that operate in time division (GSM).
- The instantaneous band is narrow (and the instantaneous interference generated is large, as in narrow-band systems).
- It does not require power control (the simultaneous use of band by different transmitters lasts for a very limited time).
- It uses non-coherent demodulation (given the limited time available to acquire the phase).
- Narrow-band interference disturbs for limited intervals of time.

Spread spectrum: DS-QPSK transmitter



- T_b: information bit duration (data)
- T_c: PN sequence chip duration
- $S_f = T_b / T_c$: spreading factor (processing gain).

Spread spectrum systems: receiver

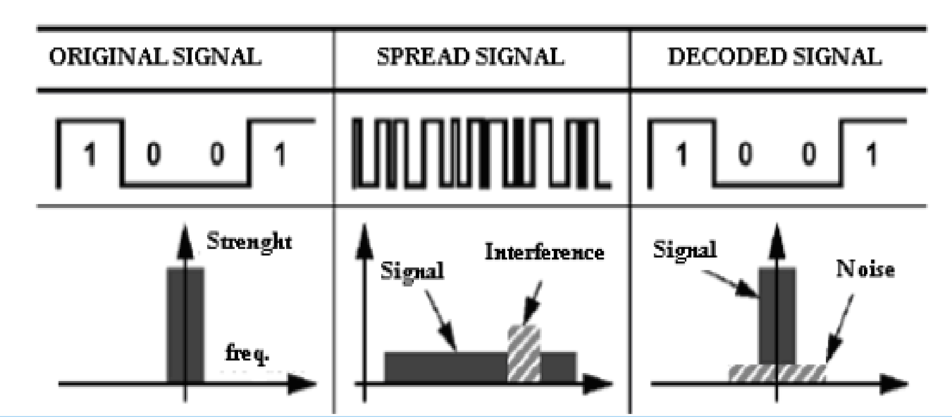


- P_r: Received signal power level
- $N_0/2$: noise power spectral density (bilateral)
- B_s: bandwidth occupied by the spread spectrum signal
- J: power of an uncorrelated interfering signal

- The output signal-to-noise ratio is given by: $\mathrm{SNR}_o = \dfrac{S_F P_r}{\dfrac{N_0 B_s}{2} + J}$

Example

- Wideband CDMA (WCDMA): Users are distinguished by orthogonal sequences. During transmission, the signal containing the information is multiplied by a spreading sequence (different for each user), thus determining a direct expansion of the spectrum (Direct Sequence Spread Spectrum – DSSS).
- During reception, the signal is remultiplied by the same sequence in order to reconstruct the transmitted signal.



Frequency selective channel (multipath)

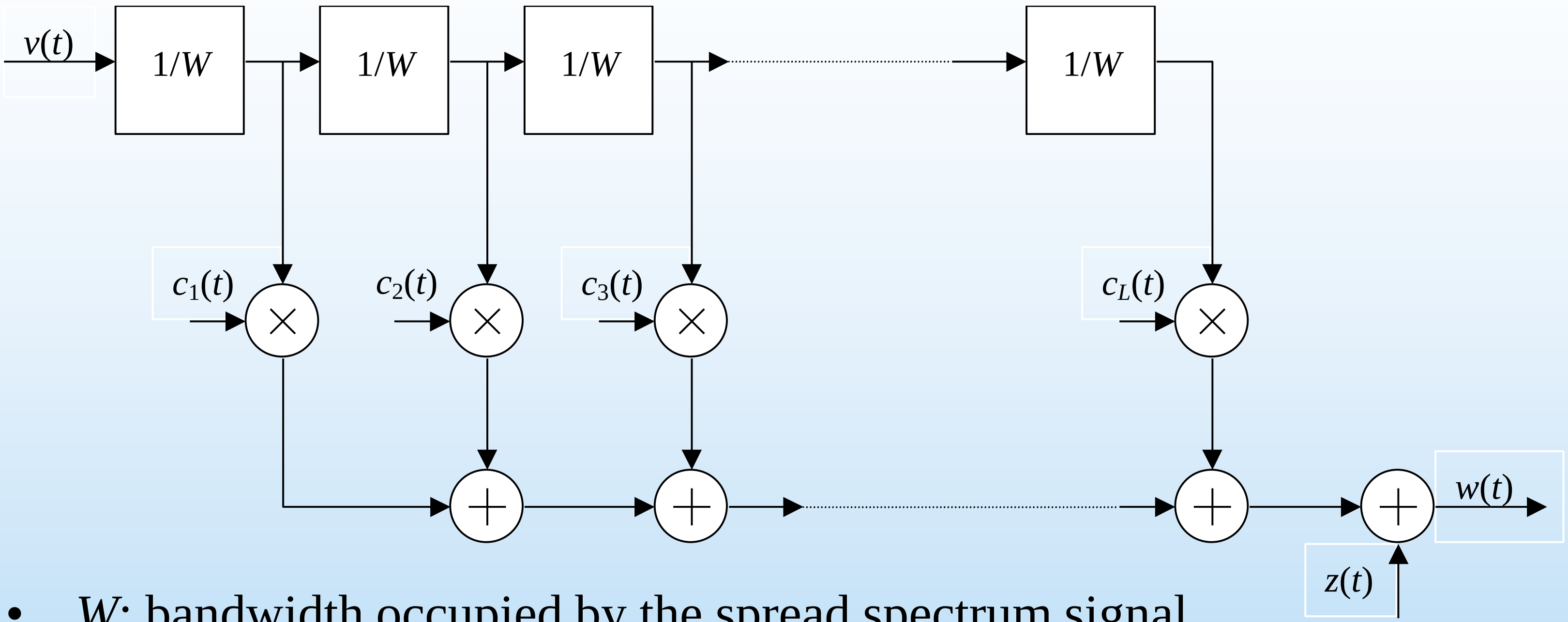
- Let W be the bandwidth of the broadband signal. The complex envelope has bandwidth $W/2$. Therefore its sampled version :

$$v_\delta(t) = \sum_n v(n/W)\delta(t - n/W) \qquad V_\delta(f) = \sum_n v(n/W)e^{j2\pi f\, n/W}$$

- Thus the output is (noting that for $|f| \le W/2$, the transform of the sampled envelope coincides with that of the envelope up to a factor of $1/W$):

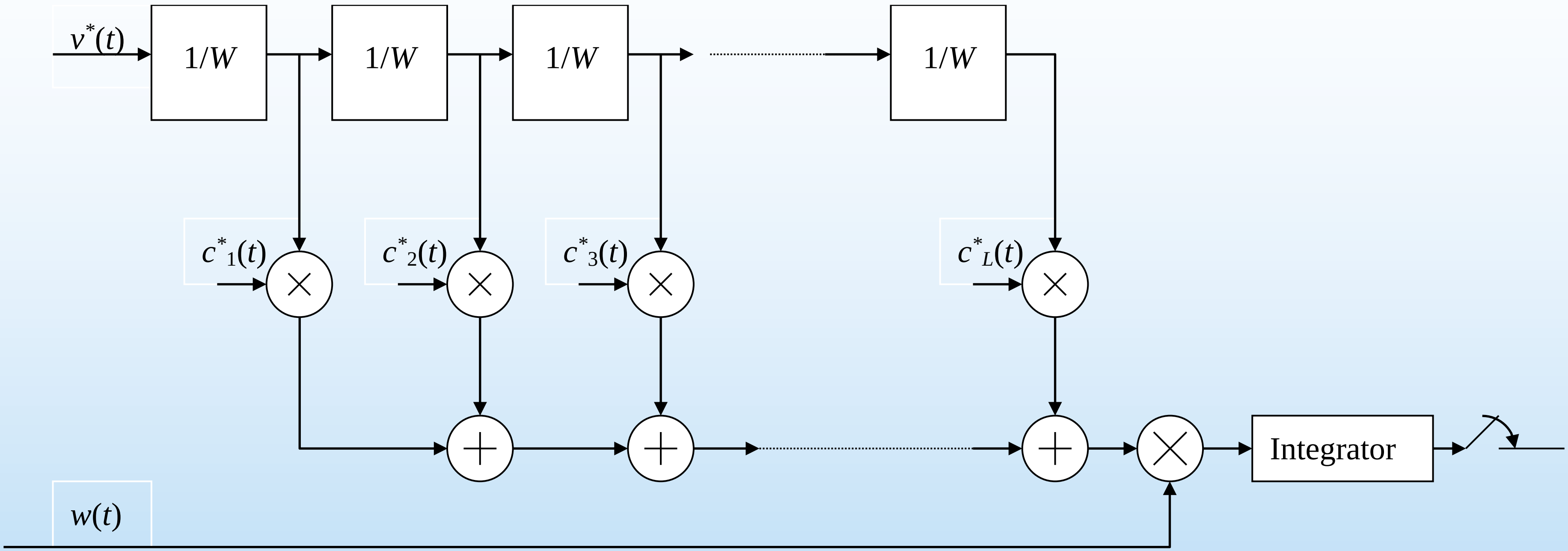
$$\begin{aligned} y(t) &= \int_{|f|\le W/2} T(f,t)\frac{1}{W}V_\delta(f)e^{j2\pi f t}\mathrm{d}f = \frac{1}{W}\sum_n v(n/W)\int_{|f|\le W/2} T(f,t)e^{j2\pi f(t-n/W)}\mathrm{d}f \\ &= \frac{1}{W}\sum_n v(n/W)c(t-n/W,t) = \frac{1}{W}\sum_n v(t-n/W)c(n/W,t) \\ &= \sum_n v(t-n/W)c_n(t) \end{aligned}$$

Frequency selective channel (multipath)



- W: bandwidth occupied by the spread spectrum signal.
- channel time-varying impulse response : $c(\tau;t)=\sum_{n=1}^{L} c_n(t)\delta(\tau-n/W)$
- Said T_m the maximum delay spread is $L=\lfloor T_m W \rfloor + 1$
- The multiplicative coefficients $\{c_n(t)\}$ are complex-valued Gaussian random processes, uncorrelated with each other, whose amplitudes follow the Rice statistics (Rayleigh in the case of absence of direct component), and whose phases are uniformly distributed. Their temporal evolution can be described by the Clarke model.

Rake receiver



- The received signal is correlated with the generic waveform, $v(t)$, filtered according to the estimated channel model (alternatively, the received signal is filtered and correlated with the generic transmitted waveform).
- The described operation performs a combination of the signals deriving from the different contributions in MRC mode.

Performance: summary (1)

- AWGN channel performance is not affected by spectrum expansion.
- The power density of the expanded signal is inversely proportional to the expansion factor.
- The protection against a narrowband interferer is proportional to the expansion factor.
- K simultaneous transmissions are possible, using uncorrelated PN sequences (CDMA – Code Division Multiple Access).

Performance: summary (2)

- If different codes are used by the same transmitter (multiplexing: downlink), the transmissions are orthogonal and do not interfere.
- Simultaneous transmissions of different transmitters (multiple access: uplink) interfere with each other. Assume that a Gaussian model is adopted for the overall interference (standard Gaussian approximation). More accurate models have been proposed (improved Gaussian approximation).
- BER (standard Gaussian approximation, BPSK, perfect power control, K simultaneous transmissions)
  - Asynchronous interference (chip and phase): $P_b \cong Q\sqrt{\dfrac{3S_f}{K-1}}$
  - Synchronous interference (chip and phase): $P_b \cong Q\sqrt{\dfrac{S_f}{K-1}}$

CDMA/SS-DS Pros

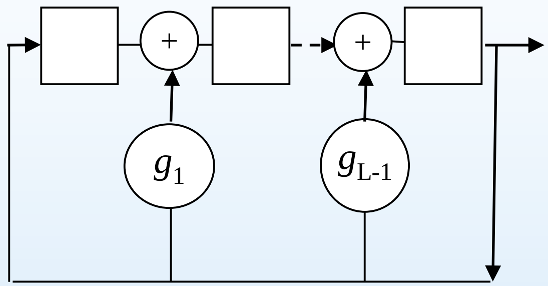
- Narrowband interference rejection.
- Intrinsic diversity.
- Intrinsic ability to exploit source inactivity periods favorably.
- Soft limitation of the maximum number of users.
- Message protection (both interception - low power density - and decoding - PN sequence are difficult).

CDMA/SS-DS Issues

- Sensitive to imperfect power control (near-far effect).
- Synchronization (high chip rate).
- PN sequence availability
  - Easy to generate.
  - Difficult to reconstruct.
  - With uncorrelated symbols.
  - Uncorrelated with sequence obtained by translation.
  - Uncorrelated with each other (multiple access).

PN sequence: ML-sequence generator

- ML sequence generator (Galois configuration) (sums modulo 2)..

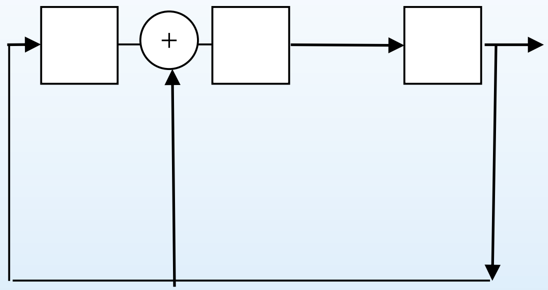


- The terms $g_i \in \{0,1\}$ are the coefficients of the generating polynomials of the dual codes of the Hamming codes. The following properties hold.
  – The sequence obtained has a maximum period $n=2^L-1$.
  – Adding modulo 2 a sequence and a translated version we obtain a further translated version.
  – In the sequence there are 2^{L-1} "ones" and $2^{L-1}-1$ "zeros".
  – Characteristics not good in terms of mutual correlation
  – Let $\{c_n\}$ be the sequence obtained by encoding the "ones" with -1, and the "zeros" with +1.

It turns out to be: $$R_c(k) = \sum_{i=1}^{n} c_h c_{h+k} = \begin{cases} n & k=0 \\ -1 & 1 \le k < n \end{cases}$$

PN sequences - examples

- Example: code of length 3, with g_2=0, g_1=1, g_0=1 (011, 3 in octal notation)



L	G ($g_{L-1}\ g_{L-2} \ldots g_0$)	G (octal)
4	0 0 1 1	0,3
5	0 0 1 0 1	0,5
6	0 0 0 0 1 1	0,3
7	0 0 1 0 0 0 1	0,2,1
8	0 0 0 1 1 1 0 1	0,3,5
9	0 0 0 0 1 0 0 0 1	0,2,1
10	0 0 0 0 0 0 1 0 0 1	0,0,1,1

PN sequence correlation

- Let $p[n]$ be the generated sequence.
- Let x=1-2$p[n]$ (The ‘1’s become -1 and the ‘0’s become +1).

$$\rho_x(k)=\frac{E[(x[h]-\mu)(x[h-k]-\mu)]}{\sigma_x^2}=\frac{E[x[h]x[h-k]]-\mu^2}{\sigma_x^2}=\rho_p(k)$$

- In a period, n, the mean value m=–1/n. Furthermore, the variance is 1–(1/n)^2.

$$x[h]x[h-k]=1-2\big(p[h]\oplus p[h-k]\big),\quad \mathrm{E}\big[x[h]x[h-k]\big]=\begin{cases}1 & k=mn\\ -1/n & \text{otherwise}\end{cases}$$

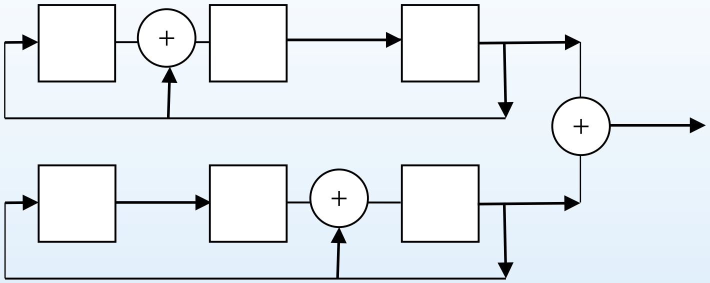
codeword

- Therefore :

$$\rho_x(k)=\begin{cases}1 & k=mn\\ -1/(n-1) & \text{otherwise}\end{cases}$$

Gold Sequences

- They are obtained by adding modulo 2, two distinct PN sequences of the same period.



- By initializing them differently (with non-zero sequences), we obtain distinct sequences, with mutual correlation values belonging to the set: {-1, -*t*(*L*), *t*(*L*)-2} (in the example *L*=3; other examples are reported in the table), being:

$$t(L)=\begin{cases} 2^{(L+1)/2}+1 & L \text{ even} \\ 2^{(L+2)/2}+1 & L \text{ odd} \end{cases}$$

L	Generator 1	Generator 2
3	3	5
4	0,3	1,1
5	0,5	2,7

Kasami sequences

- They are obtained by adding modulo 2, two distinct PN sequences, one that uses a generator of length L, the other that uses a generator of length $2L$.
- Translating the sequence obtained with the generator of length L we obtain the small Kasami set. Translating the other we obtain the large set.
- Some pairs of generators are reported in the table.

L	G 1 (length *L*)	G 2 (length 2*L*)
3	3	4,7
3	3	4,1
3	5	6,3
3	5	5,5
4	0,3	0,3,5
4	1,1	0,5,3

Orthogonal sequences

- Used in multiplexing phase
- Example: Walsh-Hadamard sequences

They are obtained from Hadamard matrices, $\mathbf{H}_M$

$$\mathbf{H}_2 = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$$

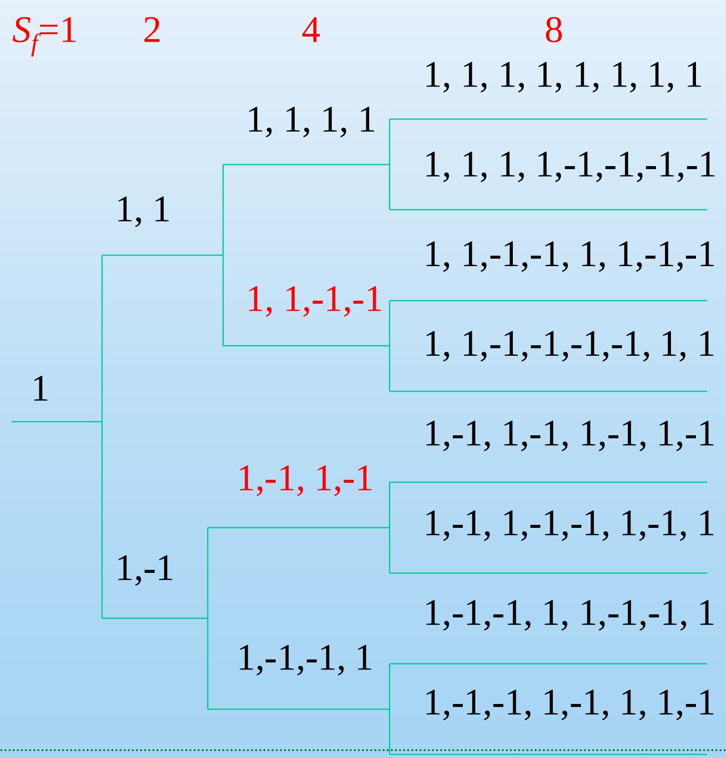
$$\mathbf{H}_{2M} = \begin{bmatrix} \mathbf{H}_M & \mathbf{H}_M \\ \mathbf{H}_M & -\mathbf{H}_M \end{bmatrix}$$

- Example

$$\mathbf{H}_4 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}$$

Channeling tree

- Each S_f corresponds to a set of orthogonal codes. In downlink they allow to obtain a certain orthogonality between the channels. If a code corresponding to a given S_f is used, all the codes that follow it in the tree are not available. In uplink, to reduce the interference it is necessary to use the PN sequences, with scrambler function.



	Example
Info A	$I_A = +1$
Spreading A	$S_A = +1\ +1\ -1\ -1$
Espanded A= $I_A \cdot S_A$	$E_A = +1\ +1\ -1\ -1$
Info B	$I_B = -1$
Spreading B	$S_B = +1\ -1\ +1\ -1$
Espanded B= $I_B \cdot S_B$	$E_B = -1\ +1\ -1\ +1$
Channel= $E_A + E_B$	C= 0 +2 -2 0
Despread A= $C \cdot S_A$	$D_A = 0\ +2\ +2\ 0$
Despread B= $C \cdot S_B$	$D_B = 0\ -2\ -2\ 0$
$Sum(D_A) > 0 \rightarrow +1$	$Sum(D_B) < 0 \rightarrow -1$

Orthogonal Frequency Division Multiplexing

- Multi-path channel (simplified expression):

$$h(t,\tau)=\sum_{l=0}^{L-1} h_l(t)\delta(\tau-\tau_l)$$

- OFDM: Let W be the available bandwidth. Divide the signal (complex envelope) into N flows that occupy a bandwidth equal to $\Delta f = W/N < B_c$ (Coherence Band). Let Sn[k] be the modulation symbols in use.

$$\tilde{s}(t)=\sum_{n=-\infty}^{\infty}\sum_{k=0}^{N-1} S_n[k]\,g(t-nT)\,e^{j2\pi(k-N/2)\Delta f t}$$

- Orthogonality condition between the carriers : $\Delta f=1/T$.
- Let $g(t)$=rect(t/T-1/2). In [0,T) we have: $\tilde{s}(t)=\sum_{k=0}^{N-1} S[k]\,e^{j2\pi(k-N/2)t/T}$
- Since the positive half-band is equal to $W/2$ we can sample with a sampling interval equal to $1/W=T/N$

$$\tilde{s}[m]=\tilde{s}\left(t=m\frac{T}{N}\right)=\sum_{k=0}^{N-1} S[k]\,e^{j2\pi(k-N/2)m/N}=\sum_{k=0}^{N-1} S_{\text{shift}}[k]\,e^{j2\pi mk/N}=N\ \text{IDFT}\{S_{\text{shift}}[k]\}$$

OFDM: transmission algorithm

- $\mathbf{S} = [S[-N/2], S[-N/2+1], ..., S[0], .., S[N/2]]$ Usually $S[0]=0$.
- Let $N_{\mathrm{FFT}}=2^K$, we add $N_{\mathrm{FFT}} - N - 1$ zeros (zero padding)

$$\mathbf{S}_{\mathrm{PADD}} = [S[-N/2], S[-N/2+1], ..., S[0]=0, .., S[N/2], 0, ..., 0]$$

$$\mathbf{S}_{\mathrm{SHIFT\text{-}PADD}} = [0, S[1], ..., S[N/2], 0, ..., 0, S[-N/2], ..., S[-1]]$$

- IFFT: $\tilde{\mathbf{s}} = [\tilde{s}[0], \tilde{s}[1], ..., \tilde{s}[N_{FFT}-1]]$
- Added cyclic prefix (CP: Cyclic Prefix), replicating at the top of the sample vector the last L-1 samples, where L is the length of the channel impulse response).

$$\tilde{\mathbf{s}}_{\mathrm{CP}} = [\tilde{s}[N_{FFT}-L+1], ..., \tilde{s}[N_{FFT}-1]]$$

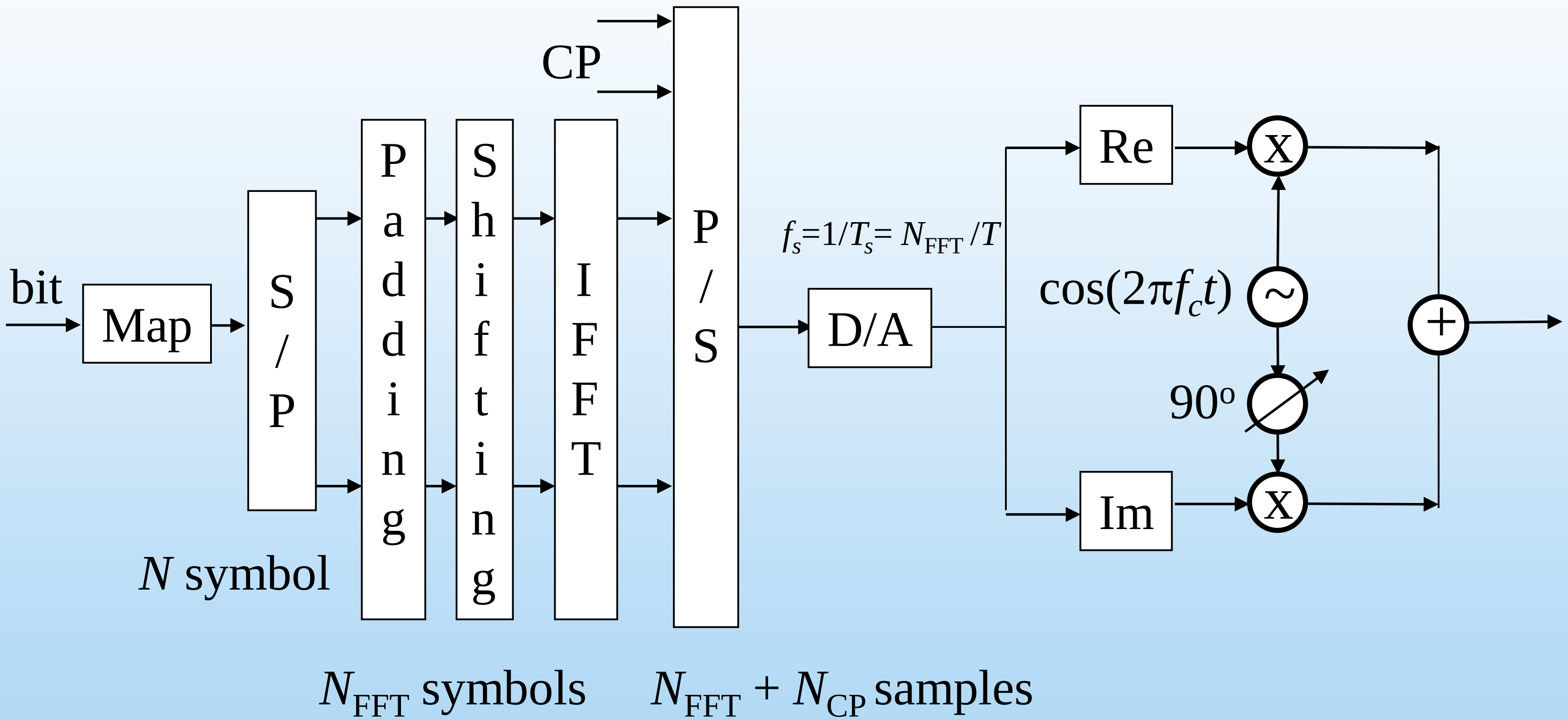
- Parallel series and transmission conversion.

Cyclic Prefix (CP)

- The receiver's task is to calculate the DFT of the received signal in NFFT values.
- By prefixing the transmitted symbol vector with the last *L*-1, the channel response (obtained by linear convolution) contains the samples that allow the direct calculation of the DFT. That is, it results in (distinct samples see distinct and independent carriers).
- Example (*L*=3, NFFT =4).

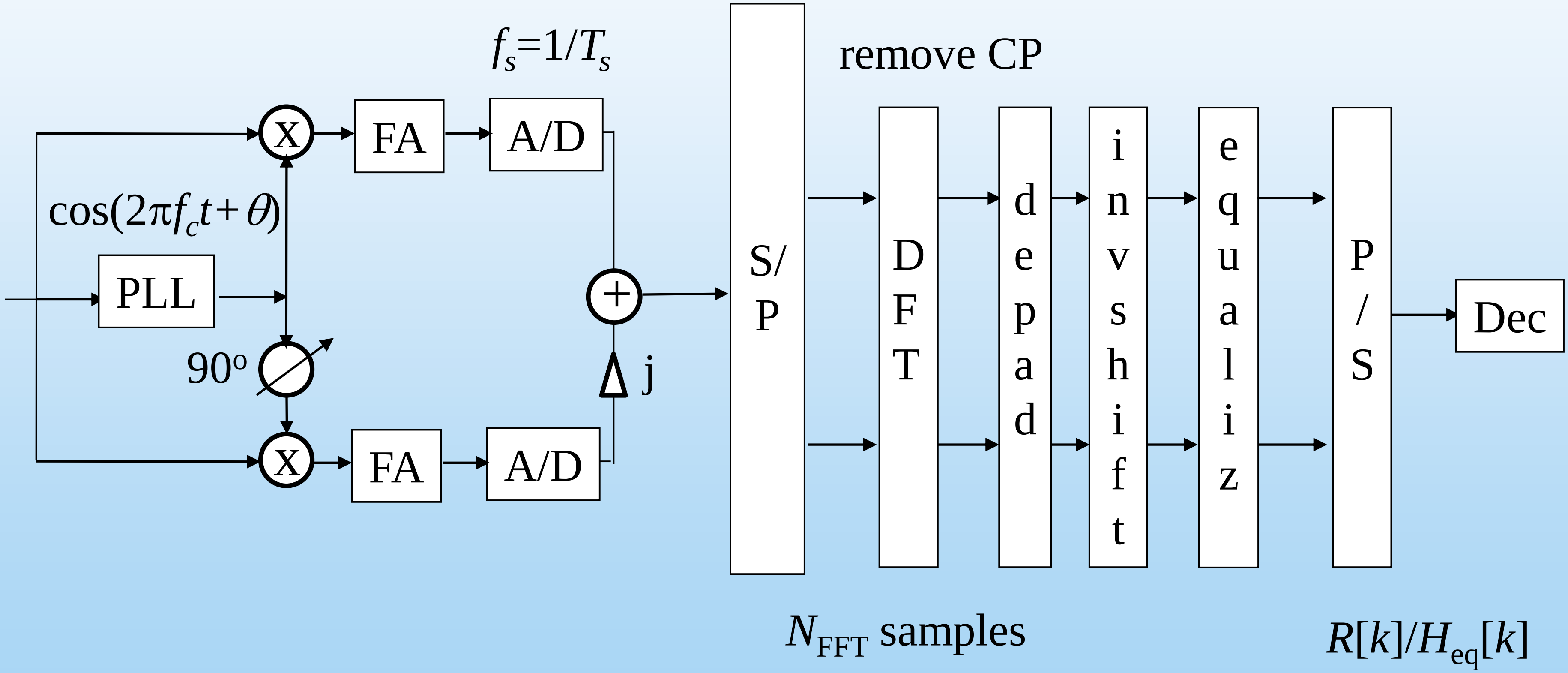
CP_{S1}		**S_1**				**CP_{S2}**		**S_2**	**Y_n**	**Cyclic convolution**
S_{12}	S_{13}	S_{10}	S_{11}	S_{12}	S_{13}	S_{22}	S_{23}	S_{20}		
h_2	h_1	h_0							y_{10}	$S_{12}\,h_2 + S_{13}\,h_1 + S_{10}\,h_0$
	h_2	h_1	h_0						y_{11}	$S_{13}\,h_2 + S_{10}\,h_1 + S_{11}\,h_0$
		h_2	h_1	h_0					y_{12}	$S_{10}\,h_2 + S_{11}\,h_1 + S_{12}\,h_0$
			h_2	h_1	h_0				y_{13}	$S_{11}\,h_2 + S_{12}\,h_1 + S_{13}\,h_0$
				h_2	h_1	h_0			CP_{R1}	
					h_2	h_1	h_0			
						h_2	h_1	h_0	y_{20}	$S_{22}\,h_2 + S_{23}\,h_1 + S_{20}\,h_0$

Transmitter



Receiver

- The equalizer block restores the starting constellation, inverting the channel (CSI), hypothetically flat in the band occupied by the sub-carrier – flat fading, coherent demodulation.



OFDM Examples

- 802.11 a/g (available bandwidth: W=20 MHz; adjacent overlapping channels)
  - $N_{\text{FFT}}=64$
  - $\Delta f = W/N_{\text{FFT}} = 312.5$ kHz
  - $T=1/\Delta f=3.2\ \mu s$
  - $T_s=T/N_{\text{FFT}}=50$ ns
  - N=52 (12 free sub-carriers)
  - $N_{\text{CP}}=12$
- LTE, available bandwidth: W=10 MHz
  - $N_{\text{FFT}}=1024$
  - $\Delta f=15$ kHz
  - $T=1/\Delta f=66.7\ \mu s$
  - $T_s=T/N_{\text{FFT}}=65$ ns
  - N=600
  - N_{CP} (normal)=72

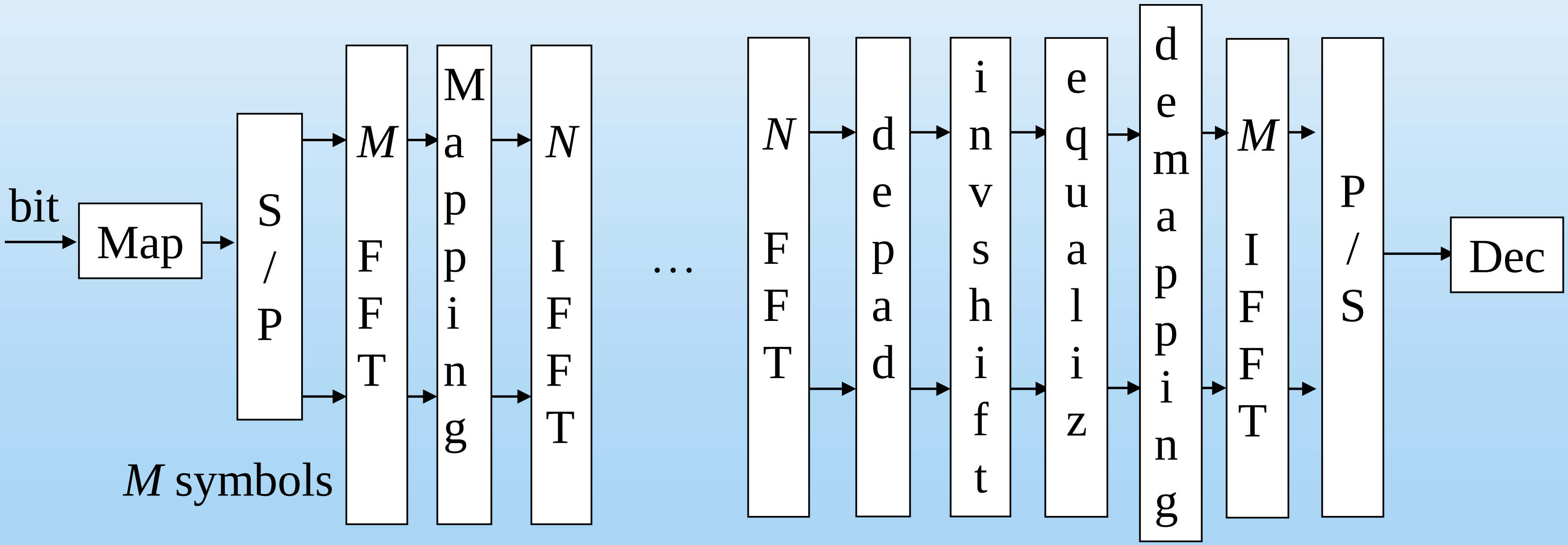
OFDM - summary

- The transmission, which is broadband overall, uses an adequate number of parallel narrowband channels, affected by flat fading.
- Channel compensation occurs in the frequency domain (coherent demodulation).
- Efficient modulation and demodulation operating in discrete time using IFFT and FFT.
- Optimal use of the band and channel using adaptive techniques (water filling).

- Observations
  - The channel must be reasonably flat in the band occupied by the subcarrier.
  - Efficiency is partially reduced by the (essential) use of the cyclic prefix.

Single Carrier - FDMA

- It uses an OFDM multiplexer, where user symbols are grouped into groups of $M<N$ (subcarriers) and transformed using the FFT, and then assigned to M subcarriers (diversity). The other subcarriers transmit 0.
- In reception, before the decision, the received symbols are reverse-transformed using the IFFT.
- Used in up-link. Different users use different subcarriers.



Orthogonal Time Frequency Space (OTFS)

- Orthogonal Time Frequency Space (OTFS) is a multiplexing technique that carries the information in the Delay-Doppler coordinate system.
- The information is transformed in the similar time-frequency domain as utilized by the traditional schemes of modulation such as TDMA, CDMA, and OFDM
- It was first used for fixed wireless, and is now a contending waveform for 6G technology due to its robustness in high-speed vehicular scenarios.
- High mobility scenarios, such as fast-moving vehicles or dynamic wireless networks, introduce severe channel impairments due to rapid time-varying fading, Doppler shift, and time dispersion. OFDM, with its fixed orthogonal subcarriers, struggles to cope with severe channel variations. As a result, the performance of OFDM degrades significantly, leading to reduced data rates and increased error rates.

- A. Monk, R. Hadani, M. Tsatsanis, S. Rakib, "OTFS - Orthogonal Time Frequency Space". arXiv:1608.02993, 2016-08-09.

Orthogonal Time Frequency Space (OTFS)

- "OTFS is a new air interface paradigm with important spectral efficiency advantages in high order MIMO and high Doppler scenarios. OTFS also provides efficiency in pilot packing of reference signals for channel estimation and prediction. All reference signals and modulation symbols are carried in the Delay-Doppler domain and experience the same channel response over the transmission /observation interval and extract the maximum diversity of the channel in both time and frequency dimensions. This allows the FEC layer to operate on a signal with a uniform Gaussian noise pattern, regardless of the particular channel structure. OTFS has a natural architectural compatibility with OFDM, based on its underlying multicarrier components. Moreover, the reference signal architecture supports any form of multicarrier modulation.
- 3GPP has identified a variety of eMBB deployment scenarios that focus on high vehicle speed and massive MIMO antenna arrays. The new radio air interface must support high spectral efficiency in high Doppler environments while supporting a large number of antennas. OTFS is ideally suited for these requirements, providing: high spectral efficiency; accurate channel estimation and prediction; and very efficient and flexible reference signals for massive MIMO applications."