Exercises For Lecture 1

March 5, 2024

Exercise 1. *Rank the following functions by increasing order of growth. That is, find any sequence $g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8$ of the functions such that $g_1 = O(g_2)$, $g_2 = O(g_3), g_3 = O(g_4), g_4 = O(g_5), g_5 = O(g_6), g_6 = O(g_7), g_7 = O(g_8)$.*

- $f_1(n) = n^{\pi}$
- $f_2(n) = \pi^n$
- $f_3(n) = \binom{n}{5}$
- $f_4(n) = \sqrt{2^{\sqrt{n}}}$
- $f_5(n) = \binom{n}{n-4}$
- $f_6(n) = 2^{\log^4 n}$
- $f_7(n) = n^{5(\log n)^2}$
- $f_8(n) = n^4 \binom{n}{4}$

Exercise 2. *For each of the following statements, decide whether it is always true, never true, or sometimes true for asymptotically nonnegative functions f and g. If it is always true or never true, explain why. If it is sometimes true, give one example for which it is true, and one for which it is false.*

1. $f(n) = O(f(n)^2)$
2. $f(n) + g(n) = \Theta(\max(f(n), g(n)))$
3. $f(n) + O(f(n)) = \Theta(f(n))$
4. $f(n) = \Omega(g(n))$ *and* $f(n) = o(g(n))$ *(little-o)*
5. $f(n) \neq O(g(n))$ *and* $g(n) \neq O(f(n))$