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Modeling theory applied: Modeling Instruction in introductory physics

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Modeling Instruction is a pedagogical approach which has been widely and successfully employed in high school physics instruction but is not commonly used at the university level. The goal of this paper is to describe the nature of Modeling Instruction at the university level and to clarify the role of models in physics instruction. A university physics class is described as it progresses through a typical modeling cycle, Introduction and Representation, Coordination of Representations, Abstraction and Generalization, and Application and Refinement. The benefits of modeling instruction are discussed.
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I. INTRODUCTION

The modeling theory of physics instruction has arguably had the greatest impact on high school physics instruction of any physics education reform.[1] Approximately ten percent of the nation's high school physics teachers have had some formal training in Modeling Instruction. Nevertheless, Modeling Instruction has had little impact on instruction at the university level. This paper describes Modeling Instruction, what it looks like at the university level, and the value of this pedagogical approach. I also discuss developments which must occur for Model Instruction to become more widely used at the university level.

Modeling Instruction is a pedagogical approach which focuses on qualitative and quantitative model development and testing. The focus is always made explicit to students. The cycle of model development in Modeling Instruction starts with qualitative features and builds to quantitative and is employed consistently and explicitly in learning about each new concept. Modeling Instruction conveys content through student investigations of various models which are consistent with observations and measurements. A modeling course focuses on model development and testing in the context of physics. Although teaching about the scientific process is a primary goal of these courses, teaching for conceptual understanding is similarly valued. Modeling Instruction has produced significant learning gains,[2] which likely result from the fact that the models that students investigate have been carefully chosen based on the underlying structure of the material.

A curricular package for Modeling Instruction appropriate for use in high school classrooms is available, but a similar curriculum for use at the college level is not. Several curricular packages exist that can be adapted for use in Modeling Instruction, including *Matter and Interactions*,[3] *Workshop Physics*,[4] and *Spiral Physics*.[5] Typically the adaptation requires framing the materials to make the modeling emphasis explicit, as well as some addition of material that increases or broadens the modeling focus.

There are several curricular packages and pedagogical approaches which assume that model development and testing should be the foundation of physics learning.[6] What sets Modeling Instruction apart from these materials and approaches is that the modeling process is continually made explicit to the students, it permeates the entire course, and there is a development cycle that is continually employed in the progression from qualitative to quantitative models and on to model testing and refinement.

II. BACKGROUND ON MODELING INSTRUCTION

A. Previous work

Hestenes has carefully described the elements of modeling theory,[7] but it has little to do with everyday practice and is known to be a difficult read.[8] A second paper[9] uses historical examples to identify elements of the theory in practice, but includes little application of the theory to actual instruction. Although Ref. 8 describes the activities of an influential teacher, it does not clearly establish the role of models and the process of modeling in the classroom. Most knowledge regarding the practice of Modeling Instruction has been disseminated via summer Modeling Instruction workshops for high school teachers. Although these workshops have been influential at the high school level, they have not been adapted in a sustained fashion for university physics instruction.

B. The theory of learning behind Modeling Instruction

The modeling theory of instruction is centered on the idea that physicists reason from mental constructs known as models. Scientists begin the process of model construction by using tools such as graphs, charts, diagrams, and formulas to represent specific physical situations. By repeated application of these representational tools and analyses scientists identify general characteristics, find common patterns of use and interpretation, and coordinate the representations into a general model that applies to a broad class of situations. In

Table I. Comparison of content in Modeling Instruction and a more standard course.

Modeling Instruction	Standard course
Models are constructs that are built in accordance with physical laws and constraints.	Laws are given in equation form and applied to solve problems.
Models are built by the application of representational tools which can then be used to solve problems.	Problem solving is primarily quantitative manipulation of equations.
Models are temporal and must be validated, refined and applied.	Content is permanent; validation has already taken place.
General models are applied to specific physical situations.	Laws apply to specific situations.
Modeling is a process that is learned through accumulating experience.	Problem solving is a game that requires tricks and is learned by solving large numbers of problems.
Models are distinct from the phenomena they represent and can include causal, descriptive, and predictive elements.	Content is indistinguishable from the phenomena.

the course of constructing situation specific models scientists accumulate experiential, declarative, and procedural knowledge which is closely associated with the model's common applications. This knowledge constitutes the modeling component of scientists' knowledge base. Examples of declarative knowledge include the laws and constraints governing a model as well as its range of applicability and scalability.[10] Procedural knowledge involves understanding how to use the model and includes experiential knowledge gained by repeated and varied experiences using it. Procedural and experiential knowledge together comprise the "tricks of the trade" and make the modeling process more efficient and fruitful.

The value of models is clear to most practicing scientists; models are the basis for theoretical and experimental research, which makes them the basis for knowledge development, reasoning, and problem solving.[11] However, the models held by scientists are dissimilar to the understanding students bring to introductory physics and to the standard content delivered in introductory physics. Students' comprehension of the physical world at the beginning of an introductory physics course is a fragmented collection of common sense generalizations which are primarily prescientific.[12] Modeling Instruction is designed to help students develop model-centered knowledge bases that resemble those of practicing scientists.

III. CONTENT ORGANIZATION: AN EXAMPLE MODEL DEVELOPMENT CYCLE

Modeling Instruction organizes the content of introductory physics around a small number of general models which can be applied in a broad array of situations. Two benefits are derived from focusing the curriculum on 6–8 *general* models. This organization matches expert knowledge organization in which a few fundamental principles are viewed as requisite for a very broad understanding. Also, students see a small number of general models as a manageable body of knowledge. In contrast, 14 or so textbook chapters likely appear to many students as hundreds of important principles and may be viewed as untenable. Table I summarizes the differences in content organization between a Modeling Instruction course and a traditionally structured course.

A. Construction of models

Central to both the pedagogy and content organization in Modeling Instruction is the process of model construction. In Modeling Instruction students employ the same general model development cycle in learning about each new topic. This cycle involves steps that I will refer to as Introduction and Representations, Coordination of Representations, Application, Abstraction and Generalization, and Refinement. To clarify these terms and the mechanics of model development, I will describe the instruction that leads to the general constant acceleration model (see Table II).

Students begin to construct general models by first learning the representational tools and building up experiential, declarative, and procedural knowledge. Instruction begins with a phenomenological introduction through inquiry-based laboratory activities. For constant acceleration the lab includes students moving in front of motion detectors and interpreting the resulting kinematic graphs. Students also begin accumulating declarative knowledge in the introductory inquiry labs by identifying the important concepts (position, velocity, distance, displacement, speed, and acceleration) and developing working definitions. These activities are all part of the first stage of model development, Introduction and Representation.

After students have been introduced to the various representations important to a topic, the instruction turns to coordinating and translating between these multiple representations. For constant acceleration students engage in conceptual activities such as interpreting graphs (for example, slope and the area under v versus t graphs) and creating corresponding motion diagrams. Learning to coordinate multiple representations happens in the second phase of the model development cycle which I call Coordination of Representations.

Along with the introduction of each representation there are applications that provide opportunities for students to use

Table II. Standard Modeling Instructional cycle applied to the development of a general constant acceleration model.

Step	Instructional goal	Example student activity
Introduction and Representation	Phenomenology—initiates the need for a new model (accelerated motion is not explained by general constant velocity model.) Introduction of kinematic graphs as useful representation.	Experimentation involving students moving with constant acceleration in front of motion detectors.
Coordination of Representations	Relate kinematic graphs to other common representations (motion maps).	Experimentation and conceptual activities.
Application	Begin to apply knowledge and tools. Develop experience, heuristics, and ability to draw conclusions based on representations.	Develop kinematic equations from kinematic graphs by analyzing velocity versus time graphs. Problem solving emphasizing use of modeling tools.
Abstraction and Generalization	Identify characteristics of representations in situations involving constant acceleration.	Review of constant acceleration and guided discussion.
Continued Incremental Development	Relate constant acceleration model to dynamical models and apply to new situations.	Continually revisit constant acceleration model, coordinate with energy and forces, apply to electricity and magnetism.

their new tools. I refer to the next step in the model development cycle as the Applications step. Quantitative problem solving is introduced in this step. The nature of problem solving in Modeling Instruction differs from that in most standard physics courses in several important ways, as is discussed in more detail in Sec. III D.

After sufficient experience with applications, the instructor leads a series of class discussions to help students organize their experiential, procedural, and declarative knowledge into a general model. This step in the model development process is referred to as Abstraction and Generalization in Table II. During a series of discussions students call on their experience in analyzing physical situations and compare situations to find similar characteristics in the application of representations. The definitions, graphs, equations, and interpretations from situation specific models are then collected and a whiteboard meeting is used to generalize the characteristics of all constant acceleration situations into a single, *general,* constant acceleration model. A wide range of representations, experiential, declarative, and procedural knowledge is abstracted into a single, coherently organized unit. An example of the characteristics general of a model is given in Table III, which describes the general characteristics of the constant acceleration model. This general model is significantly different than the situation specific models students have constructed. Generalization is critical because it groups representations and situation specific models and thereby reduces the cognitive load on the student.

The Refinement step in the model development process continues for the duration of the semester. Once the generalization of a model has taken place, it is applied and improved through repeated application in new contexts. Within each new context the applicability, validity, and scalability of the model is considered.

B. Shifting from descriptive to causal models

Once students have generalized the constant acceleration model, it is extended when the instructor presents a situation that requires the inclusion of energy. A typical sequence of classroom activities involved in refining and extending the

Table III. Preliminary tools and characteristics of a general but purely descriptive constant acceleration model.

Tool characteristic	
Kinematic graphs (for 1D motion)	Position versus time graphs are parabolic. Slope of tangent=instantaneous velocity. Velocity versus time graphs are linear Bounded area=displacement Slope=acceleration. Acceleration versus time is horizontal. Bounded area=$\Delta\mathbf{v}$
Motion maps	Velocity vectors are constantly changing. Vector subtraction gives direction of acceleration.
Kinematic equations (valid as vector equations)	$\mathbf{v_f}=\mathbf{v_0}+\mathbf{a}t$ $\mathbf{d}=\mathbf{v}_0t+\mathbf{a}t^2/2$

Table IV. Timeline of activities in the energy introduction unit of a modeling course.

Course meeting	Activity	Intent	Topic
Day 1	Ball bounce	Model Refinement Introduction and Representation	Introduction of energy conservation—qualitative
Day 2	Quantitative energy lab	Coordination of Representations Model Refinement	Application of energy conservation—quantitative
Day 3–4	Modeling physical situations	Model Application Abstraction and Generalization	Energy problem solving
Day 5	Modeling static situation	Model Refinement	Introduction of forces

constant acceleration model is shown in Table IV. Up to this point the models students have developed are purely descriptive.

Students begin their introduction to energy with a lab that builds on their kinematic models. The "Ball Bounce" lab requires the students to create descriptive models of the motion of a ball from the time it is dropped until it reaches its highest point after the first bounce. They are able to use the general constant acceleration model as a template to model this motion. Then students in small lab groups collect data to test the predictions they generated with their kinematic models. While the students use computers to collect data, the instructor asks one or two groups why the ball does not return to the original height. Students generally respond "energy is lost." Once energy has been introduced *by* a student, the instructor continues by asking what students know about energy and then engages the selected lab group in a discussion of energy conservation using common sense questions such as, "If the ball has energy at the bottom, and energy is conserved, that energy must have come from somewhere. Where could it have come from?" The instructor then uses the need to track the storage and transfer of energy to introduce energy pie charts. Energy pie charts, which are an adaptation of Van Huevelen's energy bar charts,[13] are used because they allow for a qualitative analysis of the bouncing ball. The selected lab group then is given the responsibility of introducing the new representational tool to the rest of the class during a whiteboard discussion session or "board meeting."

In this board meeting one of the essential discussion points is the relation between the existing constant velocity and constant acceleration models and the newly introduced representational tool. This discussion is critical because it validates the new representational tool and ensures coherence and self-consistency within the model. This activity introduces a new representation that can be incorporated into students' kinematic models and because it provides causal explanations, it improves the power of their models.

C. Extending the model; becoming quantitative

Modeling Instruction uses various representational tools because they enhance students' ability to reason conceptually about physical situations. Prior to spending significant time calculating the energy before and after some event, students in a Modeling Instruction course do a homework assignment and have at least one board meeting on the use of energy pie charts and their interpretation. These activities help students develop a qualitative understanding of energy without adding mathematical complexity. Although analysis and prediction are possible using only energy pie charts, their applicability is limited. Students need quantitative tools for energy.

The introduction of energy in Modeling Instruction differs from traditional curricula in three important ways and exemplifies important aspects of Modeling Instruction. First, energy is introduced prior to force and is always used in the context of energy conservation. Traditional curricula tend to introduce work, and the work-kinetic energy theorem next, which is consistent with a force-centered approach to the content. This approach has negative instructional implications and leads students to interpret force as a more important concept than energy.[14] Second, energy is used to extend the kinematic models that students have developed in the first part of the course. New content is always introduced to extend the applicability of models. In the standard approach the curriculum is organized such that new content is introduced in distinct chapters which atomizes the curriculum and leads students to miss the coherence of physics. Third, the students focus on all the various representations that are important in the model (kinematic graphs, motion maps, energy pie charts, and the associated mathematics); they are not asked to rely solely on mathematical representations. Multiple representations of physical situations are a central element in Modeling Instruction. The representations are not just introduced, and students are expected to solve problems by utilizing them.

D. Problem solving in Modeling Instruction

Once students have developed quantitative representations for energy, they practice using these representations by modeling physical situations. The first situation presented to the students is one that they have already encountered while modeling constant acceleration, but now they are instructed to include energy. Using a situation they have already modeled provides a way to discuss how the models change with new elements and how models need to be adapted to become more useful and efficient.

Although this part of the class is analogous to traditional problem solving, it is not equivalent. There are significant

Standard Problem Statement	Modeling Problem Statement
A rope is used to pull a 3.57 kg block at constant speed 4.06 m along a horizontal floor by a rope. The force on the block from the rope has a magnitude of 7.68 N and is directed 15.0° above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block floor system, and (c) the coefficient of kinetic friction between the block and floor?	Construct a complete model of the following situation: A 3.57 kg block is drawn at a constant speed of 4.06 m along a horizontal floor by a rope. The force on the block from the rope has a magnitude of 7.68 N and is directed 15.0° above the horizontal.

Fig. 1. Comparison of problem statements from a standard textbook problem (Ref. 17, Problem #43, p. 192) and a modeling problem.

philosophical differences between traditional problem solving and the application and adaptation of models. Traditionally the problem solving aspect of a course assesses students on well-defined physics problems with specific numerical answers. In contrast, students in Modeling Instruction courses need to view correct model development as the goal. To clarify this difference I will compare a standard textbook problem[15] and how the same problem would be used in a Modeling Instruction course. (see Fig. 1).

The responses to the standard problem are numeric. The response to the modeling problem is a constant velocity model which is adapted to the situation described. A complete model for this situation would include kinematic graphs, motion maps, a system schema,[16] a force diagram, and energy pie charts, as well as applications of Newton's second law and the first law of thermodynamics. All of the information required in the standard problem should be available by interpreting the representations that compose the model. The model answer would be much richer in representation and would be easy to troubleshoot by comparing interpretations across multiple representations. Often, when students create rich models, they identify problems within their reasoning, or validate their answers through redundancy within the model.

To encourage students to see the value in creating a model, the problems must be chosen so that the answer is a model. However, it takes significantly more time to create a rich model than to answer a standard physics problem. As a result, the number of assigned problems must be much smaller than in a standard course. By assigning fewer problems (on the order of 2–3/week) students see that the emphasis is on quality and the richness of the model rather than right answers. By using a small number of carefully chosen problems, students can efficiently learn the procedural aspects of modeling in each area because a well-constructed model solves many problems at once. Also, grading must change in a modeling course. The grade must reflect whether or not the student actually created a model, rather than whether an answer is achieved. In some ways grading is the most challenging aspect of such homework assignments for instructors new to modeling.

E. When a model reaches its limits

In a Modeling Instruction course a new topic is introduced via a situation for which students' existing general models are fundamentally insufficient. A static situation highlights inadequacies of constant acceleration models. Because there is no motion and no energy transfer, the students' existing models are not useful. Instead, students begin to build another model through a new cycle of experimentation, representation, and application. Setting an existing model aside and starting the cycle over as a result of a fundamental disconnect between the existing model and a specific type of situation aids students in clearly establishing at least one criterion for choosing one model over another. For the example discussed here, energy approaches are useful when energy transfers occur, but forces are the appropriate approach when modeling static cases.

IV. ADVANTAGES OF MODELING INSTRUCTION

A. Review, coherence, and focus on fundamental principles

Halloun[17] has described the differences in organization between traditional physics instruction and Modeling Instruction. The primary difference is that the former is organized into discrete topics often associated with particular textbook chapters. In contrast, Modeling Instruction is organized around a few general models that are continually revisited and refined. The cyclic reexamination of content that naturally results from Modeling Instruction's organization leads to several important advantages. These advantages include the opportunity to revisit and thereby reinforce newly learned material.

The limited number of models in Modeling Instruction courses keeps students focused on the fundamental principles of physics. In addition, because models are continually revisited and refined in response to new situations, students more naturally see the connected nature of the introductory physics course content. The opportunity to apply a general model in a given situation additionally aids in developing a student's appreciation of physics' coherence. Reif and Heller assert that optimum problem solving performance is predicated on coherent, hierarchical knowledge organization which results from model-centered curriculum organization.[18] The theory of learning behind Modeling Instruction supports their premise that the design of the introductory curriculum must mimic expert physicists' knowledge structure.

B. Reflection of authentic scientific process

A second benefit of Modeling Instruction is the relation between the curriculum design and the practice of science. Modeling Instruction provides students with a learning experience that is representative of the work of scientists. Models, like scientific knowledge, are flexible with limitations that are regularly examined. Through their work with models, students begin to see science as a process and scientific knowledge as a work in progress. Traditional curricula rarely highlight the underlying assumptions, limitations, or range of applicability of new content, whereas these elements are essential to Modeling Instruction. In contrast, traditional curricula tend to encourage the view that certain models apply in certain contexts and no model is especially global. This view is inconsistent with the view of practicing physicists.

C. Rich problem solving experiences and development of transferable skills

The third major benefit of Modeling Instruction is a shift in the nature of quantitative problem solving in the course.

As discussed in Sec. III D the types of questions that students are asked to answer and are capable of answering are different in Modeling Instruction courses. The quantitative problem solving aspect of Modeling Instruction focuses on the development and application of quantitative models and therefore is inherently nonalgorithmic. The ability to develop, test, and refine quantitative models is a skill that is very important and is readily transferable to other scientific and technical disciplines.

Quantitative problem solving in Modeling Instruction also requires use of multiple representations. Many studies have identified the benefits of using multiple representations, including qualitative or quantitative diagrams, graphs, and equations in solving problems.[19–22] By making multiple representations a continual and essential element of a Modeling Instruction course, we encourage students to develop an array of powerful tools. Although many reform physics courses include increased emphasis on representations, traditional courses almost exclusively rely on mathematical representations in solving problems. Because the Modeling Instruction approach is systematic about the use and analysis of representations, it encourages students to attend to conceptual aspects in the analysis of physical situations. Larkin *et al.*[23] related these conceptual analyses to greater success in problem solving, and Brewe has shown evidence of improved problem solving for students in a Modeling Instruction course.[24]

V. ISSUES IN COLLEGE LEVEL IMPLEMENTATION

Although Modeling Instruction has many benefits, there are a number of impediments to widespread adoption at the university level. As mentioned, Modeling Instruction endeavors to have students engage in activities that are consistent with the activities of practicing scientists. Students are treated as neophyte physicists, learning the practice of physics. As a result, Modeling Instruction is best implemented using a hands-on, inquiry-based approach. This approach makes studio-format classes an ideal learning environment because students can easily and often interact with simple physical systems, and the artificial separations between experimental and theoretical considerations are removed. Modeling Instruction would be much more difficult to implement successfully in large lecture-based courses, especially if there is no associated laboratory.

Although there are existing research based materials[3,4] which can be adapted to Modeling Instruction, many materials and most university level textbooks hinder instructors' abilities to teach a model-centered physics course. Additionally, university level modeling instructors might lack a working understanding of students' existing knowledge and its typical evolution throughout the course. As is the case with any significant shift in pedagogy, it is time-consuming and sometimes difficult for instructors to learn how to teach with this new emphasis on model development.

As with most reformed curricula, instructors are often uncomfortable with the reduced content coverage that must result if students are to be allowed time to develop a deep understanding and skill in model development. However, if we consider how much physics content is truly required and learned by certain student populations, the outcome of Modeling Instruction may make "less is more" an attractive option for many introductory physics courses. Modeling Instruction students show greater conceptual learning gains than students in traditional courses.[2] They also develop significant ability to transfer important skills, including the ability to develop and test quantitative models and work interchangeably with multiple representations. Modeling Instruction students certainly gain an understanding of the scientific process.

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