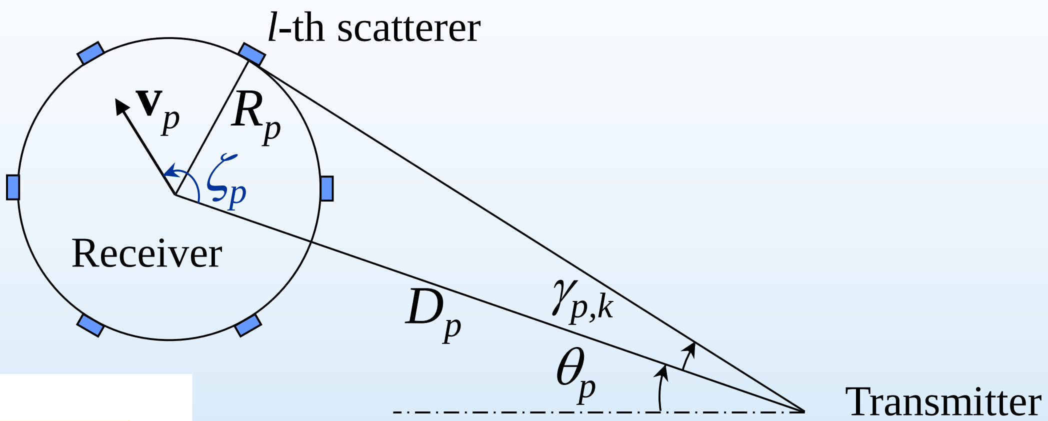
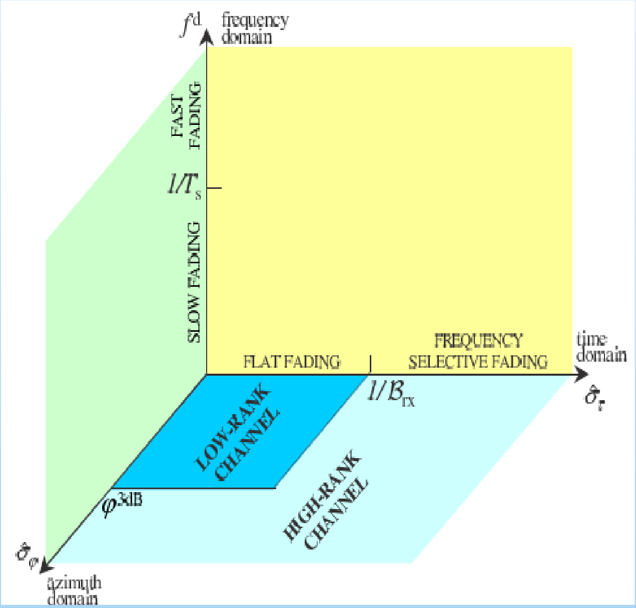
The Mobile Radio Channel

The multipath channel: an example

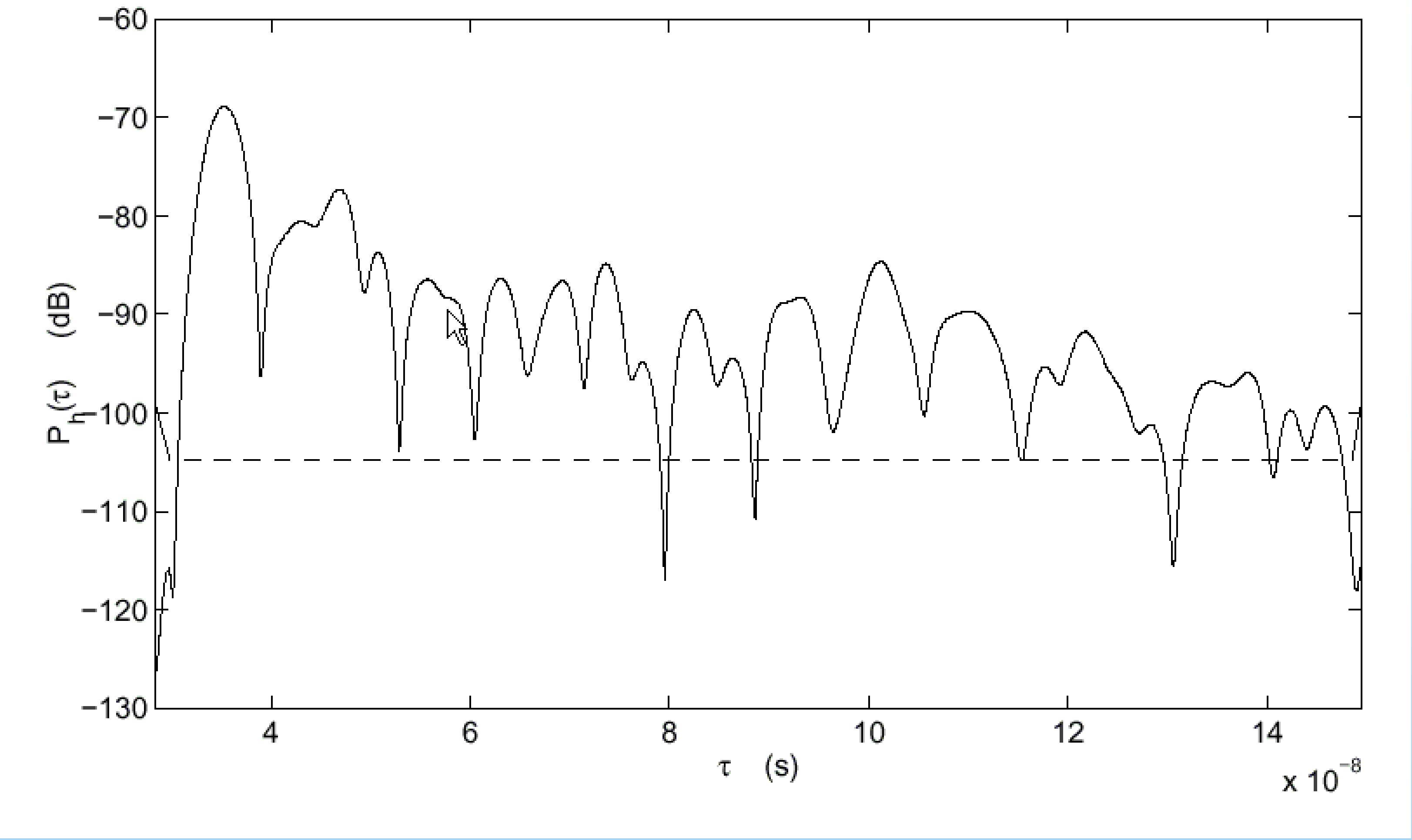




$$h(t,\tau)=\sum_{p=1}^{P}\sum_{l=1}^{L_p}\alpha_{l,p}(t)e^{j\varphi_{l,p}(t)}\,\delta(\tau-\tau_{l,p})$$

Power Delay Profile

- Assume to transmit an ideal pulse at t=0. Measured power response.



Delay spread – coherence bandwidth

- Average delay: $$\mu_\tau = \frac{\int_0^\infty \tau \phi_c(\tau)\mathrm{d}\tau}{\int_0^\infty \phi_c(\tau)\mathrm{d}\tau}$$
- Delay spread: $$\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau - \mu_\tau)^2 \phi_c(\tau)\mathrm{d}\tau}{\int_0^\infty \phi_c(\tau)\mathrm{d}\tau}}$$
- Be $\Phi_T(\Delta_f)$ function measures the frequency correlation of the channel. We define the coherence band, B_c, the frequency separation, Δf, for which $\Phi_T(\Delta f)$ assumes a prefixed value (usually 0.5). If the signal bandwidth, W, does not exceed B_c, the linear distortion due to the channel is not significant.
- The coherence band is proportional to the reciprocal of the delay spread:

$$B_c \equiv \frac{1}{\sigma_\tau}$$

Coherence Time

- Be $\Phi_H(\nu)$ the functions that provides the behavior of the output power as a function of the Doppler frequency, ν.
- The bandwidth, B_d, within which this function assumes significant values is called Doppler spread.
- The inverse of this bandwidth is called coherence time (time interval in which there is a significant correlation between the characteristics of the channel).
- Observe that B_d is proportional to the mobile speed, v, $B_d \equiv v/\lambda = v f_c / c$, being f_c the carrier frequency and c the speed light in the vacuum.

N paths channel

- Suppose to model the channel with N paths, each of which has an attenuation α_i and a delay τ_i. The time varying channel pulse response is:

$$c(t,\tau)=\sum_{i=1}^{N}\alpha_i(t)e^{-j2\pi f_c\tau_i(t)}\delta(\tau-\tau_i(t))$$

- Weight of the generic path:

$$p_i=\alpha_i^2\Big/\sum_{n=1}^{N}\alpha_n^2$$

- Average delay and delay spread:

$$\mu_\tau=\sum_{i=1}^{N}p_i\tau_i \qquad \sigma_\tau=\sqrt{\sum_{i=1}^{N}p_i(\tau_i-\mu_\tau)^2}$$

Narrow band signaling

- Signal bandwidth less than coherence bandwidth.
- Non-selective frequency fading.
- Received signal:

$$r(t) = \alpha e^{j\phi} u_m(t) + z(t), 0 \leq \mathrm{t} \leq \mathrm{T}$$

- where α e ϕ are random variables, while $z(t)$ represents the Gaussian noise.

Clarke model

- Hypotheses:
  - Incident field obtained as a result of many components coming from all directions.
- Clarke model:
  - The real and imaginary components of the field are random variables Gaussian (average zero if the transmitter and receiver are not in visibility - Rayleigh fading, not zero otherwise – Rice fading), uncorrelated.
  - The autocorrelation coefficient (normalized autocovariance) of each of the two components is expressed by: $\rho(\tau) = J_0(2\pi f_d \tau)$
    (J_0 it is the Bessel's function of first kind and of order 0, and f_d is the Doppler spread).

Power spectrum (Rayleigh fading)

- The autocorrelation coefficient (normalized autocovariance) of amplitude and power are both given by the following expression (for the amplitude it is an approximate description, while for power it is an exact expression):

$$\rho(\tau) = J_0^2(2\pi f_d \tau)$$

- The power spectrum is given by:

$$S(f) = \frac{P}{64\pi}\frac{1}{f_d}\mathrm{K}\left(\sqrt{1-\left(\frac{f}{2f_d}\right)^2}\right), \quad 0 \le |f| \le 2f_d$$

where K(x) is the complete elliptic integral of the first type.

Slow-fast fading

- In the hypothesis of non selective channel (signal bandwidth less than the coherence bandwidth), the signal undergoes a complex attenuation, $\alpha(t)$, variable over time (fading).

- Observing the autocorrelation coefficient, we note that fading values spaced by $\tau<<0.1/f_d$ are strongly correlated, while values spaced by $\tau>>0.1/f_d$ are uncorrelated.

- Defined T the time interval of interest (for example duration of a symbol or a package), fading is said to be slow if $T<<0.1/f_d$ (consecutive symbols undergo the same attenuation), and fast if $T>>0.1/f_d$ (consecutive symbols undergo independent attenuations).

Fading: amplitude model

- The amplitude of the received signal, ρ, is a random variable, the probability density function (pdf) of which is given by (Rice):

$$f_\rho(\rho|P,K)=(1+K)e^{-K}\frac{\rho}{P}e^{-\frac{1+K}{2P}\rho^2}I_0\left(\rho\sqrt{\frac{2K(1+K)}{P}}\right)$$

where K is the Rice factor, (ratio between the power of the direct path and the power received through reflections), P is the average received power, and I_0 is the modified Bessel function of first kind and order 0.

Rice fading

- The received, $p=\rho^2/2$, is a random variable the pdf of which is a non central χ^2 distribution with 2 degrees of freedom::

$$f_p\left(p\middle|P,K\right)=(1+K)\frac{e^{-K}}{P}e^{-\frac{1+K}{P}p}I_0\left(\sqrt{4K(1+K)\frac{p}{P}}\right) \qquad \sigma^2=P^2\frac{1+2K}{(K+1)^2}$$

- When K=0 (no direct path) we obtain the Rayleigh fading for which:

$$f_\rho\left(\rho\middle|P\right)=\frac{\rho}{P}e^{-\frac{\rho^2}{2P}} \qquad f_p\left(p\middle|P\right)=\frac{1}{P}e^{-\frac{p}{P}}$$

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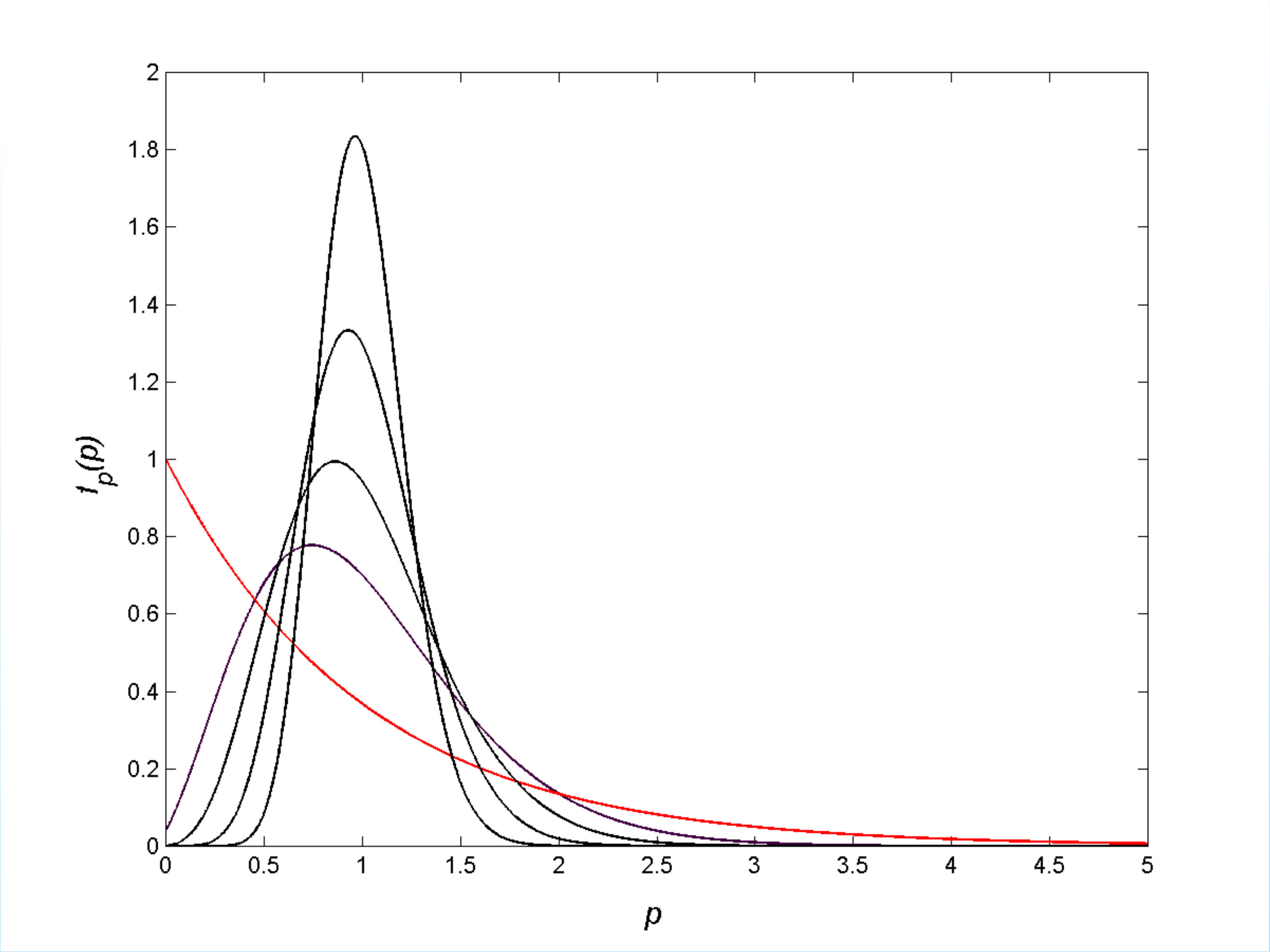
Rice fading



Power pdf for P=1 and K=0,5,10,20,40 .

Shadowing

- The attenuation due to obstacles, foliage follow a log-normal type statistic (shadowing).
- Given a random variable $V \geq 0$, it follows a log-normal statistic if the variable $U = \ln(V)$ (*or* $U = 10\log(V)$, or $U = 20\log(V)$ is a Gaussian variable.
- Average received power in dBm

$$U = 10\log_{10} P_R \qquad U_m = E[U] \qquad \sigma^2_{\log n} = E\left[(U - U_m)^2\right]$$

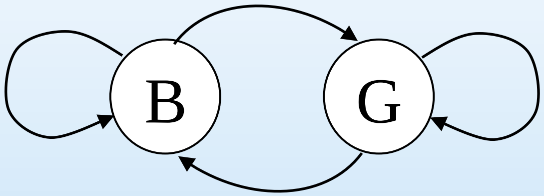
$$f_U(U) = \frac{1}{\sqrt{2\pi\sigma^2_{\log n}}} e^{-\frac{(U-U_m)^2}{2\sigma^2_{\log n}}} \Rightarrow f_{P_R}(P_R) = \frac{C}{\sqrt{2\pi\sigma^2_{\log n}}} \frac{1}{P_R} e^{-\frac{(10\log_{10} P_R - U_m)^2}{2\sigma^2_{\log n}}}$$ with $C = 10\log_{10} e$

defined $\mu = U_m / C$ and $\sigma = \sigma_{\log n} / C$, we have $f_{P_R}(P_R) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{P_R} e^{-\frac{(\log P_R - \mu)^2}{2\sigma^2}}$

from which $E[P_R] = \exp(\mu + \sigma^2/2)$ and $\mathrm{var}[P_R] = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$

Two-state ITU-R P.681-11channel (1)

- Assume that the long-term variations in the received signal may be described by a semi-Markov chain including two distinct states, Good (with a limited shadowing) and Bad (with severe shadowing) [Rec. ITU-R P.681-11].



- The duration of each state is considered to be log-normally distributed.

$$f_X(x) = \frac{1}{x\sqrt{\pi\sigma_i^2}} e^{-\frac{(\log x - \mu_i)^2}{2\sigma_i^2}}$$

where: i = G for Good states

i = B for Bad states

μ_G and σ_G: mean and standard deviation for Good state

μ_B and σ_B : mean and standard deviation for Bad state.

Two-state ITU-R P.681-11 channel (2)

- The signal envelope in the Good and Bad states follows a Loo distribution taking into account both fading and shadowing.

$$f_{\text{Loo}}(x) = x \frac{20\log_{10} e}{\sigma_i^2 \Sigma_{A_i} \sqrt{2\pi}} \int_0^\infty \frac{1}{a} \exp\left[-\frac{\left(20\log_{10} a - M_{A_i}\right)^2}{2\Sigma_{A_i}^2} - \frac{x^2 + a^2}{2\sigma_i^2} \right] I_0\left(\frac{ax}{\sigma_i^2}\right) \mathrm{d}a$$

- where M_{Ai}: mean of the direct signal
  Σ_{Ai}: standard deviation of the direct signal
  $M_{Pi}=10\log_{10}(2\sigma_i^2)$ dB: mean of the reflections (multipath)

- Without shadowing, $a=A$, we obtain the Rice fading

$$f_{\text{Rice}}(x) = \frac{x}{\sigma_i^2} \exp\left[-\frac{x^2 + A^2}{2\sigma_i^2} \right] I_0\left(\frac{Ax}{\sigma_i^2}\right)$$

with $A = \sqrt{\dfrac{2KP}{1+K}}$, $\sigma_i^2 = \dfrac{P}{1+K}$, from which $K = \dfrac{A^2}{2\sigma_i^2}$ $P = 2\sigma_i^2 + A^2$

Two-state ITU-R P.681-11 channel (3)

- Without the reflections (σ_i=0) we obtain the lognormal density probability

$$f_{\text{lognormal}}(x) = \frac{1}{x} \frac{20\log_{10} e}{\Sigma_{A_i}\sqrt{2\pi}} \exp\left[-\frac{\left(20\log_{10} x - M_{A_i}\right)^2}{2\Sigma_{A_i}^2} \right]$$

with moments

$$E[x] = 10^{M_{A_i}/20 + \left(\Sigma_{A_i}/20\right)^2 \log(10)/2}$$

$$\mathrm{var}[x] = 10^{M_{A_i}/20 + \left(\Sigma_{A_i}/20\right)^2 \log(10)} \left(10^{\left(\Sigma_{A_i}/20\right)^2 \log(10)} - 1 \right)$$

Two-state ITU-R P.681-11 channel (4)

- The data sets provided for ITU-R P.681-11 channel are applicable for frequency range of 1.5 GHz to 20 GHz.
- Matlab release 2024a includes a channel simulator for the ITU channel.
- These parameters are requested to model a specific scenario

- Environment (values: "Urban" (default) | "Suburban" | "RuralWooded" | "Village" | "Residential" | "Highway" | "Rural" | "Train" | "Custom“)

- Carrier frequency

- Elevation angle

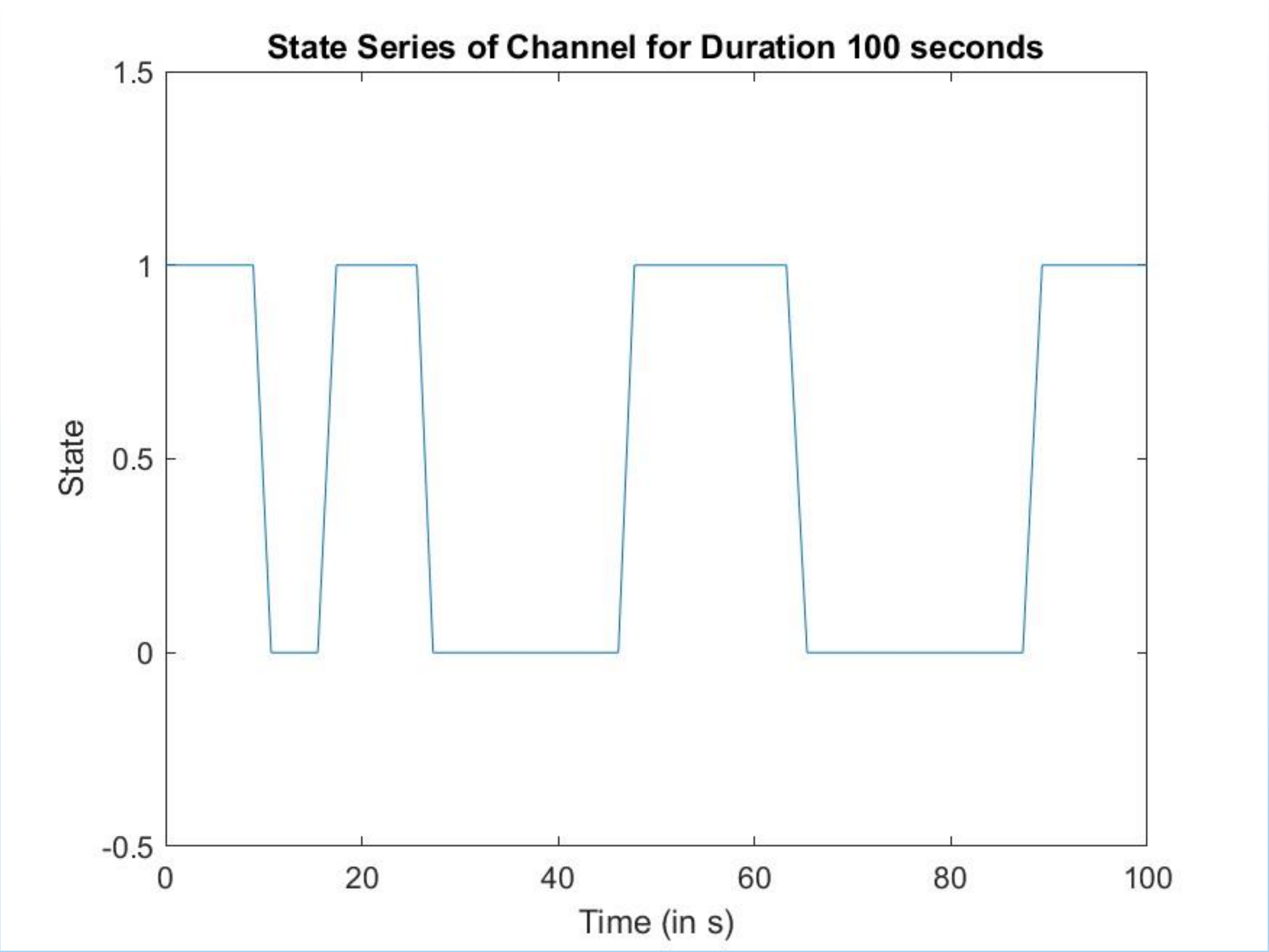
- Speed of the ground terminal

- Azimuth orientation of the ground terminal

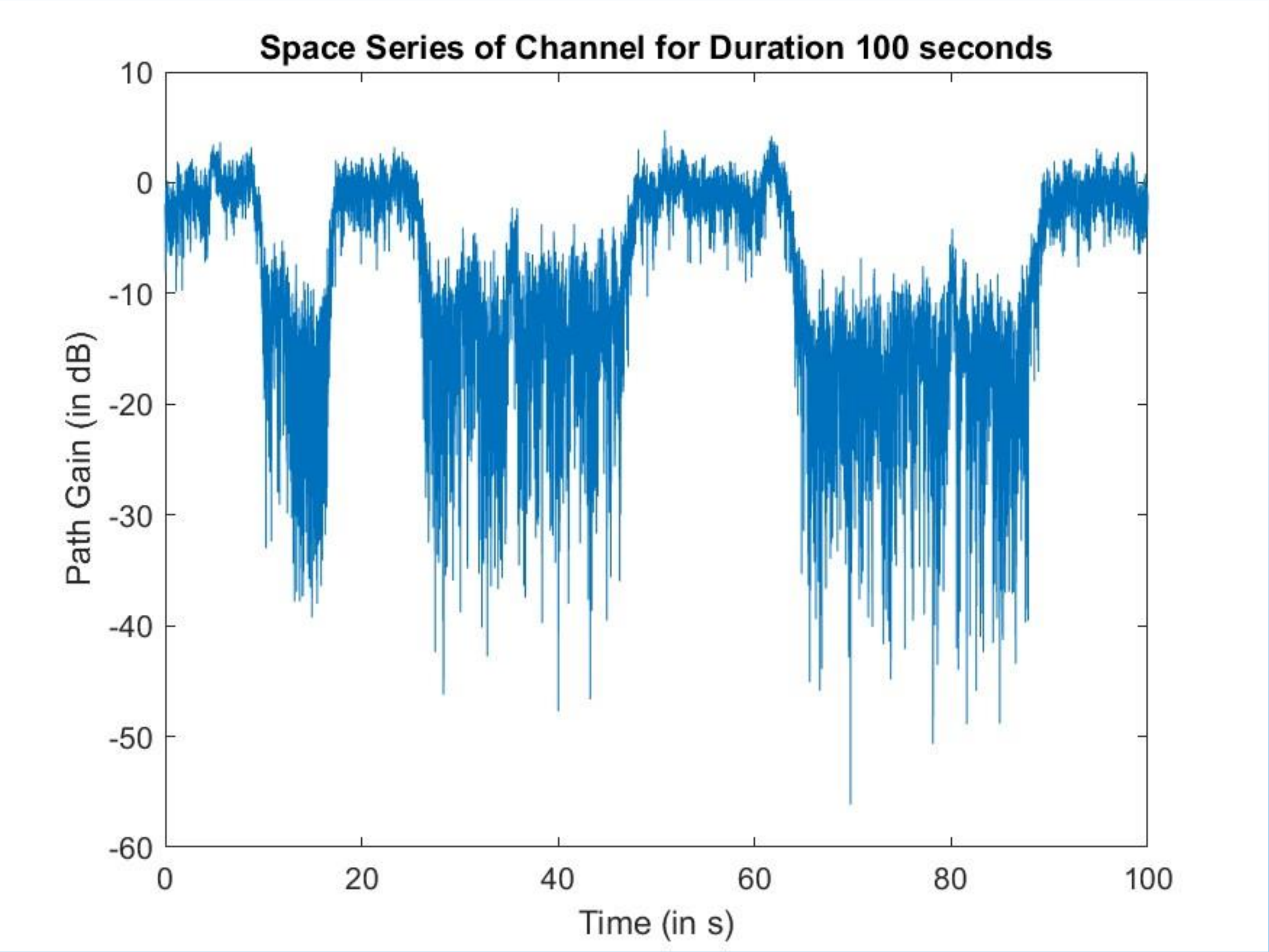
Two-state ITU-R P.681-11 channel (5)

- Example
  - Environment type: “Urban”.
  - Carrier frequency: 3.8e9 Hz.
  - Elevation angle: 45°
  - Speed of the ground terminal: 2 m/s
  - Azimuth orientation of the ground terminal: 0°

Two-state ITU-R P.681-11 channel (6)



Two-state ITU-R P.681-11 channel (7)



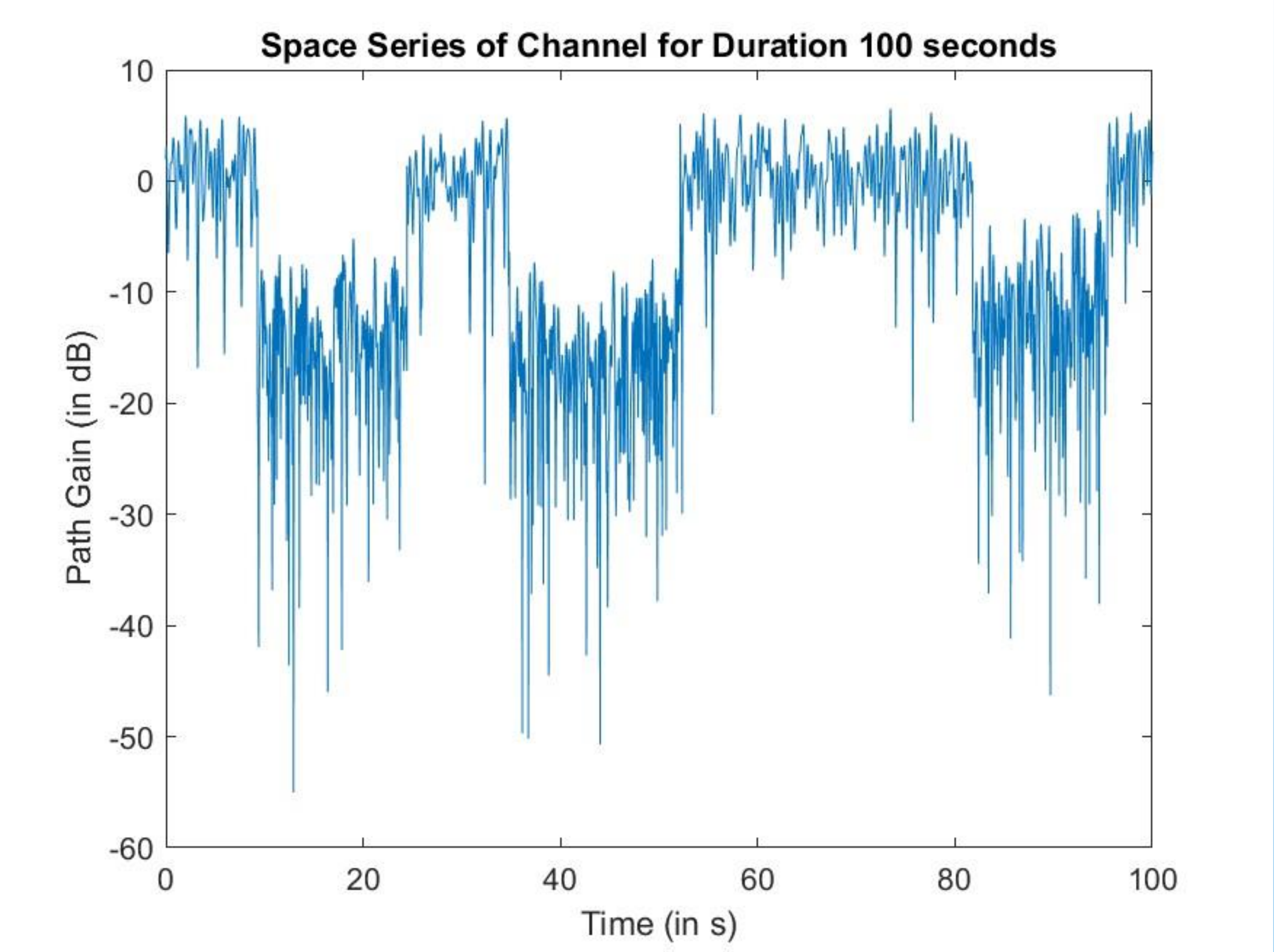
Two-state Lutz channel (1)

- E. Lutz, D. Cygan, M. Dippold, F. Dolainsky, and W. Papke, "The land mobile-satellite communication channel-recording, statistics, and channel model", IEEE Trans. Veh. Technol., vol 40, no. 2, pp. 375-386, 1991.
- Suitable for L band (1.54 GHz).
- Uses Rician distribution in good state and Rayleigh with log-normal distribution in bad state.
- The state duration distribution has to be specified.
- Available in Matlab with the following parameters
  - Rician K-factor
  - Lognormal fading parameters
  - State duration distribution
  - Mean state duration
  - Maximum Doppler shift

Two-state Lutz channel (2)

- Example
  - Rician K-factor: 5.5 dB
  - Lognormal fading parameters: [-13.6 3.8]
  - State duration distribution: "Exponential"
  - Mean state duration: [21 24.5]
  - Maximum Doppler shift: 2.8538 Hz.

Two-state Lutz channel (3)



Two-state channel

- Fading depth determined by Matlab simulator