



COMPUTER ENGINEERING



Orbits and Constellations

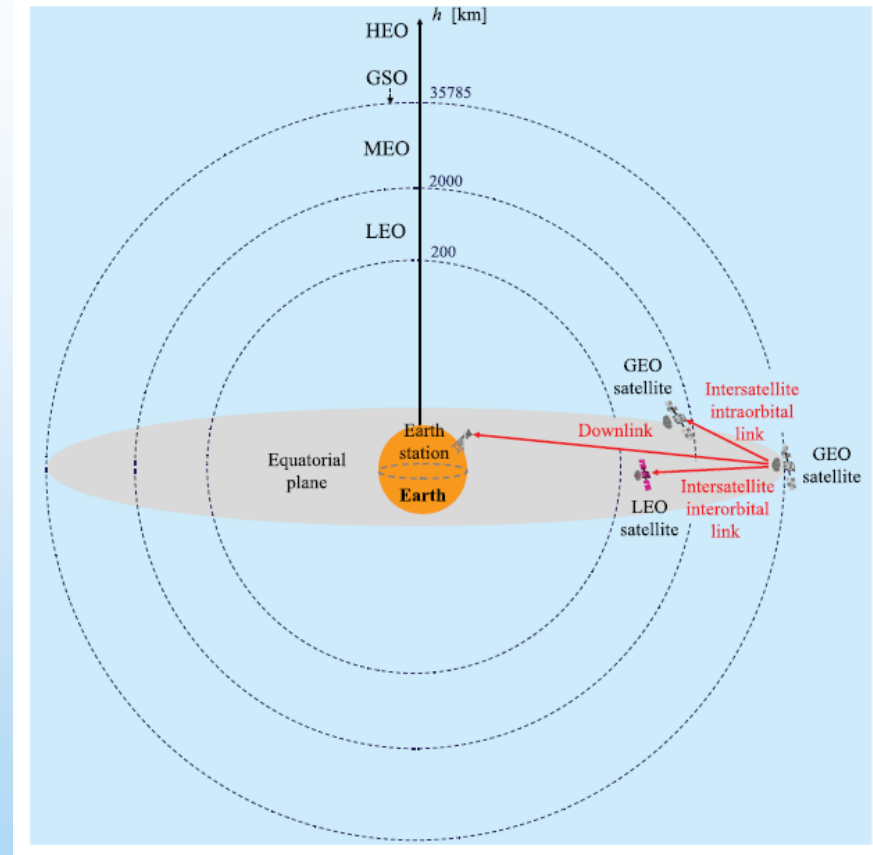
- **Height, h** , with respect to the earth's surface, based on which artificial satellites can be distinguished between

LEO ($200 \leq h \leq 2000$ km),

MEO ($2000 \leq h < 35785$ km),

GSO ($h = 35785$ km) and

HEO ($h > 35785$ km).



- **Inclination** λ_0 , i.e. the angle between the plane containing the orbit (plane orbital) and the Earth's equatorial plane. $0 \leq \lambda_0 \leq 180^\circ$.
- The motion of the satellite can be distinguished in

prograde when $0 \leq \lambda_0 < 90^\circ$, i.e. when the satellite rotates in the same direction (counterclockwise) as the Earth's rotation;

retrograde when $90 < \lambda_0 < 180^\circ$, i.e. when the satellite rotates in opposite direction (clockwise) of the Earth's rotation.

Special values: $\lambda_0 = 0$ (prograde equatorial), $\lambda_0 = 90^\circ$ (polar) and $\lambda_0 = 180^\circ$ (retrograde equatorial).

Third Kepler's law

- A simplified description of the orbital motion can be formulated by adopting the circular hypothesis and modeling both the Earth and the satellite as point-like bodies characterized by a mass $m_T=5.97 \cdot 10^{24}$ kg and m_S , respectively.
- Therefore, identifying with $G=6.67 \cdot 10^{-11}$ N m²/kg² the universal gravitational constant and with $\hat{\mathbf{r}}$ the unit vector of the radial coordinate of the spherical reference system with origin at the center of the Earth, the motion of a satellite rotating with respect to the center of the Earth at a distance R_T+h with angular velocity ω_s is determined by the equilibrium between the vector of the Earth's

gravitational attraction force
$$\mathbf{F}_T = -\frac{Gm_T m_S}{(R_T + h)^2} \hat{\mathbf{r}}$$

which would tend to make the satellite fall onto the Earth, and the vector of its centrifugal force:

$$\mathbf{F}_C = m_S \omega_s^2 (R_T + h) \hat{\mathbf{r}}$$

which would instead tend to move it away from the Earth.

- By imposing that the resultant $\mathbf{F}_T + \mathbf{F}_C$ between the two forces is zero, the module of the angular velocity of the satellite can be obtained as:

$$\omega_s = \sqrt{\frac{Gm_T}{(R_T + h)^3}}$$

- We have: $(Gm_T)^{1/2} = (6.67 \cdot 10^{-11} \cdot 10^{-9} \cdot 5.97 \cdot 10^{24})^{1/2} = 631.35 \frac{\text{km}^{3/2}}{\text{s}}$

By assuming $\omega_e = 2\pi / (24 \cdot 3600)$, from $\omega_s = \omega_e$ and $R_T = 6371 \text{ km}$ we obtain $h = 35.785 \cdot 10^3 \text{ km}$.

- A farther parameter for characterizing the orbit is the eccentricity of the ellipse which represents the trajectory. Identifying with a and b ($b < a$) the semi-axes of the ellipse, eccentricity is defined as:

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

so it takes on values $0 \leq e < 1$. In the case of circular orbit we have $e = 0$, while eccentricity values gradually closer to one indicate an ever greater departure from circular geometry.

- A specific orbit that is of considerable importance for various applications is the geostationary one (GEostationary Orbit - GEO). This is a geosynchronous orbit prograde equatorial circular, i.e. characterized by the parameters $h = 35785$ km, $e = 0$ and $\lambda = 0$. An observer positioned on Earth would see a satellite GEO always stops at the same point in the sky, i.e. with latitude and longitude fixed.

- Distance Satellite - Ground Station

$$d(t) = \sqrt{R^2 + (R+h)^2 - 2R(R+h)\cos\alpha(t)}$$

$$\omega = \omega_s - \omega_e \cos\lambda_0$$

- Visibility interval

From the sin rule

$$\frac{R+h}{\sin\left(\frac{\pi}{2} + \theta\right)} = \frac{R}{\sin\left(\frac{\pi}{2} - \theta - \alpha\right)}$$

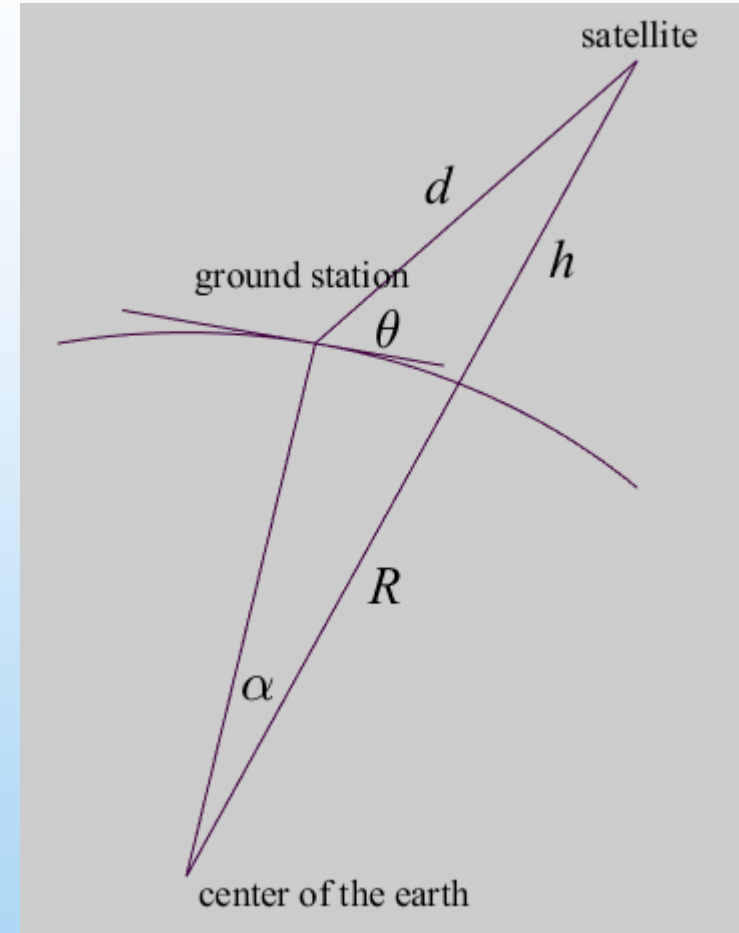
we have

$$\alpha(\theta) = \arccos\left(\frac{R}{R+h}\cos\theta\right) - \theta$$

defining θ_m and θ_M the minimum and maximum visibility angle, from the geometry of spherical triangles, half of visibility angle $\Delta\psi$ is given by

$$\Delta\psi = \arccos\left(\frac{\cos\alpha(\theta_m)}{\cos\alpha(\theta_M)}\right)$$

and the visibility interval is given by

$$t_v \cong 2 \frac{\Delta\psi}{\omega}$$


- Consider a spherical triangle.
- It is characterized by 3 inner angles and 3 outer angles characterized by the relationships:

$$\cos \alpha_1 = \cos \alpha_2 \cos \alpha_3 + \sin \alpha_2 \sin \alpha_3 \cos A_1$$

In our scenario

$$\alpha_1 = \alpha(\theta_m)$$

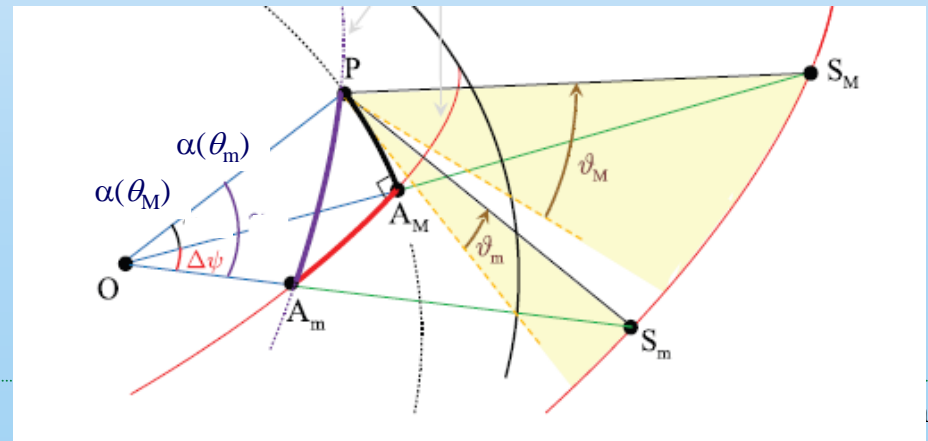
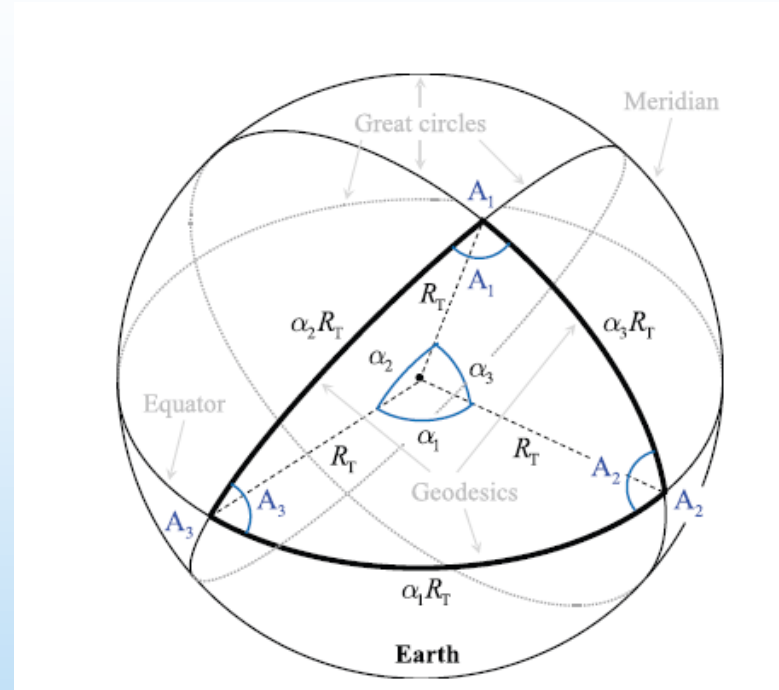
$$\alpha_2 = \alpha(\theta_M)$$

$$\alpha_3 = \Delta\psi$$

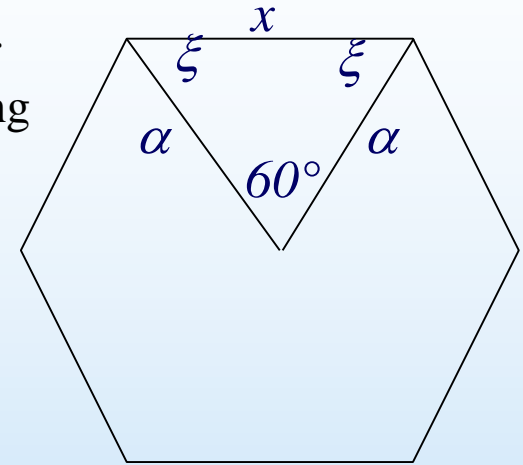
$$A_1 = \frac{\pi}{2}$$

We obtain:

$$\cos \Delta\psi = \frac{\cos \alpha(\theta_m)}{\cos \alpha(\theta_M)}$$



- The footprint (the ground area that its transponders offer coverage) may be schematized by an hexagon, consisting of 6 spherical isosceles triangles with inner angles α , α and x , and outer angles ξ , ξ , and 60° .



We have
$$\cos x = \cos^2 \alpha + \frac{1}{2} \sin^2 \alpha = 1 - \frac{1}{2} \sin^2 \alpha$$

$$\cos \alpha = \cos \alpha \cos x + \sin \alpha \sin x \cos \xi$$

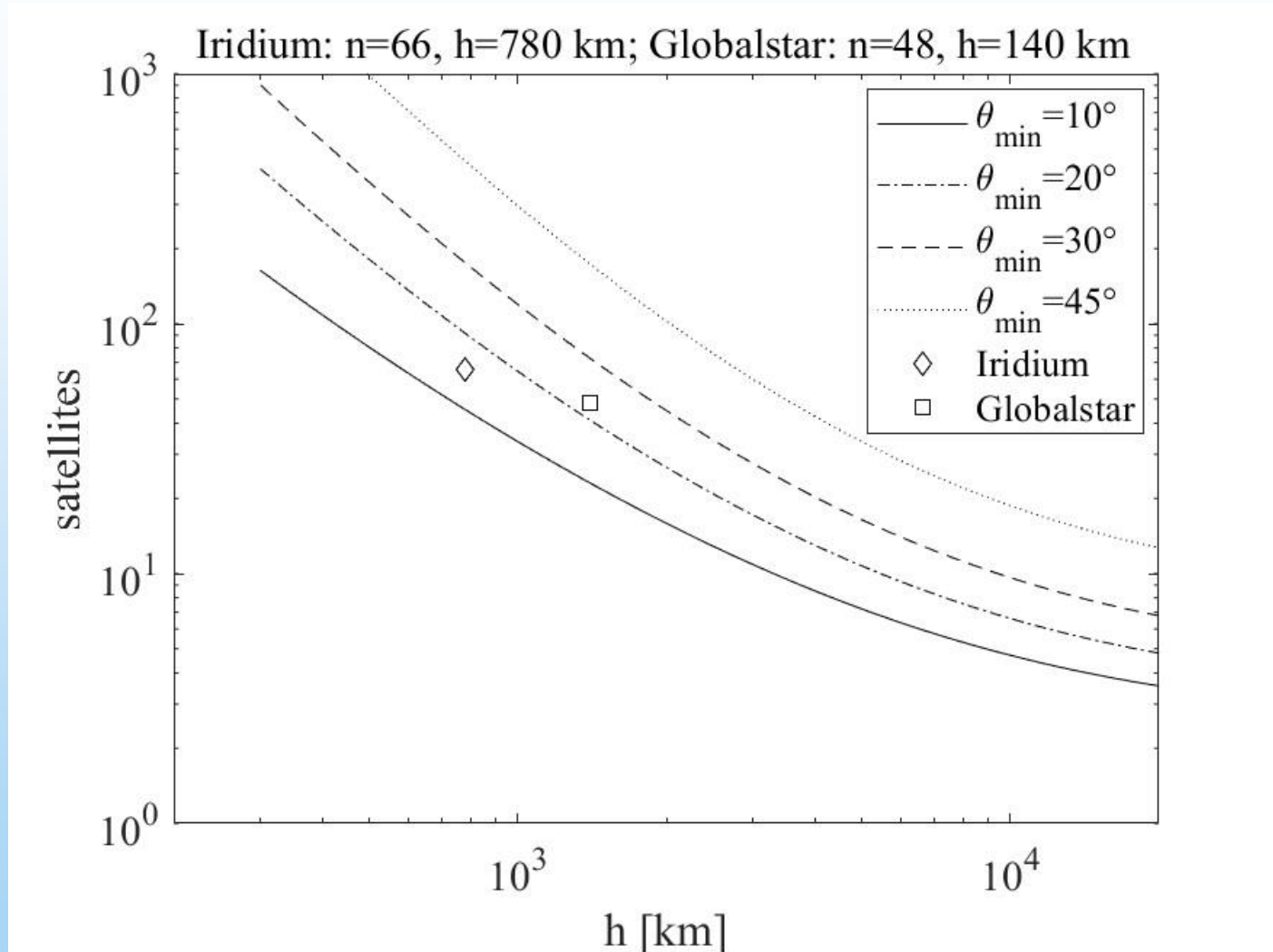
obtaining
$$\cos \xi = \frac{\cos \alpha \sin \alpha}{2 \sin x}$$

From the
$$\frac{\sin x}{\sqrt{3}/2} = \frac{\sin \alpha}{\sin \xi}$$
 we obtain
$$\tan \xi = \frac{\sqrt{3}}{\cos \alpha}.$$

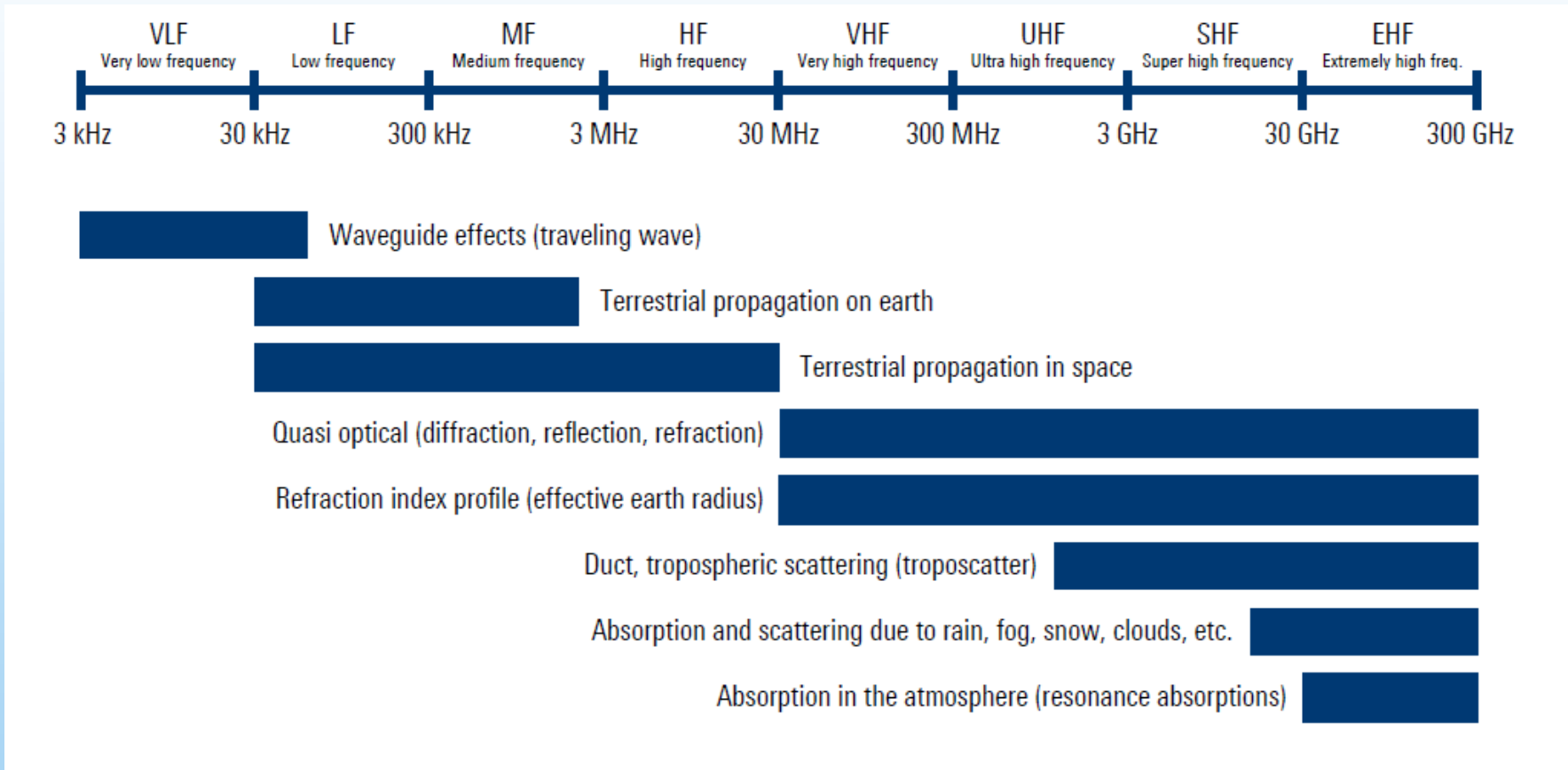
The area of the hexagon is given by
$$A = 6R^2 \left(2\xi + \frac{\pi}{3} - \pi \right) = 6R^2 \left(2\xi - 2\frac{\pi}{3} \right)$$

and the number of satellites required for the full heart coverage is
$$n = \frac{1}{(3\xi/\pi - 1)}$$

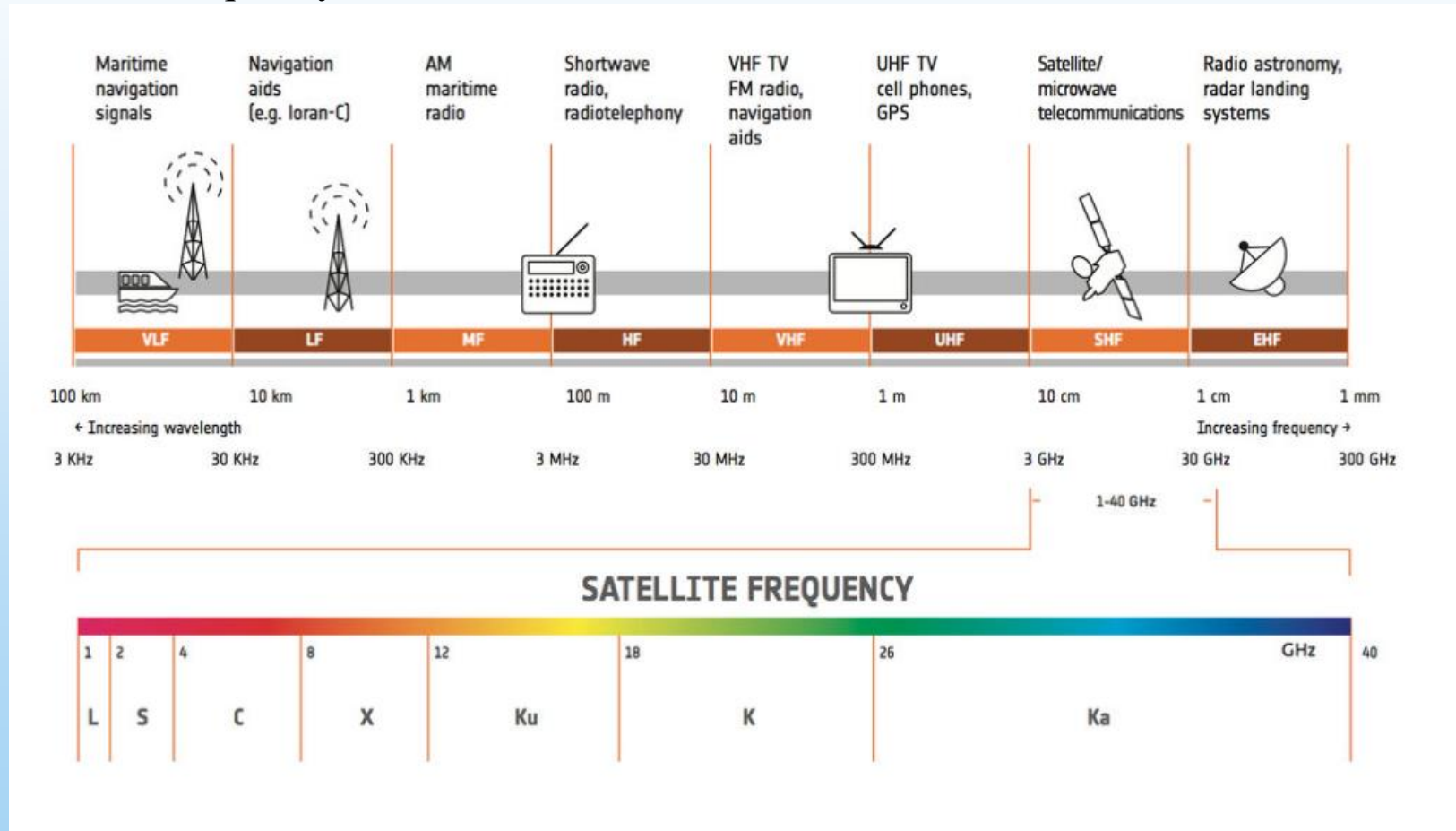
Required number of satellites



- Dominant propagation characteristics versus frequencies (ITU)



- https://www.esa.int/Applications/Connectivity_and_Secure_Communications/Satellite_frequency_bands



Satellite frequency bands

- **L-band (1-2 GHz)**, is primarily used for satellite communications such as global positioning systems (GPS), mobile satellite services, and aircraft surveillance. Used also by satellite mobile phones, such as Iridium, Inmarsat providing communications at sea, land and air. With its lower frequency, L-band signals are resilient to weather variations and can penetrate obstacles such as foliage, making it ideal for terrestrial mobile communication.
- **S-band (2-4 GHz)**, is commonly used for weather radar, surface ship radar, and some communications satellites, especially those used by NASA to communicate with the International Space Station. The S-band is also employed in mobile satellite services like the Globalstar network, as well as for WiFi and other technologies.
- **C-band (4-8 GHz)**, is widely used for satellite communications, television broadcasting, and weather radar systems. Its susceptibility to rain fade (signal loss during heavy rainfall) is lower compared to bands with higher frequencies, such as Ku band, making it popular in regions with high rainfall. However, it requires larger antennas for reception, which makes its use less practical for direct-to-home TV broadcasting.

Satellite frequency bands

- **X-band (8-12 GHz)**, is used in military communications, radar, and satellite communications. It is particularly favoured for deep space telecommunications, due to its moderate susceptibility to rain fade and its ability to maintain a strong signal. It is used for air traffic control, maritime vessel traffic control, defence tracking and vehicle speed detection for law enforcement.
- **Ku-band (12-18 GHz)**, is frequently used for satellite television broadcasting. It allows for smaller, more compact satellite dishes compared to C-band. However, it's more susceptible to rain fade, limiting its effectiveness in tropical regions.
- **Ka-band (26.5-40 GHz)**, has become increasingly popular in recent years for broadband and satellite communication services. It offers higher bandwidth capacity, enabling the provision of high-speed data services such as video streaming and internet access. However, similar to the Ku-band, it's highly susceptible to rain fade.
- **V-band (40-75 GHz)**, is a relatively new player in the world of satellite communication. It offers even greater bandwidth capacity than the Ka-band, making it a promising option for future broadband services. However, it still faces considerable technological hurdles due to its high susceptibility to atmospheric absorption and rain fade.

- Long-term channel effects (first-order statistics):
 - Attenuation due to precipitation (rain, snow, hail, ...)
 - Gaseous absorption (oxygen, vapor, ...)
 - Cloud attenuation (liquid water models from ITU)
 - Tropospheric scintillations (fast fading, varying refractive index)
 - Signal depolarization (Faraday rotation)
 - Increase in sky noise (previous effects may cause an increase in noise)
 - Total attenuation (e.g. free space path loss)
- Dynamic channel effects:
 - AWGN dynamics due to rain fading
 - Statistic models for second-order statistics like fade slope, fade duration
 - Movement of LEO: combinations of statistical distributions, no constant statistical channel model
 - Doppler shift

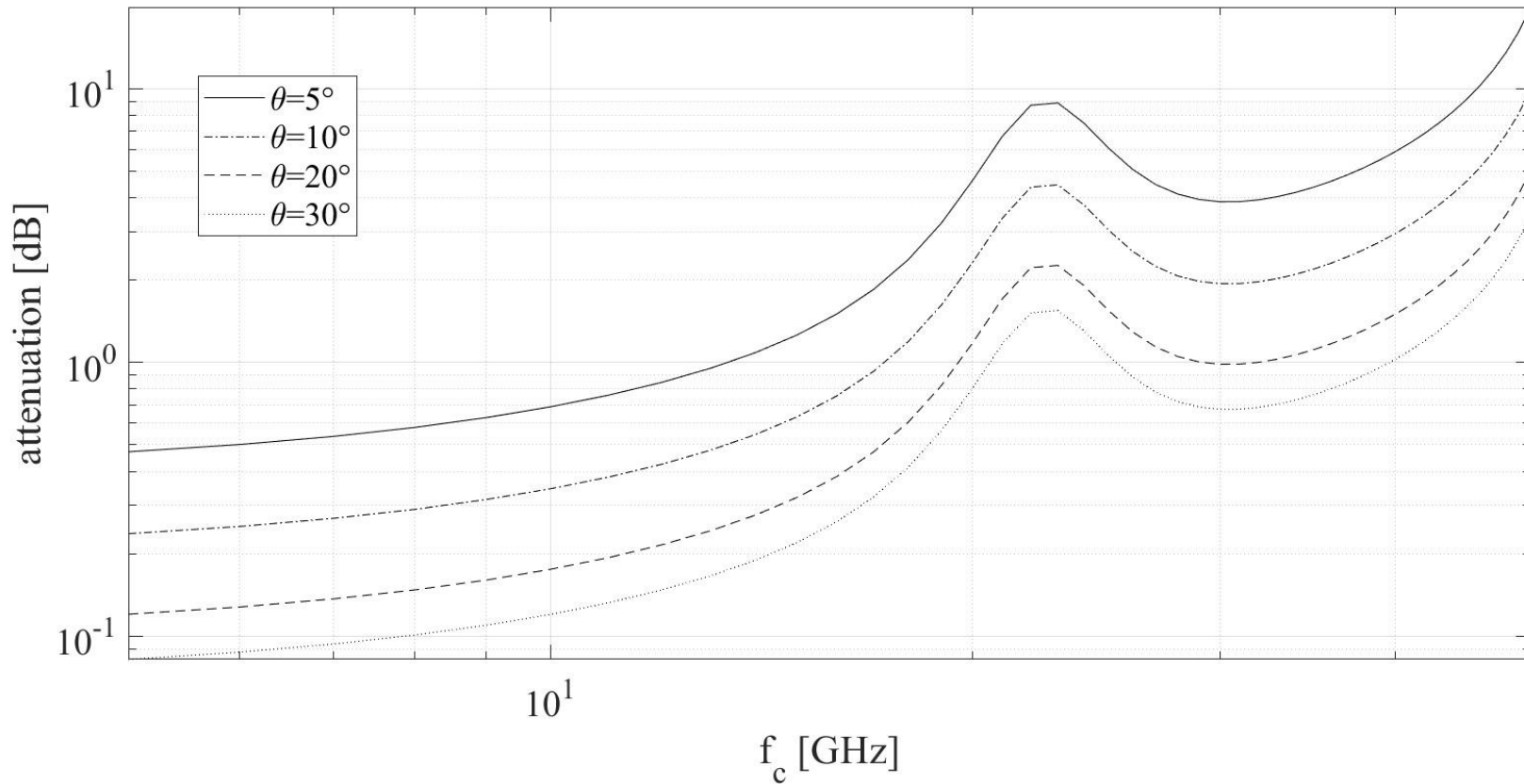
- The amount of scintillation attenuation depends on several factors, since the conductive properties of the ionosphere are strongly influenced by the time of day, latitude and solar phases.
- Note that the space wave is never reflected by the ionosphere, but always passes through it, although undergoing some degradation. This is due to the operating frequencies. The space wave adopts frequencies between 1 and 40 GHz (inside the microwave band). Therefore, there is no ionospheric reflection, which would be completely undesirable in this case.
- At such frequencies the ionosphere becomes traversable despite some disturbance phenomena, such as scintillation.

- Propagation issues have been widely investigated by 3GPP, while defining the interaction among 5G and satellite networks
- In its study item [TR 38.811], 3GPP considers channel models based on those two ITU recommendations:

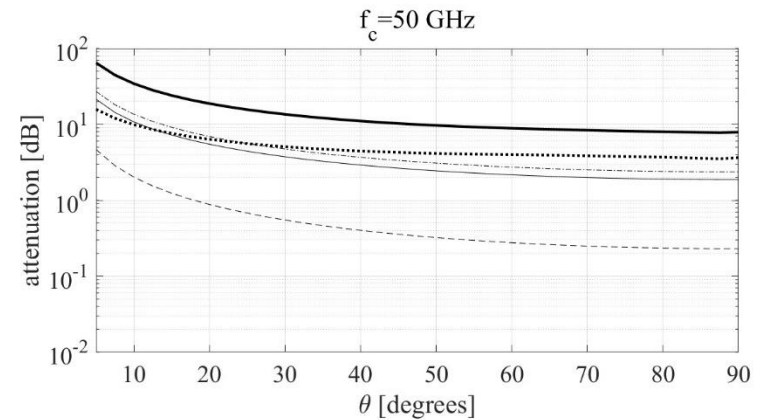
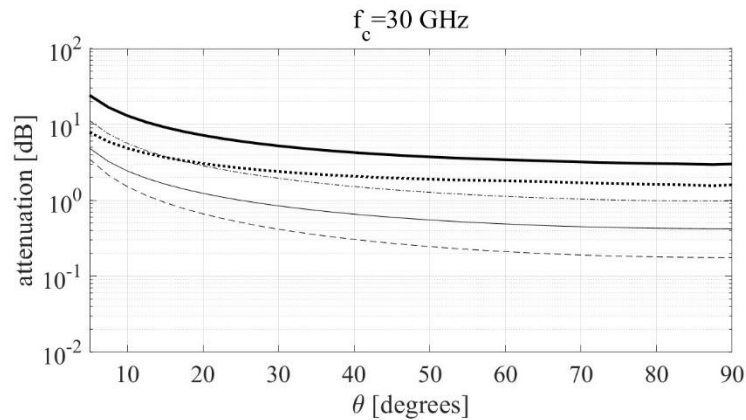
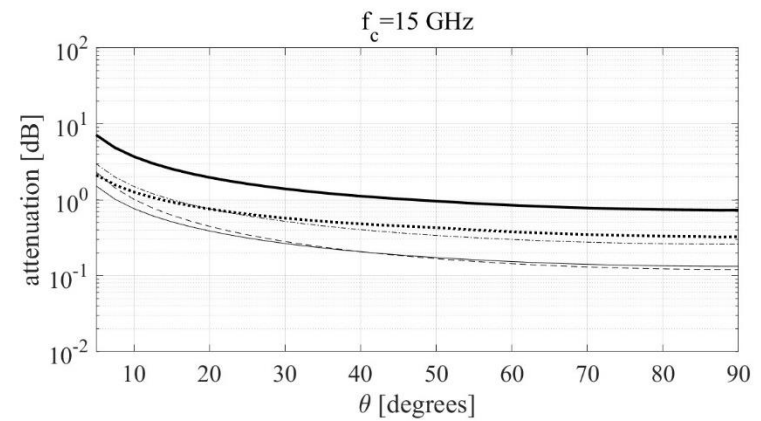
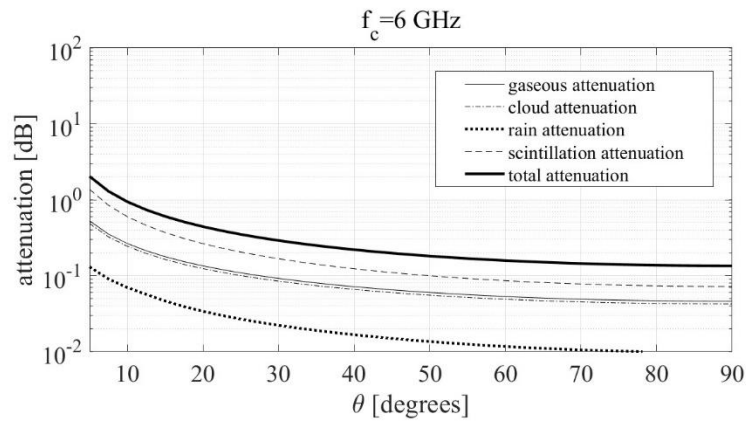
ITU-R P.681 that defines the Land Mobile Satellite channel with measurements up to 20 GHz

ITU-R P.618 that describes atmospheric effects such as gas attenuation, scintillation

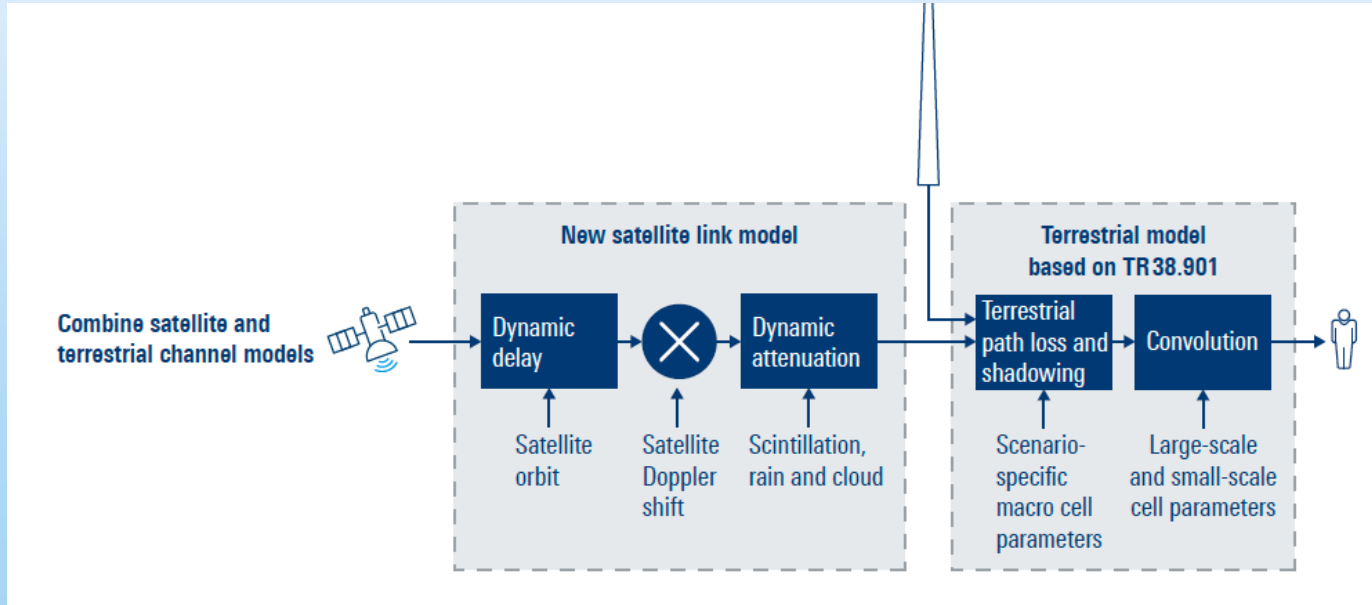
- Attenuation in dB, according to Recommendation ITU-R P.618



- Attenuation in dB, according to Recommendation ITU-R P.618



- NTN assumes primarily Line of Sight (LOS) scenarios with Multi Path Propagation (MPP) resulting from nearby objects. However Non-LOS situations may occur.
- The 3GPP study item [TR 38.811] is motivated by the main objective to develop NTN channel models based on the existing legacy terrestrial channel models. The study investigates channel models focusing on NTN deployment scenarios including urban, suburban and rural.



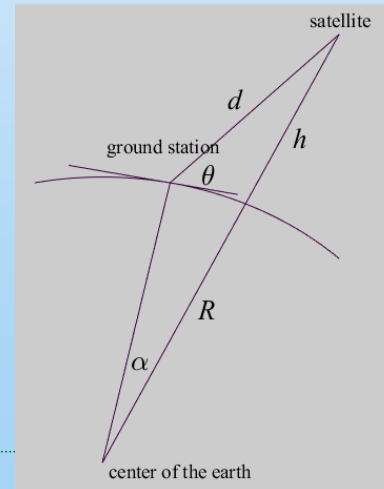
- The performance of the satellite is characterized by the transmitted power, P_{TX} , and the gain of the transmitting antenna, G_{TX} , taking into account a pointing error and a certain depolarization. All these elements contribute to quantifying the effective isotropic radiated power (EIRP), which represents the figure of merit of the satellite.
- The performance of the Earth station is also influenced by the gain of the receiving antenna, G_{RX} , net of the pointing error and depolarization, which determine the effective gain of the antenna.
- For evaluating the performance it is now necessary to estimate the noise power of the receiver, which can be calculated as $N = kTB$ in which $k \approx 1.38 \cdot 10^{-23}$ J/K is the Boltzman constant, B represents the bandwidth, and T is the total temperature given by $T = T_A + T_R$, being T_A the antenna temperature and T_R the temperature of the receiver.

- As far as the path loss is concerned, this is mainly influenced by the free-space path loss (FSPL) component, but additionally, there is an influence by the nearby surroundings like buildings, hills, tree foliage, etc., as they may result in components responsible for clutter loss and shadow fading.
- The path loss can be mathematically described as a function of the elevation angle. This elevation angle will also play a major role in future channel models assuming HAPS or UAVs as airborne gNBs instead of higher altitude satellites, as the elevation angle will be much different in such situations. The elevation angle of the LOS path of the satellite/HAPS versus the ground horizon from the UE perspective will be the most relevant parameter in such channel models.
- From

$$d^2 + R^2 - 2dR \cos(\pi/2 + \theta) = (R + h)^2$$

we obtain

$$d = \sqrt{R^2 \sin^2 \theta + h^2 + 2hR} - R \sin \theta$$



- Free space received power

$$P_{RX} = P_{TX} G_{TX} G_{RX} \left(\frac{\lambda}{4\pi d} \right)^2$$

being P_{TX} the transmitted power, G_{TX} the transmit antenna gain, G_{RX} the receive antenna gain, and λ the wavelength.

- We obtain that the free space path loss is given by

$$FS_{PL} = -10 \log_{10} \left(\frac{\lambda}{4\pi d} \right)^2 = -20 \log_{10} \left(\frac{c}{4\pi f_c d} \right) = 92.44 + 20 \log_{10} f_c + 20 \log_{10} d \text{ dB}$$

in which f_c is the carrier frequency in GHz e d is in km.

- Example: $f_c = 20$ GHz, $d = 35.8 \cdot 10^3$ km; we obtain $FS_{PL} = 209.5$ dB

- Summarizing, the receiver *CNR* (in dB) is given by

$$\text{CNR} = \text{EIRP} + G_{RX} - FS_{PL} - PL_g - PL_s - PL_e - PL_{AD} - k - B - T$$

in which $\text{EIRP} = P_{TX} + G_{TX} - L_P - L_T$ [dBW] is the effective isotropic radiated power, being P_{TX} the transmitted power in dBw, G_{TX} the transmitting antenna gain, L_P the polarization loss, and L_T the tracking loss. FS_{PL} is the free space path loss, PL_g the attenuation due to atmospheric gases, PL_s the attenuation due to atmospheric scintillation, PL_e a shadowing margin, PL_{AD} an optional degradation due to feeder link losses in transparent architecture and B the channel bandwidth (in logarithmic value), k the Boltzmann constant (-228.6 dBW / (K Hz)), and T is the noise temperature (in dB) [TR 38.811]

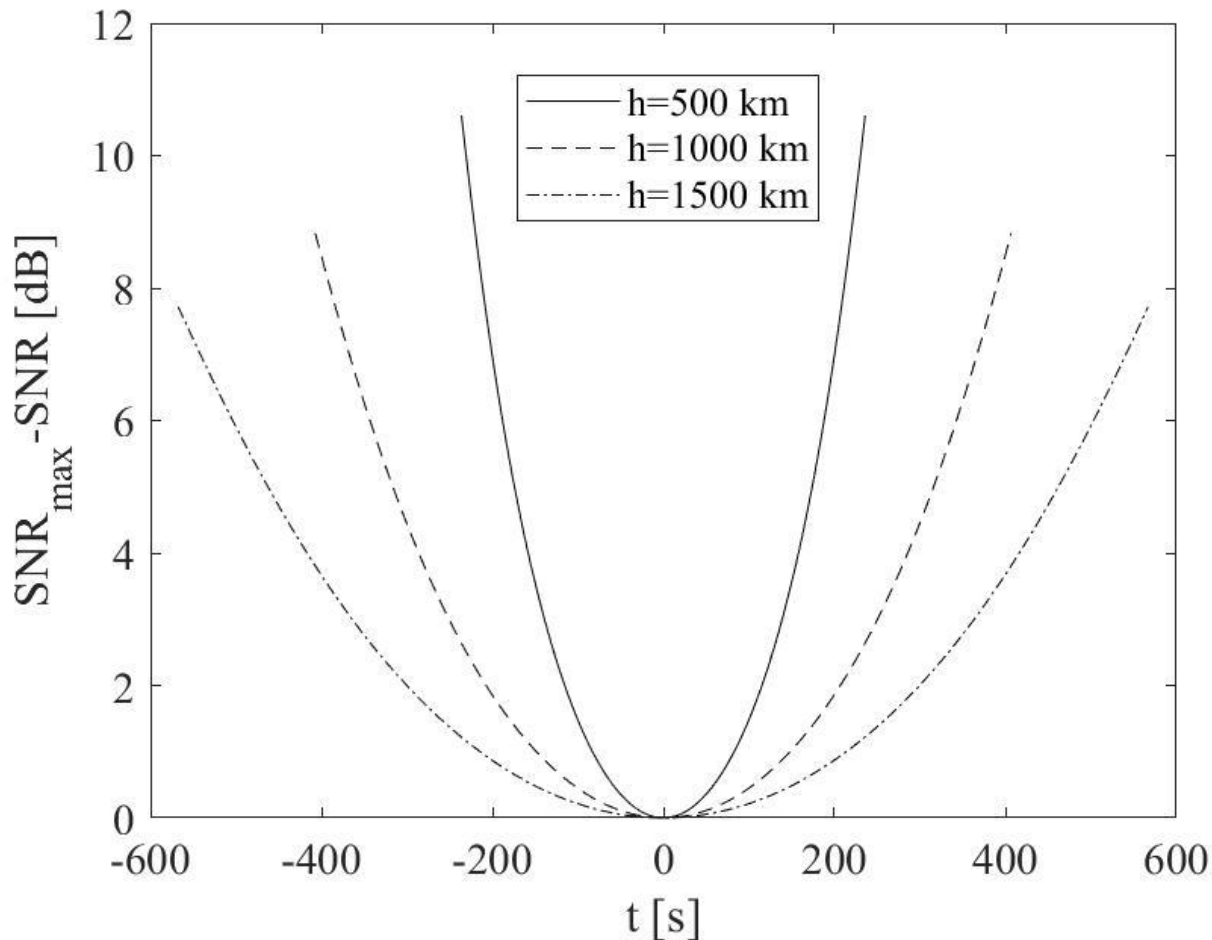
$$T = NF + 10 \log_{10} \left(T_0 + \frac{T_A - T_0}{10^{NF/10}} \right) = 10 \log_{10} \left[T_A + T_0 \left(10^{NF/10} - 1 \right) \right] + 10 \log_{10} (T_A + T_R)$$

in which NF is the receiver noise figure in dB, T_0 the ambient temperature (290 K), T_R is the receiver noise temperature, and T_A the antenna temperature.

Link Budget

Link budget		log		log
P_{Tx} [W - dBW]	0.20	-6.99	0.20	-6.99
Antenna diameter [m]	0.90		0.90	
Frequency, f_c [GHz]	1.00		1.60	
Wavelength, λ (m)	0.30		0.19	
Antenna area [m ²]	0.64		0.64	
Efficiency	0.25		0.50	
A_{eff} = Eff. antenna area [m ²]	0.16		0.32	
$G_{Tx}=4 \pi A_{\text{eff}} / \lambda^2$	22.21	13.46	113.70	20.56
L_p+L_t		0.3		0.3
EIRP [dBW]		6.48		13.27
Distance, d [km]	1400.00		780.00	
Path loss $FS_{PL}=(4\pi d/\lambda)^2$	3.4E+15	155.36	2.7E+15	154.37
Losses [dB]		5.00		5.00
P_{RX} [dBW]		-153.89		-146.10
G_{Rx} [dB]		43.00		10.00
Useful signal [dBW]		-110.89		-136.10
Receiver Noise Figure NF [dB]		10.00		4.00
T_R , Noise temperature receiver= $T_0(10^{NF/10}-1)$	2610.00		438.45	
T_A , Antenna temperature	15.00		290.00	
$k(T_R+T_A)$ [dBW]	3.57E-20	-194.47	9.91E-21	-200.04
Bandwidth, B [MHz]	40.00		1.00	
Noise power [dBW]	1.43E-12	-118.45	9.91E-15	-140.04
Carrier to noise ratio, CNR, [dB]		7.56		3.94

- $\theta_{\min}=10^\circ$, $\theta_{\max}=90^\circ$, $\lambda=20^\circ$



- Consider a GEO-GEO infra-orbital intersatellite link.
- The free space attenuation, the only relevant one in the exosphere, where gases are highly rarefied and conductive effects are very low, may be calculated by assuming an intersatellite distance $d = 10000$ km, and a carrier frequency $f_c = 20$ GHz.

we obtain
$$FS_{PL} = -20 \log_{10} \left(\frac{c}{4\pi f_c d} \right) = 198.5 \text{ dB}$$

- The noise power is evaluated using $N = kTB$, assuming an absolute temperature of $T = 300$ K of the receiver on board the second GEO satellite, and a bandwidth $B = 50$ MHz, obtaining $N = -126.8$ dBW
- Assuming a transmission power $P_{TX} = 10$ dBW, an antenna gain $G_{TX} = G_{RX} = 40$ dB, a Polarization Loss $L_P = 0.2$ dB, and a Tracking Loss: $L_T = 0.1$ dB we obtain the results which are shown in the table.

Transmitting GEO Satellite	
Transmission power: P_{TX} [dBW]	10
Antenna Gain: G_{TX} , dB	40
Polarization Loss: L_P [dB]	0.2
Tracking Loss: L_T [dB]	0.1
EIRP=$P_{TX}+G_{TX}-L_P-L_T$ [dBW]	49.7
Path Loss: FS_{PL} [dB]	198.5
Receiving GEO Satellite	
Antenna Gain: G_{RX} , dB	40
Polarization Loss: L_P [dB]	0.2
Tracking Loss: L_T [dB]	0.1
Noise Power, N : dBW	-128.6
Balance	
CNR=$EIRP-FS_{PL}+G_{RX}-L_P-L_T-N$ [dB]	17.8

- Consider now a GEO-LEO inter-orbital intersatellite link.
- Assuming an intersatellite distance $d=34000$ km, and a carrier frequency $f_c=20$ GHz.

we obtain $FS_{PL} = -20 \log_{10} \left(\frac{c}{4\pi f_c d} \right) = 209.1$ dB

- Assuming an absolute temperature of $T = 200$ K of the receiver on board the LEO satellite, and a bandwidth $B=50$ MHz, we obtain $N=-128.6$ dBW
- Assuming a transmission power $P_{TX}=10$ dBW, an antenna gain $G_{TX}= 40$ dB $G_{RX}=30$ dB, a Polarization Loss $L_p =0.2$ dB, and a Tracking Loss: $L_T =0.1$ dB we obtain the results which are shown in the table.
- Observe that now the CNR value is significantly lower, with severe constraints on system (modulation, channel coding, ...) design.

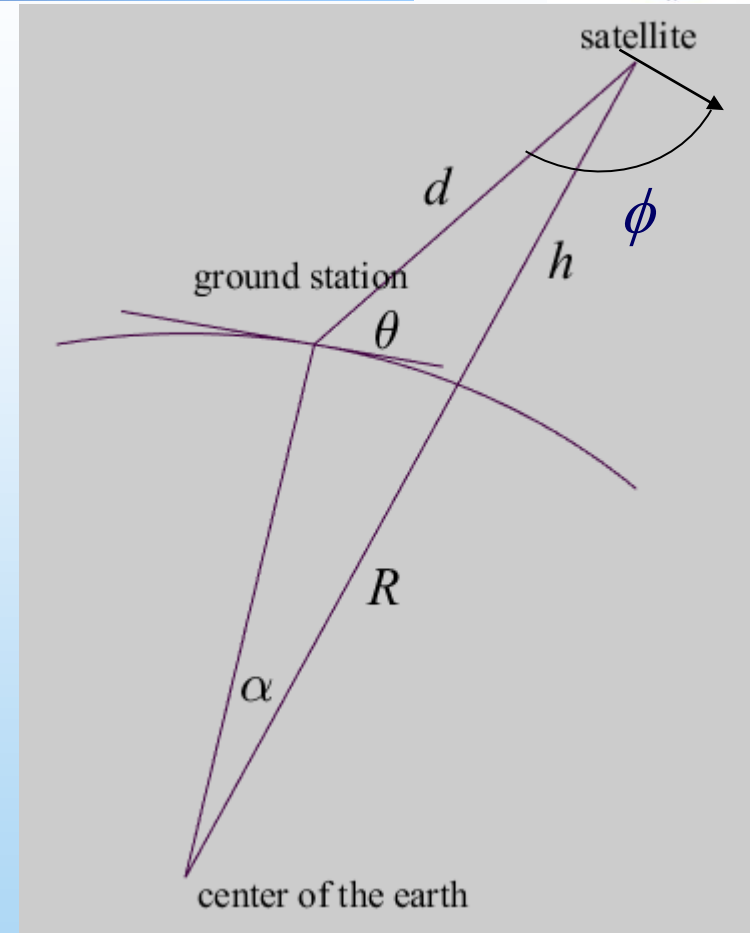
Transmitting GEO Satellite	
Transmission power: P_{TX} [dBW]	10
Antenna Gain: G_{TX} , dB	40
Polarization Loss: L_P [dB]	0.2
Tracking Loss: L_T [dB]	0.1
EIRP=$P_{TX}+G_{TX}-L_P-L_T$ [dBW]	49.7
Path Loss: FS_{PL} [dB]	209.1
Receiving GEO Satellite	
Antenna Gain: G_{RX} , dB	30
Polarization Loss: L_P [dB]	0.2
Tracking Loss: L_T [dB]	0.1
Noise Power, N : dBW	-126.8
Balance	
CNR=$EIRP-FS_{PL}+G_{RX}-L_P-L_T-N$ [dB]	-1.1

- The Doppler shift occurring is calculated based on the velocity \mathbf{v} (relative velocity between UE and satellite), the carrier frequency f_c and the angle ϕ between the velocity vector and the direction of propagation of the signal.

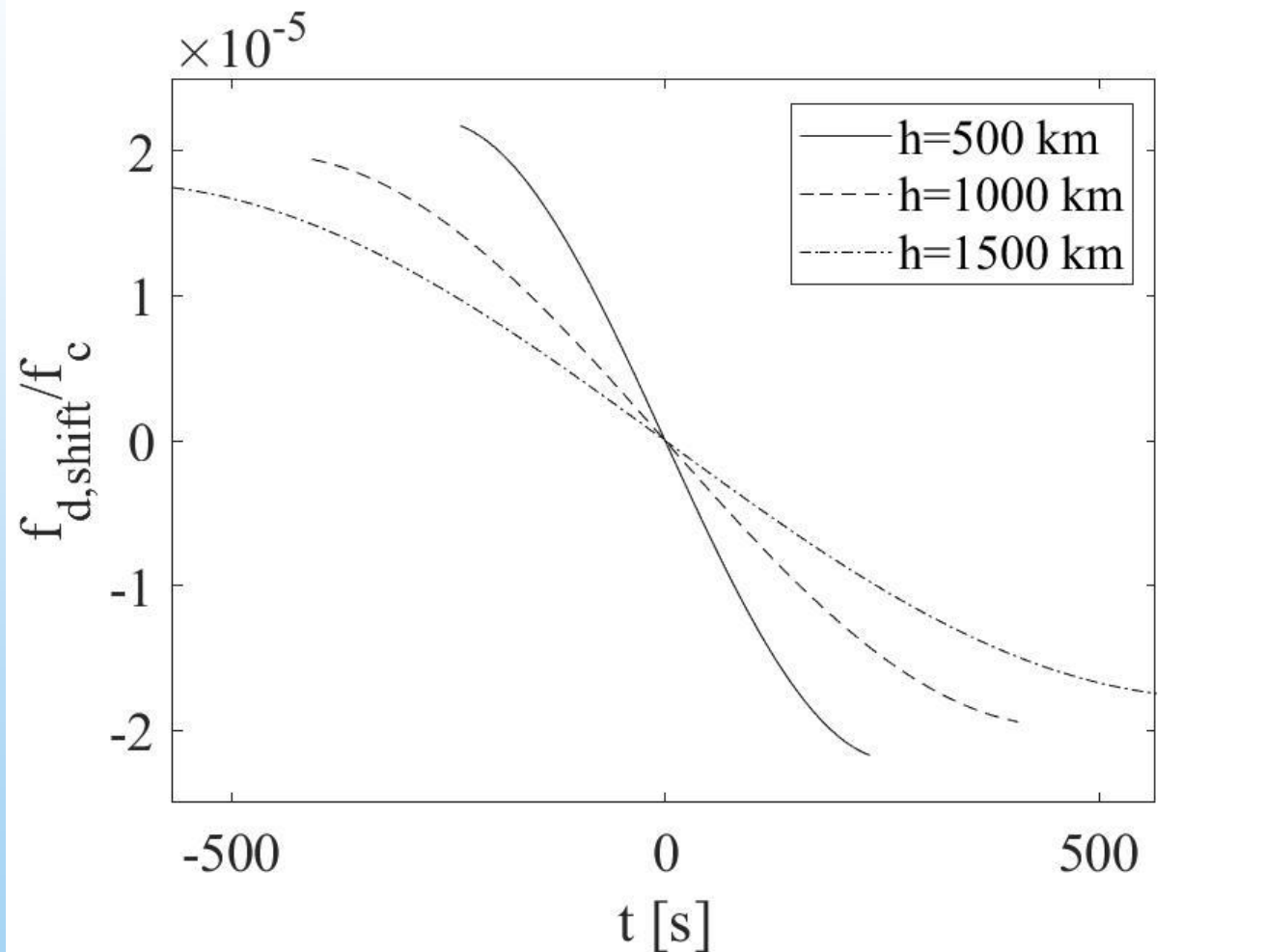
- Being
$$\frac{R+h}{\sin\left(\frac{\pi}{2}+\theta\right)} = \frac{R}{\sin\left(\frac{\pi}{2}+\phi\right)}$$

we have
$$\frac{f_{d,\text{shift}}}{f_c} = \frac{v_{\text{sat}}}{c} \frac{R}{R+h} \cos\theta$$

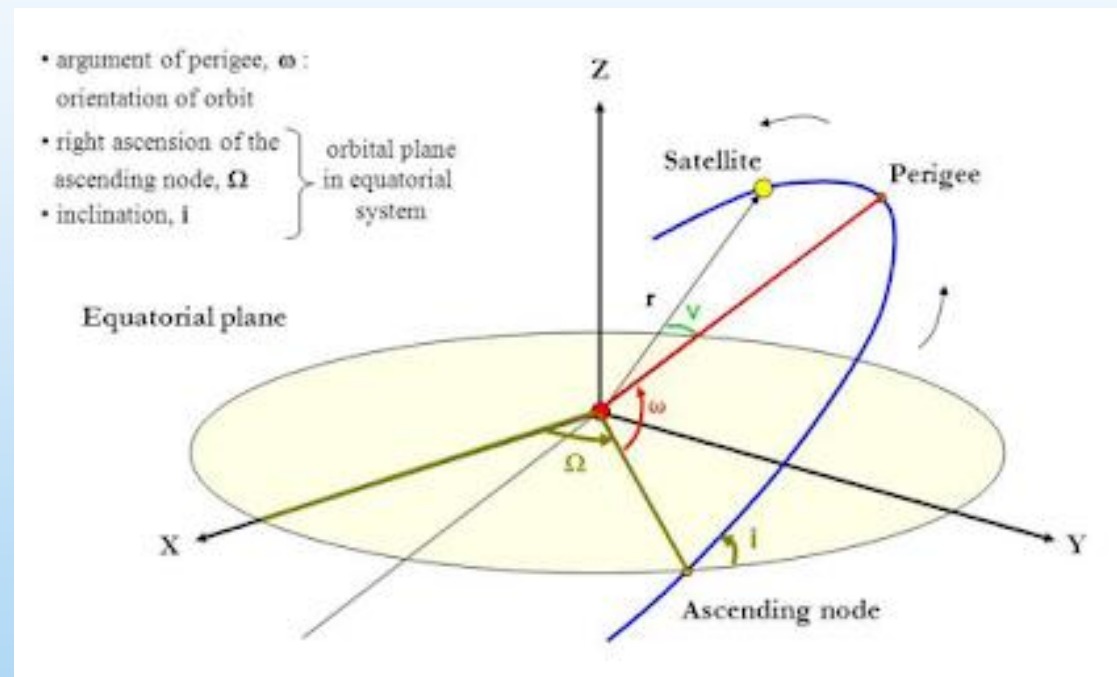
- As a methodology to compensate the Doppler shift in UL, the UE estimates its location e.g. based on GNSS positioning, and the UE receives system information (SIB) together with satellite ephemeris information. As a result, the UE will then adapt its uplink carrier frequency to adjust the carrier frequency at the satellite receiver in such a way that the Doppler shift impact in uplink direction can be disregarded



- $\theta_{\min}=10^\circ$, $\theta_{\max}=90^\circ$, $\lambda=20^\circ$



- <https://www.astronomicalreturns.com/p/section-43-six-orbital-elements.html>
- Any orbit can be defined by 6 parameters, known as the 6 orbital elements:
 - a: length of the semi-major axis
 - ε : eccentricity
 - i: inclination
 - Ω : right ascension of the ascending node
 - ω : argument of periapsis
 - v: true anomaly / time of periapsis passage



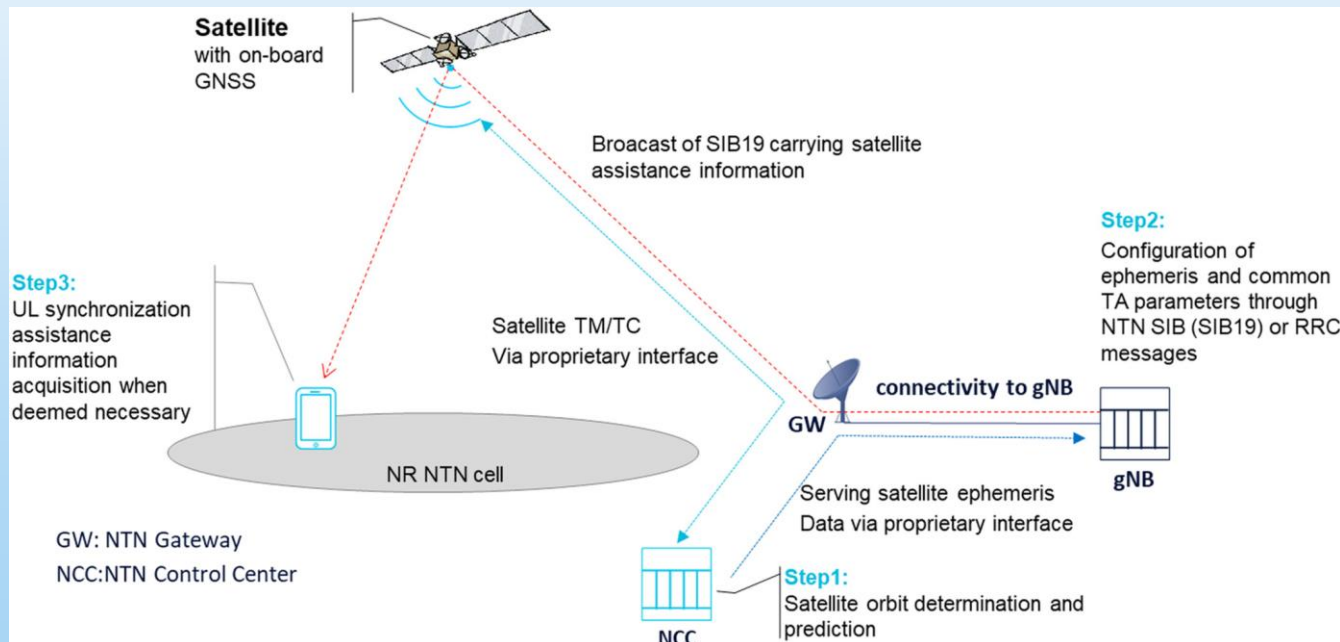
- **Right ascension of the ascending node:** the satellite's orbit defines two nodes which are the two points where the orbit intersects the Earth's equatorial plane. One is called the ascending node (when the satellite is travelling from the southern to the northern hemisphere) and the other is called the descending node. Consider the Earth-centered, Earth-fixed coordinate system (**ECEF**) which has the following parameters:
 - The origin is center of mass of the Earth.
 - The Z axis is the line between the North and South Poles, with positive values increasing northward.
 - The X axis is in the plane of the equator, passing through the origin and extending from 180° longitude (negative) to the prime meridian (positive).
 - The Y axis is also in the plane of the equator, passing through extending from 90° W longitude (negative) to 90° E longitude (positive).

The right ascension of the ascending node is the angle between the X axis and the ascending node. It's always measured eastward on the Earth's equatorial plane.

- **Argument of periapsis:** it is the angle between the ascending node and the point of periapsis (passage at perigee). It's always measured on the orbital plane in the direction of spacecraft motion.
- **True anomaly / time of periapsis passage:** true anomaly is the angle between periapsis and the satellite's current position, measured in the direction of motion.
- Summarizing
 - Two parameters for the shape of the orbit, e.g. its semi-major axis a and eccentricity ε
 - Three parameters for its orientation relative to earth, e.g. inclination i , the right ascension of the ascending node Ω , and argument of periapsis ω .
 - One parameter as a reference point in time, the true anomaly / time of periapsis passage.
- Alternatively, the ECEF coordinates specify the actual satellite system.

Example (1)

- In LEO systems, the satellite assistance or ephemeris information, together with the UE terrestrial location information can be used to help UEs to perform measurements supporting cell selection and reselection. In 5G NTN the ephemeris information included may be transmitted in addition to the physical cell ID (PCI) and frequency information.



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- Ephemeris data may be provided with three other parameters: TACCommon, TACCommonDrift, and TACCommonDriftVariation, which are required to derive the common one way propagation delay at a given time and from there estimate the Timing Advance.
- The epoch time is a reference time for which assistance information (i.e., serving satellite ephemeris and common TA parameters) is valid for. When it is not explicitly indicated in SIB19, this epoch time of assistance information is implicitly known as the end of the system information (SI) window during which the NTN-specific SIB (i.e. SIB19) message is transmitted. When provided through dedicated signaling, epoch time of assistance information is the starting time of a DL sub-frame, which is explicitly indicated by a system frame number (SFN) and a sub-frame number.