PROBLEMS

1. *Poisson Distribution*

In the Drude model the probability of an electron suffering a collision in any infinitesimal interval dt is just dt/τ.

(a) Show that an electron picked at random at a given moment had no collision during the preceding t seconds with probability $e^{-t/\tau}$. Show that it will have no collision during the next t seconds with the same probability.

(b) Show that the probability that the time interval between two successive collisions of an electron falls in the range between t and $t + dt$ is $(dt/\tau)e^{-t/\tau}$.

(c) Show as a consequence of (a) that at any moment the mean time back to the last collision (or up to the next collision) averaged over all electrons is τ.

(d) Show as a consequence of (b) that the mean time between successive collisions of an electron is τ.

(e) Part (c) implies that at any moment the time T between the last and next collision averaged over all electrons is 2τ. Explain why this is not inconsistent with the result in (d). (A thorough explanation should include a derivation of the probability distribution for T.) A failure to appreciate this subtlety led Drude to a conductivity only half of (1.6). He did not make the same mistake in the thermal conductivity, whence the factor of two in his calculation of the Lorenz number (see page 23).

2. *Joule Heating*

Consider a metal at uniform temperature in a static uniform electric field **E**. An electron experiences a collision, and then, after a time t, a second collision. In the Drude model, energy is not conserved in collisions, for the mean speed of an electron emerging from a collision does not depend on the energy that the electron acquired from the field since the time of the preceding collision (assumption 4, page 6).

(a) Show that the average energy lost to the ions in the second of two collisions separated by a time t is $(eEt)^2/2m$. (The average is over all directions in which the electron emerged from the first collision.)

(b) Show, using the result of Problem 1(b), that the average energy loss to the ions per electron per collision is $(eE\tau)^2/m$, and hence that the average loss per cubic centimeter per second is $(ne^2\tau/m)E^2 = \sigma E^2$. Deduce that the power loss in a wire of length L and cross section A is I^2R, where I is the current flowing and R is the resistance of the wire.

3. *Thomson Effect*

Suppose that in addition to the applied electric field in Problem 2 there is also a uniform temperature gradient ∇T in the metal. Since an electron emerges from a collision at an energy determined by the local temperature, the energy lost in collisions will depend on how far down the temperature gradient the electron travels between collisions, as well as on how much energy it has gained from the electric field. Consequently the power lost will contain a term proportional to $\mathbf{E} \cdot \nabla T$ (which is easily isolated from other terms since it is the only term in the second-order energy loss that changes sign when the sign of **E** is reversed). Show that this contribution is given in the Drude model by a term of order $(ne\tau/m)(d\mathcal{E}/dT)(\mathbf{E} \cdot \nabla T)$, where $\mathcal{E}$ is the mean thermal energy per electron. (Calculate the energy lost by a typical electron colliding at $\mathbf{r}$, which made its last collision at $\mathbf{r} - \mathbf{d}$. Assuming a fixed (that is, energy-independent) relaxation time τ, $\mathbf{d}$ can be found to linear order in the field and temperature gradient by simple kinematic arguments, which is enough to give the energy loss to second order.)