Corso di Laurea in Fisica - UNITS

ISTITUZIONI DI FISICA PER IL SISTEMA TERRA

SEISMIC RAYS

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Heterogeneities

.. What happens if we have heterogeneities ?

Depending on the kind of reflection part or all of the signal is reflected or transmitted.

- What happens at a free surface?
- Can a P wave be converted in an S wave or vice versa?
- How big are the amplitudes of the reflected waves?

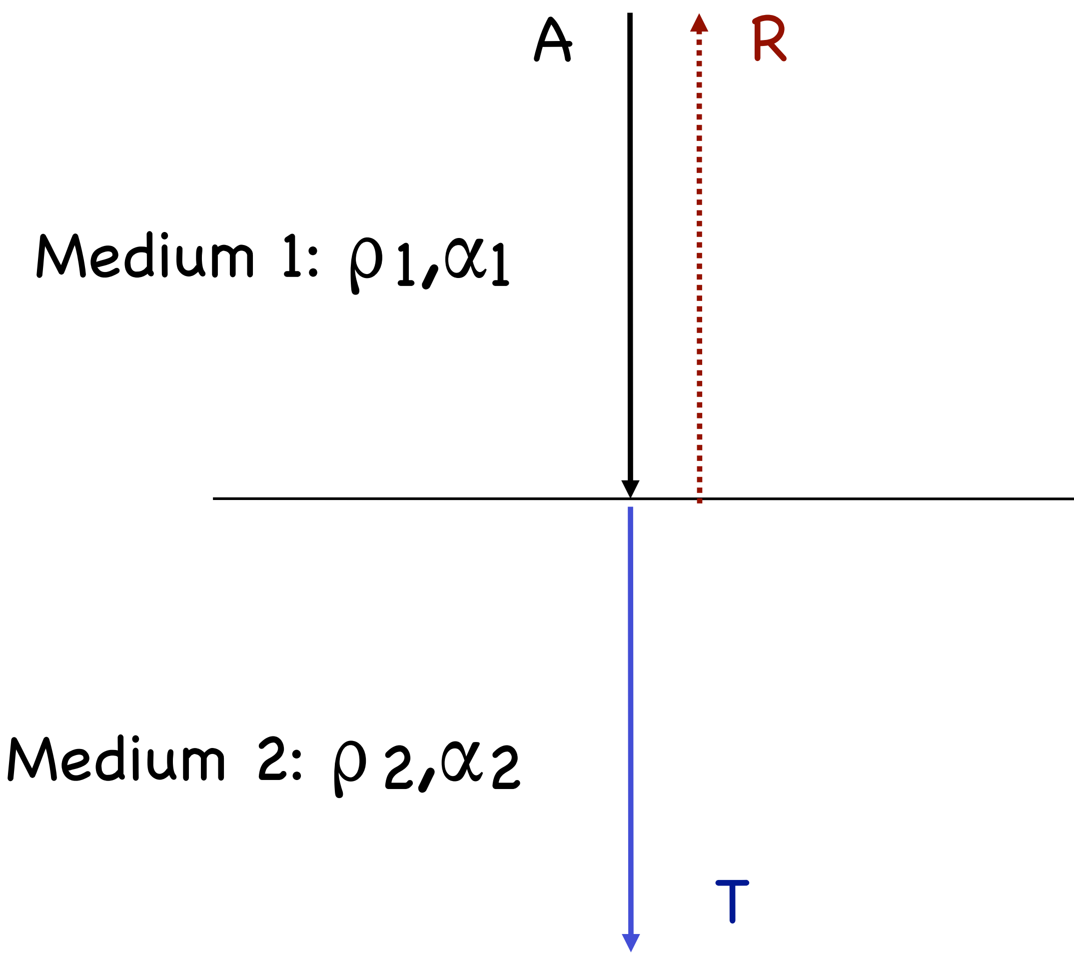
Boundary Conditions

What happens when the material parameters change at a discontinuity interface?

Continuity of displacement and traction fields is required

Kinematic (displacement continuity) gives **Snell's law**,
but how much is reflected, how much transmitted?

Let's take the most simple example: P-waves with **normal** incidence on a material interface. Dynamic conditions give:



$$\frac{R}{A} = \frac{\rho_2\alpha_2 - \rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}$$

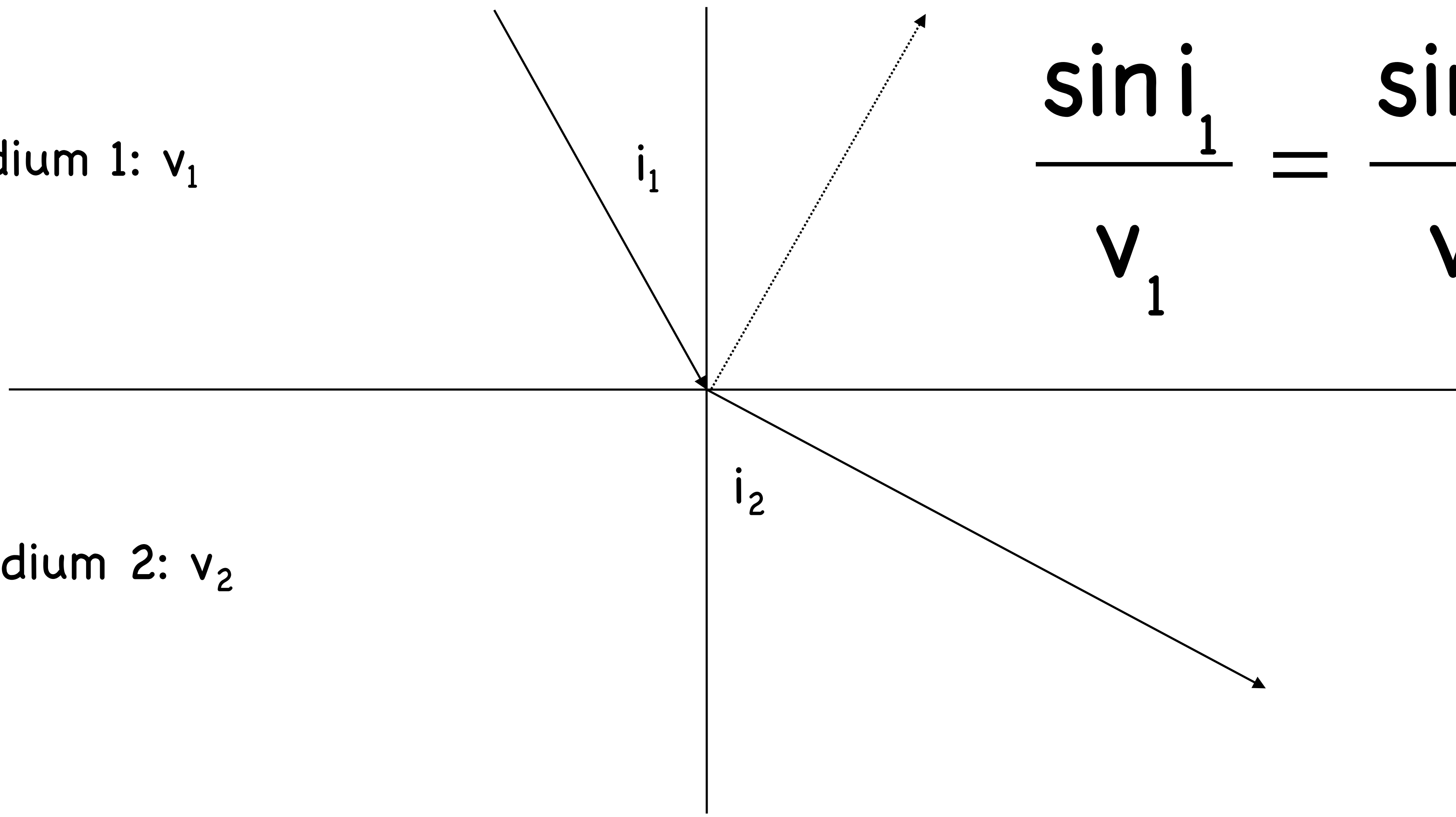
$$\frac{T}{A} = \frac{2\rho_1\alpha_1}{\rho_2\alpha_2 + \rho_1\alpha_1}$$

At oblique angles conversions from S-P, P-S have to be considered.

Reflection & Transmission–Snell's Law

What happens at a plane material discontinuity?

Medium 1: v_1



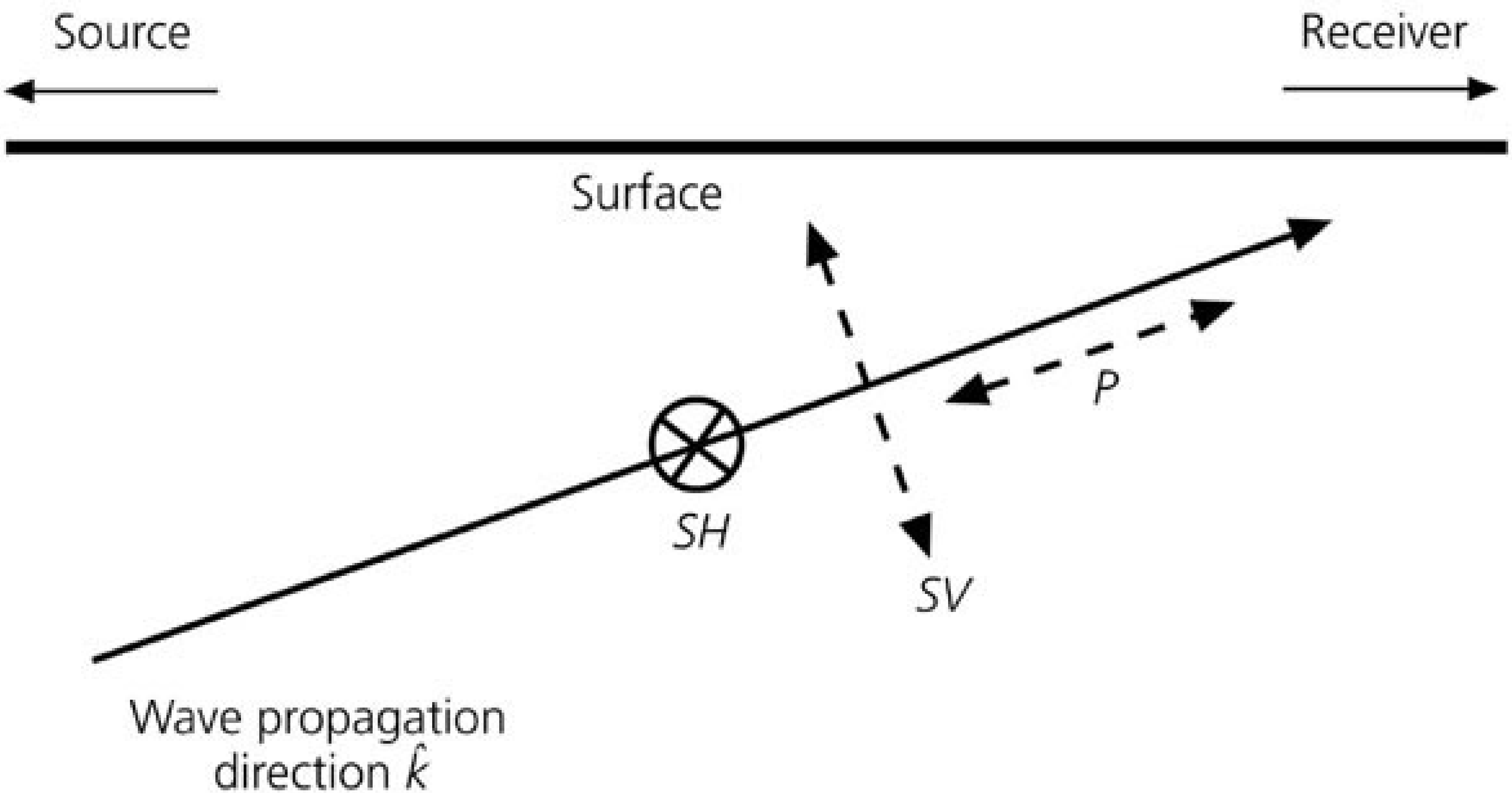
$$\frac{\sin i_1}{v_1} = \frac{\sin i_2}{v_2}$$

Medium 2: v_2

A special case is the **free surface** condition, where the surface tractions are zero.

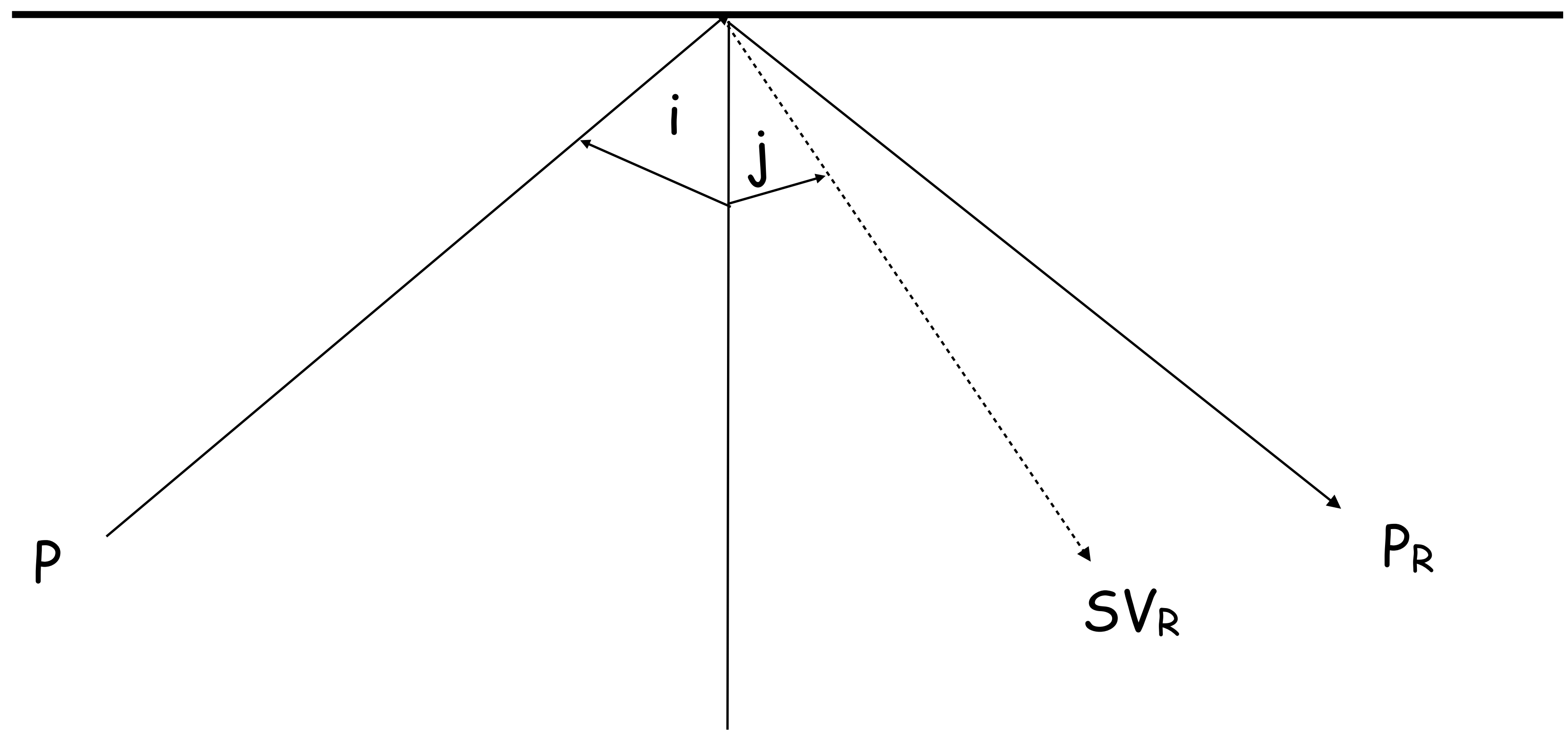
Free surface: P-SV-SH

Figure 2.4-4: Displacements for *P*, *SV*, and *SH*.



Case 1: Reflections at a free surface

A P wave is incident at the free surface ...



In general (also for an S incident wave) the reflected amplitudes can be described by the **scattering matrix** S

$$S = \begin{pmatrix} P_u P_d & S_u P_d \\ P_u S_d & S_u S_d \end{pmatrix}$$

Free surface: apparent velocity

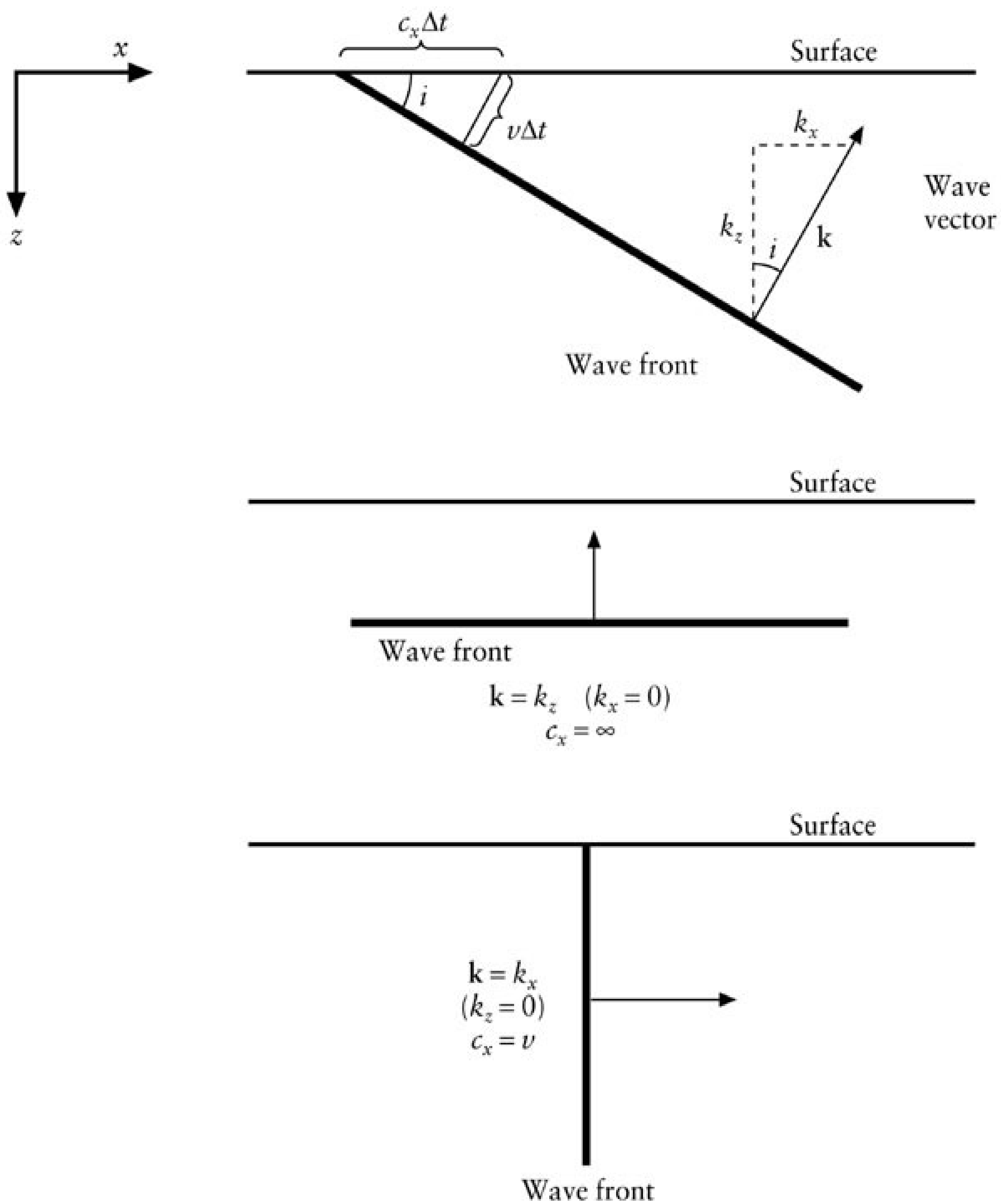
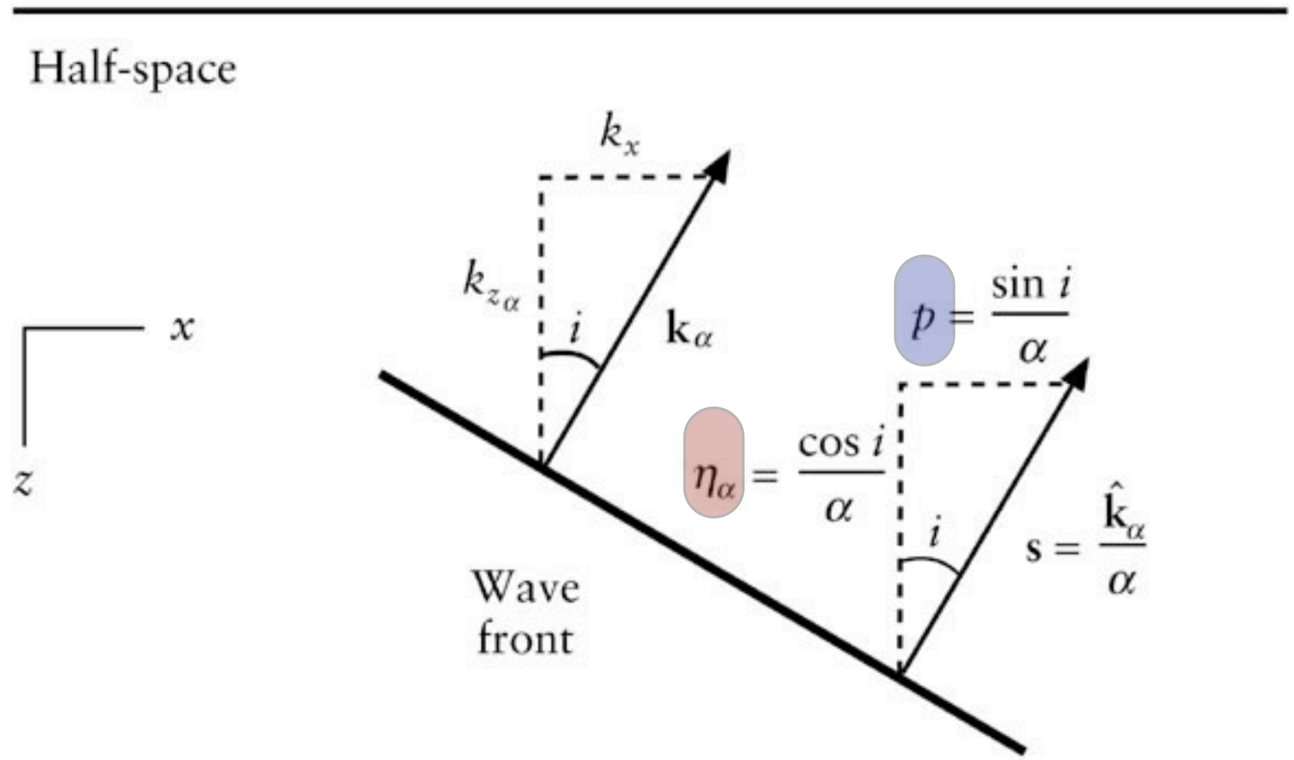


Figure 2.5-9: Definition of the slowness vector.



$$\Phi = A_0 e^{i(k_j x_j - \omega t)} = $$

$$= A_0 e^{i\omega(px - \eta z - t)} =$$

$$= A_0 e^{i\frac{\omega}{\alpha}(a_j x_j - \alpha t)} =$$

$$= A_0 e^{i(k_x x - k_x r_\alpha z - \omega t)}$$

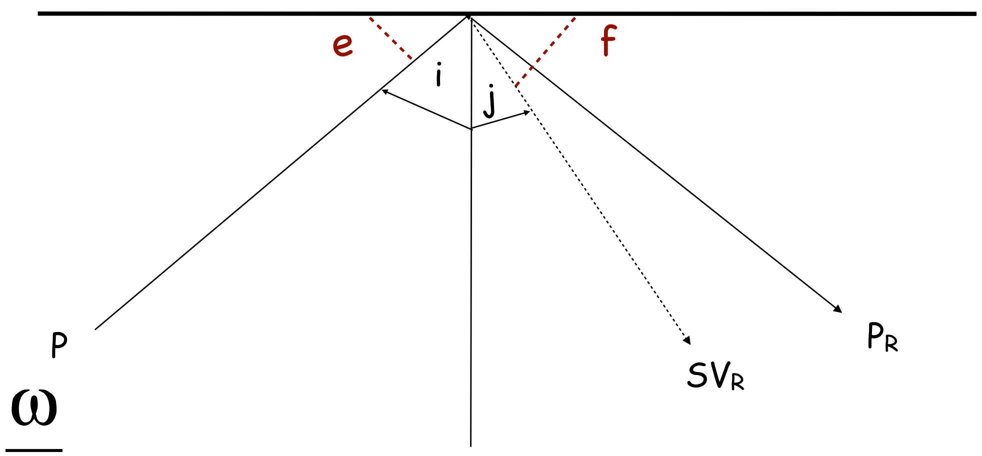
$$k_x = \omega p; k_z = \omega \eta = \omega \frac{\sqrt{1 - \sin^2 i}}{\alpha} = \omega p \sqrt{\left(\frac{c_x}{\alpha}\right)^2 - 1} = \omega p r_\alpha$$

R&T – Ansatz at a free surface

… here a_i are the components of the vector normal to the wavefront : a_i=(sin i, 0, -cos i)=(cos e, 0, -sin e), where e is the angle between surface and ray direction, so that for the **free surface**

$$\Phi = A_0 \exp\left[ik(x - zr_\alpha - ct)\right] + A\exp\left[ik(x + zr_\alpha - ct)\right]$$

$$\Psi = B\exp\left[ik'(x + zr_\beta - c't)\right]$$



where

$$c = \frac{\alpha}{\cos e} = \frac{\alpha}{\sin i} \qquad c' = \frac{\beta}{\cos f} = \frac{\beta}{\sin j}$$

$$k = \frac{\omega}{\alpha}\cos e = \frac{\omega}{\alpha}\sin i = \frac{\omega}{c} \qquad k' = \frac{\omega}{\beta}\cos f = \frac{\omega}{\beta}\sin j = \frac{\omega}{c}$$

what we know is that z=0 is a **free** surface, i.e.

$$\sigma_{xz}\big|_{z=0} = 0$$

$$\sigma_{zz}\big|_{z=0} = 0$$

Reflection and Transmission - Coeffs

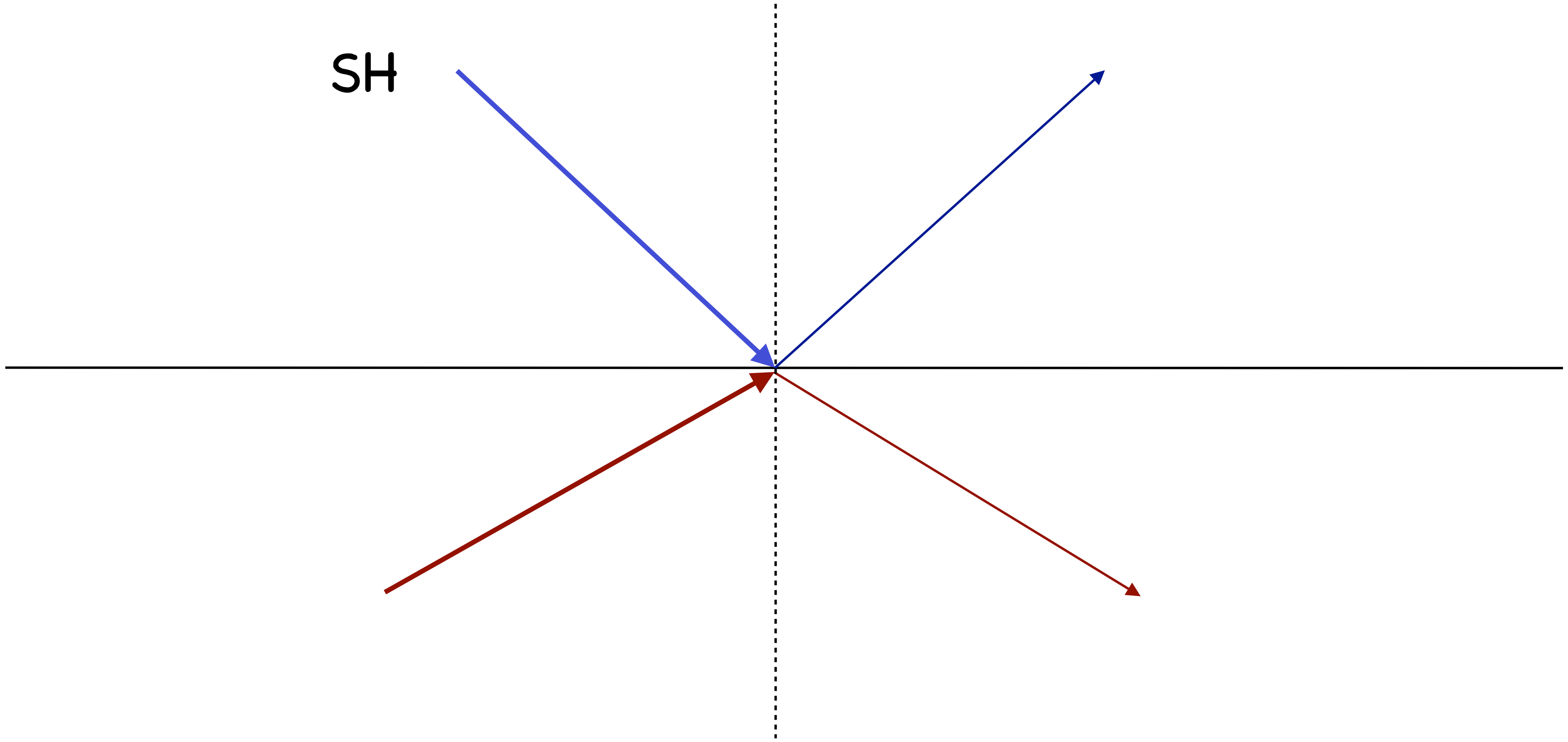
... putting the equations for the potentials (displacements) into these equations leads to a relation between incident and reflected (transmitted) amplitudes

$$R_{PP} = \frac{A}{A_0} = \frac{4 r_\alpha r_\beta - (r_\beta^2 - 1)^2}{4 r_\alpha r_\beta + (r_\beta^2 - 1)^2}$$

$$R_{PS_V} = \frac{B}{A_0} = \frac{4 r_\alpha (1 - r_\beta^2)}{4 r_\alpha r_\beta + (r_\beta^2 - 1)^2}$$

These are the reflection coefficients for a plane P wave incident on a free surface, and reflected P and SV waves.

For layered media SH waves are completely decoupled from P and SV waves

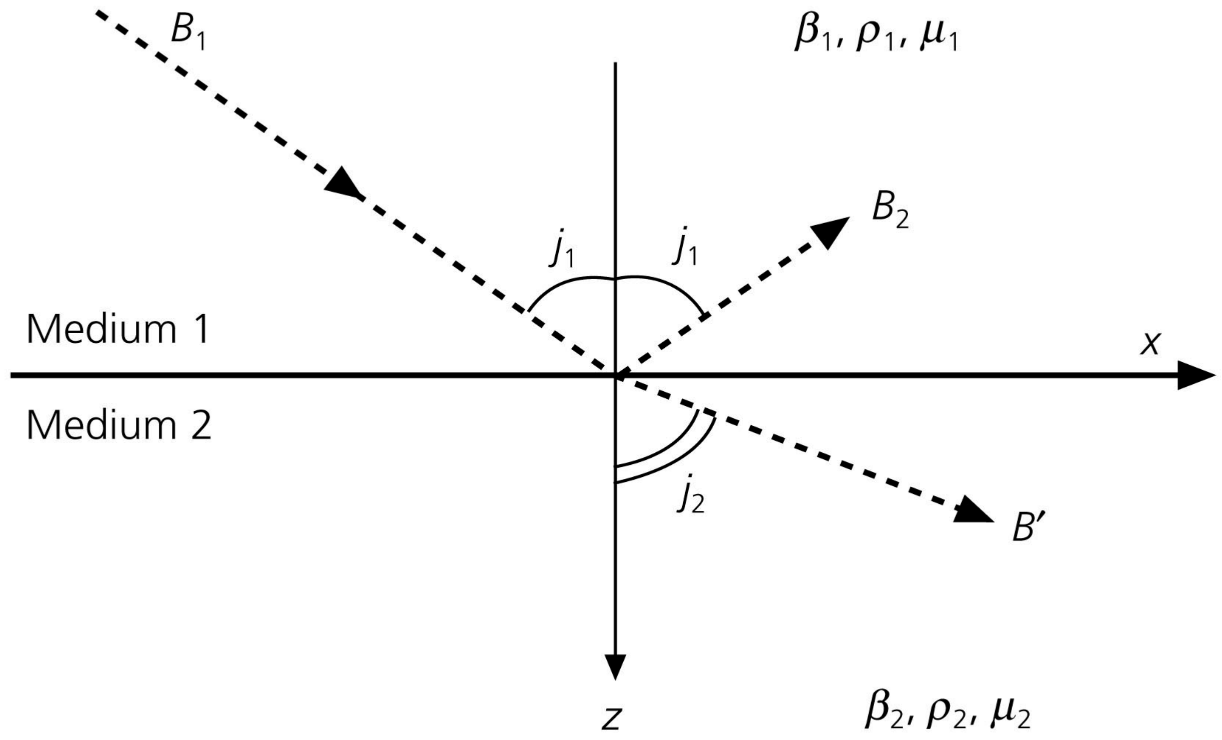


There is no conversion:
only SH waves are reflected or transmitted
In general we can write a scattering matrix:

$$S = \begin{pmatrix} S_u S_u & S_u S_d \\ S_d S_u & S_d S_d \end{pmatrix}$$

Case 2: SH waves

Figure 2.6-2: *SH* wave incident on a solid-solid boundary.



In medium 1: $u_y^-(x, z, t) = B_1 \exp(i(\omega t - k_x x - k_x r_{\beta_1} z)) + B_2 \exp(i(\omega t - k_x x + k_x r_{\beta_1} z))$

In medium 2: $u_y^+(x, z, t) = B' \exp(i(\omega t - k_x x - k_x r_{\beta_2} z))$

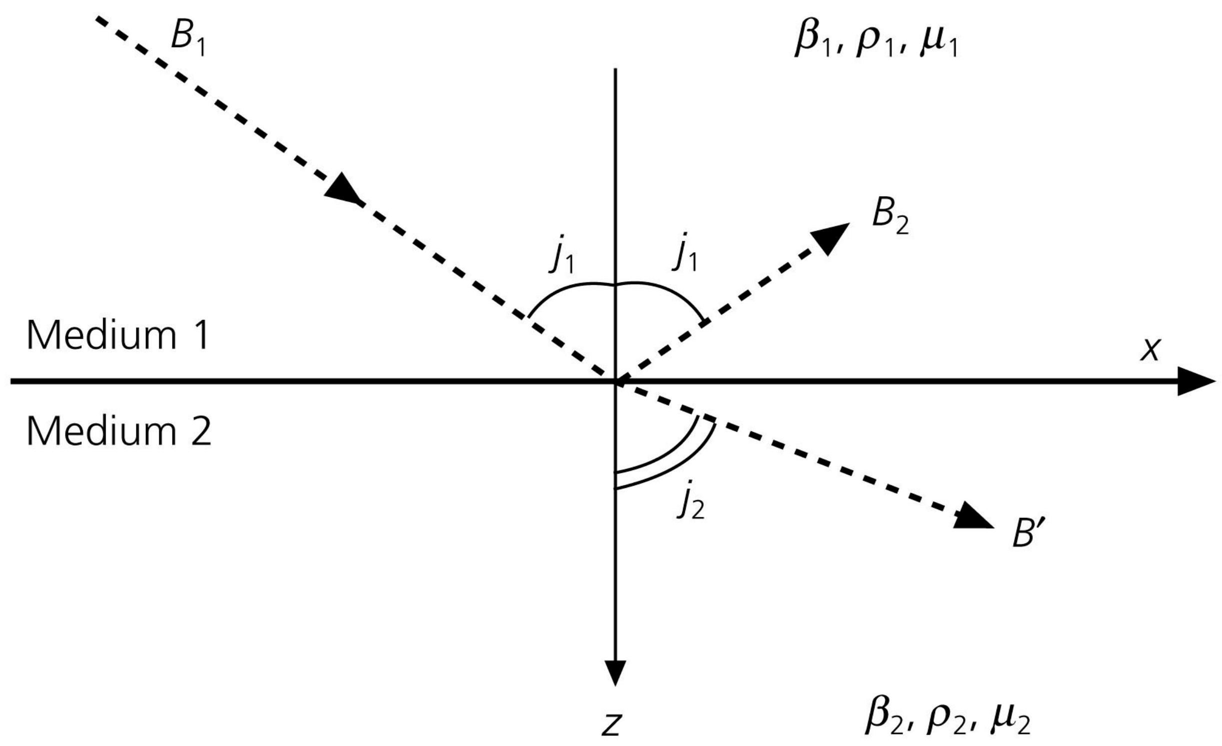
Boundary condition: continuity of displacement: $u_y^-(x, 0, t) = u_y^+(x, 0, t)$

$$(B_1 + B_2) \exp(i(\omega t - k_x x)) = B' \exp(i(\omega t - k_x x))$$

$$\boxed{B_1 + B_2 = B'}$$

Case 2: SH waves

Figure 2.6-2: *SH* wave incident on a solid-solid boundary.



Boundary condition: traction σ_{yz} is continuous: $\sigma_{yz} = 2\mu e_{yz} = \mu\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) = \mu\left(\frac{\partial u_y}{\partial z}\right)$.

(in this case, u_x and u_z are zero, so $\sigma_{xz} = \sigma_{zz} = 0$):

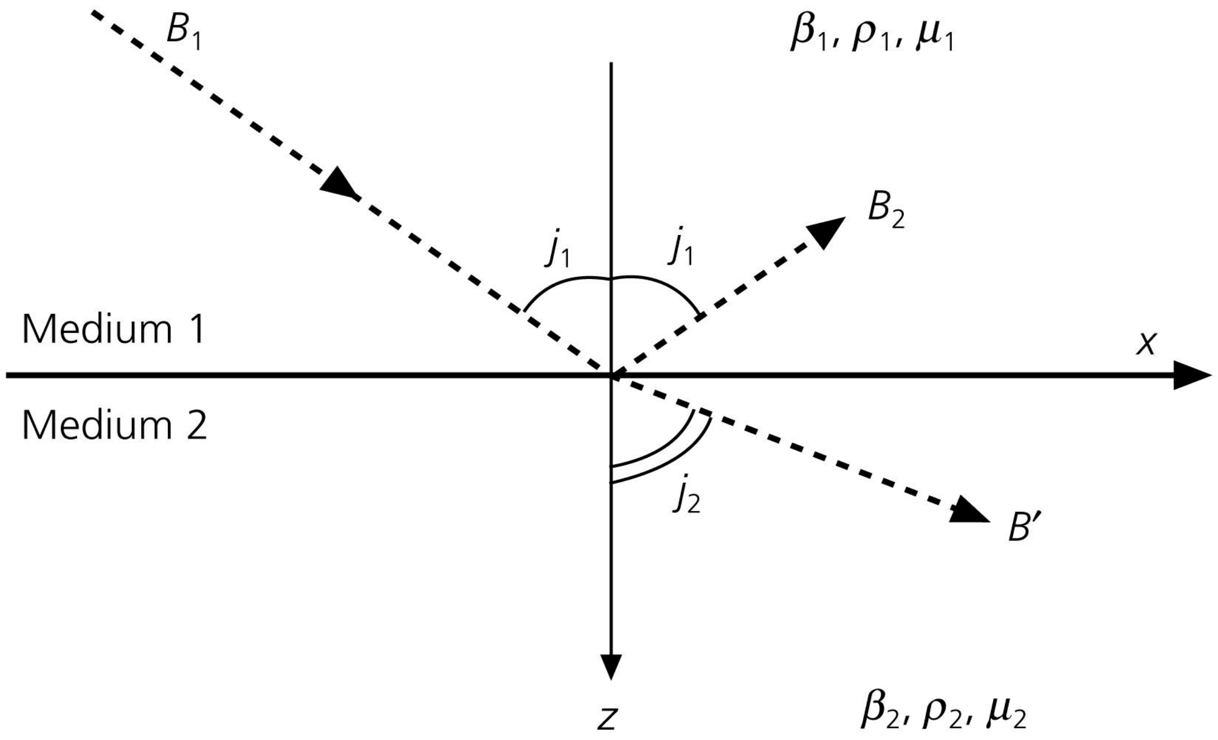
$$\sigma_{yz}^{-}(x, 0, t) = \sigma_{yz}^{+}(x, 0, t)$$

$$\mu_1 i k_x r_{\beta_1}(B_2 - B_1)\exp(i(\omega t - k_x x)) = -\mu_2 i k_x r_{\beta_2} B' \exp(i(\omega t - k_x x))$$

$$\boxed{(B_1 - B_2) = B'(\mu_2 r_{\beta_2})/(\mu_1 r_{\beta_1})}$$

Case 2: SH waves

Figure 2.6-2: *SH* wave incident on a solid-solid boundary.

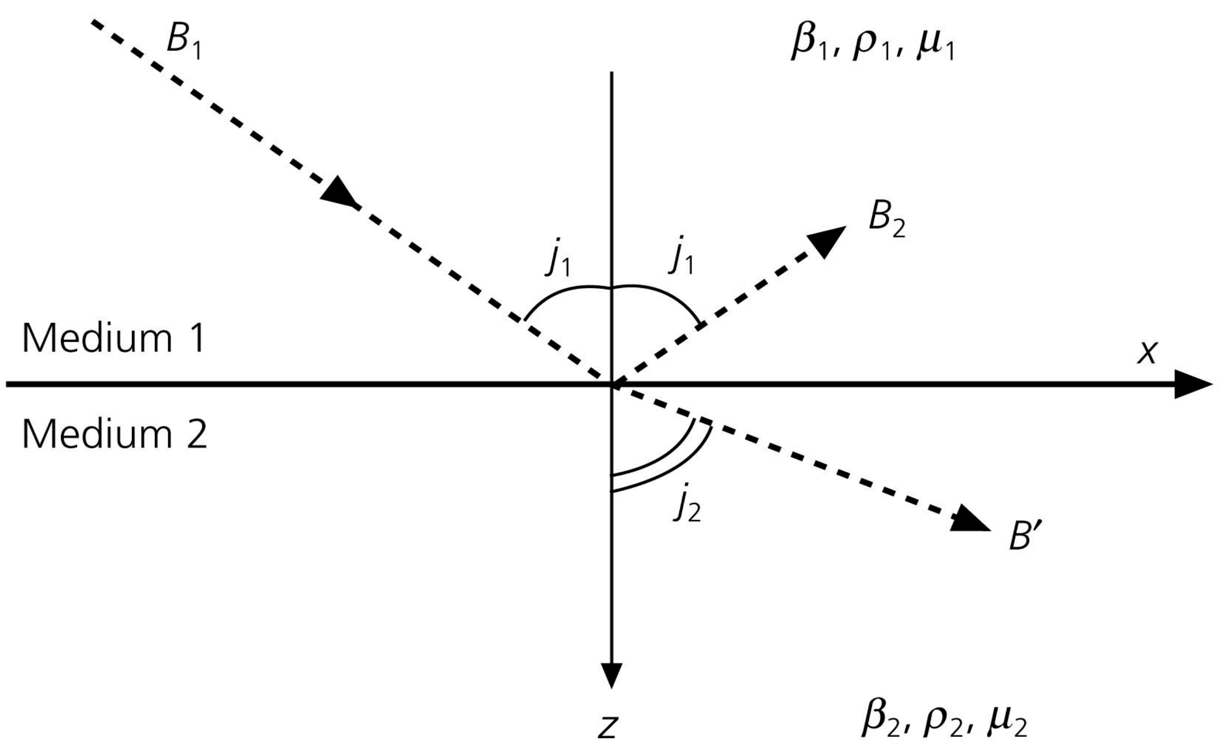


$$B_1 + B_2 = B' \qquad (B_1 - B_2) = B'(\mu_2 r_{\beta_2})/(\mu_1 r_{\beta_1})$$

$$T_{12} = \frac{B'}{B_1} = \frac{2\mu_1 r_{\beta_1}}{\mu_1 r_{\beta_1} + \mu_2 r_{\beta_2}} \qquad R_{12} = \frac{B_2}{B_1} = \frac{\mu_1 r_{\beta_1} - \mu_2 r_{\beta_2}}{\mu_1 r_{\beta_1} + \mu_2 r_{\beta_2}}$$

Case 2: SH waves

Figure 2.6-2: *SH* wave incident on a solid-solid boundary.



Using $r_{\beta_i} = c_x \cos j_i / \beta_i$

$$T_{12} = \frac{2\rho_1\beta_1 \cos j_1}{\rho_1\beta_1 \cos j_1 + \rho_2\beta_2 \cos j_2} \qquad R_{12} = \frac{\rho_1\beta_1 \cos j_1 - \rho_2\beta_2 \cos j_2}{\rho_1\beta_1 \cos j_1 + \rho_2\beta_2 \cos j_2}$$

$$R_{12} = -R_{21} \qquad T_{12} + T_{21} = 2 \qquad 1 + R_{12} = T_{12}$$

$R_{12} = 1$ at surface and CMB.

At vertical incidence ($j_1 = j_2 = 0$): $$T_{12} = \frac{2\rho_1\beta_1}{\rho_1\beta_1 + \rho_2\beta_2} \qquad R_{12} = \frac{\rho_1\beta_1 - \rho_2\beta_2}{\rho_1\beta_1 + \rho_2\beta_2}$$

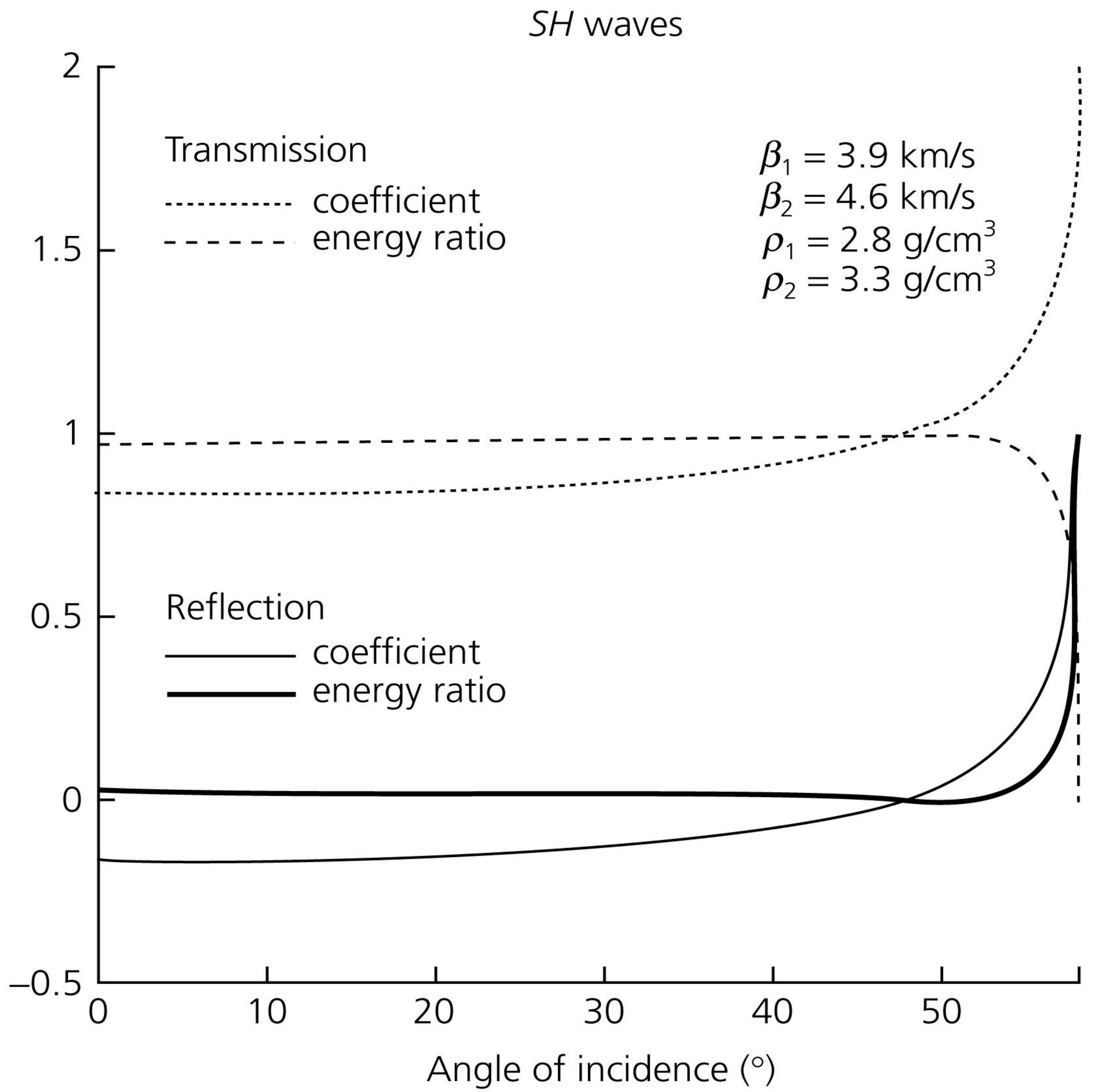
Case 2: SH waves

$$\frac{\dot{\mathbf{E}}_R}{\dot{\mathbf{E}}_I} = R_{12}^2$$

$$\frac{\dot{\mathbf{E}}_T}{\dot{\mathbf{E}}_I} = T_{12}^2 \frac{\rho_2 \beta_2 \cos j_2}{\rho_1 \beta_1 \cos j_1}$$

For example,
if $R_{12} = 0.1$,
then the energy ratio is
$\dot{\mathbf{E}}_R / \dot{\mathbf{E}}_I = 0.01$.

Figure 2.6-4: Reflection and transmission coefficients for incident *SH* waves.



At the critical angle, $c_x = \dfrac{\beta_1}{\sin j_1} = \dfrac{\beta_2}{\sin j_2} = \dfrac{\beta_2}{1} = \beta_2$

For angles i_1 that are GREATER than the critical angle, we have the unusual situation that

$c_x = \dfrac{\beta_1}{\sin j_1} < \beta_2$ If $c_x < \beta_2$, then $r_{\beta_2} = (c_x^2/\beta_2^2 - 1)^{1/2}$ becomes an imaginary number!!

This means that the transmitted wave $u_y(x, z, t) = B' \exp(i(\omega t - k_x x - k_x r_{\beta_2} z))$ has a *real* exponent!

Pick the negative sign of the square root of -1 (Why?) to define $r_{\beta_2} = -ir^*_{\beta_2}$ $r^*_{\beta_2} = (1 - c_x^2/\beta_2^2)^{1/2}$

so that the z term in the displacement, $\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$, decays exponentially away from the interface in medium 2 as $z \to \infty$.

The transmitted wave becomes an *evanescent* or *inhomogeneous* wave "trapped" near the interface.

Case 2: SH waves

$$\exp(-ik_x r_{\beta_2} z) = \exp(-k_x r^*_{\beta_2} z)$$

$$R_{12} = \frac{\mu_1 r_{\beta_1} + i\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1} - i\mu_2 r^*_{\beta_2}}$$

This a complex number divided by its conjugate, so the magnitude of the reflection coefficient is one, but there is a phase shift of 2ε:

$$R_{12} = e^{i2\varepsilon} \qquad \varepsilon = \tan^{-1} \frac{\mu_2 r^*_{\beta_2}}{\mu_1 r_{\beta_1}}$$

At critical incidence,

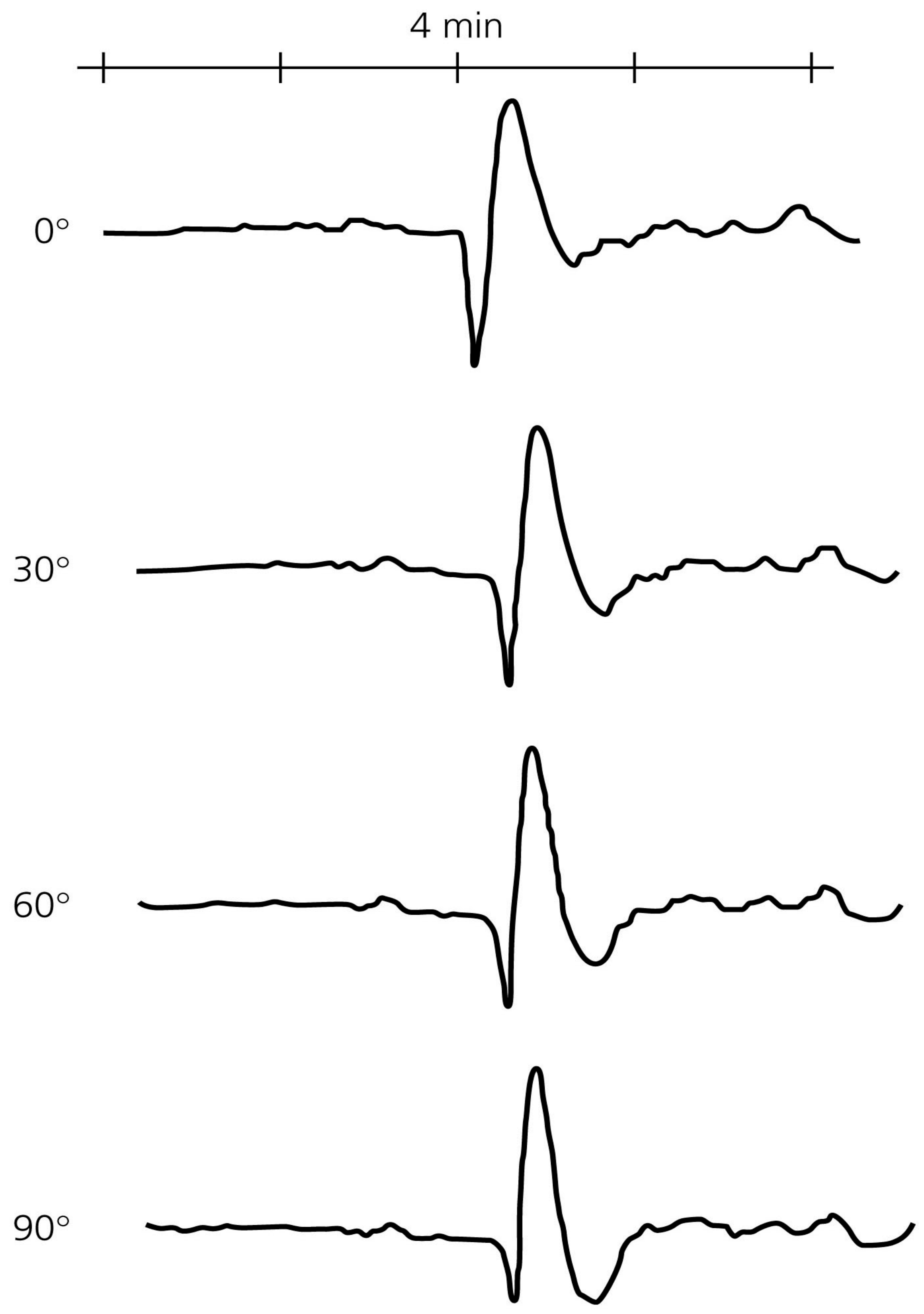
$c_x = \beta_2$, so $r^*_{\beta_2} = 0$ and $\varepsilon = 0°$

As the angle of incidence increases beyond critical, ε increases.

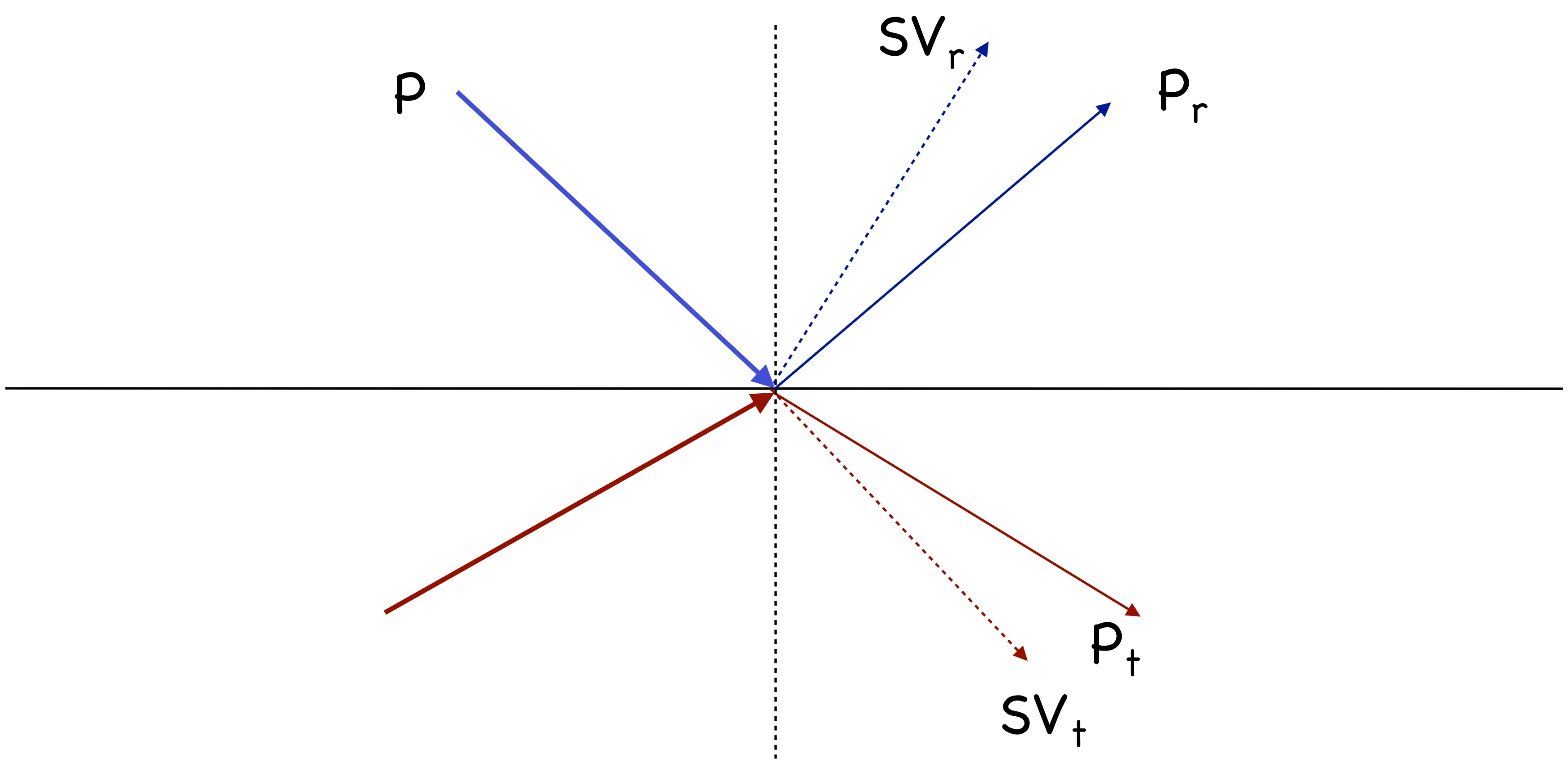
At grazing incidence, $j_1 = 90°$, we have

$c_x = \beta_1$, $r_{\beta_1} = 0$ and $\varepsilon = 90°$

Figure 2.6-5: Effect of phase shifts on a seismic waveform.

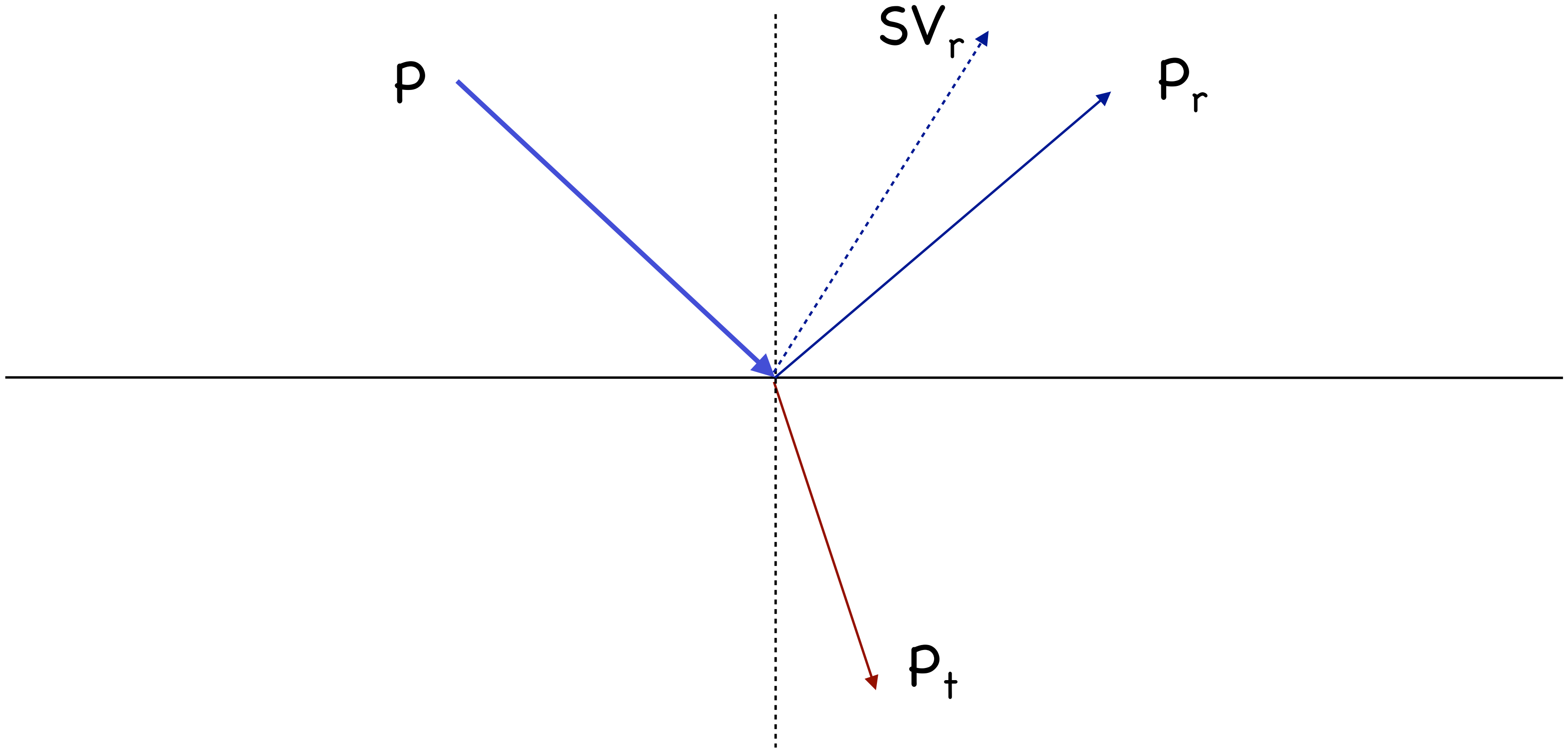


Case 3: Solid-solid interface



To account for all possible reflections and transmissions we need 16 coefficients, described by a 4x4 scattering matrix.

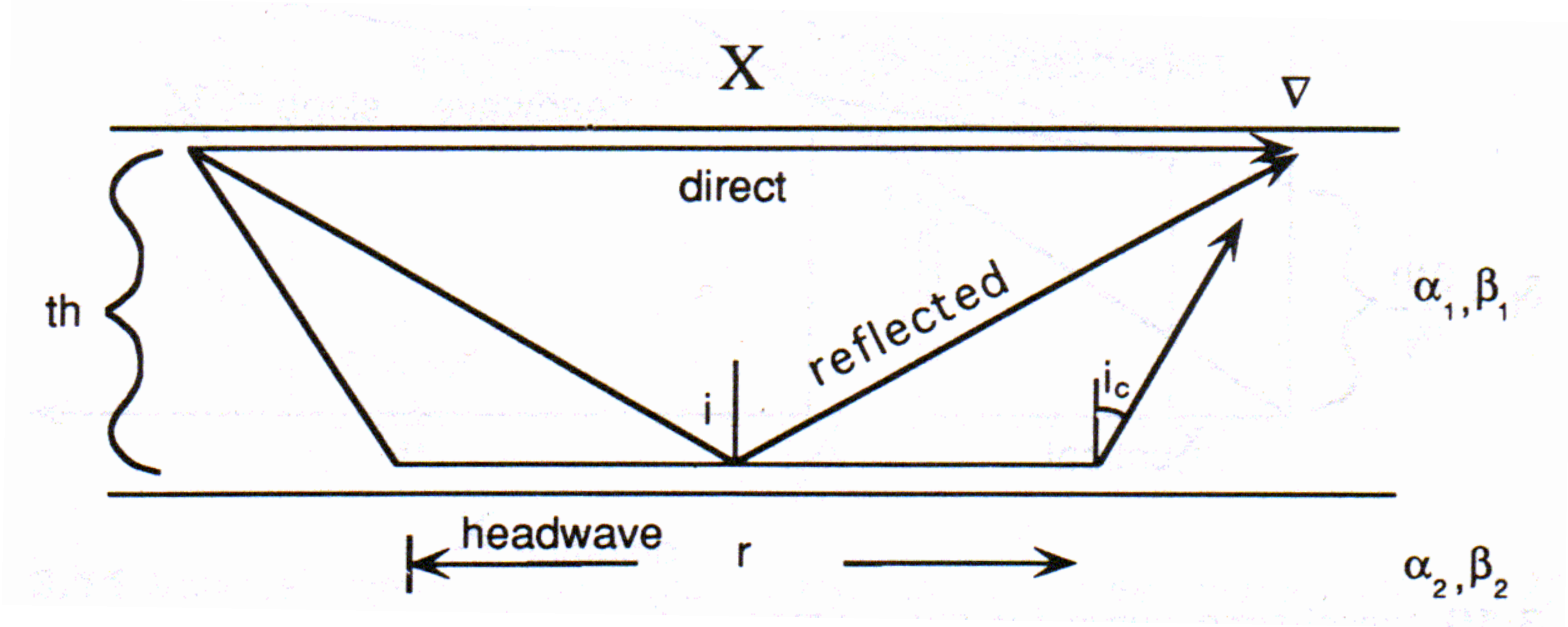
Case 4: Solid-Fluid interface



At a solid-fluid interface there is no conversion to SV in the lower medium.

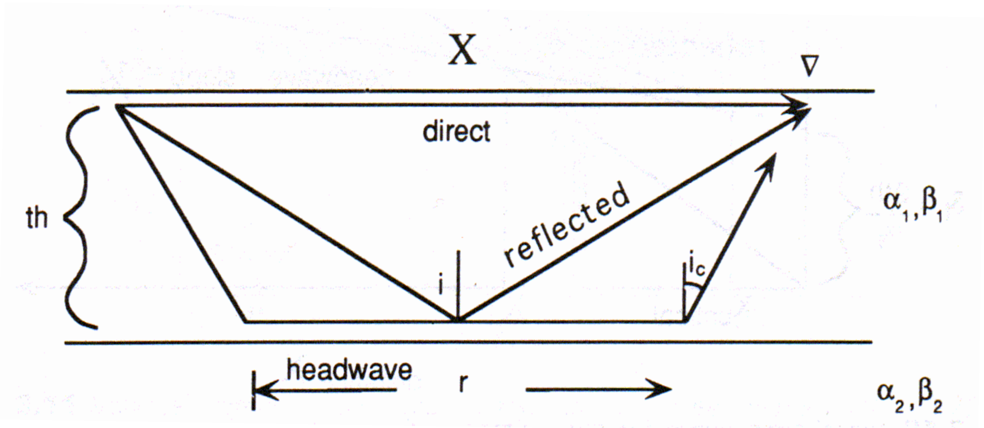
Rays in flat layered Media

Much information can be learned by analysing recorded seismic signals in terms of layered structured (e.g. crust and Moho).
We need to be able to predict the arrival times of reflected and refracted signals ...



Travel Times in Layered Media

Let us calculate the arrival times for reflected and refracted waves as a function of layer depth, h, and velocities α_i, i denoting the i-th layer:



We find that the travel time for the reflection is:

$$T_{refl} = \frac{2h}{\alpha_1 \cos i} = \frac{2\sqrt{h^2 + X^2/4}}{\alpha_1}$$

And for the the refraction:

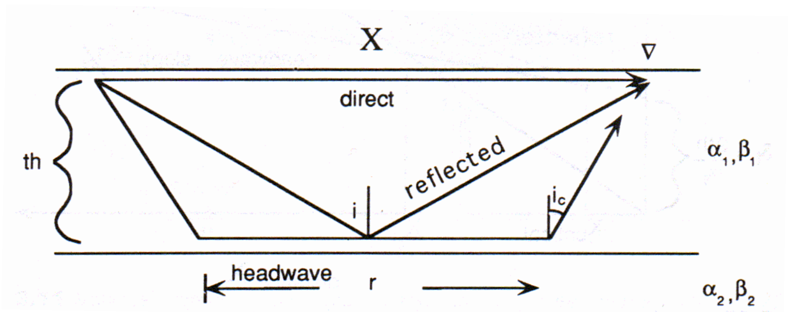
$$T_{refr} = \frac{2h}{\alpha_1 \cos i_c} + \frac{r}{\alpha_2}$$

$$r = X - 2h \tan i_c$$

where i_c is the critical angle:

$$\frac{\sin(i_1)}{\alpha_1} = \frac{\sin(r_2)}{\alpha_2} \Rightarrow i_c = \arcsin\left(\frac{\alpha_1}{\alpha_2}\right)$$

Travel Times in Layered Media



Thus the refracted wave arrival is

$$T_{refr} = \frac{2h}{\alpha_1 \cos i_c} + \frac{1}{\alpha_2}\left(X - \frac{2h\alpha_1}{\alpha_2 \cos i_c}\right)$$

where we have made use of Snell's Law.

We can rewrite this using

$$\frac{1}{\alpha_2} = \frac{\sin i_c}{\alpha_1} = p$$

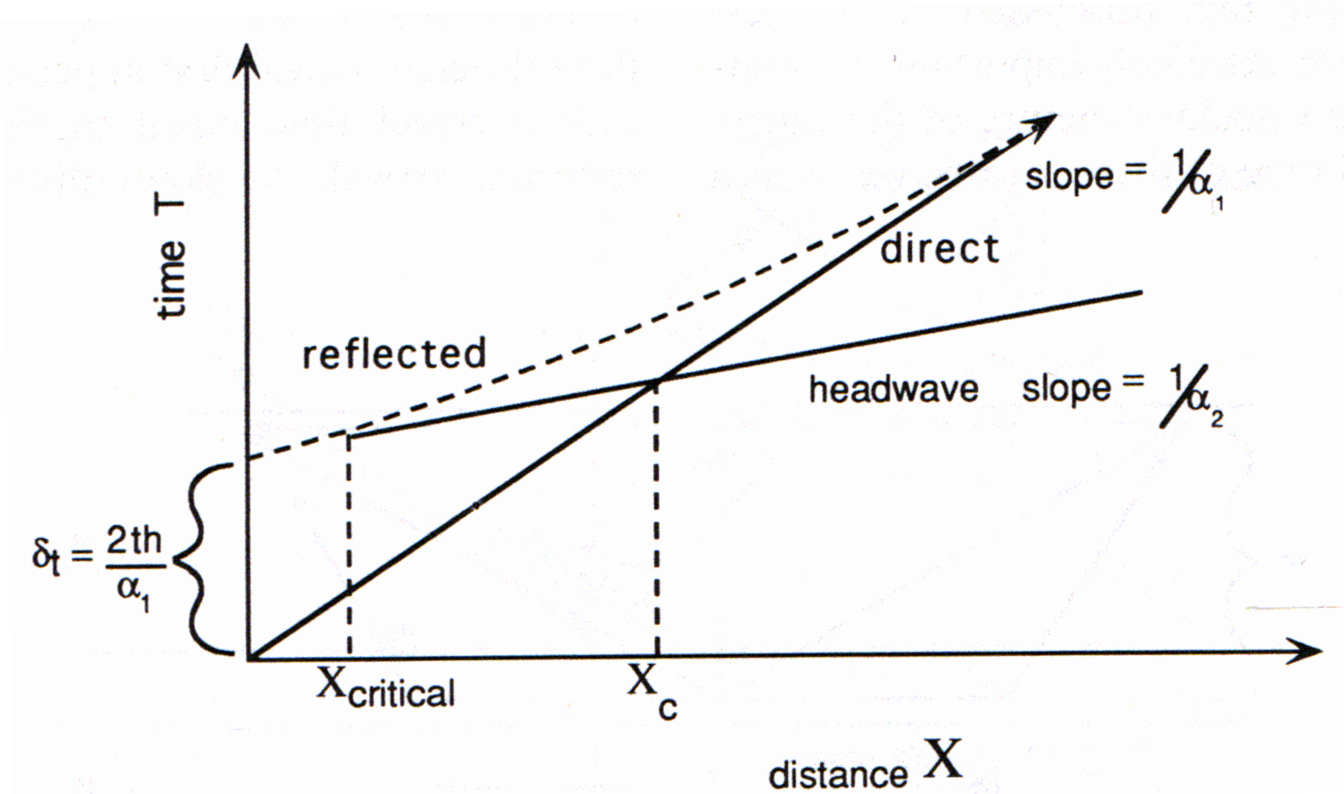
$$\cos i_c = (1 - \sin^2 i_c)^{1/2} = (1 - p^2\alpha_1^2)^{1/2} = \alpha_1\left(\frac{1}{\alpha_1^2} - p^2\right)^{1/2} = \alpha_1 \eta_1$$

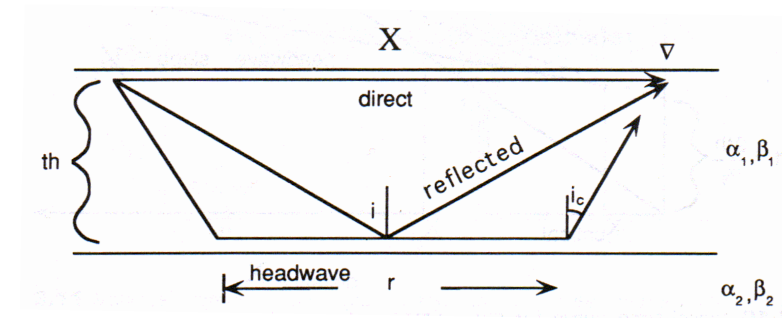
to obtain

$$T_{refr} = Xp + 2h\eta_1$$

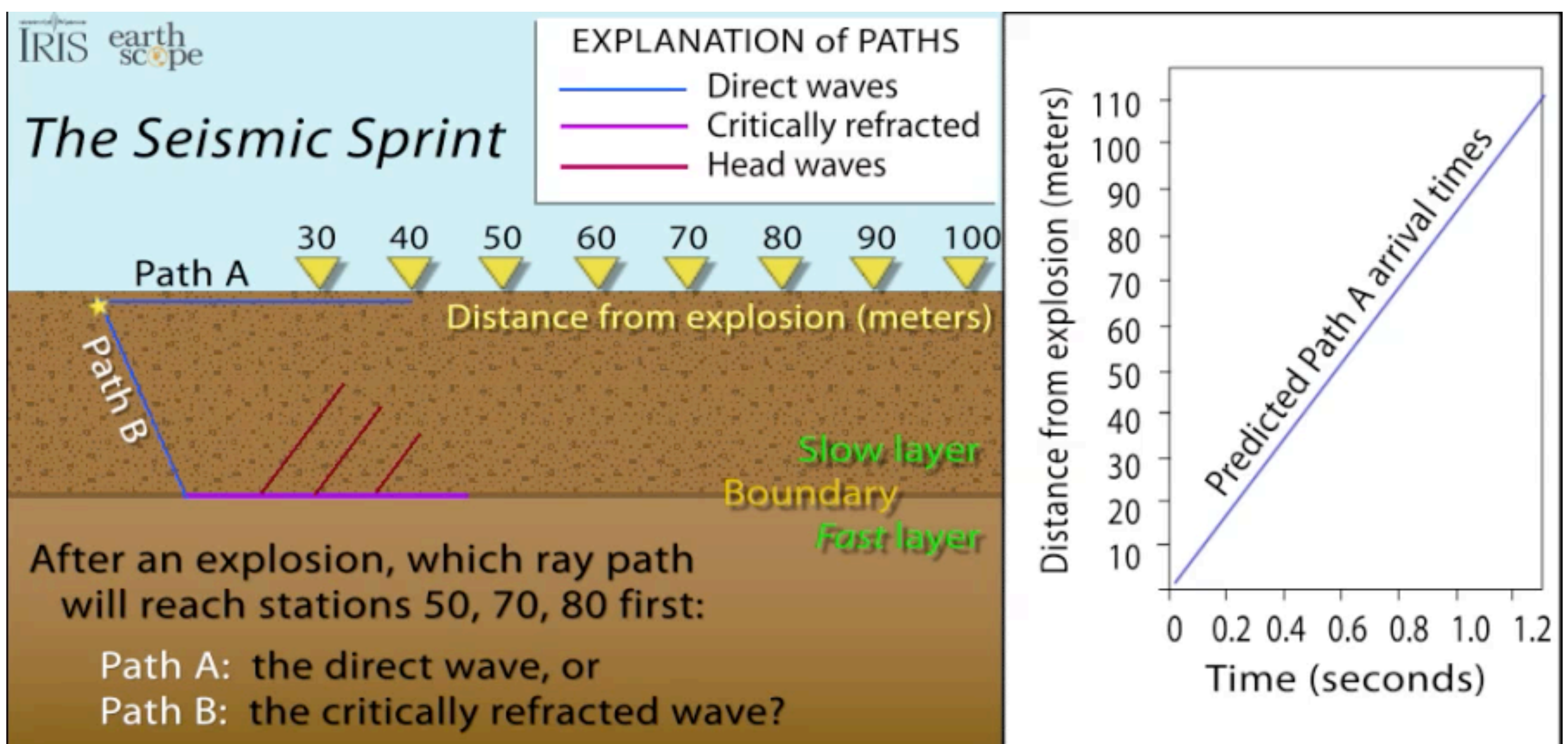
Which is very useful as we have separated the result into a vertical and horizontal term.

Travel time curves





What can we determine if we have recorded the following travel time curves?

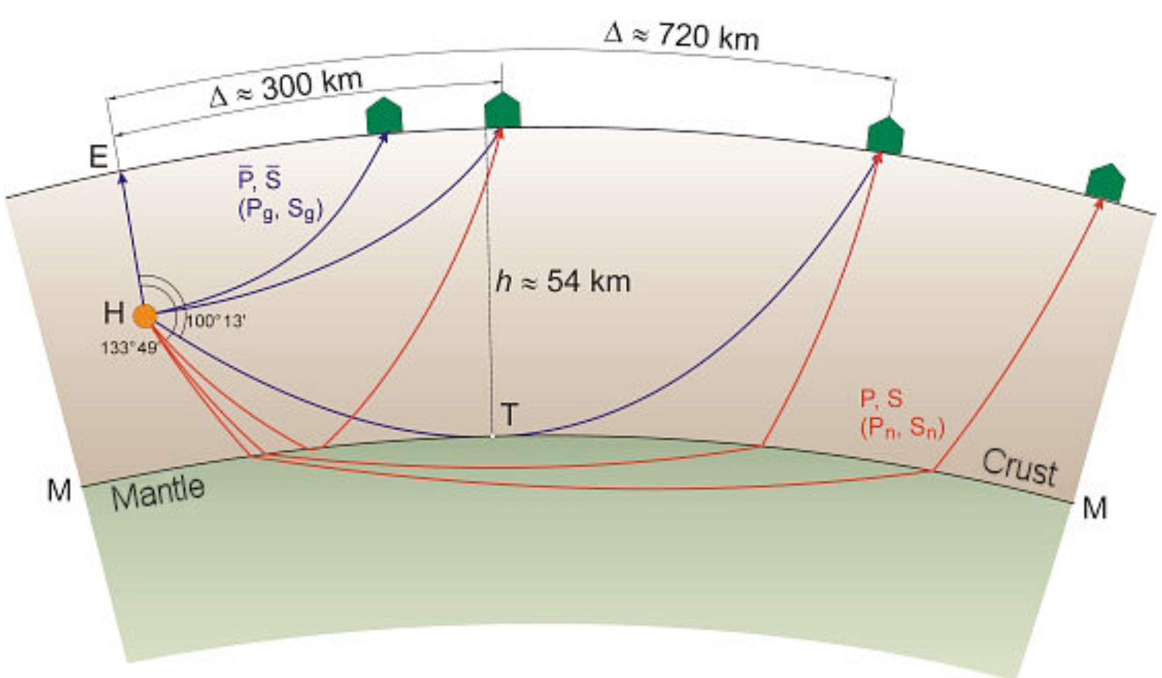


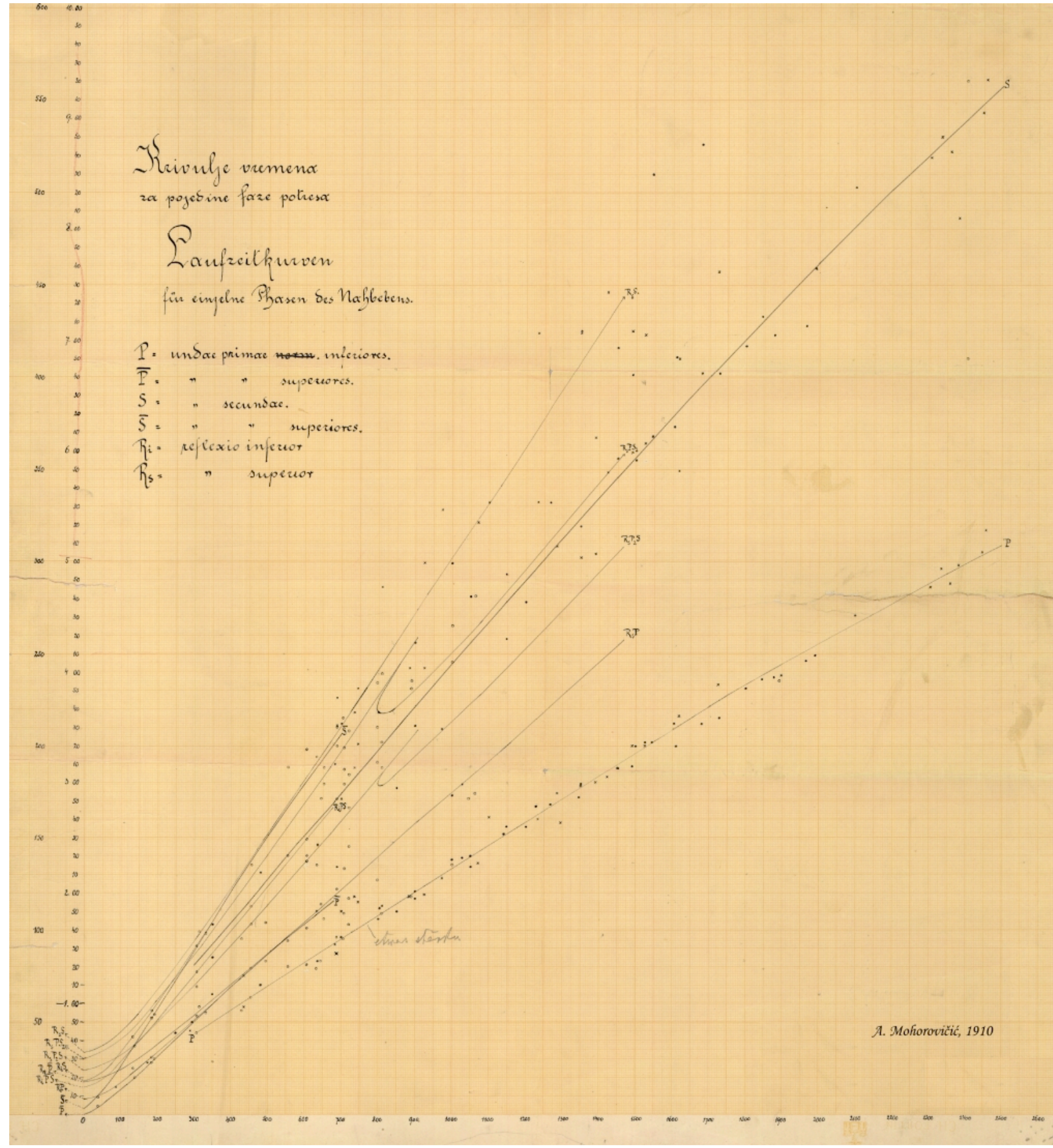
http://www.iris.edu/hq/programs/education_and_outreach/animations/13

http://home.chpc.utah.edu/~thorne/animations.html

Andrija MOHOROVIČIĆ

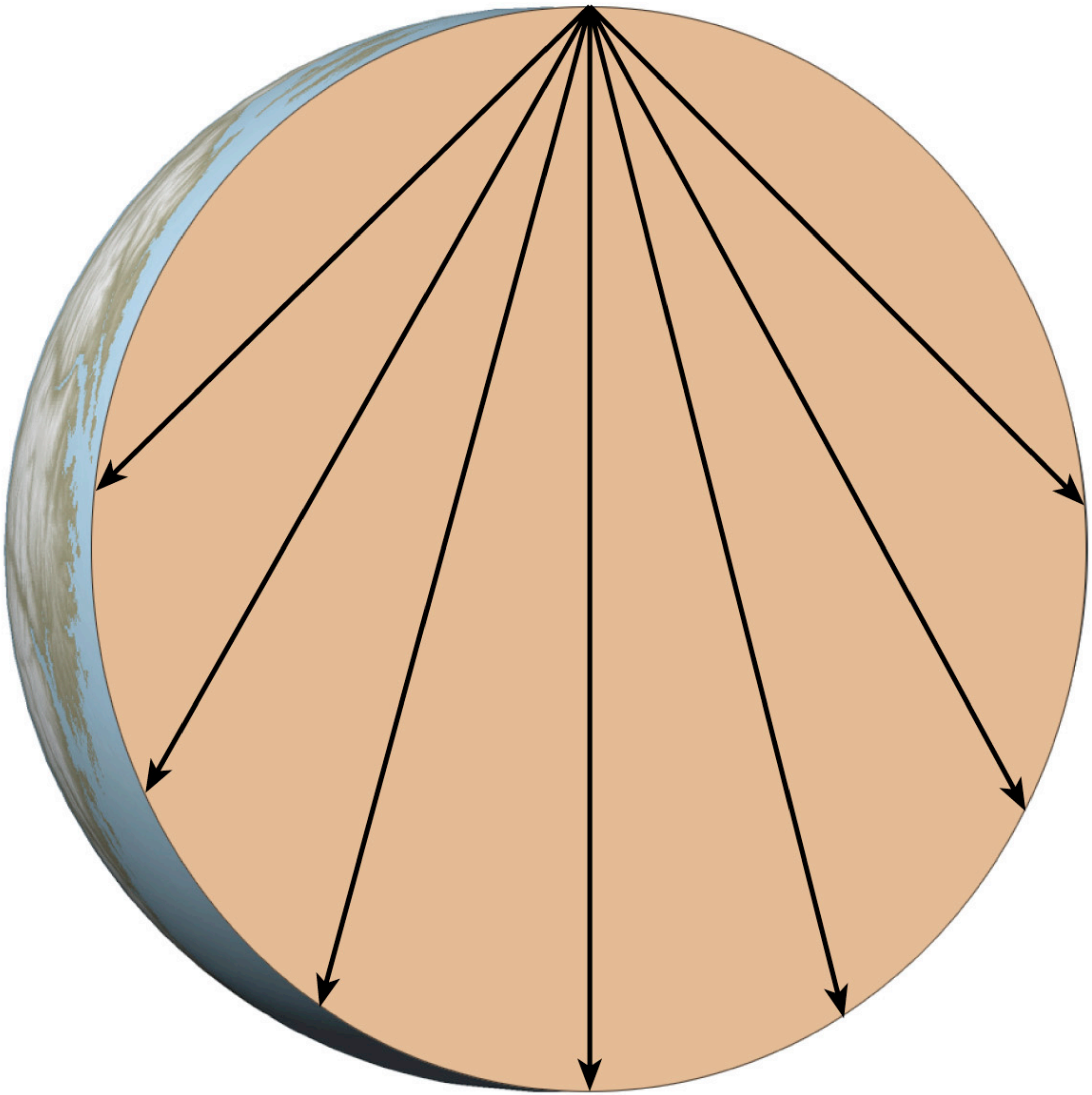
Godišnje izvješće zagrebačkog meteorološkog opservatorija za godinu 1909. Godina IX, dio IV. - polovina 1. Potres od 8. X. 1909



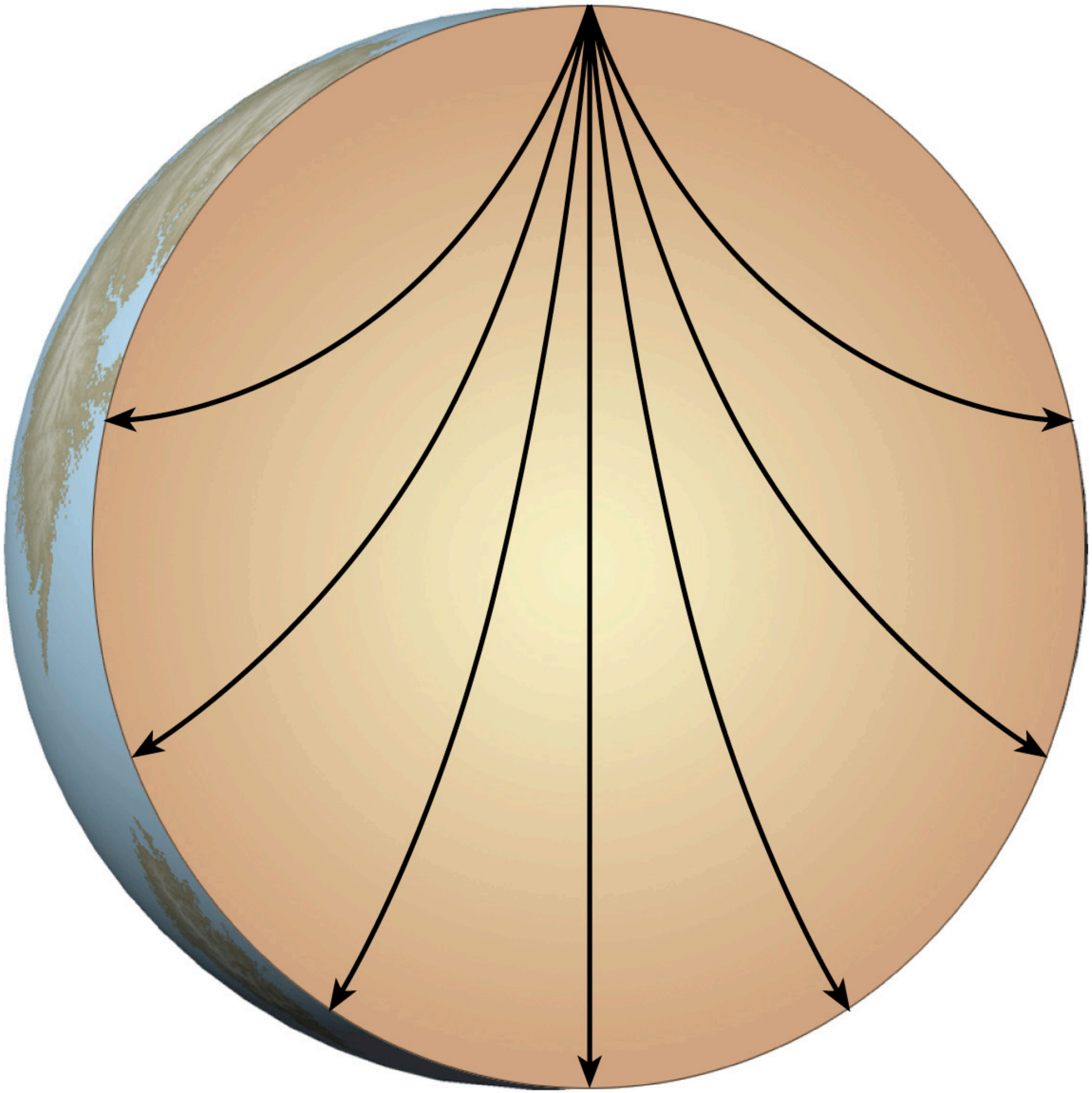


http://www.gfz.hr/sobe-en/discontinuity.htm

Rays in homogeneous sphere



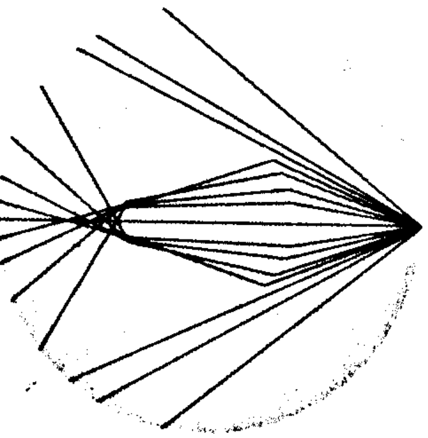
Sphere with increasing velocity…



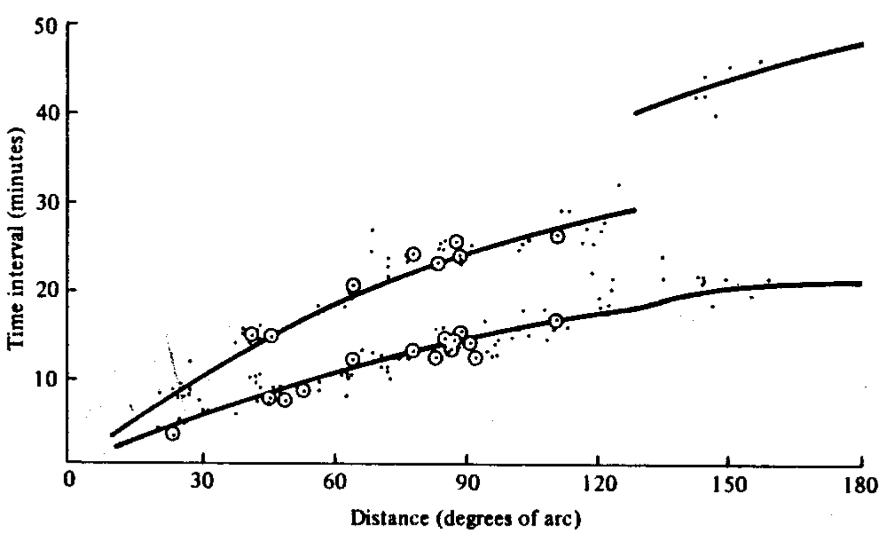
Earth’s core

Richard Dixon Oldham

The Constitution of the Earth as revealed by earthquakes, Quart. J. Geological Soc. Lond., 62, 456-475, 1906



Paths of seismic waves through the Earth assuming a core of radius 0.4*R*, in which the speed is 3 km/sec, while the speed outside it is 6 km/sec. [From Oldham, 1906.]

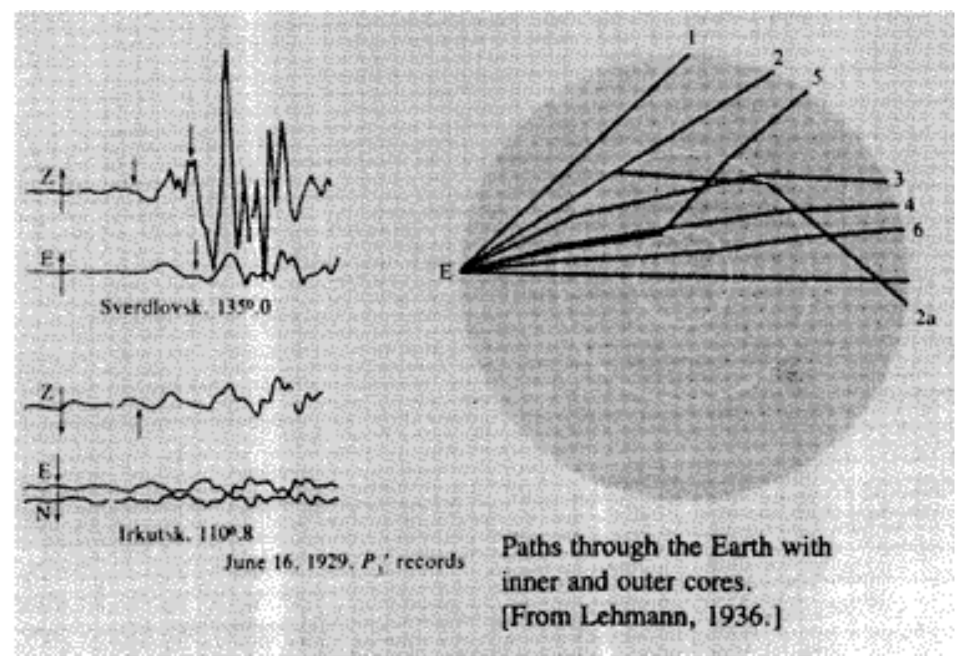


Time curves of first and second phases of preliminary tremors. The marks surrounded by circles are averages. [From Oldham, 1906.]

Beno Gutenberg

1914 Über Erdbebenwellen VIIA. Nachr. Ges. Wiss. Göttingen Math. Physik. Kl, 166.

who calculated depth of the core as 2900km or 0.545R



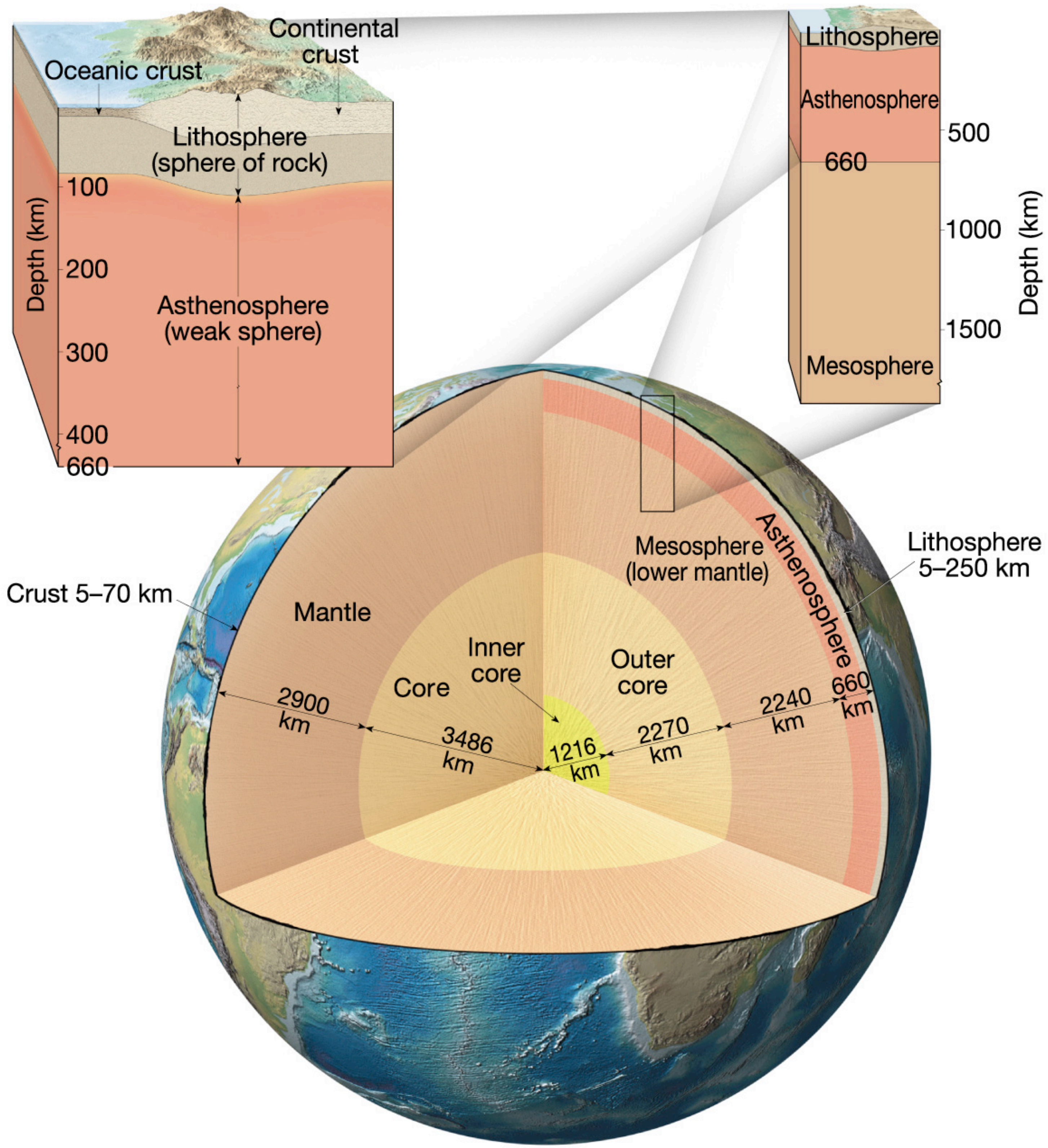
Paths through the Earth with inner and outer cores. [From Lehmann, 1936.]

Inge Lehmann

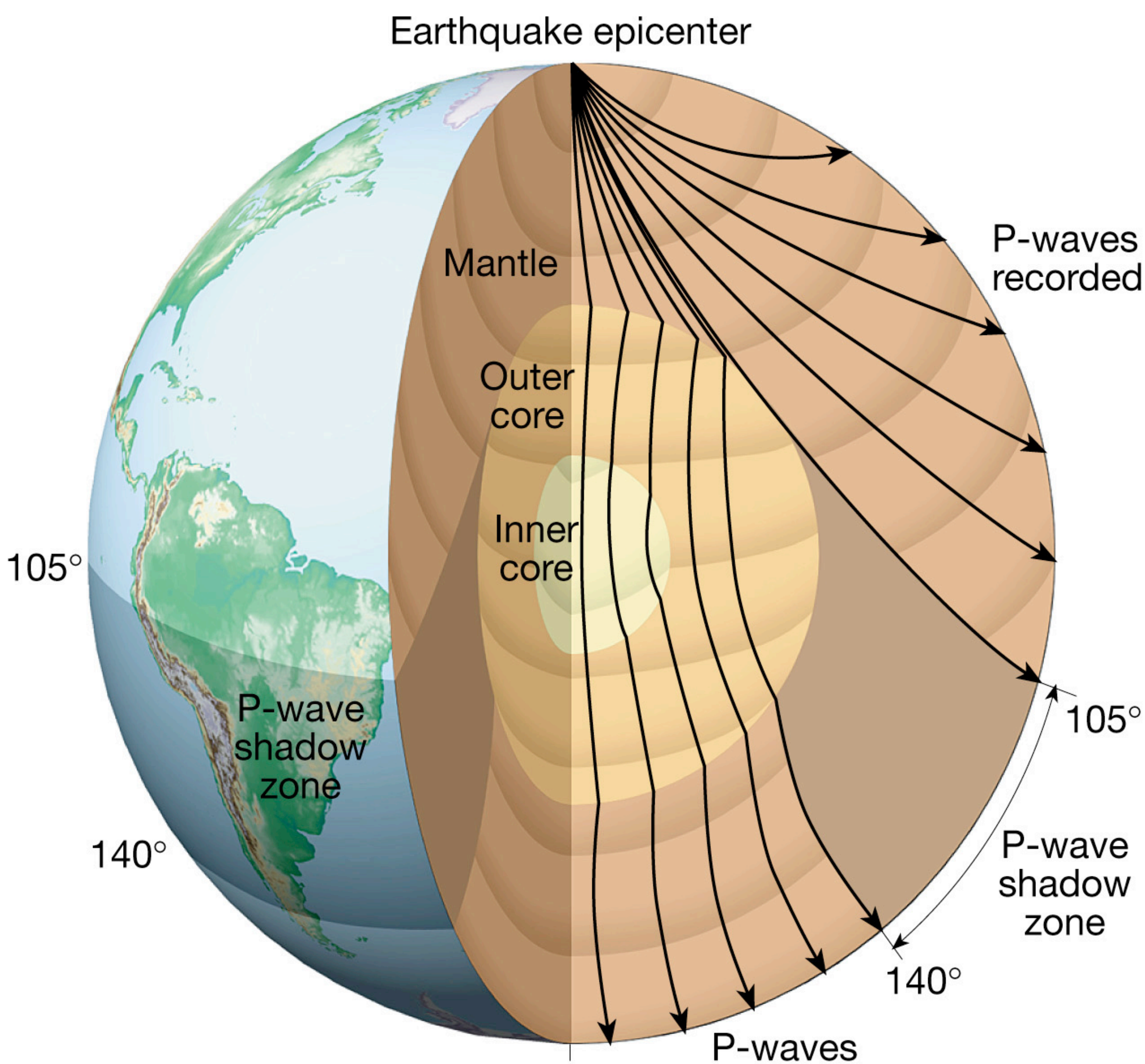
Bureau Central Seismologique International, Series A, Travaux Scientifiques, 14, 88, 1936.

who discovered of the earth's inner core.

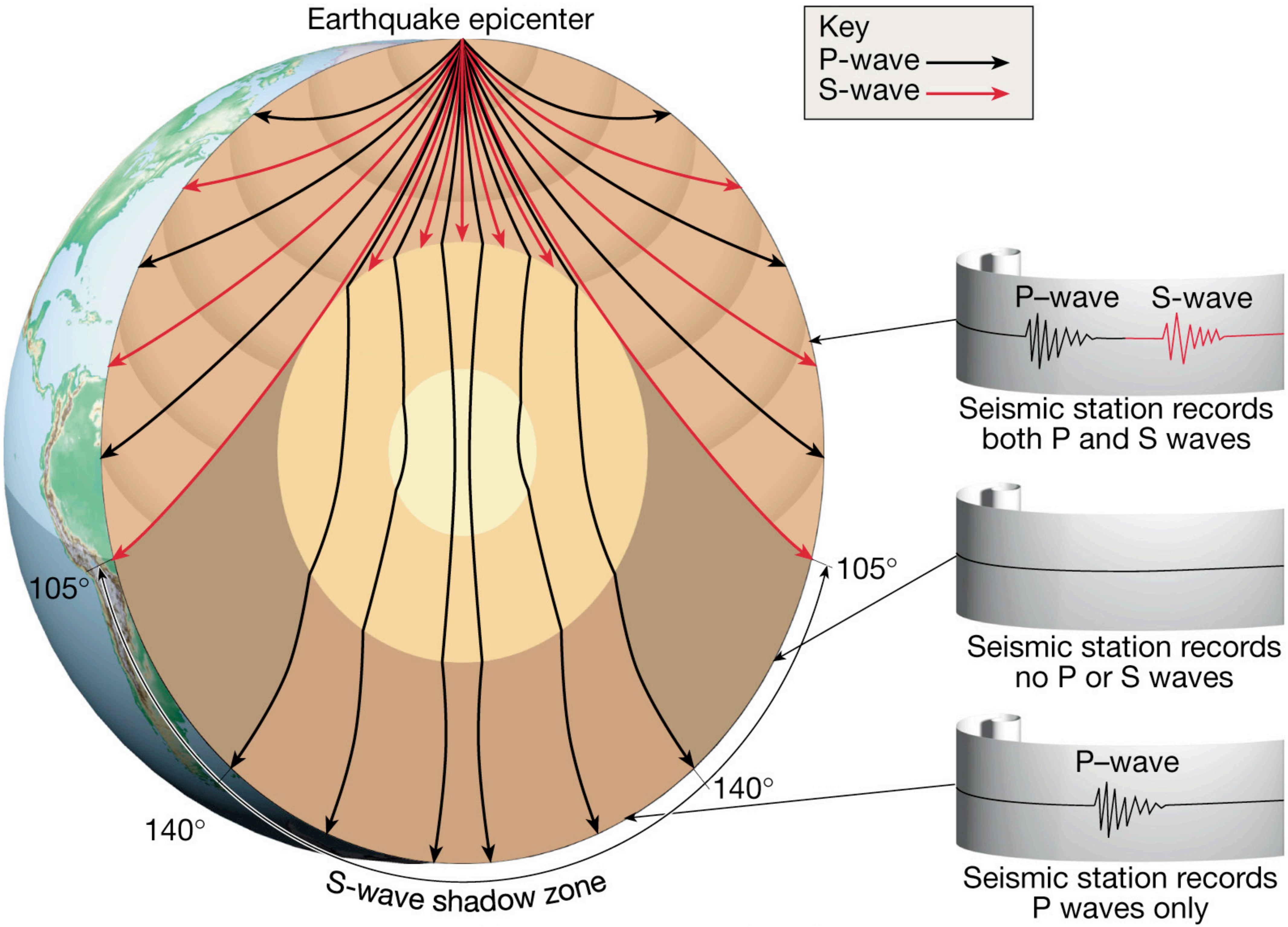
Earth layered structure



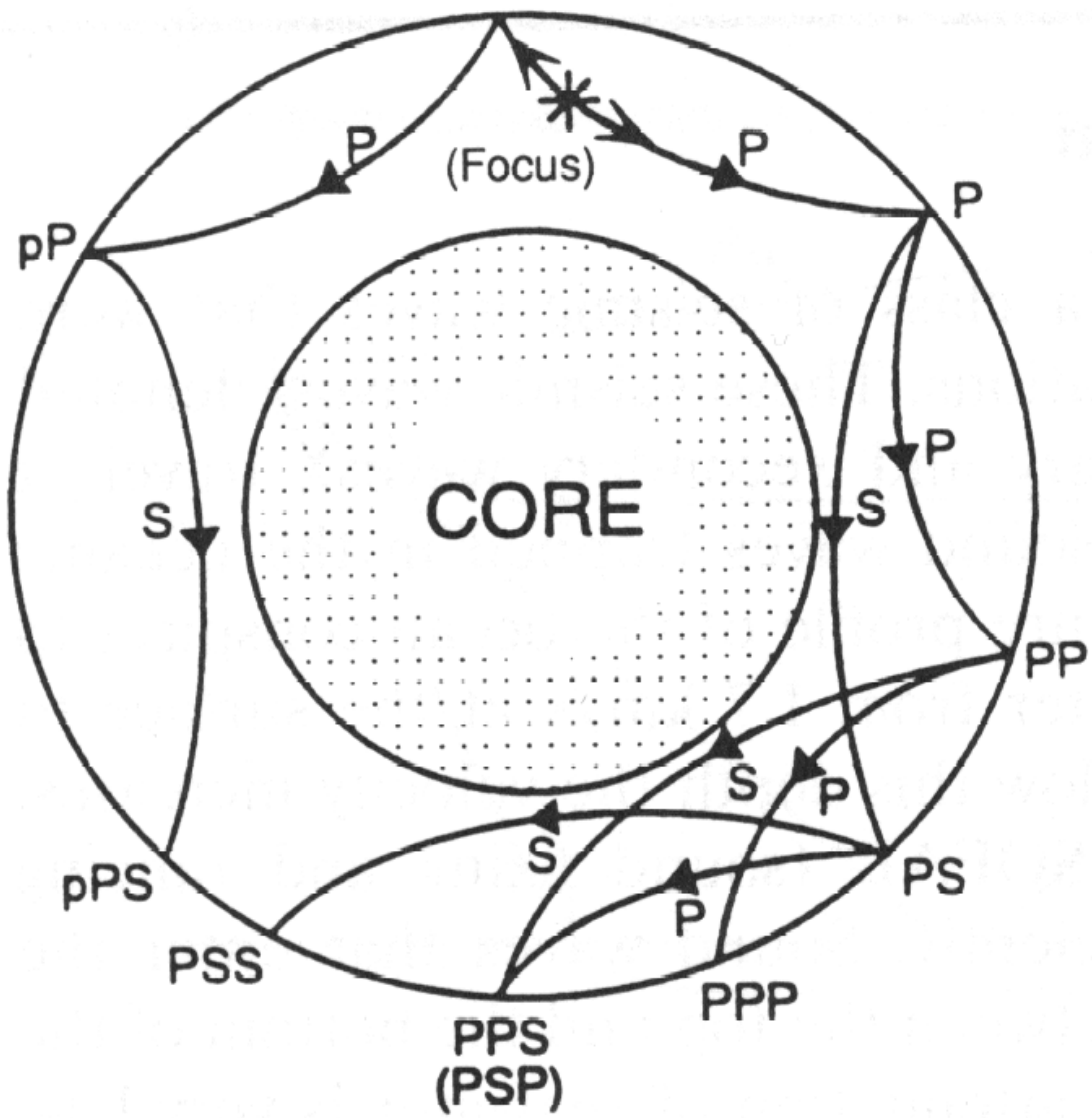
Ray Paths in the Earth (1)



Ray Paths in the Earth (2)

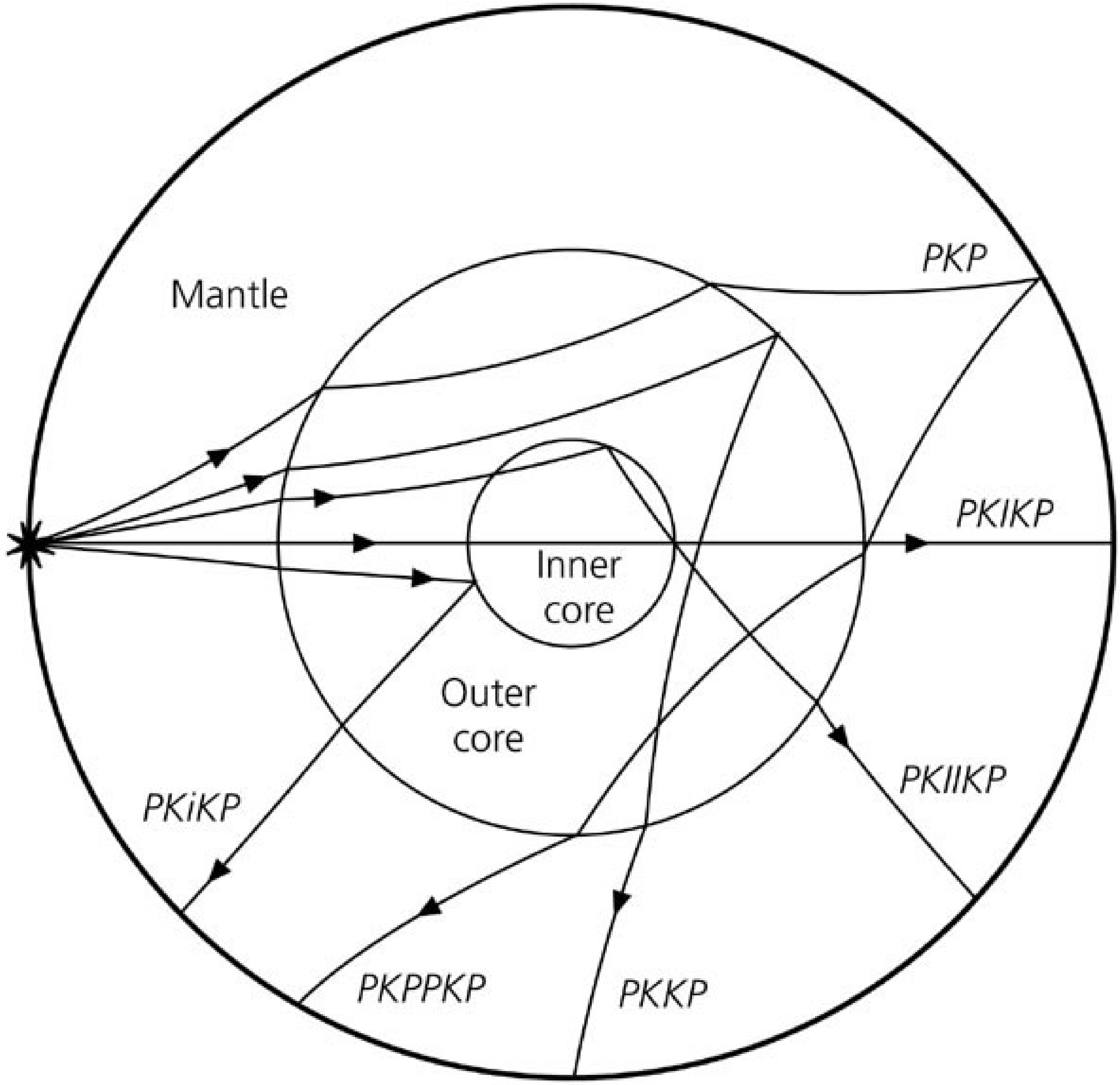


Ray Paths in the Earth (3)



Ray Paths in the Earth (4)

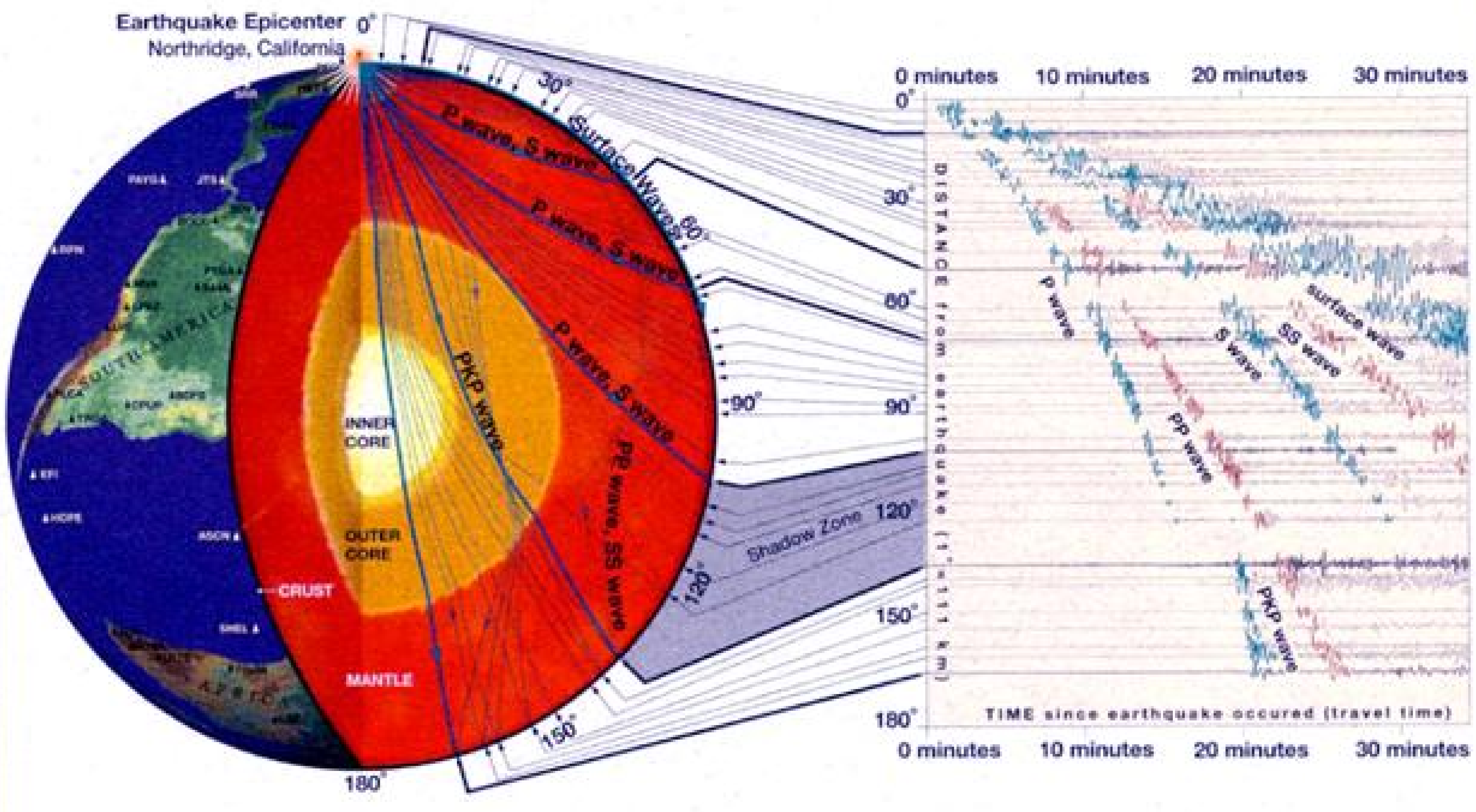
Figure 3.5-10: Ray paths for additional core phases.



Ray Paths in the Earth - Names

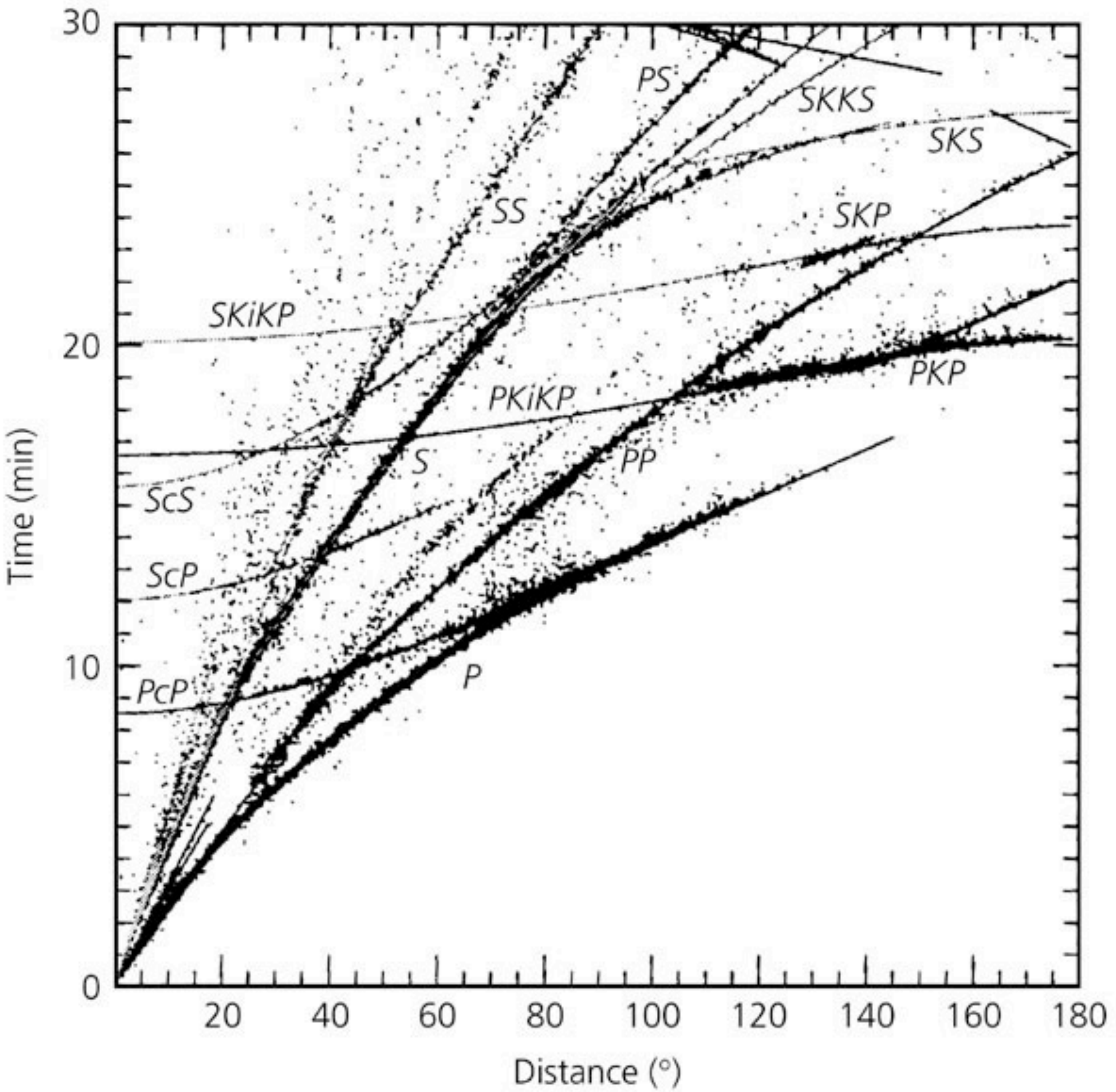
P	P waves
S	S waves
small p	depth phases (P)
small s	depth phases (S)
c	Reflection from CMB
K	wave inside core
i	Reflection from Inner core boundary
I	wave through inner core

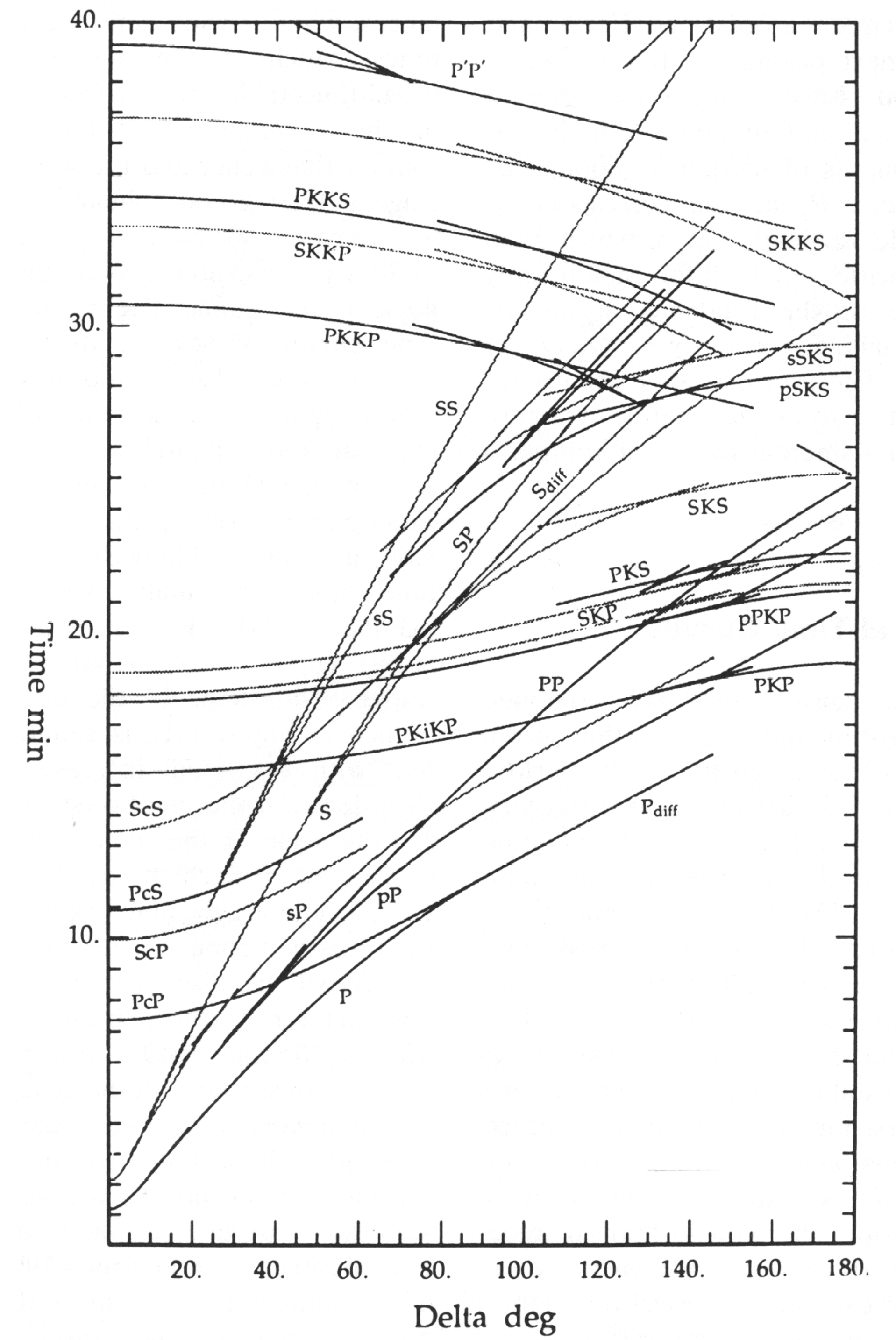
Travel times in the real Earth



Travel times in the Earth

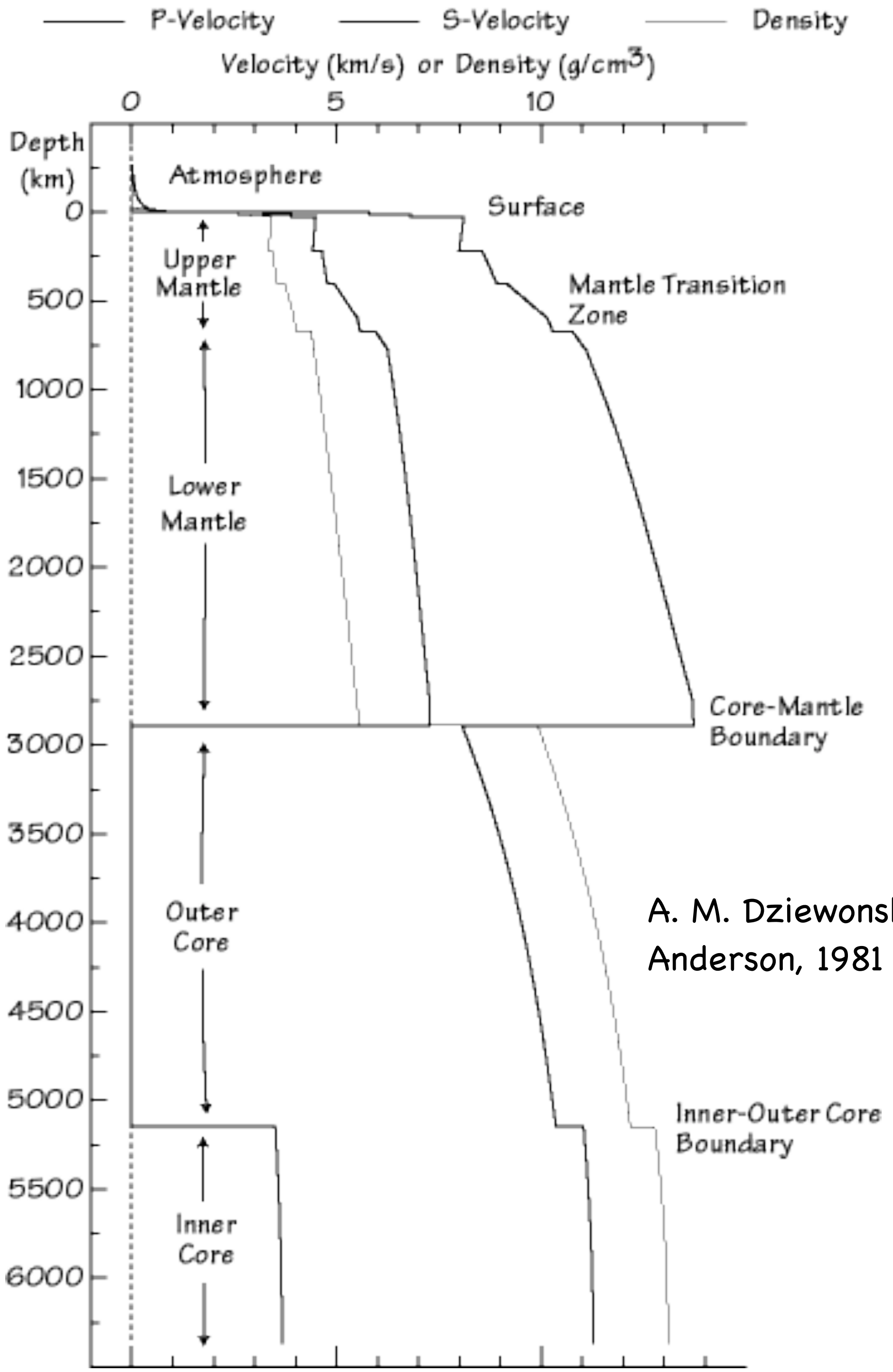
Figure 3.5-3: Travel time data and curves for the IASP91 model.



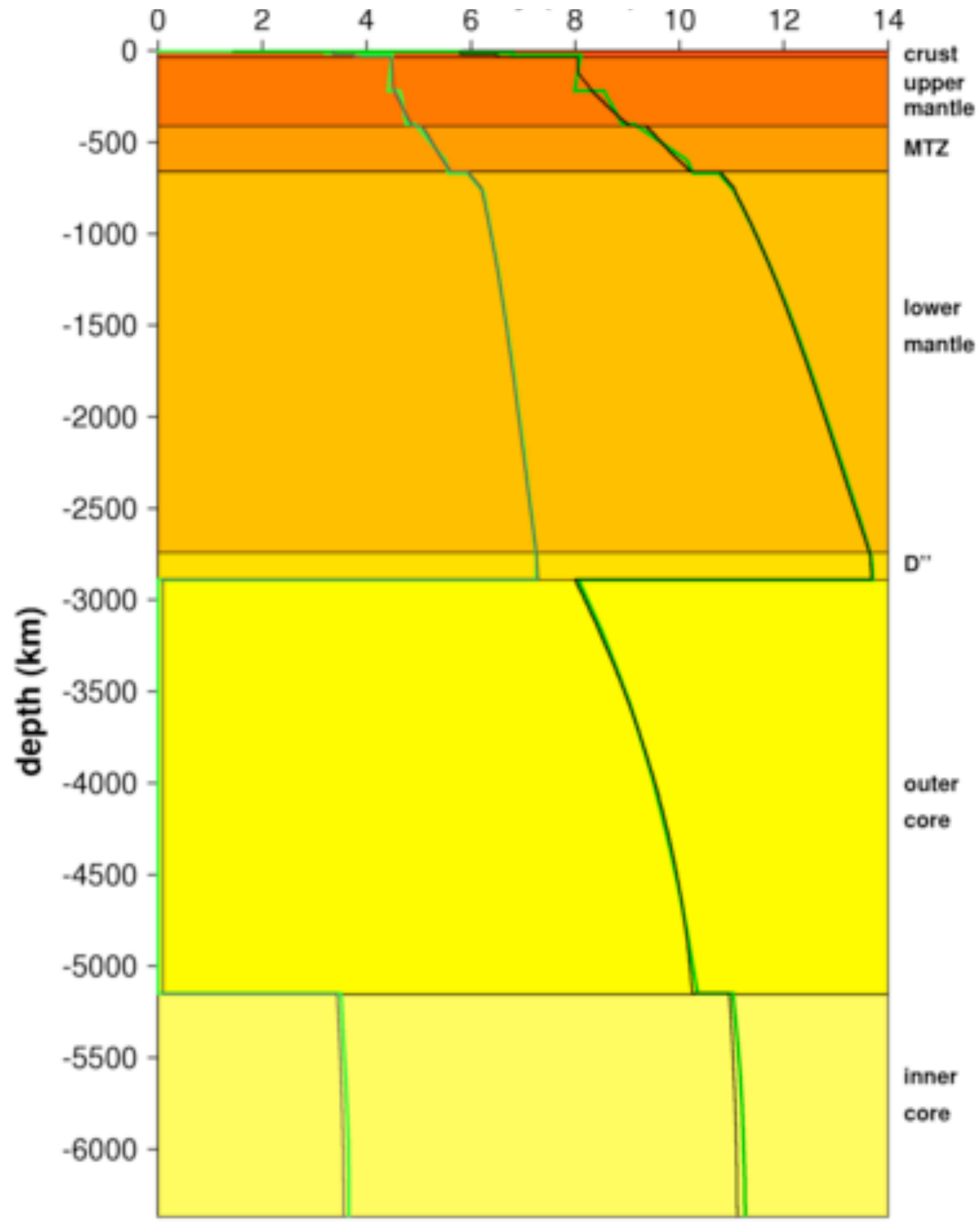


Kennett, B. L. N., and E. R. Engdahl (1991). Traveltimes for global earthquake location and phase identification. Geophysical Journal International 122, 429–465.

Spherically symmetric models



Velocity and density variations within Earth based on seismic observations. The main regions of Earth and important boundaries are labeled. This model was developed in the early 1980's and is called **PREM** for Preliminary Earth Reference Model.



Model **PREM** giving S and P wave velocities (light and dark green lines) in the earth's interior in comparison with the younger **IASP91** model (thin grey and black lines)

http://www.iris.edu/ds/products/emc-referencemodels/