

PAPER • OPEN ACCESS

## Monitoring PCK physics teachers' strategies for Math and Physics Languages Integration: the teacher footprint.

To cite this article: V Bologna *et al* 2022 *J. Phys.: Conf. Ser.* **2297** 012034

View the [article online](#) for updates and enhancements.

You may also like

- [Generating pedagogical content knowledge in teacher education students](#)  
Ed van den Berg
- [What makes a good physics teacher? Views from the English stakeholder community](#)  
James de Winter and John Airey
- [Identification of Pedagogical Content Knowledge \(PCK\) for Prospective Chemistry Teachers: Efforts to Build Teachers' Professional Knowledge](#)  
Y Sri, A A Mardiyah, M Mohammad et al.



The Electrochemical Society  
Advancing solid state & electrochemical science & technology

243rd ECS Meeting with SOFC-XVIII

**More than 50 symposia are available!**

Present your research and accelerate science

Boston, MA • May 28 – June 2, 2023

[Learn more and submit!](#)

## Monitoring PCK physics teachers' strategies for Math and Physics Languages Integration: the teacher footprint.

V Bologna<sup>1,a</sup>, F Longo<sup>1,2,b</sup>, M Peressi<sup>1,c</sup> and P Sorzio<sup>3,d</sup>

<sup>1</sup>Physics Department, University of Trieste, Italy

<sup>2</sup>INFN -Trieste, Italy

<sup>3</sup>Department of Humanities, University of Trieste, Italy

<sup>a</sup> email: [valentina.bologna@phd.units.it](mailto:valentina.bologna@phd.units.it)

<sup>b</sup> email: [francesco.longo@ts.infn.it](mailto:francesco.longo@ts.infn.it)

<sup>c</sup> email: [peressi@units.it](mailto:peressi@units.it)

<sup>d</sup> email: [psorzio@units.it](mailto:psorzio@units.it)

**Abstract.** The last twenty years of research in Physics Education have underlined the important role of the so-called Phys-Math interplay. The lack of students' ability, at different ages and levels, to construct the mathematical model of physical processes or to describe the physical meaning of mathematical constructs has been emphasized. The main research work is currently attributing this difficulty to the way in which these subjects are taught, with a direct connection to the teachers' Pedagogical Content Knowledge (PCK). The purpose of the present investigation is to examine the role of the physics teachers' PCK in this interplay at the early stage of the physics study. To this aim, teachers' and classes' activities in different high schools have been observed. We report here the results of such observations, focusing in particular to the integration of the two disciplines' languages.

### 1. Introduction

Teachers' backgrounds and experiences give an appreciable footprint to their class activities. This footprint is substantially depicted in the process of each Pedagogical Content Knowledge (PCK).

It is widely known and extremely focused by the research in Physics Education that the interplay between Math and Physics follows and can be described in different patterns [1,2,3].

As Pospiech and her collaborators well described [2], teachers employ Phys-Math interplay as part of their practice, fostering a better understanding of physics by the analysis of extreme cases, examining functional relations between physical quantities and the laws they describe. Teachers also commonly create webs of concepts, exploring and sometimes forcing similarities between phenomena referred to different physical situations.

In other cases the interplay is emphasized in the process of problem solving or in exercises of multiple representations of physical situation.

The main characteristic of the teachers' PCK of the Phys-Math interplay directly descends from the PCK model suggested by Magnusson et al. [4], adapted by Etkina [5] and developed by Lehavi [3] in focusing different patterns following different "steps" between physics and mathematics and within each domain.

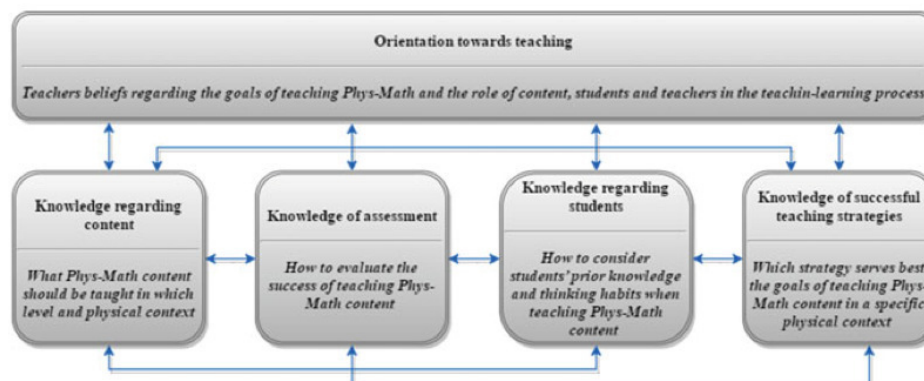
By referring to Magnusson, teachers' PCK is devoted to foster the following goals:

- Helping students develop the 'science process' skills;
- Representing a particular body of knowledge;
- Transmitting the facts of science;



- Facilitating the development of scientific knowledge by confronting students with contexts to explain that challenge their naïve concepts;
- Involving students in investigating solutions to problems;
- Representing science as inquiry;
- Constituting a community of learners whose members share responsibility for understanding the physical world, particularly with respect to the use of tools for science.

All these features are well designed in the most particular PCK depicted by Etkina and dedicated not to all science teachers but to the physics ones in particular, as shown in Figure 1, well declined and interpreted by Lehavi et al. [3].



**Figure 1.** Aspects of the Pedagogical Content Knowledge (PCK), as Etkina [5] suggested, analyzed in correlation with the Math/Phys interplay by Lehavi [3] (reproduced with the permission from Lehavi).

Observing teachers' practices and class activities and covering different content areas of the physics curriculum Lehavi et al. [3] had identified the Phys-Math patterns (Table 1).

**Table 1.** Phys-Math patterns, teaching goals and teaching practices from Lehavi et al. [3], in close relation to Etkina [5] and Magnusson [4] PCK model (reproduced with the permission from Lehavi).

Patterns	Teaching goal	Teaching practices
A. Exploration	To demonstrate how Phys-Math is used to explore the behavior of physical systems	Exploring within Math ramifications for the physical system: limits (of validity, of approximation), extreme cases, etc.
B. Construction	To demonstrate how Phys-Math is used in constructing a model for physical systems	Constructing and developing (from experiments or from first principles) mathematical tools to describe and analyze physical phenomena.
C. Broadening	To demonstrate how Phys-Math can be used in broadening the scope of a physical context	Adopting a bird's-eye view and employing general laws of physics, symmetries, similarities and analogies
D. Application	To demonstrate how Phys-Math provides aid in problem solving	Employing already known laws and mathematical representations in problem solving

These patterns clearly fit into the PCK framework categories: orientation towards teaching, knowledge regarding content and knowledge of successful teaching strategies.

In order to validate and confirm the identified patterns by Lehavi et al. we have monitored an Italian group of high school physics teachers. Our observations have been analyzed in pursuing the goal of finding in which way the five points of Etkina's PCK model are declined and to deeply explore which are the main characteristics of the teachers' orientations towards teaching the Phys-Math interplay.

We recognized that the patterns proposed are internationally valid, and also somehow curriculum independent (because we monitored different high schools, with different physics curriculum).

Furthermore we compared the patterns observed monitoring teachers' in the first year of physics curriculum study with those that are prevalent in the last year. We didn't analyze the observations to identify if the teachers were master or not, although all of them had a lot of teaching experience. We think that the patterns are strictly connected with the disciplinary language integration that the teacher does as a part of his/her knowledge of instructional strategies to scaffold students' learning of key concepts and practices in physics.

## 2. Monitoring Features

All the teachers we monitored were involved in classes where the students are at their first year of curricular physics studies: this is an important point in our investigation, because it could be relevant whether difficulties in learning and studying physics arise from a particular teachers' PCK at this specific educational step, immediately following previous studies characterized by a basic knowledge of mathematics. For the comparison of the patterns between the first and the last year of the curriculum studies we collected data from some classes involved in testing educational experience.

### 2.1. Monitoring Method

In order to perform a deep investigation of teachers' PCK our observations consisted in a sequence, which improved the way to explore our findings.

We suggest that this sequence collects, in a more detailed way, the information we need in order to analyze the correlation between the different PCK aspects and the role of the Phys-Math interplay in each one. This sequence consists of three parts.

**Table 2.** The monitoring method scheduled at three different times and actions.

Time	Action
BEFORE	Teachers' monitoring in their lessons planning.
	Teachers' interviewing for collecting information about students and class educational trends.
DURING	Observations during class activities in presence and online (in the first COVID lockdown period) for the extension of a learning module or a testing case study.
	In some cases, preparing evaluation tests together with teachers at the end of the learning module, with particular attention to the integration of math and physics languages.
AFTER	Feedback discussions with teachers on monitoring activities.
	Test revisions and corrections trying to identify the most frequent mistakes and to classify them in terms of mastery and knowledge of physics languages.
	Collecting students' interviews about their difficulties in that learning module and final evaluation test.

First of all we collected information **before** the observations of the lesson activity about teacher's beliefs, methodologies, physics insights and then about class and students' skills, attitudes and

everything is concerning learning strategies, emerging and recurrent difficulties, assessments. Then as a second step, we gathered information **during** the observations of lesson activities, from explanations to evaluation time. In the end, we got information, **after** the observations, of lesson activities from the teachers' point of view to the students' self-reflection about their performance and learning (Table 2). All these actions gave us a complete and exhaustive way to investigate teachers' PCK.

### 2.2. Monitoring sample

The main feature of this sample is the age difference among students at the first year of physics studies in Italian high schools. Some groups of students start studying curricular Physics at 14 years of age and some others start later (at 16). Of course, this is relevant for the stage of cognitive development, in terms of abstraction functions and metacognitive processes, and it also could be relevant in terms of content building processes for obtaining successful learning.

This sample consists of thirteen classes: five from scientific high secondary school ("Liceo Scientifico"), three from technical institute ("Istituto Tecnico Commerciale e Turistico") and three from high school devoted to humanities ("Liceo Classico e Linguistico") in Trieste (Italy).

At variance the sample for comparing observations of the prevalent patterns of Math/Phys interplay in the last year of the curricular studies (students 18-19 years old) only corresponds to two classes from scientific high secondary school ("Liceo Scientifico").

### 2.3. Monitoring data collection

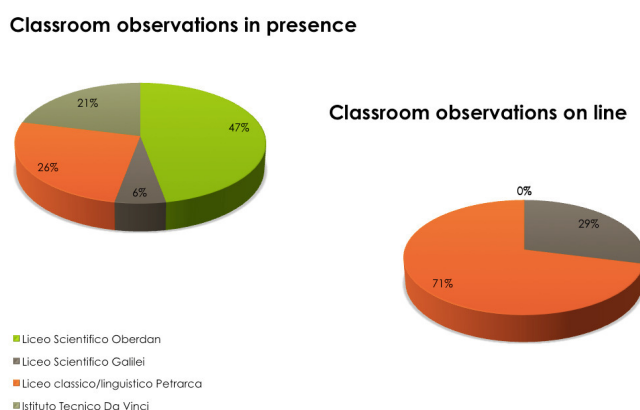
We observed teachers in classroom activities for about 153 hours as it is shown in Table 3. We emphasize in the table the hours of the first years of physics studies, referred as monitoring action, and those of the last year in the context of the testing of educational case studies.

In Figure 2 we reported the distribution percentage of observations divided into the different high schools involved.

**Table 3.** Hours of direct observations in classroom activities.

	Monitoring		Testing	
	Hours in presence	Hours online	Hours of case studies	
High school for Scientific Studies "Oberdan"	40	0	30	
High school for Scientific Studies "Galilei"	5	11	0	
High school for Ancient and Modern Languages Studies "Petrarca"	22	27	0	
High School for Technical Studies "Da Vinci"	18	0	0	TOTAL HOURS
	85	38	30	153

The major percentage of our observations was made in high school for scientific studies: the curriculum consists in two hours per week in the first two years while the last three years have three curricular hours per week. In all the other schools the hours per week are two and the physics study is covered for two or three years at most.



**Figure 2.** Comparison of hours of observation (in percentage) between different high schools involved and between in presence and online monitoring.

During the monitoring phase we collected a lot of information about teachers' way of conducting lessons, how many time they spent for contents' explanations and what is the time spent in mathematical demonstration of physics laws with respect to the total. We focused the attention on their dialogic teaching [9] and if there were implemented scaffolding [10] or other kind of students' supports in learning process.

We then devoted particular interest to problem solving strategy adopted by teachers and the corresponding students' response in question times and resolving tasks (in terms of requested formulas, applied procedures rather than argumentations, explanations and conceptual inferences asked to students).

We tried to notice which kind of interaction between Math and Phys was drawn by teachers in their activities and if they could be referred to the Lehari's patterns [3], observing the use of graphs, equations, modelling structures and laws applications in problem solving situations. In the meanwhile we took particular attention to the time spent in argumentations [11], as a way for promoting dialogic learning process (how many times teachers ask students phenomenological causes, evidences and explanations, even in the resolution of simple exercises).

### 3. Monitoring analysis and results

The PCK is the teacher's footprint. So, according to all the information we collected and the monitoring method we used, we were able to depict one PCK for each of the teachers that took part to our study. Remarkably, we observed that even if each teacher has his/her own PCK, which is strictly connected to academic degree and pre-service and in-service training, we found a great number of analogies and similarities among them that are not dependent on the specific school. These similarities are referred to the way of conducting lessons, the weight of math demonstrations and the short time reserved to ask students a deep argumentation of their results (in problem solving) or to extend their conceptual understanding.

This finding guided our analysis to catalogue a certain numbers of these matching features and to identify them as part of the educational process involved in the first year of physics teaching, and of course not directly dependent on the teacher's footprint.

On the contrary we could think that physics teaching has some independent features that moves the teacher to create a matter-dependent footprint.

This means that some of the following results coming from our analysis of the PCK correspond to a specific domain discipline feature and not to a specific teacher. We could also conclude that the teachers' orientation toward the discipline is influenced by the application pattern often adopted.

In our opinion by determining analogies and similarities we could identify some teachers' needs, some factors reducing their efficacy in learning processes, what could make teachers aware of their footprints.

More specifically in our analysis we distinguished two steps: the first examines the analogies and the similarities according to the Etkina's PCK model [5], while in the second step we tried to identify the prevalent pattern used by teachers at the beginning and at the end of curricular studies according to the Lehavi et al. [1].

### 3.1. First step analysis and results

Going into details of our observations, in Table 4 we summarized what are the common features characterizing the monitored teachers.

**Table 4.** Monitoring results correspondent to specific aspect of the PCK.

Aspect of PCK	Results from monitoring analysis
Orientation to Science Teaching	It would be very important to carry out laboratory activities but these require a large amount of school time.
	To understand a physical law, students need to solve many problems.
Knowledge of curricula	Students should arrive in the first year of learning physics with more consolidated mathematical skills.
	To facilitate students' study, curricular activities almost always follow the sequence proposed in the textbook.
Knowledge of students' prior understandings and difficulties with key concepts and practices in science.	Sometimes complex phenomena and processes (including extreme cases and borders of validity and approximation), difficult to be described mathematically, are omitted in the curricular treatment.
	Students' mathematical difficulties clearly emerge when considering the use of procedures toward solution, the manipulation of formulas (solving equations, using the properties of power laws, using the vector representation of quantities), the simple use of the calculator.
Knowledge of instructional strategies to scaffold students' learning of key concepts and practices in science.	Students' intuitive knowledge is generally not used for the conceptual construction of a new knowledge.
	In practical and laboratory activities, the teacher's support aspect (scaffolding) by the teacher is more structured and calibrated. The student is guided step by step through the activity; in the "theoretical" lesson in the classroom this guiding role is attributed to the mathematical treatment.
Knowledge of what to assess and specific strategies to assess students' understandings of key concepts and practices.	A lot of attention is given to the mathematical demonstration of a physical law, using mathematics also for an argumentative support.
	Written / oral tests and class exercises based on problem solving with the following characteristics: <ul style="list-style-type: none"> <li>• computational</li> <li>• descriptive</li> <li>• procedural / manipulative of formulas</li> <li>• with limited requests for argumentation</li> </ul>
	Control questions during explanations frequently aimed at clarifying mathematical rather than physical aspects

All of these have been collected during the three different times illustrated in the section above. They are teachers' sentences during their interviews or in the recorded discussions pre and post lessons. We tried to catalogue them referring to the five points of Etkina's PCK model for physics teaching.

### 3.2. Second step analysis and results

A close reading of all the information, the discussions with the teachers and the attempt of grouping the observations according to teachers' features in the five points of their PCK, gave us some suggestions about what could be revealed which is the prevalent pattern used by teachers in the first year of the Physics studies. We weren't surprised to find that teachers' preference is for the application pattern, with some weak integration with the construction one. In both patterns Math controls the process even in an early Physics learning experience. Therefore the Phys-Math interplay is reduced to these two aspects and only at the end of the studies the other patterns (the exploration and the broadening one) are sometimes used in teaching activities.

By searching the analogies between teachers' footprints and their prevalent application pattern, we found other interesting results.

First of all, if the teachers are aware of students' difficulties in Math, or of the absence of Phys-Math interplay, they try to support their learning process focusing on mathematical languages. This causes a large use of math in the demonstration of physics laws and a great number of math exercises applied to physics phenomena, with the consequence that even the evaluation tests seem to be mathematically rather than physically oriented, for the lack of a requested argumentation.

On the other hand, where the lack of students' math skills is relevant, the teacher adopts strategies converging to the strong use of observation/description language for conceptualization. What happens is that some teachers try to resolve the students' difficulties in Math using improperly the different language structures (formulas, graphs...) making a conceptual reduction of Physics, separating what comes from mathematics results and what corresponds to a physical phenomenological observation.

This kind of approach tends to amplify the distance between the two disciplines instead of favouring their interplay and integration also in a form of interdisciplinarity to be thought and to be taught.

## 4. Conclusion and Perspectives

According to the evidence suggested by Pospiech [6], it might be useful to introduce students right from the beginning in the interplay of physical phenomena of objects, a physical model and a mathematical model. This is possible if teachers are encouraged to give a variety of different tasks which require flexible use of mathematics as well as physics concepts.

From our analysis it emerges that currently the monitored teachers adopt indeed a use of mathematics less flexible than what could be desired to obtain a better integration between the two disciplines. In this sense we could identify a need to provide further in service training to achieve the main goals of the Phys/Math interplay in the classroom's practices.

In order to make teachers integrate in their strategies the Phys-Math patterns interplay, we propose to create a stable community for the professional learning, where the teachers discover and become more aware of their PCK and try to design his/her own footprint with a particular attention to the use of Math in Physics, and Physics in Math, taking also into account the different students' age, and the corresponding different cognitive skills, when Physics studies start. During the observation and monitoring activities, the discussion with the teachers, proved already the benefit of such a kind of in service community among teachers and researchers in pedagogy and physics education.

It is also clear that teachers need to be scaffolded in this process: they have to discover that the interplay of those two disciplines is strictly connected with a more nested integration of the two characteristic languages. The Research in Educational Physics is now suggesting some interesting ways to achieve these goals, from Multiple Representation [7, 8] to Content Representation [12].

A preliminary work with some teachers has started along this direction, aimed at their professional in service improvement, which is a warranty for a footprint, meaningful and successful for the effective learning of Physics.



## 5. Acknowledgements

We are grateful to teachers and students of Italian schools for their hospitality and great willingness to take part in the *Early Physics Research Project*:

- Liceo Scientifico Statale G. Oberdan, Trieste;
- Liceo Scientifico Statale G. Galilei, Trieste;
- Liceo Classico e Linguistico F. Petrarca, Trieste;
- ISIS Carli, Da Vinci, Sandrinelli, Trieste.

## 6. Authors' ORCID iD

Valentina Bologna, 0000-0003-4390-4950

Francesco Longo, 0000-0003-2501-2270

Maria Peressi, 0000-0001-6142-776X

Paolo Sorzio, 0000-0003-0853-0199

## 7. References

- [1] Lehavi Y, Bagno E, Eylon B, Mualem R, Pospiech G, Böhm U and others 2015 Towards a PCK of physics and mathematics interplay *The GIREP MPTL 2014 Conference Proceedings* ed C Fazio, S Mineo and R Maria (Palermo: Università degli Studi di Palermo) pp 843–853
- [2] Pospiech G, Eylon B, Bagno E and Lehavi Y 2019 Role of Teachers as Facilitators of the Interplay Physics and Mathematics *Mathematics in Physics Education* ed G Pospiech, M Michelini and B Eylon (Springer Nature Switzerland) pp 269-291
- [3] Lehavi Y, Bagno E, Eylon B, Mualem R, Pospiech G, Böhm U, Krey O and Karam R 2017 Classroom Evidence of Teachers' PCK of the Interplay of Physics and Mathematics *Key Competences in Physics Teaching and Learning (Springer Proceedings in Physics vol 190)* ed T Greczylo and E Debowska (Springer International Publishing Switzerland) pp 95-103
- [4] Magnusson S, Krajcik J and Borko H 1999 Nature, sources and development of pedagogical content knowledge for science teaching *Examining pedagogical content knowledge: The construct and its implications for science education* ed J Gess-Newsome and N G Lederman (Dordrecht: Kluwer Academic Publishers) pp 95 -133
- [5] Etkina E 2010 Pedagogical content knowledge and preparation of high school physics teachers *Phys. Rev. Sp. T.: Phys. Ed. Res.* **6** 1
- [6] Pospiech G 2019 *Framework in Mathematization in Physics form a Teaching Perspectives Mathematics in Physics Education* ed G Pospiech, M Michelini and B Eylon (Springer Nature Switzerland) pp 1-36
- [7] Opfermann M, Schmeck A, Fischer H E 2017 Multiple Representations in Physics and Science Education – Why Should We Use Them? *Multiple Representations in Physics Education. Models and Modeling in Science Education* ed D Treagust, R Duit and Fischer H (Springer, Cham) pp 1-22
- [8] Etkina E, Warren A and Gentile M 2006 The Role of Models in Physics Instruction *The Physics Teacher* **44** p 34
- [9] Robin A 2018 Developing dialogic teaching: genesis, process, trial, *Res. Pap. in Ed.* **33(5)**, 561
- [10] Levrini O, Tasquier G, Branchetti L and Barelli E 2019 Developing future-scaffolding skills through science education *Int. J. of Sc. Ed.* **41(18)** p 2647
- [11] Wang J and Buck G A 2016 Understanding a High School Physics Teacher's Pedagogical Content Knowledge of Argumentation *Journal of Science Teacher Education* **27(5)** p 577
- [12] Juhler M V 2016 The use of lesson study combined with content representation in the planning of physics lessons during field practice to develop pedagogical content knowledge *J. Sc. Teac. Ed.* **27(5)** p 533