Quantum Mechanics

Courtesy by S.Faletic

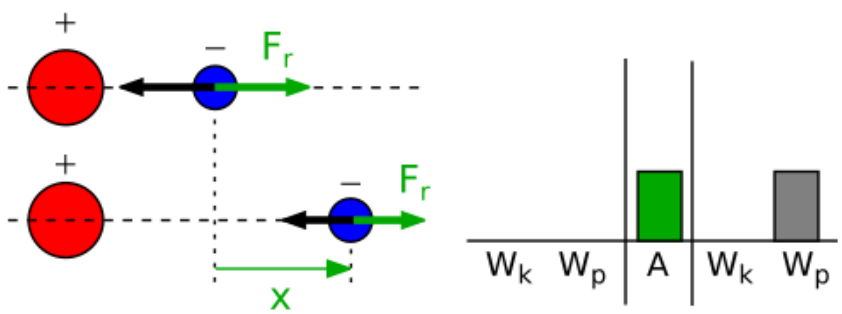
1st meeting

We have seen that an electron in an atom can only have certain values of energy, that its energy is quantised. Our ultimate goal is how to explain this, or what model we can build for the electron that incorporates this property.

So let's start with a model of the electron in an atom and see what else we can learn.

Potential well

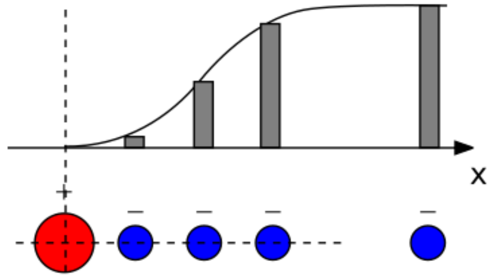
First, what does an electron "sense" in an atom?
It feels the attractive force of the core. But in quantum mechanics, it is more convenient to describe it in terms of energies. To move the electron further away from the nucleus, we have to overcome the attractive force. In doing so, we do work and increase some form of energy. Since the kinetic energy is zero at the beginning and zero at the end, it must be some other energy. We call it **electric potential energy** (W_p).[1] For the sake of brevity, we will call it potential energy, as we will ignore gravitational potential energy.

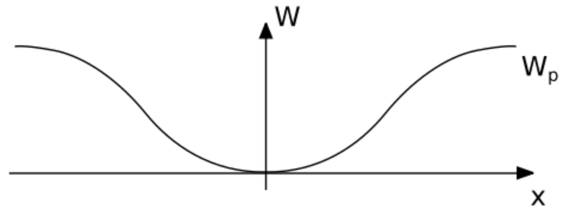


If we think about it a little, we can guess how this energy varies approximately with distance from the positive charge. If we plot the values of W_p at different positions and connect them, we

[1] Let us be clear that a single body cannot have potential energy. Potential energy is always the energy of a system of two or more bodies. It is a kind of energy of the configuration of the system, of the relative position of the bodies in the system, if this affects the energy. In our example, the system is an electron and a proton. In the case of gravity, a body and the Earth. If we consider potential energy, then we have to consider all the energies of all the bodies in the system. In our case, because of the large mass, we assume that the proton is stationary and therefore has no kinetic energy. The potential energy is the energy of both bodies anyway.

can obtain a graph of W_p versus position. From somewhere onwards, the effect is so small that W_p hardly changes any more.

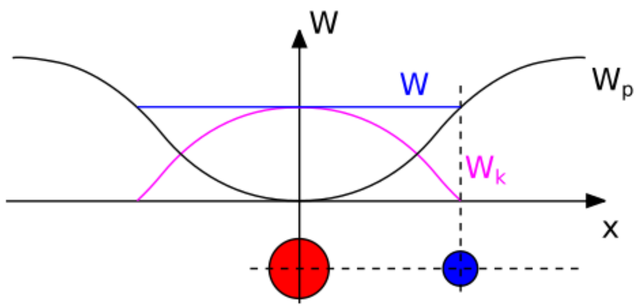




Since the potential energy is indeterminate up to an additive constant (i.e. zero can be chosen anywhere), it is sometimes convenient to choose zero at the lowest value, but usually in quantum mechanics it is chosen at the highest value, i.e. at the flat part where the electron and the proton no longer feel each other. The valley then has negative potential energy values. For obvious reasons, this form of potential energy is called a **potential cavity**.

Full energy of an electron

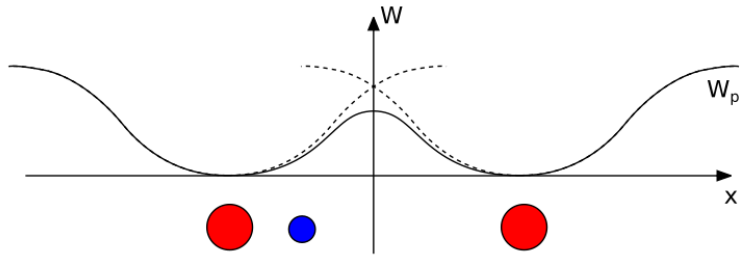
The energy we have mentioned in previous meetings, which is quantised, is the total energy of the electron-proton system. Let us now place the electron at such a distance from the nucleus as to raise its potential energy slightly. Instead of "holding" it in this position, we drop it. The electron will start to oscillate around the nucleus (assuming it does not collide with the proton). On the other side, it will reach the same level of potential energy as it did at the beginning, and this will repeat indefinitely until the electron loses energy. This can only be done by colliding with the nucleus or by emitting light.[2] Throughout the process, the potential energy will change to kinetic energy, and the full energy (the sum of all energies) of the electron will be constant.



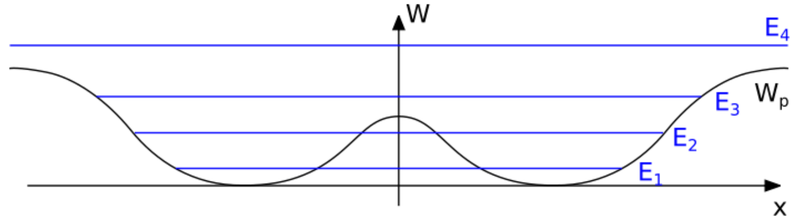
Double potential well

What happens when two protons are brought close enough that their potential cavities start to overlap. Then, in some area in the middle, the electron feels the influence of both protons, i.e. both potentials. The two potentials are added (subtracted) and the potential at the centre is slightly lower than it would be for each well separately.

[2] There is no friction or drag, because these are macroscopic quantities that represent the forces between atoms and molecules, and here we are well within one atom.



The energy in one atom is quantised. An atom is a potential cavity. So we can assume (and so it turns out) that the energy is quantised in each potential cavity. We do not yet know which values are possible. So let us assume some hypothetical values, as in the figure below.



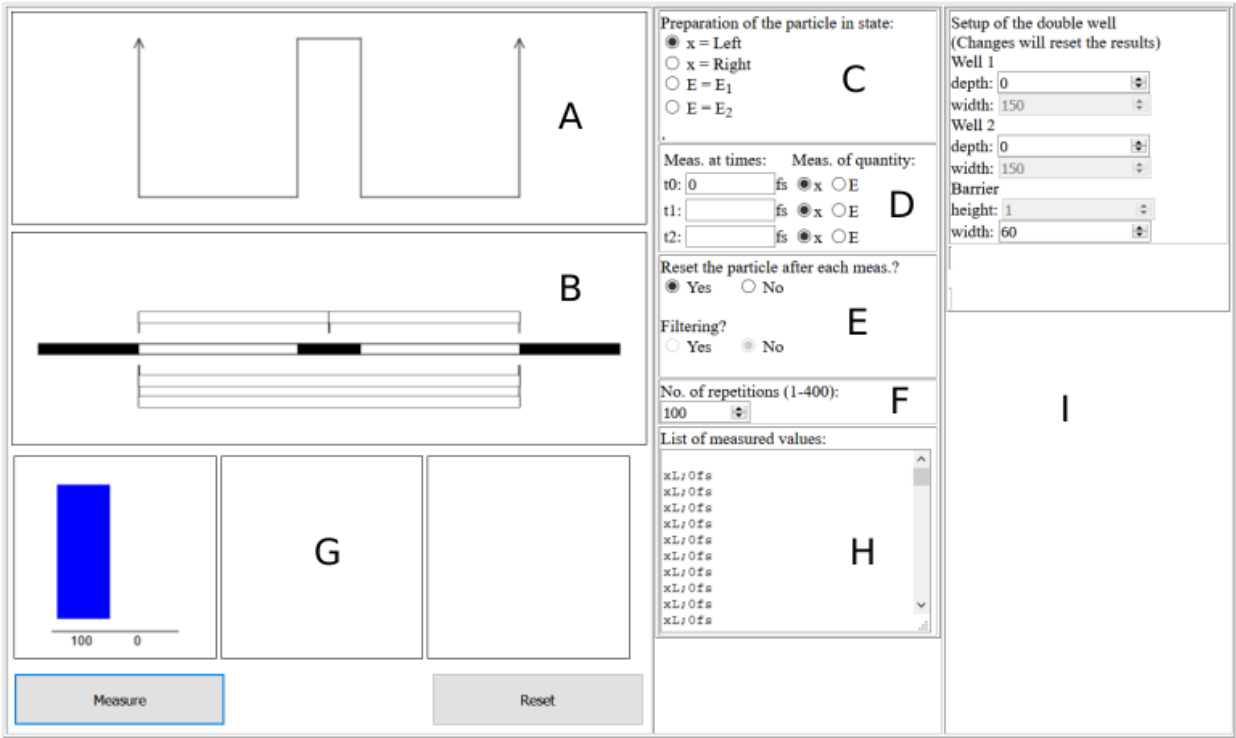
We can see that some values may be such that, in principle, they lock the electron into one of the cavities. E_1 and E_2 in the figure are such. The value of E_3 is such that it confines the electron to both wells. Such an electron is therefore a "property" of both atoms. This is a covalent bond. But there are certainly values outside the cavity, such as E_4 . Such an electron is no longer bound to the cavity and represents a knocked-out electron, as we have seen in the photoeffect.

In the following, we will deal with electrons with energies E_1 and E_2 . The double well is a system that will allow us to discover some important laws of quantum mechanics. In fact, it has two very well defined possible values of energy (E_1 and E_2) and two possible values of position, which we will denote by L - left well and R - right well.

Simulation of a double well

Our double well will be somewhat idealised. Instead of having horizontal edges, it will have vertical edges. What is more, these edges will be infinitely high. The middle partition will also be vertical. To simulate the measurement, we will use the simulation that can be found at the link below

- Simulation



The graphical interface of the simulation is shown in the figure above.

- Area A represents a potential well. We can see the vertical edges and the vertical central barrier. If we were to change the settings of the well (I), which we will not, we would also see changes in A.
- Area B shows what such a potential well might physically look like. It would be a wire (it must be one-dimensional), with areas where the electron can move freely (white) and areas where it cannot (black). The intermediate barrier would need to be slightly brighter, as the potential there is not infinitely high. With enough energy, the electron could in principle reach this region. If we change the settings (I), this would also be seen here. If we increase the potential in the white area it would become more grey, if we lower the middle barrier it would become more grey (less black).
- Area C allows you to set the initial state of the particle. In the example above, it is set to x=Left, which means that at time 0 the particle will be in the left well. We can also select the right well, one or the other energy.
- Area D allows you to set the measurement time. Three measurements can be taken. You choose which times and which quantity to measure in these settings.
- Area E allows us to perform the measurements one after the other (selection "No"), or to return the system to the initial state set in C (selection "Yes") before each of the three measurements set in D. If "Yes" is selected, the simulation returns the system to the initial state after the first measurement and after the second measurement. If "No" is selected, the simulation does not return the system to its initial state.
- Area F allows you to select multiple repetitions of the experiment. It is used instead of pressing the "Measure" button. When "Measure" is pressed, all measurements set in D are performed according to the settings of E. If three measurements are set, three are

performed. Pressing again puts the system in the initial state (C) and repeats all three measurements.

- Panel G shows the statistics of the results in the form of a histogram. How many times we got a left well and how many times a right well. Or how many times the energy value E_1 and how many times E_2 . For each measurement set in D, we have our own histogram.
- Area H extracts each result for possible later processing in Excel or a similar editor.
- Area I allows you to change the well settings. This will not be used.

In the example above, we set the particle in the left well at time 0 (C) and measure the position at time 0 (D). Only once. Then repeat.

2nd Meeting

[Simulation of a double well]

Introducing terms:
Quantity: e.g. position, velocity, energy

x - quantity: location

A line can take two values: x = L or x = R

L, R - values

We'll introduce another notation: x:L, x:R. It means practically the same as x = L or x = R, but we call it a property. An electron has the property that its position is in the left cavity, or the property that its position is in the right cavity.

x:L, x:R - properties: the property that the location is in the left well and the property that the location is in the right well.

Activity 1

Activity 1: (Un)predictability of the measurement outcome
Purpose: Find out what quantum mechanics can predict about the outcome of a measurement.
Tasks:

<table>
<tr><td colspan="2">Prepare the particle in the left well. Set the position measurement (*x*) at 30 fs. Take each measurement several times. Try to answer the questions as you go along. When you feel that it is no longer necessary to take time to think about each outcome, you can set the number of repetitions to 100 and perform 100 repetitions of the experiment. You can also repeat this measurement several times if necessary.</td></tr>
<tr><td>a) Have both sensors ever flashed at the same time? What does this mean in terms of the possible values that can be measured for each quantity?</td><td>When we observe the beats, we find that the sensors never both blink at the same time. This means that only one value can be measured for each quantity.</td></tr>
<tr><td>b) Is there a pattern in the order in which the individual values of x:L and x:R appear? Describe it if it exists. Write down the results of the measurement from which your answer is derived.</td><td>Five consecutive measurements, five repetitions each
<table>
<tr><td>xL;30fs</td><td>xL;30fs</td><td>xL;30fs</td><td>xL;30fs</td><td>xL;30fs</td></tr>
<tr><td>xR;30fs</td><td>xL;30fs</td><td>xR;30fs</td><td>xL;30fs</td><td>xL;30fs</td></tr>
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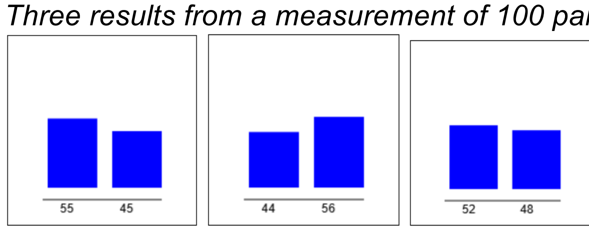
<tr><td>xL;30fs</td><td>xL;30fs</td><td>xL;30fs</td><td>xR;30fs</td><td>xR;30fs</td></tr>
</table>
The table shows that there is no obvious pattern. The values appear randomly.</td></tr>
<tr><td>c) Is there a pattern in the distribution of outcomes between x:L and x:R? If there is, describe it. Write down the results of the measurement from which your answer is derived.</td><td>Three results from a measurement of 100 particles:


The distributions are similar. We can also calculate the average number of particles in the left and right wells: 50:50.</td></tr>
<tr><td>d) Can you predict with certainty whether the next measurement on one particle will give x:L or x:R? Explain.</td><td>The observation that there is no pattern in the sequence of outcomes implies that we cannot predict the next outcome with certainty. Even if the probability of x:R is smaller, it is possible that this outcome will occur in the very next measurement.</td></tr>
<tr><td>e) What can you predict about the future outcomes of experiments with the same settings? Explain.</td><td>The most we can predict is obviously the probability of a particular outcome. Specifically, in this case, we can predict that, given the same settings, there will be a 50% probability that the outcome of the measurement will be x:L, and a 50% probability that the outcome will be x:R.</td></tr>
</table>

We measured the position of the electron.
Findings of question a):

U3) We always measure a value (there is never no value).
U4) Never measure both values at the same time.

Findings of question b):

U5) The values of L or R occur randomly.

No pattern was observed in the order of occurrence. Below are some examples of the first few measurement results:

xR;30fs xR;30fs xL;30fs xR;30fs xL;30fs xR;30fs xR;30fs xL;30fs xR;30fs xR;30fs	xL;30fs xL;30fs xL;30fs xL;30fs xL;30fs xL;30fs xL;30fs xL;30fs xR;30fs xR;30fs	xR;30fs xL;30fs xR;30fs xL;30fs xR;30fs xR;30fs xR;30fs xL;30fs xR;30fs xL;30fs	xR;30fs xR;30fs xR;30fs xR;30fs xR;30fs xL;30fs xR;30fs xL;30fs xL;30fs xL;30fs	xL;30fs xL;30fs xL;30fs xR;30fs xL;30fs xR;30fs xR;30fs xR;30fs xR;30fs xL;30fs

Findings of question c):
The class did not reach consensus at first. Part of the class felt that there was no pattern, part felt that the distribution was skewed towards 1:1. There were some other ideas: 1:3, 40:60.
We cannot detect a pattern from a single measurement. The *distribution* measurement must be repeated several times.
I promised a few more iterations of the distribution measurement. You are certainly welcome to do them yourself and see if you get similar distributions.

48:52
58:42
53:47
51:49
50:50
38:62 (copied from your data)
48:52
44:56
54:46
52:48

The ratio looks like it is closer to 1:1, but it is not always exactly 1:1. Given the random nature of each measurement result, it is expected that the distribution will have some random variation. If we flip a fair coin once, despite a theoretical distribution of 1:1, we may only get 1:0 or 0:1. If we flip it twice, it is quite possible that we get 2:0 or 0:2. If we roll three times, it is still quite possible to get 3:0 or 0:3, but the probability is much lower. Mathematically, we have shown that the expected deviation from the mean of the distribution is the square root of the number of measurements. That is $\sqrt{N}$if N is the number of measurements. Thus, for a total of 100 tosses of a fair coin, we expect a random deviation of the order of magnitude $\sqrt{100} = 10$. This means that we expect 2/3 of all measurements to be between 50:50 ± 10. So 2/3 of all measurements should be between 40:60 and 60:40. The remaining 1/3 may deviate even further from 50:50.

The above position measurement results are therefore perfectly consistent with a 50:50 distribution. Only one measurement out of ten (less than 1/3) is outside the range between 40:60 and 60:40. You are welcome to make these measurements yourself and see if you get similar results.

The conclusion of question c) is therefore:

> **U6) There is a pattern in the distribution.** It looks like about 50:50 +- 10.

3rd Meeting

We repeated the findings of U3), U4), U5) and U6) from the previous meeting. We asked about the predictability of the result of the experiment when placing the particle in the left well and then measuring its position 30 fs later. Since the result of each such measurement is random, we cannot predict the result of any particular measurement. However, we can predict the probability of the result, since the distribution of results is reproducible in the same way as a coin toss. So two observations follow from the previous ones:

> **U5a) The outcome of each experiment is generally not predictable.**
> **U6a) The probability of the outcome of each experiment is predictable.**

Then we remembered that measuring a particle placed in the left well immediately after it is placed there (at time 0 fs) always gives x = L. So the probability of getting a particle in the left well is 100%. So:

> **U6b) Quantum mechanics predicts probabilities of events, and these probabilities can be 100% in special cases, but most of the time they are not.**

What is the 50:50 situation?

We continued with the same observation experiment as the previous logging. We watched how each particle "moved" between the two wells.

0 fs 30 fs
xL -> xR
xL -> xR
xL -> xL
xL -> xR
xL -> xL
xL -> xR

Our next task was to propose explanations, "bold ideas", about what might be happening to the particle between 0 and 30 fs, or how the particle ends up in the left and the right well. We asked the two questions below and the answers were:

1) What are the possible explanations?
    A) Get some energy and jump into another well. (6)
        A.i) If it has enough energy, it goes to another well, if not, it stays.(4)
        A.ii) In any case, it jumps, but sometimes into the same well, sometimes into a different one.(1)
    B) It jumps between the two wells. (2)
    C) It oscillates in an orderly way, but each particle has a different frequency, which is why we get them in different wells. (1)
    D) A random event moves a particle (not energy).(1)
    E) The two cavities come together and when they go apart again, the particle stays in one or the other.(1)
2) Which well is the particle in just before the second measurement?
    A) In one of the wells. (7)
        A.i) The opposite of where it is then found. (2)
    B) In both wells. (6)

Next time we will test these hypotheses. In particular, the answer to question 2) can only be tested by simulation and the rules of quantum mechanics that we know so far.

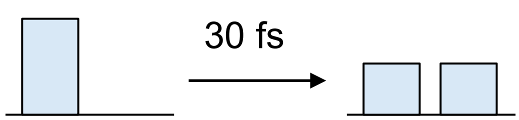
4th Meeting

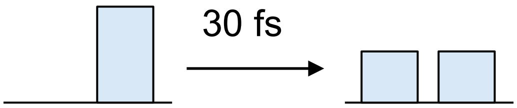
In the previous meeting, we hypothesised what happens to particles between 0 and 30 fs and where each particle is before the measurement at 30 fs.

At this meeting, we test a hypothesis about where the particle is located.

Hypothesis
We have two hypotheses:
A) The particle is in one of the wells.
B) The particle is in both wells.

Known pattern
The following pattern was discovered some time ago for particles in one of the wells. This applies to both wells.





For a more typing-friendly notation, we can write:

100:0 --30 fs--> 50:50
0:100 --30 fs--> 50:50

Test experiment

Place the particle in the left well. Wait 30 fs. Do not measure. Wait another 30 fs and measure the position.

Prediction based on hypothesis A

If a particle is in one of the wells just before the measurement at 30 fs, then there will be about 50 particles in the left well and about 50 particles in the right well. The 50:50 situation at 30 fs can therefore be decomposed into 50:0 and 0:50. If this is true, what do we expect after a further 30 fs?

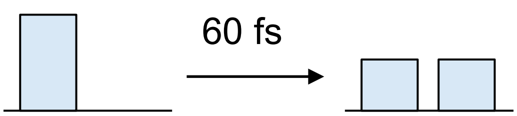
According to the sample above:

50:0 --30 fs--> 25:25
0:50 --30 fs--> 25:25

So

50:50 --30 fs--> 50:50

After the second 30 fs of waiting (i.e. after a total of 60 fs of waiting), we expect a 50:50 split.

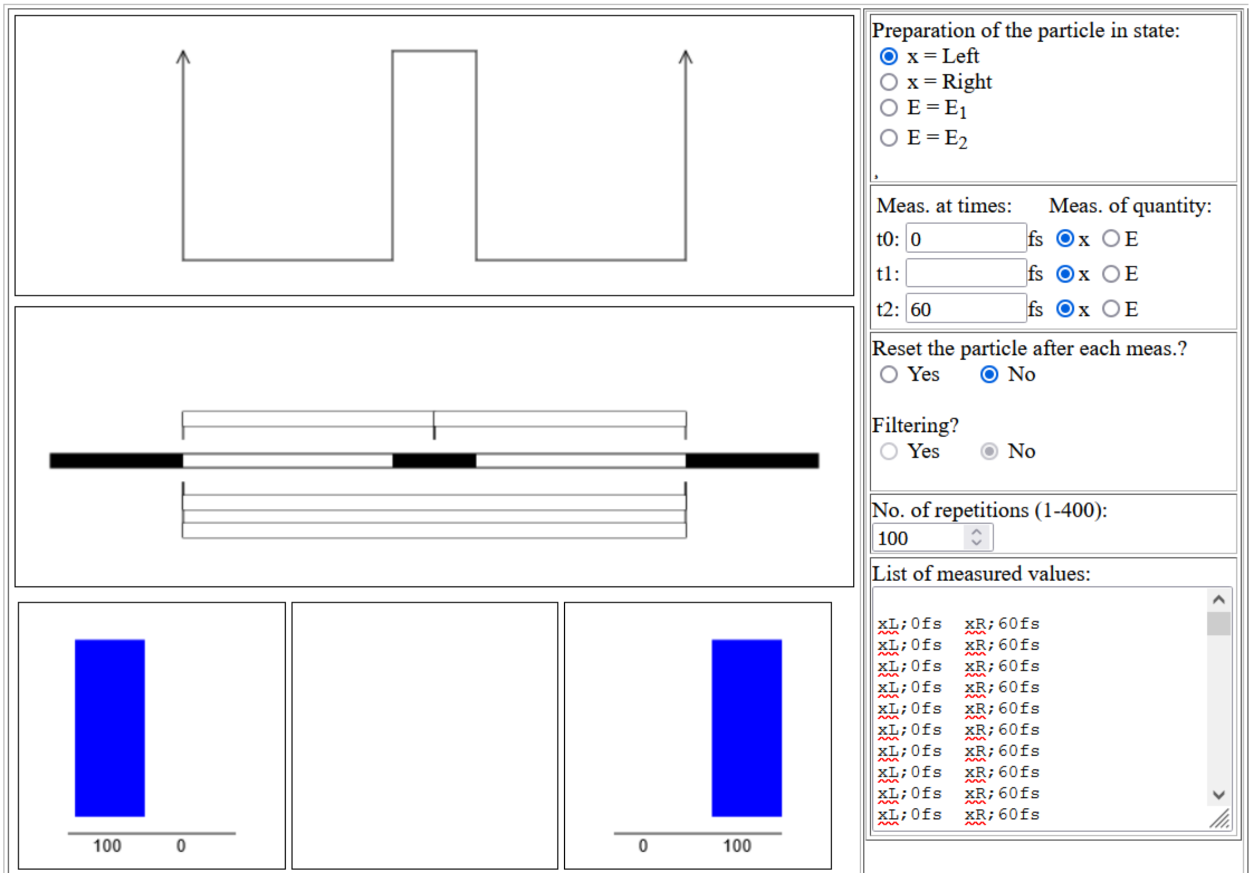


(By the way: congratulations on your correct predictions. Practically everyone who made a prediction made the right prediction :))

Prediction based on hypothesis B

Unfortunately, we have not yet identified any pattern in the behaviour of the 50:50 situation, so we cannot make any predictions based on this hypothesis at this time.

The outcome of the test experiment

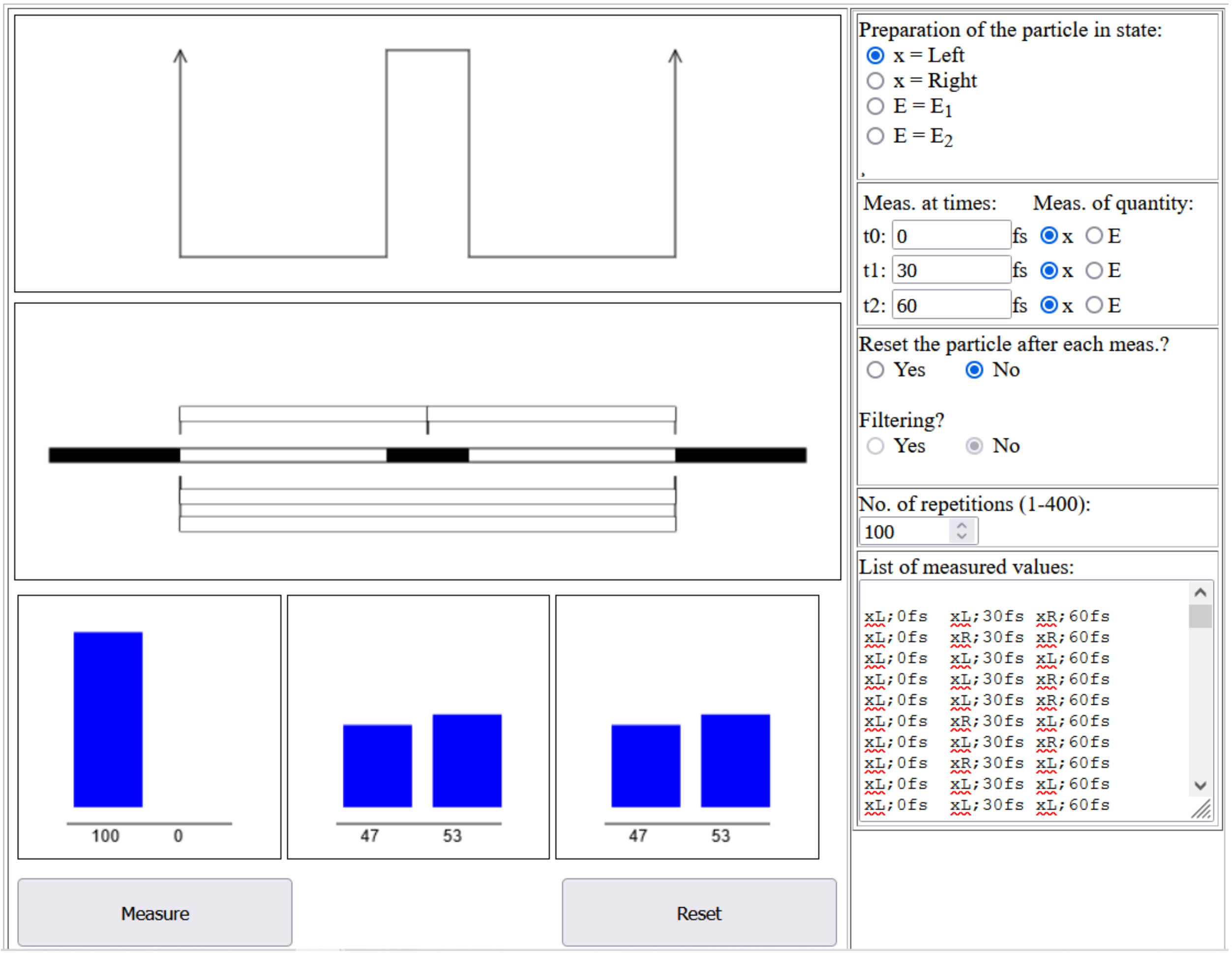


Judgment
The outcome of the experiment does not match the prediction based on hypothesis A, so **hypothesis A is not correct**.
We cannot judge hypothesis B because we do not have a prediction based on it.

We have found that a 50:50 situation should not be imagined as 50 particles in the left well and 50 particles in the right well. It is a different situation, which is a different combination of the two options (L and R).

Teaser observational experiment: experiment with intermediate measurement.

We did another very similar experiment.
Place the particle in the left well. Wait 30 fs. Measure the position. Wait another 30 fs and measure the position again.



In such an experiment, the outcome is exactly as predicted. So in this case, we do not reject hypothesis A. The measurement clearly makes a difference.

We conclude the "first season" with the conclusion that the 50:50 situation is not 50 in one well and 50 in the other, but some other combination of the two possibilities. We have a longer break for tests, etc. For the "second season" we have now discovered two important issues.

1) What can we say about this strange situation, which is not either left or right, but some other combination of the two?

2) What exactly is the impact of the measurement on the state of the system?

* Correct predictions, incorrect hypotheses

We have said a few words about how your prediction was correct, even though it does not match the outcome of the experiment. We are talking about so-called **hypothetico-deductive reasoning**, which we know from research is under-emphasised in school. It is about form reasoning:

If [hypothesis] is correct and I do [test experiment], then I expect [prediction].

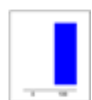
From such a structure, the following follows:

If the prediction does not come true, then the hypothesis is not correct.

Example: if the hypothesis that I can fly is correct, and I jump out of the window, then I will fly. If I had actually carried out this experiment, I would have landed on the ground. This does not mean that my prediction based on the hypothesis was wrong. It was the hypothesis itself that was wrong.

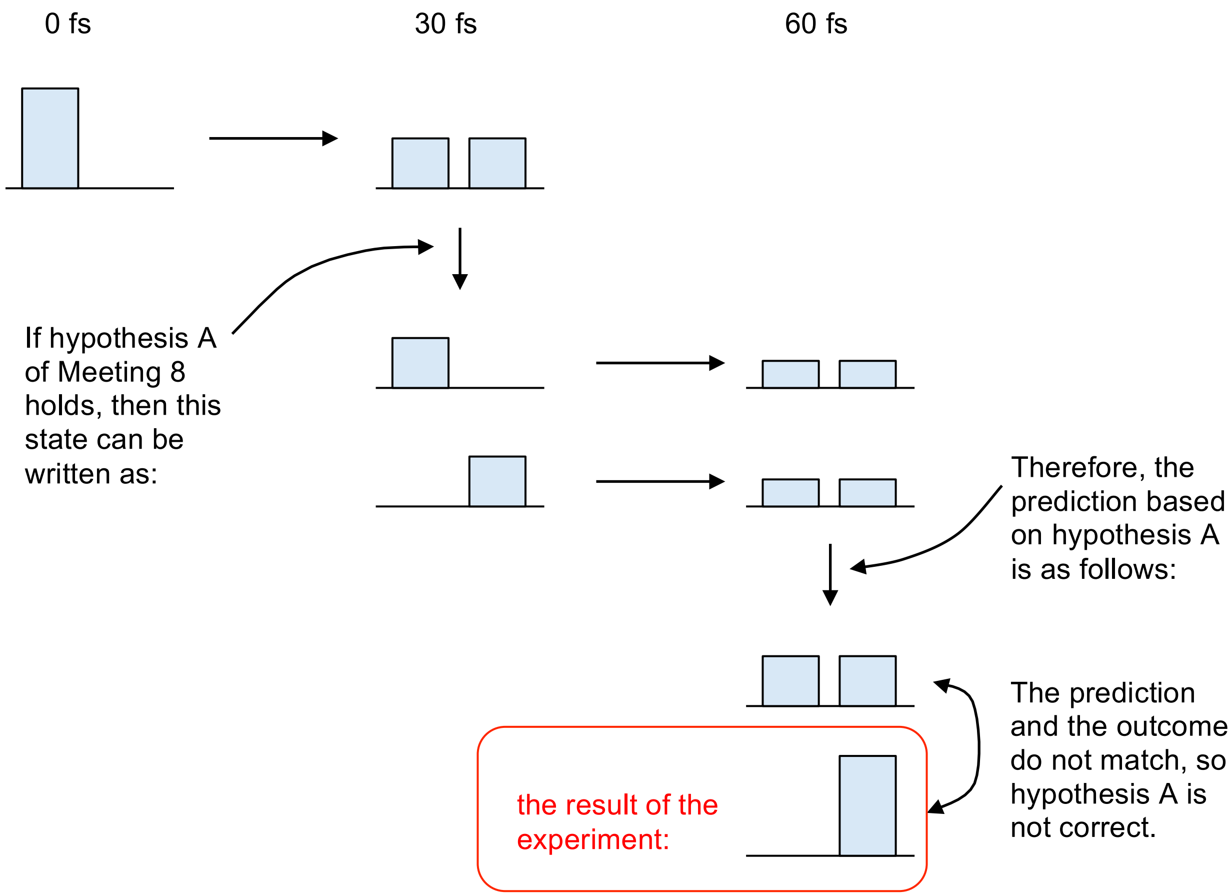
Time evolution of states

In the rest of the time, we have investigated in more detail the pattern of how the states of L and R change with time. This is called the *time evolution of* the states. In the table, the empty spaces indicate that we do not make measurements in between. This evolution is only observed if the first and the last are the only two measurements. We will investigate what happens if we take measurements in between after the break.

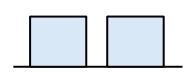
0 fs	20 fs	30 fs	40 fs	60 fs	80 fs	90 fs
						
						
						
						
						
						

5th Meeting

Repeating the process that led us to realise that the state at 30 fs is not the case, that some particles are in the L state and some in the R state.



We have run another experiment: we take a measurement at 30 fs and then again at 60 fs. The result of this experiment (figure right) agrees with the prediction of hypothesis A. **So the measurement changes the state of the system**, and now we are trying to find out how.



We have also introduced a **notation for the state of a system**. A state covers several quantities, in a way everything that can be said about a system. We gave the example of a table and gas in a room. For example, the state of a gas can be defined by five quantities: p, T, V, m and M. We have introduced a notation for the state **|...>** called **"ket"**. Just as a vector arrow indicates that a quantity has a direction as well as a magnitude, a ket indicates that it is a state that encompasses a whole set of quantities.

(The term "ket" comes from the mathematical operation <...|...>. P. A. M. Dirac gave the left part, <...|, the name "bra" and the right part, |...>, the name "ket", so that together they form bra(c)ket - brackets in English.)

We have also given a name for the unusual state on 30 fs, which is not composed of particles in the left and right wells. We called it **superposition**.

So far, we have recognised three states. In addition, we will introduce a |Y> label for each state that is not one of these three, until we have learned more about them:

- |The particle is certainly in the L well.
- |R> the particle is definitely in the R well.
- |A> superposition state at 30 fs, where the particle is neither in the left nor in the right cavity.
- |Y> a state that is not one of the above. (In class we called it |B>)

Let's introduce a few more terms:

- **Eigenstate**: a state for which we can **predict** with **100% certainty the outcome of the measurement of** the selected quantity. The eigenstates of the position are |L> and |R>.
- **Superposition**: a state for which we can only **predict the probability of** a particular measurement outcome (less than 100%). For a lego, a superposition state is e.g. |A>.

In the next activity, we looked for hypotheses about what the measurement does to the state of the system. The question was what do you think the state is after the measurement (indicated by a question mark). The table below represents an experiment carried out on six particles. Each row represents an experiment on one particle. In the table, the labels 30- fs are used for the time just before the measurement at 30 fs and 30+ fs for the time just after the measurement at 30 fs.

<table>
<tr><th>Situation at time 0</th><th>Situation at time 30- fs</th><th>Result of the measurement at 30 fs</th><th>Situation at time 30+ fs</th></tr>
<tr><td>|L></td><td>|A></td><td>x = L</td><td>?</td></tr>
<tr><td>|L></td><td>|A></td><td>x = R</td><td>?</td></tr>
<tr><td>|L></td><td>|A></td><td>x = R</td><td>?</td></tr>
<tr><td>|L></td><td>|A></td><td>x = L</td><td>?</td></tr>
<tr><td>|L></td><td>|A></td><td>x = R</td><td>?</td></tr>
<tr><td>|L></td><td>|A></td><td>x = L</td><td>?</td></tr>
</table>

We have a set of suggestions (hypotheses):

<table>
<tr><th></th><th>H1</th><th>H2</th><th>H3</th><th>H4</th><th>H5</th><th>H6</th><th>H7</th></tr>
<tr><td>x=L</td><td>|R></td><td>|A></td><td>|R></td><td>|R></td><td>|L></td><td>|Y></td><td>|A></td></tr>
<tr><td>x=R</td><td>|L></td><td>|A></td><td>|Y></td><td>|R></td><td>|R></td><td>|Y></td><td>|Y></td></tr>
<tr><td>N</td><td>2</td><td>14**</td><td>5*</td><td>1*</td><td>3</td><td>1</td><td>1</td></tr>
</table>

*One suggestion was x=L -> |R> x=R -> |R>/|Y>. This is counted under |R>, |R> and |R>, |Y>
** One proposal has all |A> except one. I count this one under |A> because there is no recognisable pattern.

Now we have different hypotheses. Our next task is to find out which one survives the test experiments.

6th Meeting

Hypothesis testing since last time.

In the first part of the table we have the hypotheses. If **the first measurement shows what we see in the first column, then immediately afterwards the state of the particle in question will be**: [as it appears in each column]. Each column represents one hypothesis. E.g. column 5 represents the hypothesis that, whatever the outcome of the first measurement, the state after this measurement will be |R>.
In the second part of the table, you have made **predictions for the results of the second measurement** if each hypothesis in the first part is correct. So if the hypothesis in column 5 is correct (|R>, |R>), then the second measurement will necessarily give a value of x=R for each measurement (column 5 in the second part of the table). This is by definition a property of the |R> condition. And so on. We cannot make a prediction for the state |Y> because we know nothing about this state. Let us look at state |A>. This is manifested by a random distribution of measurement results between x=L and x=R. Random means that it is uncorrelated with the result of the first measurement - the result of the first measurement has no effect on the result of the second. Some x=R will be followed by x=R and other x=R will be followed by x=L. And the same for x=L. One possible example of this is given in the third column under hypothesis |A>, |A>.

<table>
<tr><th colspan="8">HYPOTHESES</th><th></th></tr>
<tr><td></td><td>H1</td><td>H2</td><td>H3</td><td>H4</td><td>H5</td><td>H6</td><td>H7</td><td></td></tr>
<tr><td>x=L</td><td>|R></td><td>|A></td><td>|R></td><td>|R></td><td>|L></td><td>|Y></td><td>|A></td><td></td></tr>
<tr><td>x=R</td><td>|L></td><td>|A></td><td>|Y></td><td>|R></td><td>|R></td><td>|Y></td><td>|Y></td><td></td></tr>
<tr><th colspan="8">PREDICTIONS</th><th>OUTCOME</th></tr>
<tr><td>x=L</td><td>R</td><td>L</td><td>R</td><td>R</td><td>L</td><td></td><td>L</td><td>L</td></tr>
<tr><td>x=R</td><td>L</td><td>L</td><td></td><td>R</td><td>R</td><td></td><td></td><td>R</td></tr>
<tr><td>x=R</td><td>L</td><td>R</td><td></td><td>R</td><td>R</td><td></td><td></td><td>R</td></tr>
<tr><td>x=L</td><td>R</td><td>R</td><td>R</td><td>R</td><td>L</td><td></td><td>R</td><td>L</td></tr>
<tr><td>x=R</td><td>L</td><td>L</td><td></td><td>R</td><td>R</td><td></td><td></td><td>R</td></tr>
<tr><td>x=L</td><td>R</td><td>R</td><td>R</td><td>R</td><td>L</td><td></td><td>R</td><td>L</td></tr>
<tr><th colspan="8">JUDGEMENT</th><th></th></tr>
<tr><td>Judgements:</td><td>Rejected</td><td>Rejected</td><td>Rejected</td><td>Rejected</td><td>not refuted</td><td>we can't judge</td><td>Rejected</td><td></td></tr>
</table>

Finally, we run the experiment and find that the outcome is only consistent with the |L>, |R> hypothesis (column 6). It is therefore the only one that is not refuted, apart from |Y>, |Y>, for which we cannot make any prediction at all.

We have discovered a new rule of quantum mechanics:

- The measurement changes the state of the system to one of the eigenstates of the quantity being measured. This property is called **collapse**. The initial state (in general) had the possibility of both outcomes, but after the measurement the state collapsed (collapsed) into only one of these possibilities.

7th meeting

Exercise

First, we stopped for a moment to review all the rules of quantum mechanics (which I'll list below). When the collapse was repeated, a question came from the class asking what would happen if we measured the position at 0, 30 and 90 fs. We immediately picked up our pens and predicted the outcome of such an experiment:

0 fs	30 fs	90 fs

You have been extremely successful. Out of 11 groups, 9 groups predicted the outcome correctly. Congratulations. This means that you have understood and can apply the rules of quantum mechanics correctly. Congratulations again!

Additional question: if the ratio L:R = 46:54 in the second measurement (at 30 fs), what will the ratio be in the third measurement (at 90 fs)? Again, this was mostly answered correctly: L:R = 54:46. Why? Because all the particles that ended up in the |R,> state after the collapse at 30 fs have switched to the |L> state in the subsequent 60 fs and vice versa.

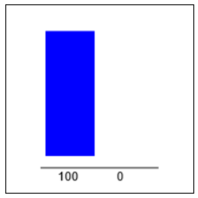
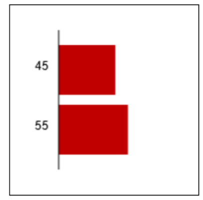
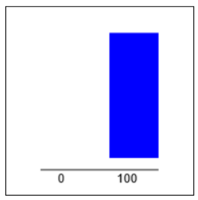
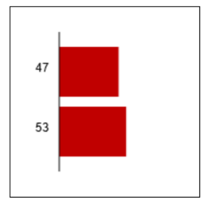
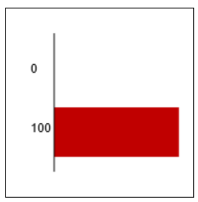
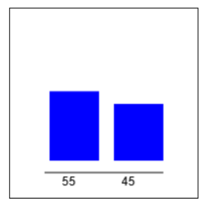
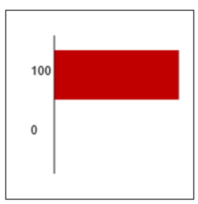
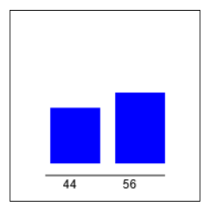
One additional question: does it matter whether or not we take a position measurement at 0 fs if we set the particles in the left well? How does the measurement affect or not the state of the particles?

Heisenberg's uncertainty principle

Later in the meeting, we learned the last rule of quantum mechanics. Then we repeated it all again.

I mentioned quantum teleportation. We will teleport a particle if we can measure all its properties at the transmitter and create an identical particle with identical properties at the receiver. The principle is simple. But that is why we need to measure more than one quantity.

So far, we have only measured lego. But now we've added the measurement of another quantity: energy. First, we determined the eigenstates |E1> (lower energy) and |E2> (higher energy). Then we measured the energy of the particle in the position eigenstates and the position of the particle in the energy eigenstates. Below is a table of the results.

|L>		
|R>		
|E1>		
|E2>		

Surprisingly, we have discovered that one's own state of position is never one's own state of energy, and vice versa. In short, we cannot know both quantities at the same time with certainty. If we measure the energy, the state collapses into one of the eigenstates of energy, which is the superposition state of the position. If we measure the position, the state collapses into one of the eigenstates of the position, which is the superposition state of energy. This makes it impossible to teleport in the way mentioned above. We will have to be twisted.

We call this phenomenon

- **Heisenberg's principle of indeterminacy**. Some pairs of quantities are such that we cannot know the values of both with certainty at the same time. In general, this is the **position and velocity of** a particle. In the case of a double cavity, so are position and energy.

We can now write down all the rules of quantum mechanics that we have discovered:

1. **Measurement always gives a result. Just one.** If we capture all possible outcomes, measurement will certainly "show" one of them.
2. **Own states.** There are states where the outcome of a measurement of a quantity is 100% predictable. These are called eigenstates of that quantity.
3. **Superposition.** There are states where more than one outcome of a measurement is possible. These are called superpositions.
4. **The unpredictability of a particular outcome** (indeterminism, randomness). In a state of superposition, it is not possible to be unpredictable which of the outcomes will be obtained in the next measurement (God is not a cube.)
5. **Predictability of the probability of the outcome.** In principle, we cannot predict the probability of a particular outcome, but we can always predict the probability of a particular outcome.
6. **Superposition is not a statistical mixture.** A particle in superposition is not in one eigenstate or another, we just don't know which one. The state of the superposition is different from one eigenstate and from another eigenstate. (Schrödinger's cat.)
7. **Collapse.** When the measurement is taken, the particle is in one of its eigenstates immediately after the measurement. One that is consistent with the result of the measurement (which would give 100% the same result if another measurement were taken immediately).
8. **Heisenberg's principle of indeterminacy.** There are quantities such that their values cannot be known at the same time. The measurement of one makes the superposition of the other and vice versa.