Multiple representations of work–energy processes

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An energy process can be represented by verbal, pictorial, bar chart, and mathematical representations. This multiple-representation method for work–energy processes has been introduced and used in the work–energy part of introductory college physics courses. Assessment indicates that the method, especially the qualitative work–energy bar charts, serves as a useful visual tool to help students understand work–energy concepts and to solve related problems. This paper reports how the method has been used to teach work–energy concepts, student attitudes toward this approach, and their performance on work–energy problems.

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I. INTRODUCTION

Experiments in cognitive science and physics education indicate that experts often apply qualitative representations such as pictures, graphs, and diagrams to help themselves understand problems before they use equations to solve them quantitatively. In contrast, novices use formula-centered methods to solve problems.[1] Studies in physics education also have found that student problem-solving achievement improves when greater emphasis is placed on qualitative representations of physical processes.[2–10] In this method, a typical physics problem is considered as a physical process. The process is first described in words—the verbal representation of the process. Next, a sketch or a picture, called a pictorial representation, is used to represent the process. This is followed by a physical representation that involves more physics-like quantities and descriptions such as free-body diagrams and graphs. Finally, the process is represented mathematically by using basic physics principles to describe the process. The pictorial and physical representations are often called qualitative representations, in contrast to the quantitative mathematical representation. In this paper, the use of verbal, pictorial, physical, and mathematical representations is called multiple-representation problem solving. An example of multiple representations for a kinematics process is shown in Fig. 1.

In terms of multiple representations, the goal of solving physics problems is to represent physical processes in different ways—words, sketches, diagrams, graphs, and equations. The abstract verbal description is linked to the abstract mathematical representation by the more intuitive pictorial and diagrammatic physical representations. First, these qualitative representations foster students' understanding of the problems since, as visual aids, they automatically enhance human perceptual reasoning.[12] Second, qualitative representations, especially physical representations, build a bridge between the verbal and the mathematical representations. They help students move in smaller and easier steps from words to equations. Third, qualitative representations help students develop images that give the mathematical symbols meaning—much as the picture of an apple gives meaning to the word ''apple.'' After representing the process, students can obtain a quantitative answer to the problem using the mathematical representation. However, the main goal is to represent a process in multiple ways rather than solving for some unknown quantity (see the example in Fig. 1).

If students understand the meaning of the symbols in the equations and the meaning of the diagram, they can work backward to invent a process in the form of a sketch or in words that is consistent with the equations or with the diagram. For example, they should be able to construct diagrammatic, pictorial, and verbal representations of the dynamics process that is described mathematically in Fig. 2. As Howard Gardner says, ''Genuine understanding is most likely to emerge...if people possess a number of ways of representing knowledge of a concept or skill and can move readily back and forth among these forms of knowing.''[13] Thus we might say that an important goal of physics education is to help students learn to construct verbal, pictorial, physical, and mathematical representations of physical processes, and to learn to move in any direction between these representations.

The examples shown in Figs. 1 and 2 involve kinematics and dynamics. As we know, most physics professors and teachers solving dynamics problems rely on diagrammatic force representations—a free-body diagram or a force diagram. There is, however, no similar representation for solving work–energy problems. This paper describes qualitative work–energy bar charts that serve the same role for analyzing work–energy processes as motion diagrams and force diagrams serve when analyzing kinematics and dynamics problems. We find that use of these bar charts helps students think more about the physics of a work–energy process rather than relying on formula-centered techniques that lack qualitative understanding.

In the remainder of the paper, a multiple-representation strategy for helping students analyze work–energy processes is described along with several examples of its use. We first look at strategies that are important for a successful analysis of a work–energy process. We then introduce the energy equivalent of a force diagram—a qualitative work–energy bar chart. Finally, we analyze student attitudes toward this method, their problem performance, and their actual use of this strategy.

II. WORK–ENERGY PROCESSES AND WORK–ENERGY BAR CHARTS

There is considerable diversity in the way that physics faculty solve work–energy problems. The procedure used in this paper has its roots in a book by P. W. Bridgman.[14] Much attention is paid to a system, to its changing character, and to its interaction with its environment.

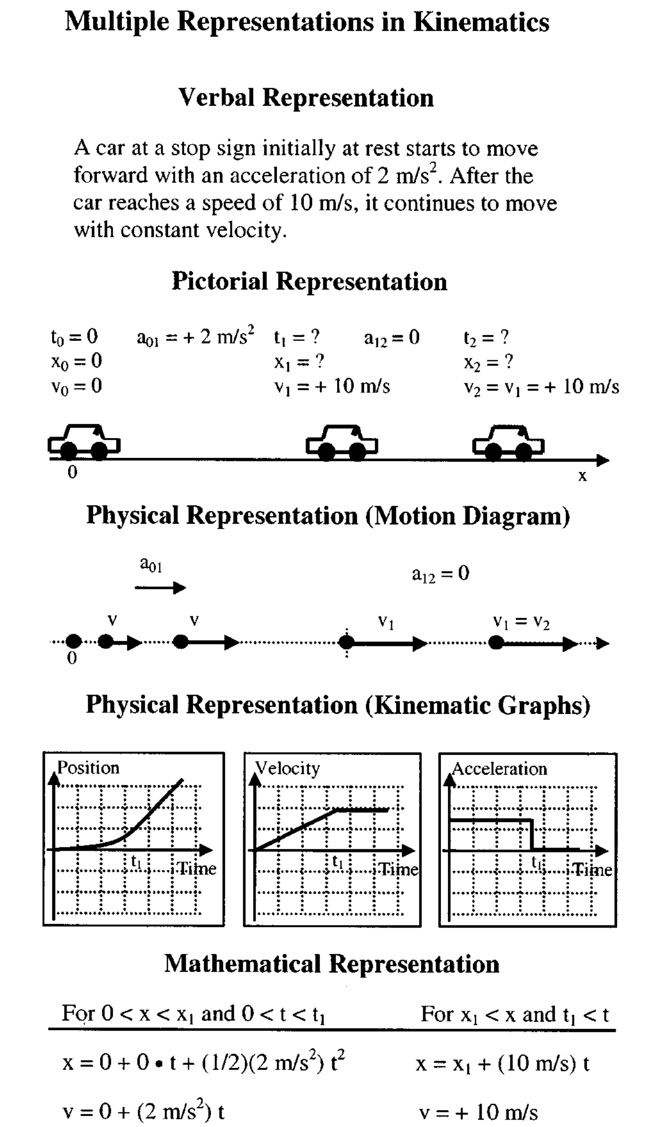

Fig. 1. The kinematics process described in the problem can be represented by qualitative sketches and diagrams that contribute to understanding. The sketches and diagrams can then be used to help construct with understanding the mathematical representation.

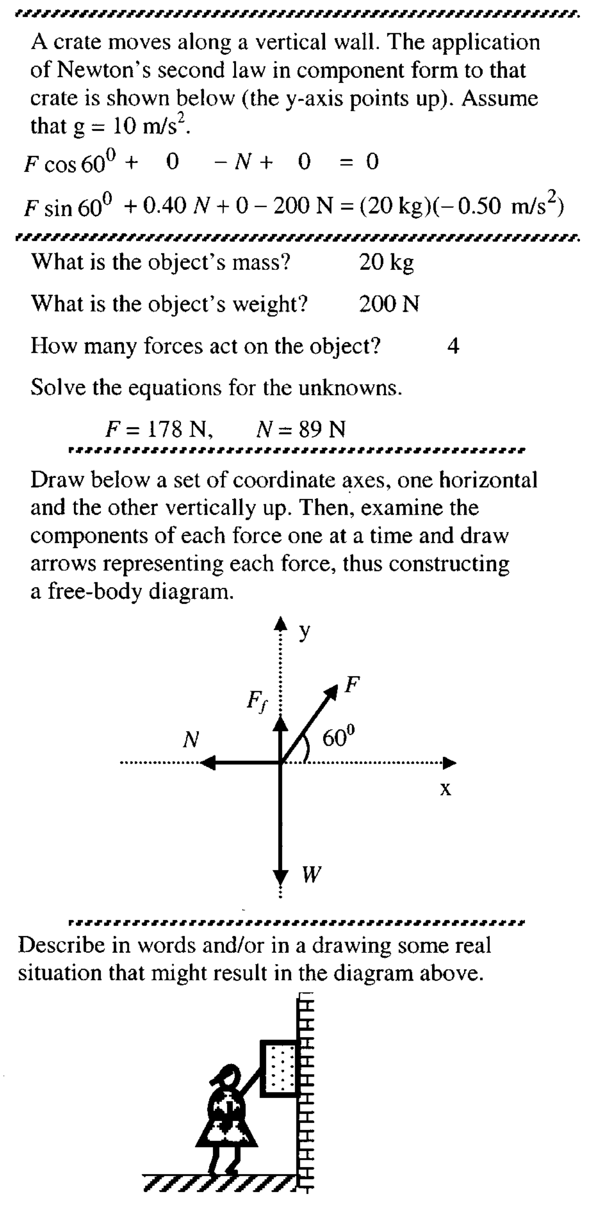

Fig. 2. The physical process described in the mathematical equations can be represented by diagrams, sketches, and words. The diagrams and sketches aid in understanding the symbolic notations, and help give meaning to the abstract mathematical symbols. (There could be more than one diagram and sketch consistent with the mathematical equations.)

To use this approach with the concepts of work and energy, students must learn certain conceptual ideas and skills, including:

- choosing a system—the object or objects of interest for the process being considered;
- characterizing the initial state and the final state of the process;
- identifying the types of energy that change as the system moves from its initial state to its final state and the signs of the initial and final energies of each type;
- deciding if work is done on the system by one or more objects outside the system as the system changes states;
- developing the idea that the initial energy of the system plus the work done on the system leads to the final energy of the system—the energy of the universe remains constant;
- constructing an energy bar chart—a qualitative representation of the work–energy process; and
- converting the bar chart to a mathematical representation that leads to a problem solution.

The qualitative work–energy bar charts illustrated in this and subsequent sections play a significant role in filling the gap between words and equations when using the concepts of work and energy to solve physics problems. Students are provided with a series of problems in which their only task is to convert words and sketches into qualitative work–energy bar charts and then into a generalized form of the work–energy equation. As shown in Fig. 3, a bar is placed in the chart for each type of energy that is not zero. For this process (neglecting friction), the system includes the spring, the block, and Earth. The initial energy of the system is the elastic potential energy of the compressed spring. This is

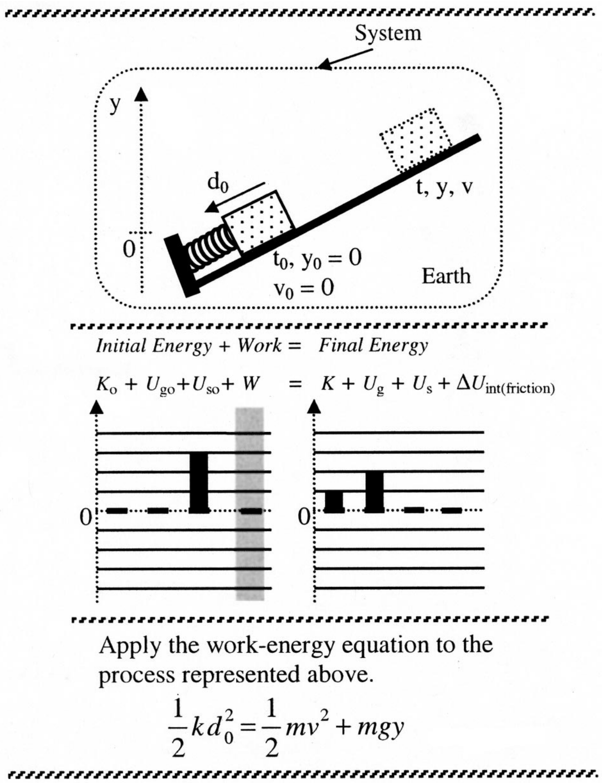

Fig. 3. The work–energy problem is originally described in the detailed sketch. Students are asked to convert the sketch into a qualitative bar chart—a bar is placed in the chart for each type of energy that is not zero, and the sum of the bars on the left is the same as that of the bars on the right. Then the generalized mathematical work–energy equation without any numbers is set up with one energy expression for each bar on the chart. (Notice that the work part in the bar chart is shaded so as to distinguish conceptually between work and energy, that is, work is a process quantity, but energy is a state quantity.)

converted into the final energy of the system, that is, the kinetic energy of the block and the gravitational potential energy due to the separation of the block and Earth. Since no external forces affect the process, no external work is done on the system. Hence no bar is drawn for the work part of the chart. To conceptually distinguish between work and energy (e.g., work is a process quantity, but energy is a state quantity), we shade the work part in the bar chart. (In teaching this work–energy bar chart method, we could either color the work part differently or show it as a separate region to indicate that work is conceptually different from energy.)

The relative magnitudes of the different types of energy in the bar chart are initially unknown, just as the magnitudes of forces in a free-body diagram are often initially unknown. The chart does, however, allow us to qualitatively conserve energy—the sum of the bars on the left equals the sum of the bars on the right.

We can also reason qualitatively about physical processes using the charts. For example, for the process represented in Fig. 3, we might ask students what changes occur in the process and in the chart if the final position of the block is at a higher or lower elevation. What changes occur in the process and in the chart if friction is added? What happens to the process and the chart if the block's mass is increased while keeping the other quantities constant?

III. SYSTEMS AND WORK

How do we decide for a given problem whether to calculate the work done by a force (for example, the work done by the gravitational force that Earth's mass exerts on an object) or to calculate a change in energy (for example, a change in gravitational potential energy)? A system approach provides a reason for one choice or the other. The book by Bridgman provides a nice introduction to the effect of system choice on the description of the work–energy process.[15] Burkhardt,[16] Sherwood,[17] Sherwood and Bernard,[18] Arons,[19] and Chabay and Sherwood[20] also discuss the importance of system choice. A system includes an object or objects of interest in a region defined by an imaginary surface that separates it from its surroundings. Choosing a system is the key to deciding what energy changes occur and what work is done. Work is done only if an object outside the system exerts a force on an object in the system and consequently does work on the system as the internal object moves.

This idea is illustrated in Fig. 4 where the same process (neglecting friction) is analyzed by using three different systems (an idea provided by Bob Sledz at Garfield High School in Cleveland). Notice that in Fig. 4(a) the cart and the spring are in the system, as is Earth. The process is not affected by objects outside the system. Thus no external work is done on the system. Potential energy change occurs when objects in a system change their shape or their position relative to other objects in the system. For example, the change in separation of a mass relative to Earth's mass in the system causes a gravitational potential energy change. The change in separation of two electric charges in a system causes a change in electrical potential energy. The change in the shape of a spring when it compresses or stretches causes a change in its elastic potential energy. In Fig. 4(a), the system's initial energy, the elastic potential energy of the compressed spring, is converted to the system's final energy, the kinetic energy of the cart and the gravitational potential energy due to the separation of the cart and Earth.

In Fig. 4(b), the system has been chosen with Earth outside the system. Thus, the Earth's mass exerts an external gravitational force on the cart and consequently does work on the cart as it moves to higher elevation. In this case, we do not count the system's gravitational potential energy as changing because Earth is not in the system. As we know, the negative work done by Earth's gravitational force on the left-hand side of the chart in Fig. 4(b) has the same effect as the positive final gravitational potential energy on the right-hand side of the chart in Fig. 4(a).

For the system in Fig. 4(c), the cart alone is in the system. Thus we now include the effect of the spring by analyzing the work done by the external force of the spring on the cart and the work done by the external Earth's gravitational force. The sum of these two work terms causes the system's kinetic energy to increase. There is no elastic potential energy change in the system since the spring is now outside the system.

In the above example, the system chosen in Fig. 4(a) is probably easiest to use in physics instruction. Gravitational potential energy is usually emphasized in high school and college introductory physics courses and is an easier concept for students to understand. It may, in practice, be easier for students to solve problems if they choose systems that include Earth. Similarly, it is more difficult for students to

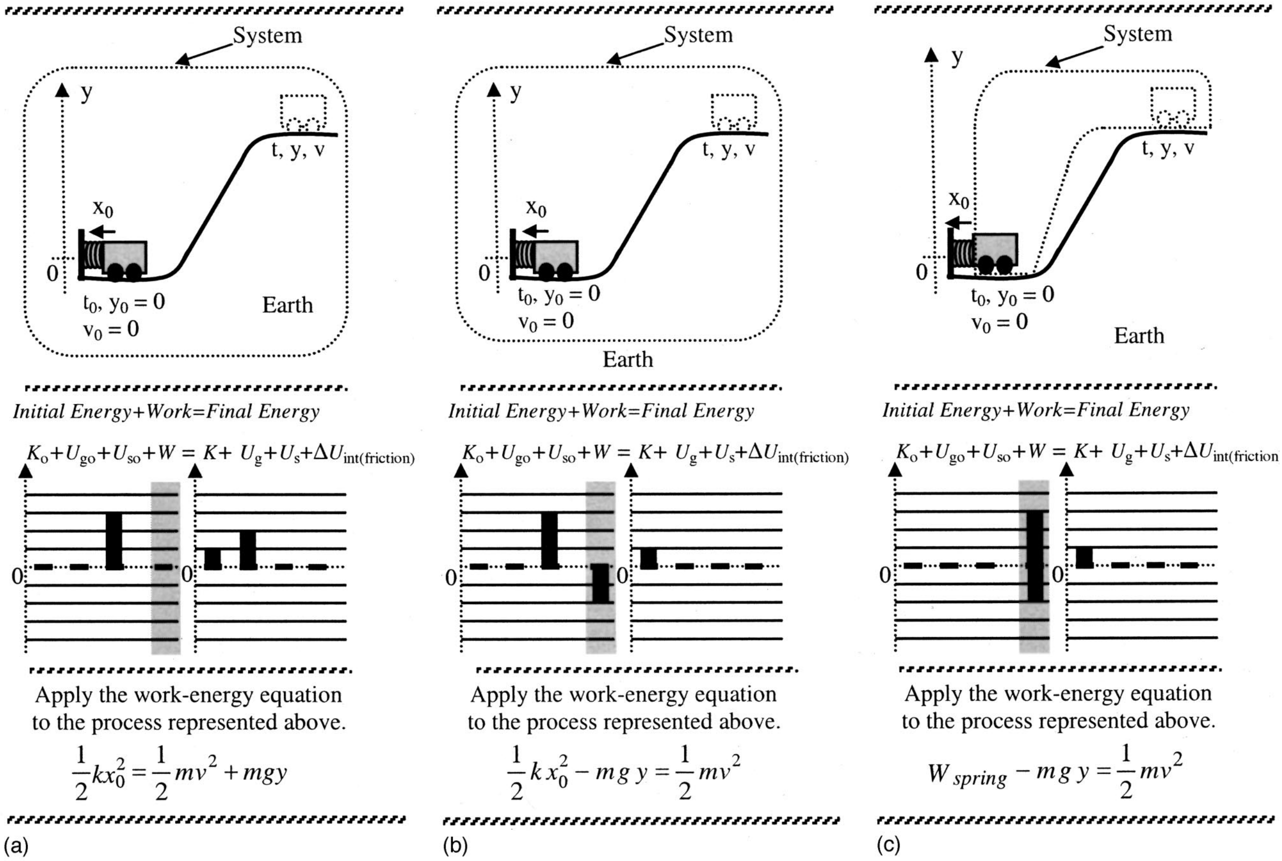

Fig. 4. The different systems are chosen for the same physical process. (a) The cart, the spring, and Earth are in the system. (b) The cart and the spring are in the system, but not Earth. (c) The system includes only the cart. For each chosen system there is one work–energy bar chart and the corresponding generalized work–energy equation. In practice, it would be easy for students to use a system that includes Earth and the spring, although the choice of the system does not affect the physical results.

calculate the work done by a spring than to calculate the elastic potential energy $\frac{1}{2}kx^2$ at the beginning and end of a process.

What about friction? Bridgman,[21] Sherwood,[17] Sherwood and Bernard,[18] Arons,[22] and Chabay and Sherwood[20] have discussed examples such as those shown in Fig. 5—one object sliding along a surface. Let us discuss a nonreal but simple situation first. In Fig. 5(a), we imagine the block is a point particle, and this point-particle block is moving until it stops on the frictional floor. The system includes only the point-particle block. So the floor exerts on the point-particle block an external frictional force which points in the opposite direction of the motion. This frictional force does a negative amount of work. This negative work in stopping the point-particle block has the same magnitude as the block's initial kinetic energy. But in a real and complex situation, such as the car skidding to stop on a rough road shown in Fig. 5(b), the car is a real object. If the car is the only object in the system, the road that touches the car causes an external frictional force. The work done by this frictional force is very difficult to calculate, if we look carefully at what really happens at the boundary between the car tires and the road. As Arons says (see Ref. 19, p. 151):

> What happens at the interface is a very complicated mess: We have abrasion, bending of ''aspirates,'' welding and unwelding of regions of ''contact,'' as well as shear stresses and strains in both the block and the floor. In this situation, it is very difficult to deal with the formal definition of work done by the frictional force. Ruth Chabay and Bruce Sherwood in their new textbook give a nice and detailed explanation about how to calculate the work done by the frictional force in such a situation—see *Matter and Interactions* (see Ref. 20, pp. 236–241). According to Chabay and Sherwood's arguments, the work done by this frictional force is still negative, but the magnitude of this work is less than the car's initial kinetic energy (and not equal to $-F_{\text{friction}}d$ either); the remaining amount of the car's initial kinetic energy is converted into internal energy of the car (see the following for detailed discussions about the internal energy).

Rather than dealing with this difficult work calculation, Bridgman[14] and Arons[19] recommend that the two touching surfaces, such as the car tires and the road, be included in the system as in Fig. 5(c). Since friction is no longer an external force, we look for energy changes in the system. Friction causes objects rubbing against each other to become warmer—the random kinetic energy of the atoms and molecules in the touching surfaces increases. Friction can also cause atomic and molecular bonds to break. This causes the potential energy holding atoms and molecules together to change. This happens, for example, when snow melts as a person's skis rub against the snow or when a meteorite burns due to air friction. A large number of bonds between rubber molecules are broken when a car skids to stop. Thus, if sur-

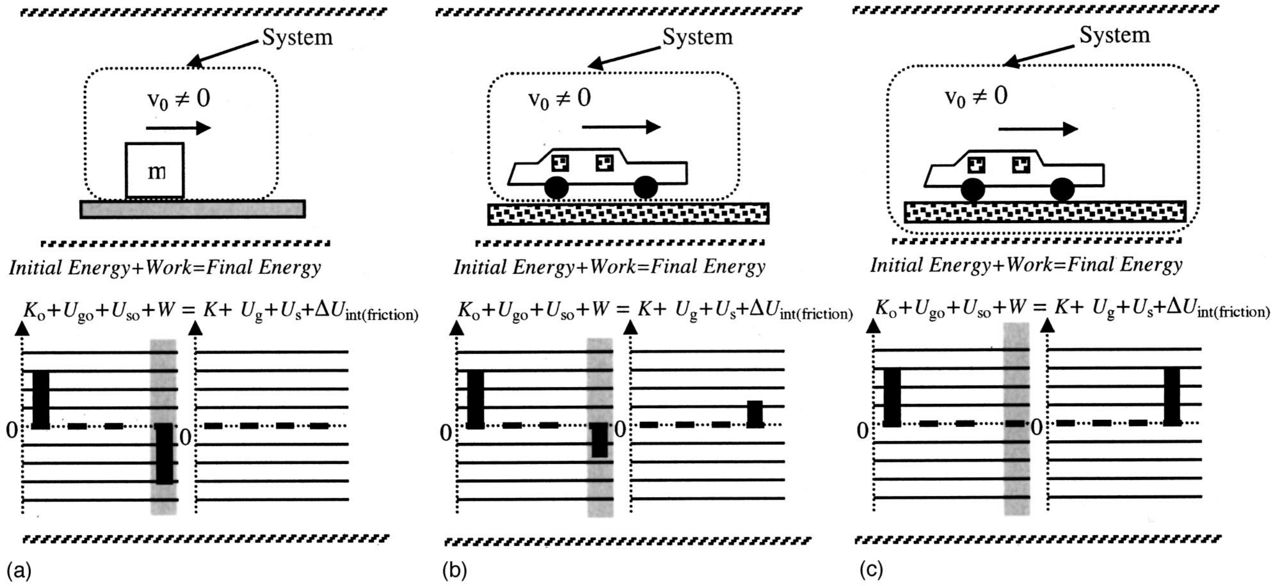

Fig. 5. The physical processes involve friction. (a) A point-particle block slides to a stop on a floor with friction. The system includes only the point-particle block. So the floor exerts an external frictional force on the point-particle block, and this frictional force does a negative amount of work, which has the same magnitude as the block's initial kinetic energy. (b) A real car skids to a stop on a rough road. The car is the only object in the system. Thus the road that touches the car causes an external frictional force and a difficult work calculation. Chabay and Sherwood argue that for such a real system that includes the car, the amount of (negative) work done on the car by road friction is less than the initial kinetic energy of the car. The remaining amount of the car's kinetic energy is converted into internal energy of the car (see Ref. 20, pp. 236–241 for detailed discussions about this advanced topic). (c) One sees a recommended system choice that includes the objects and the frictional interfaces between the objects in the system. In this way we can readily include the system's internal energy change due to friction rather than dealing with a complex work calculation.

faces with friction are in the system, the system's internal energy increases due to friction. The internal energy of the system is expressed by the last term, ΔU_{int}(friction), in the qualitative work–energy bar chart. Helping students visualize this form of energy leads naturally to an introduction to internal energy in thermodynamics.

IV. QUALITATIVE REPRESENTATIONS OF WORK–ENERGY PROCESSES

How do we use the bar charts with students? Figures 6–9 are examples of worksheets used by students to qualitatively analyze work–energy processes. Usually, the processes shown in the worksheets are demonstrated with real objects in the lecture or laboratory. In Fig. 6(a), a work–energy process is described in words and in a sketch. Students are asked to construct a detailed sketch that identifies the system and its initial and final states, includes a coordinate system, and indicates the values of relevant quantities in the system's initial and final states. Construction of a sketch such as that in Fig. 6(b), a pictorial representation of the process, is perhaps the most difficult task for students.

Having completed a pictorial representation of the process, students next construct a qualitative work–energy bar chart for the process (as in Fig. 7). The student looks at the initial situation and decides whether the system has kinetic energy, K_0. If so, the student places a short bar above the initial kinetic energy slot. If there is no initial kinetic energy, no bar is drawn. The bar for initial gravitational potential energy, U_{g0}, depends on the initial location of an object in the system relative to the origin of the vertical coordinate system. The student draws a positive bar if the object is at a higher position than the origin of the y axis, no bar if at the same elevation as the origin, or a negative bar if lower than the origin. Similarly, the bar for the initial elastic potential energy, U_{s0}, depends on whether compressed or stretched elastic objects are initially in the system. The work W bar depends on the presence of objects outside the system that

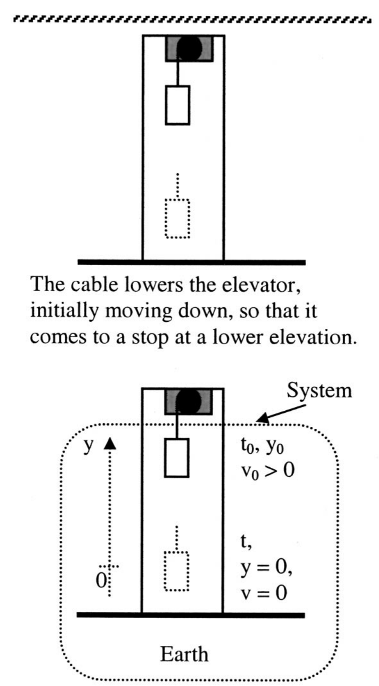

Fig. 6. The work–energy process is described in words and in a sketch. Students are asked to construct the pictorial representation, including a system choice and a coordinate system, and indicating the values of the quantities in the system's initial and final states.

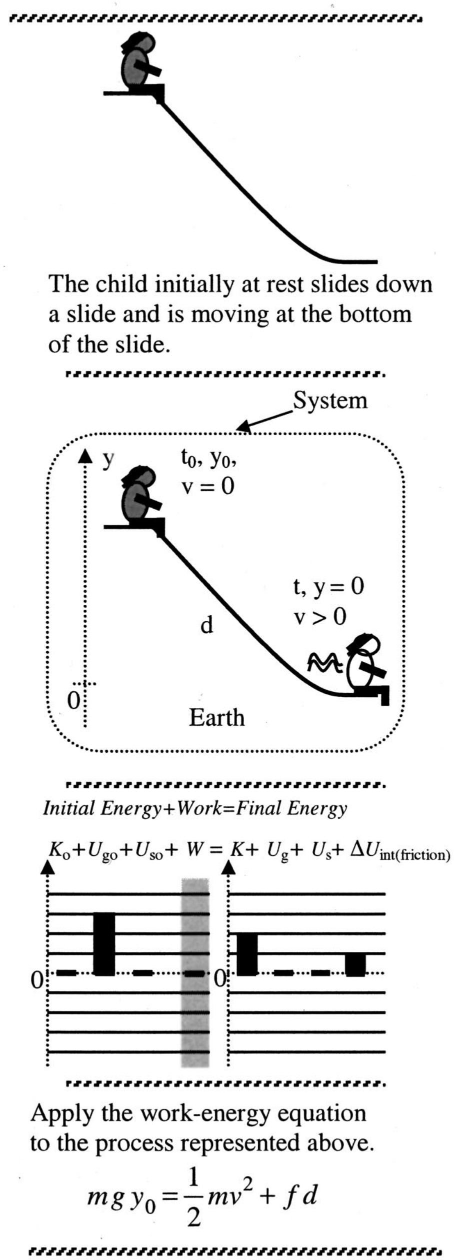

Fig. 7. For the given work–energy process, students are asked to construct the detailed sketch, and then convert it to an energy bar chart. Finally, they use the bar chart to apply the generalized work–energy equation.

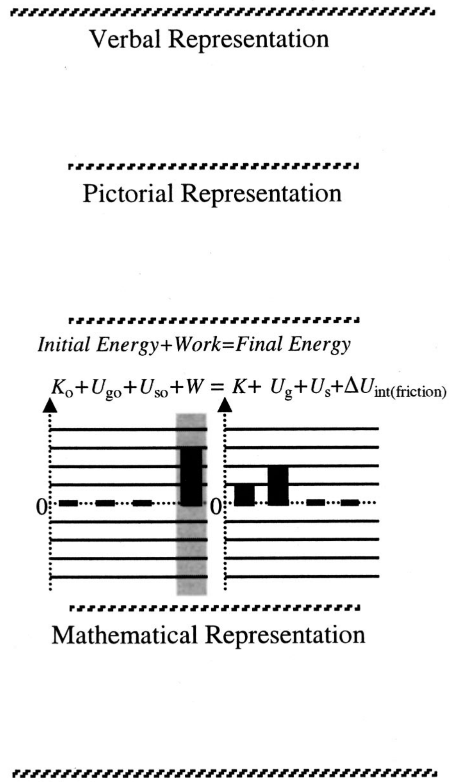

Fig. 8. A work–energy process is described by a work–energy bar chart. Students start with the bar chart and invent a sketch, a real-world situation in words, and a generalized work–energy equation that is consistent with the bar chart.

exert forces on objects inside the system as the system moves from its initial state to its final state. The sign of the work depends on the direction of the external force relative to the direction of the displacement of the object in the system. The final energy of the system is analyzed in the same way as the initial energy.

Having identified nonzero energy terms by placing short bars in the chart, we can now emphasize the conservation of energy principle by making the sum of the lengths of bars on the left equal to the sum of the bars on the right. The relative magnitudes of the bars on one side are usually unknown, just as the relative magnitudes of force arrows in a force diagram are often unknown.

Having constructed the bar chart, it is now a simple task to construct the generalized work–energy equation to describe this process. There is one term in the equation for each term in the bar chart. When detailed expressions for the types of energy are developed, students can include these expressions in the equations, as in the bottom mathematical representation in Fig. 7. For the problem in Fig. 7, students could be asked to think about how the chart and the actual process would change if the coefficient of kinetic friction was doubled.

In Fig. 8, students start with a complete bar chart and are asked to invent a verbal and pictorial description of a physical process that would lead to that bar chart, and to construct the mathematical representation of the physical process. (There are many processes that could lead to a particular chart.) In Fig. 9, students start with the mathematical representation of a process and are asked to construct a bar chart that is consistent with the equation, and to invent a process that would produce the equation and the chart—a so-called Jeopardy problem.[23] Students should now have a good qualitative understanding of work–energy processes—at least much better than when the processes are introduced first using a formal mathematical approach.

V. QUANTITATIVE WORK–ENERGY PROBLEM SOLVING

Students next solve quantitative problems using this multiple-representation strategy. They go from words, to sketches and symbols, qualitative bar graphs, a generalized work–energy equation, a solution, and finally an evaluation to see if their solution is reasonable. An example is shown in Fig. 10. A set of Active Learning Problem Sheets[24] (the

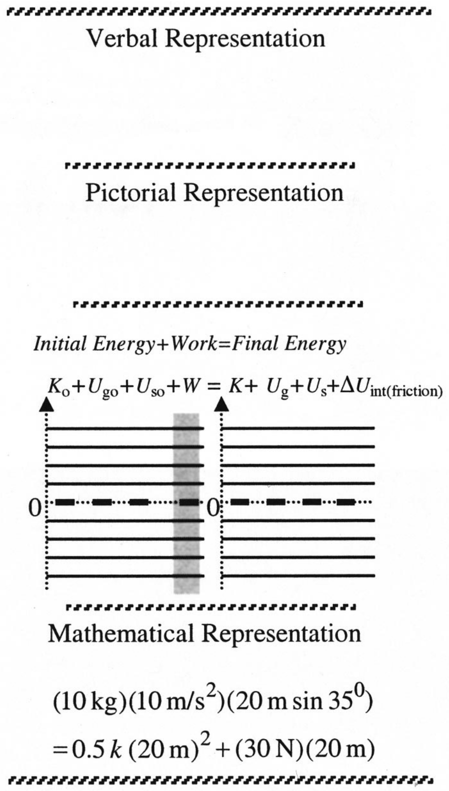

Fig. 9. The so-called Jeopardy problem starts with the mathematical equation for a work–energy process. Students are asked to construct a bar chart that is consistent with the equation, to draw a sketch, and to invent a process that would produce the equation and the chart.

ALPS Kits) has a Work–Energy Kit that includes 38 qualitative questions [such as illustrated in Figs. 6(a) and 8] and 16 quantitative problems (such as illustrated in Fig. 10). Students buy the ALPS Kits at the beginning of the semester or quarter. They solve some of these problems during lectures, during recitations, and for homework. Problems from the text are assigned with the proviso that the same format should be used on these problems. In addition, a set of 14 qualitative and quantitative work–energy problems (included as part of the ActivPhysics1 CD and workbook)[25] is used as active learning activities during lectures and laboratories. The problems are computer simulations, which include the dynamic work–energy bar charts to help students visualize energy transformations and conservation during physical processes.

VI. STUDENT ACHIEVEMENT

Do the energy bar charts and the problem-solving method involving the multiple representations of a work–energy process help students understand energy concepts and solve related problems? Do students have better performance on energy problems using this method than using other approaches? Do students use these multiple-representation strategies when rushing through an hour-long exam or a final exam? In the following, we try to answer these questions from a study done during the Fall 1997 and Winter 1998 quarters of the calculus-based introductory physics courses at The Ohio State University (OSU).

A. Student attitudes toward the energy bar charts and the multiple representations of work–energy processes

Students responded very positively on a three-question free-response survey concerning the energy bar charts and the method of the multiple representations. The survey was administered in the last week of the fall quarter 1997 to 67 honors engineering freshmen who learned this approach in a calculus-based introductory physics class.[26] This ten-week class covered the regular concepts of mechanics: kinematics, Newton's dynamics, momentum, work and energy, and some

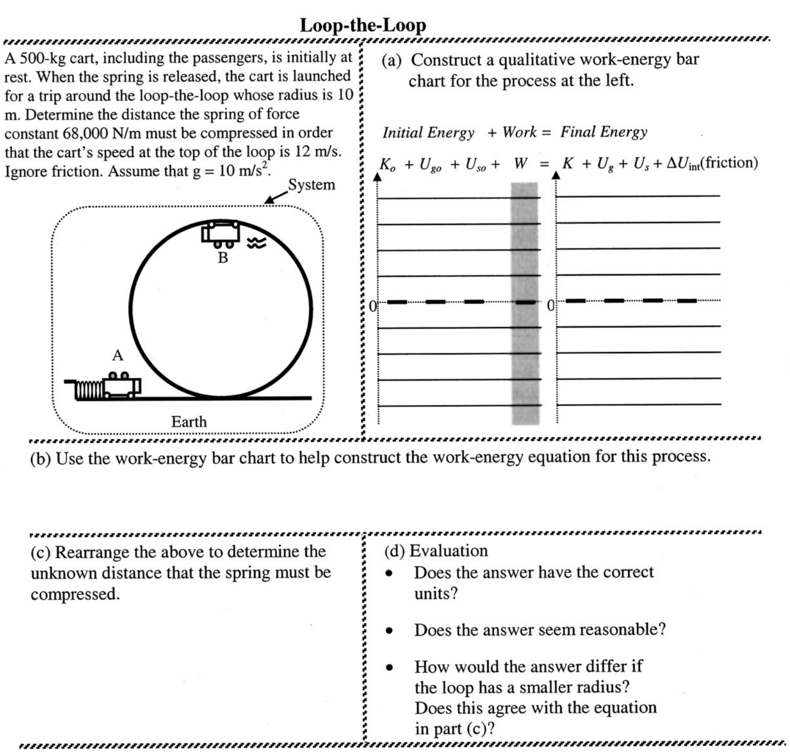

Fig. 10. One of the quantitative problems included in the Active Learning Problem Sheets. Students solve these problems using the multiple-representation strategy after having developed skills to construct qualitative representations. These multiple-representation problems help students develop qualitative understanding about the physical processes and develop problem-solving expertise, instead of using only an equation-centered method.

Table I. Students' responses for question 1 on the survey; Did using the energy bar charts help you learn energy concepts and solve work–energy problems? Explain why they were useful or not useful.

	92% Useful ($N=67$)
64%	64% of the students thought that the energy bar charts helped them visualize what is happening to different types of energy in energy problems, and set up the right equations to solve the problems easily. *Two examples of the students' responses:* • Greatly helped, because they provided visual representation of what's going on and made figuring out the equations easier. • The energy bar charts were extremely useful because they provided direction for each problem. You were able to see what types of energy were being used at different stages of the problems and what individual energy equations to apply to the problems.
15%	15% of the students thought that the energy bar charts helped them understand the abstract energy concepts and the energy conservation better. *Two examples of the students' responses:* • Yes, they were very helpful. They allowed me to see how and in what forms energy was conserved. • These were extremely helpful. Doing this made me change my way of think [*sic*] from thinking in terms of equation to concepts. Once I had the concepts down, then I can choose the right equations.
10%	10% of the students thought that the energy bar charts were helpful in learning the energy concepts at the beginning, and after a while they could create the bar charts mentally rather write them out. *One example of the students' responses:* • They were useful since they gave a visual way to go about the problems. After a while I got to the point that I automatically did them in my head did not need to really write them out.
3%	3% of the students thought that the energy bar charts were helpful, but they did not explain their reasons.
	8% Not useful ($N=67$)
8%	8% of the students preferred using equations directly rather the energy bar charts. *One example of the students' responses:* • No. I had trouble finding relationships between the energies involved using the energy bar charts. I was better off using the equations. The energy bar charts only showed before, after, which increased, or decreased, and which energy was more. Personally, I needed exact values, which working equations provided. I did not gain a better understanding of energy using the charts.

rotational dynamics. The students worked with the work and energy concepts for about six lecture periods, three recitation periods, one lab period, and in their homework assignments. Students' answers on each of the three free-response survey questions are summarized in Tables I, II, and III. From the results in Tables I and II (the results in Table III will be discussed in Sec. VI C), we can see that energy bar charts as a visual representation play a crucial role in helping students understand energy concepts and solve related problems. Most students thought the work–energy multiple representation technique was helpful for learning physics. They also felt it helped them develop problem-solving expertise, instead of using a formula-centered method.

Table II. Students' responses for question 2 on the survey; Did representing the work–energy processes in multiple ways—words, sketches, bar charts and equations—help you learn energy concepts and solve energy problems? Explain why they were useful or not useful.

	84% Useful ($N=67$)
84%	84% of the students thought that the multiple-representation strategy was helpful to understand the concepts better, and to set up the problems correctly, and was a good teaching method to help a variety of students learn physics. *Two examples of the students' responses:* • Doing these problems in many different ways with different descriptions helped to visualize the deeper basic concepts underlying them. Seeing a problem only one way, or learning a concept only one way, causes your knowledge of that concept to be very one-dimensional. Representing multiple ways helps me to see all sides of a concept. • They did help me by letting me see the problem different ways. Also I would like to note that this is a good teaching method to help a variety of students learn physics. Often times a student doesn't understand one method but sees another.
	9% Useful and not useful ($N=67$)
9%	9% of the students thought that the multiple representations were helpful to understand the energy concepts, but were sometimes a waste of time for quantitative problem-solving. *One example of the students' responses:* • Sometimes they were useful but doing each type of description for each problem was overkill and wasted a lot of time.
	7% Not useful ($N=67$)
7%	7% of the students thought that the multiple representations were not helpful except equations. *One example of the students' responses:* • No. Only the equations were of any use to me. The rest just seemed to get in the way.

B. Student performance on a tutorial-type problem

Although the energy bar charts and the multiple-representation approach are developed as a problem-solving strategy to help students acquire problem-solving expertise, they emphasize the development of qualitative understanding and reasoning about work–energy concepts and processes. Does performing this type of qualitative analysis improve student scores on conceptual work–energy questions? To address this question, we examined our students using a tutorial-type work–energy problem (shown in Fig. 11) developed by the Physics Education Group at The University of Washington (UW).[27,28]

The problem in Fig. 11 asks students to compare the final kinetic energy of two pucks having different masses pushed by the same force across the same distance. The OSU honors engineering freshmen were given this problem on a survey test at the end of winter quarter 1998—one quarter after they had studied the introduction of the work–energy method in fall quarter 1997. Out of the 56 students, about 60% of them gave correct answers with correct reasoning in words or by using equations. The same problem was given on the final exam to a regular OSU engineering calculus-based physics class after standard instruction in which the bar charts and the multiple representation strategy had not been used. About 20% of 147 students in this class provided correct explanations for the problem.

Table III. Students' responses for question 3 on the survey: Did you (or how did you) use the energy bar charts and the multiple representations of work–energy processes to solve energy problems while doing homework, group problems in recitation, problems in lab, and/or on exam problems? Did you use them when first becoming familiar with the concepts and then less as you become more expert at solving work–energy problems?

	95% Use in problem solving ($N=67$)
46%	46% of the students used the energy bar charts and the multiple representations to solve problems at the beginning, but less and less as they became more familiar with the problems. *Two examples of the students' responses:* • I used them less as I became more of an expert. I began to go straight to writing out the formulas (for each type of energy) and the equations. But, I kept the bar chart ideas in the back of my mind. • I did use them. I first used the bar chart just to see what parts I wanted in the equations, such as work from friction or potential spring energy. Then I would write the equation down and solve. Now I've become more able to solve problems. I still use the bars whenever I get confused. They really help to limit mistakes in writing the initial equation.
33%	33% of the students used the energy bar charts and the multiple representations most of the time when solving energy problems. *Two examples of the students' responses:* • I used the charts and representations throughout the process to be sure I was setting up the problem right without overlooking important details. Converting one representation to another was just one more way I double checked my work. • I used the charts every time, although I did't always write it down on paper. It's a safe guard [*sic*] that you are not neglecting any form of energy.
16%	16% of the students used the energy bar charts and the multiple representations in other ways. *Two examples of the students' responses:* • I personally only used the bar charts if I felt the situation was difficult enough to require a further analysis. However, at the beginning I used the bar charts for every problem and they really helped me grasp the concepts. • I used them very little in the very beginning, but then I realized their uses. Once I became familiar with them, I was able to be more comfortable solving problems. Now I have worked with many different problems, I use them to check my work.
	5% Never use in problem solving ($N=67$ in total)
5%	5% of the students never used this method, and they only used equations or got help from the textbook.

The paper by O'Brien Pride, Vokos, and McDermott (see Ref. 28) reports that after standard lecturing for 985 regular and honors calculus-based physics students from UW, 15% got correct answers with correct reasoning on this question. In addition, for 74 UW tutorial instructors (most of them graduate teaching associates) before teaching the Washington work–energy tutorial sections, 65% answered correctly with correct explanations. Furthermore, it is interesting to see from the paper that before the tutorials, 65% of 137 physics faculty from the national tutorial workshops successfully produced correct answers to the question shown in Fig. 11. The percentages of these different groups successfully

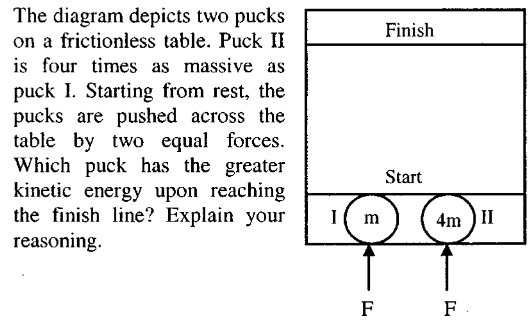

Fig. 11. The problem originally developed by the Physics Education Group at UW was administered to the OSU honors engineering freshmen one quarter after they had learned the work–energy method. The students were told that their scores on the problem did not affect their class grades. This problem was also given to 147 OSU regular calculus-based introductory physics students after standard instruction in which the bar charts and the multiple-representation strategy had not been used.

answering this question are summarized in the bar graph in Fig. 12. The OSU calculus-based honors engineering freshmen learning the multiple-representation strategy and using the energy bar charts performed much better on this problem than regular and honors calculus-based physics students both from OSU and from UW, and they did almost as well as physics faculty and physics graduate students.

C. Student use of the multiple-representation strategy in problem solving

How do students use the multiple-representation method to solve energy problems? In the survey with the three free-response questions administered to the 67 honors engineering freshmen, the last question asked how the students used the energy bar charts and the multiple representations for solving problems. A summary of the student responses is reported in Table III. We find that most of the students used this method to solve problems. Although about half of the students used it less and less when becoming more familiar with energy problems, many of them still kept the energy bar charts in their mind as a useful problem-solving tool to evaluate their solutions.

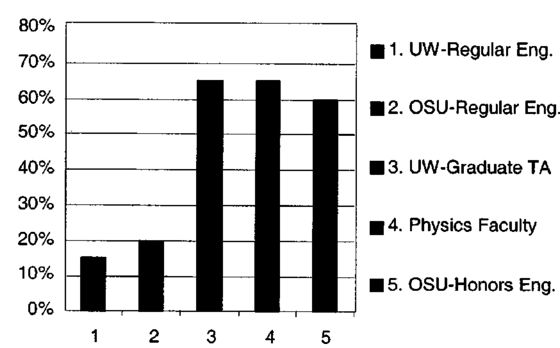

Fig. 12. The graph shows the percentage of each group that correctly answered the question shown in Fig. 11. Bars 1 and 2 indicate that no more than 20% of over 1000 calculus-based physics students from UW and OSU answered and explained the problem correctly after standard instruction. Bars 3 and 4 indicate that 65% of about 200 physics graduate students and faculty correctly provided the answers and reasoning for the problem on pretests before the UW tutorials. Bar 5 indicates that 60% of more than 50 OSU honors engineering freshmen successfully answered the question with correct reasoning.

But did the students actually apply the multiple-representation method when rushing through their exams? On the final exam problem given in this calculus-based honors engineering freshmen class, 66% of the students constructed a pictorial representation of the process. 57% of the students solved the problem using the work–energy method, but 43% of them applied Newton's second law. Among the 57% of students who used the work–energy approach, about 16% of them drew a qualitative work–energy bar chart, and 79% of them correctly constructed the generalized work–energy equation.

From the above results, we see that many students made a pictorial sketch to help them understand the problem, but not many actually constructed a work–energy bar chart on the final exam problem. There could be several reasons for this. With the time pressure needed to complete an exam, students skip steps that they feel are not needed to complete the solution. Students' responses on the survey question in Table III indicate that many students had become more familiar with work–energy problems and had learned to draw the bar charts mentally—they no longer needed to draw them explicitly. The similar case can be found for the students using the motion diagrams in solving kinematics problems. The students explicitly drew the motion diagrams less and less when they could mentally use the motion diagrams. This final exam problem may not have been difficult enough for most of the students to need to use the work–energy bar charts to complete the solution. Finally, it might be that some students felt the bar charts were not useful in problem solving, and so there was no reason to use them.

For a future study, we plan a more detailed experiment to investigate how the energy bar charts and the multiple-representation strategy help introductory physics students to understand qualitatively work–energy processes and to develop problem-solving expertise compared to similar students learning other work–energy approaches.

VII. SUMMARY

It is well known that students attempt to solve problems by matching quantities listed in the problem statement to special equations that have been used to solve similar problems. Students move between words and equations, which are very abstract representations of the world, with no attempt to connect either representation to more qualitative representations that improve understanding and intuition.

In an alternative strategy proposed by others and used here, we view physics problems as descriptions of physical processes. We ask students to represent these processes in various ways. Our preliminary study indicates several advantages for this method. For example, the linkage between the word description of the process and its pictorial and bar chart representations helps students produce mental images for the different energy quantities. The bar charts help students visualize the conservation of energy principle—the students can ''see'' the conservation of energy. A sketch with its system choice combined with the qualitative bar chart helps students reason about physical processes without using mathematics and helps them predict conceptually how changes in various factors will affect the process. A bar chart functions like a bridge, helping students move from the abstract realm of words, or from real-life sketches with surface features, to the abstract realm of scientific and mathematical notations. Additionally, this strategy aids students in overcoming formula-centered naïve methods and in developing expertise in problem solving.

The actual classroom strategies are very important. Students learn to learn better if they understand the reasons for various pedagogical strategies. We find, for example, that students in some classes regard the construction of sketches and bar charts as activities that are independent and unrelated to applying the conservation of energy principle to a problem. We have observed the same response when students are asked to construct free-body diagrams and to apply the component form of Newton's second law—these are considered as unrelated independent activities. For an experienced physicist, a qualitative representation is important for solving a challenging problem. For instance, a Feynman diagram provides a more visual representation of a scattering process. These more qualitative representations are used to help apply the more abstract mathematical representations correctly, for example, an *S* matrix for a scattering process. Students must understand why they use the qualitative representations.

However, an expert may subconsciously use a qualitative representation for an easier problem—for example, construct a mental free-body diagram. Students may avoid using qualitative representations early in their study because they do not understand what is being represented. It makes no sense for a student to draw a free-body diagram if the student does not understand the concept of force or the nature of different types of forces or how the free-body diagram is to be used to help solve a problem. Similarly, a bar chart is not helpful if a student does not understand the different types of energy or the conservation of energy principle or how the bar charts can be used to solve problems. Students must understand why they are learning to represent processes in the more qualitative ways and how these qualitative representations can be used to increase their success in quantitative problem solving.

We feel that students accept qualitative representations more easily, understand them better, and use them more effectively for qualitative reasoning and problem solving if the qualitative representations are introduced before the corresponding mathematical equations are introduced. There is less ''noise'' in the mind.

Many students have experienced only formula-centered didactic instruction. For these students, it may be difficult to apply this new multiple-representation method in their problem solving. Some students like only equations and think it wastes time or is a redundant task to represent a problem in different ways. For a novice with little conceptual understanding, this is not true. However, as students acquire understanding, some qualitative representations become mental schema and constructing paper versions is less necessary.

Finally, when students have learned all of the representation types, their understanding improves if they learn to move among representations in any direction. For example, they might be given an equation that is the application of the conservation of energy principle for some process. Their task is to construct other representations of that process—playing Equation Jeopardy.[23] They learn to ''read'' the symbolic mathematics language of physics with understanding.

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TURNING TO PHILOSOPHY

It is not uncommon for distinguished scientists in the twilight of their careers to turn their hand to philosophy. Unfortunately, the failures among such endeavors are generally acknowledged to outnumber the successes, and . . . 's contribution to the genre must on the whole be consigned to the majority.

John Dupré, in a book review in Science **280**, 1395 (1996).