Augmented Reality for Learning Mathematics

A Systematic Literature Review

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Abstract—In the last decade, the usage of Augmented Reality (AR) has proliferated especially in the education sector. However, only limited articles have systematically reviewed the research trends in the implementation of AR for learning mathematics. Thus, this paper presents a systematic and comprehensive analysis of the research trend for the period between the year 2015–2019. From a leading indexing Scopus database, this review has identified only 19 journal articles based on the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) framework. These works of literature were analyzed and grouped into eight themes to illustrate the trends of these research focus: AR apps types, AR apps development tools, research contribution categories, benefits of AR for math, learning problems, testing methods, and math subtopics. Moreover, this synthesized review on learning of mathematics through AR could benefit researchers and educators, thereby suggesting avenues for future research.

Keywords—Current trend, augmented reality, math education, systematic review.

1 Introduction

The objective of this paper is to review, analyze and classify the existing work of literature that is related to learning of mathematics through Augmented Reality (AR). This systematic review investigates research trends and identifies similar themes, frameworks and samples of research. More specifically, Table 1 shows the research questions (RQ) of this study.

Table 1. Research questions

RQ	Research question
1	What are the most popular types of AR development tools used for learning mathematics?
2	What are the popular types of AR tools to develop mathematics applications?
3	What did the research say on the implementation, development and effectiveness of AR?
4	What are the significant benefits of AR in math learning?
5	What are the major problems of AR in math learning?
6	What are the research approaches used to study AR in math learning?
7	What are the methods used to evaluate the effectiveness of AR math tools?
8	What are the main topics in mathematics that implement AR for learning?

In the last decade, the usage of Augmented Reality (AR) has grown rapidly due to the popularity of AR in different fields—aligned with the fast explosion of mobile technology. AR is a mixed reality technology that contains virtual objects that are implemented or 'augmented' with the real world. AR provides a new experience for learners by enabling them to 'see' the existing environment overlaid by the digital learning content [1]. AR has also been applied as the blended instructional strategy to strengthen the learning process due to its attractiveness. It provides opportunities for the students to explore and interact with the 3D models of learning objects [2]. AR starts between the real world and augmented virtuality [3]. The usage of AR in education has increased as it has opened new possibilities in the teaching and learning process [4]. As AR simplifies the complex information, it provides a lot of benefits in education. The usage of AR for mathematics education is also currently increasing.

However, there are only a few efforts to systematically review the relevant works of literature in AR implementation for mathematics education. A few attempts have been made to provide systematic literature review, but none has presented a thorough analysis of the themes in a comprehensive picture. Saidin, Halim, and Yahaya [5] included 11 sets of final articles, but their review was not specific for mathematics subject. Ibáñez and Delgado-kloos [6] limited their study to empirical research focusing on STEM and not mathematics. Another study by da Silva et al. [7] consists of non-empirical and empirical research but did not focus on any subjects. Table 2 compares other related reviews with the present paper.

Table 2. Comparison with existing work

Factors	Saidin, Halim, and Yahaya [5]	Ibáñez and Delgado-kloos [6]	da Silva et al. [7]	Present study
Review methodology	Meta-analysis	PRISMA	PRISMA	PRISMA [8]
Year of publication	2015	2018	2019	2020
No. of search database	-	7	4	1
Final set of articles	11	28	45	19
Type of reviewed articles	Non-empirical and empirical	Empirical	Non-empirical and empirical	Non-empirical and empirical
Benefits and problems	Yes	-	Yes	Yes
Subject	Medical	STEM	-	Mathematics

This paper is organized as follows. Section 2 describes the review method. Meanwhile, the review results are presented in section 3. Section 4 discusses the analysis and reviews the research trends. Lastly, section 5 concludes the paper and presents future work.

2 Review Methodology

In order to present a comprehensive picture of the existing literature on AR for learning mathematics, we conducted a systematic literature review according to the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines. The aim is to analyze the content of 19 selected articles to answer our research questions. However, only articles that meet the following inclusion criteria were selected:

- Published in refereed academic journals (to ensure high-quality refereeing).
- Discussed AR implementation for learning mathematics.
- Published during the last five years, i.e. 2015-2019 (to study the publication trends).
- Indexed in Scopus database (for quality of indexing and citation).
- Written in English.

To accurately identify the targeted articles, we established these exclusion criteria for the types of documents: conference proceedings, books, book reviews, magazines, short surveys, short communications, correspondences, newsletters, discussions, product reviews, editorials, publisher’s notes and erratum. The identified articles were read and analyzed. Fig. 1 shows the PRISMA flow diagram [8].

In order to identify the themes related to the research patterns and trends in the study, a brief thematic analysis was carried out. All 19 articles were analysed to extract useful data that answered our research questions and grouped into eight main themes.

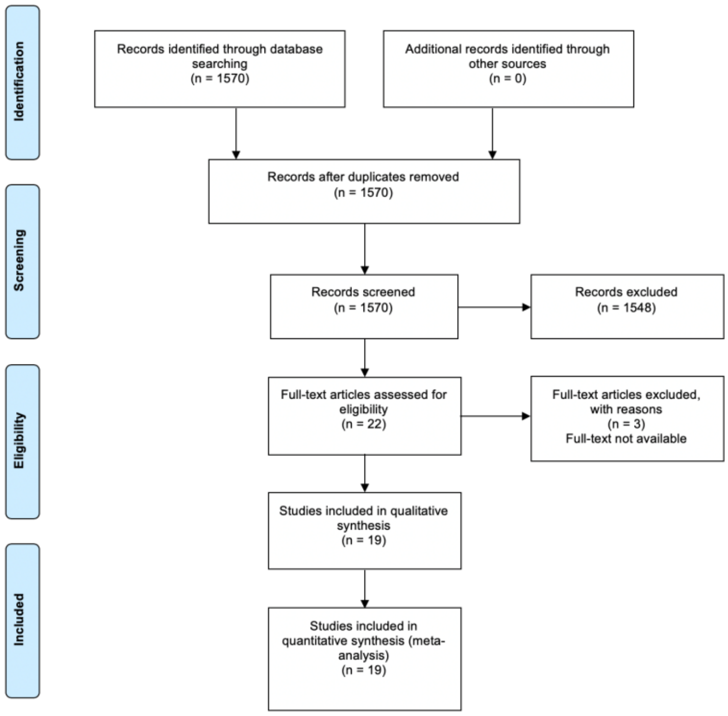

Fig. 1. PRISMA Flow Diagram [8]

3 Results and Analysis

3.1 Summary of findings

A search was performed on the Scopus database. As a result, 1570 journal articles were found based on Title-Abstract-Keywords search: "Augmented Reality" AND "Mathematics" OR "Geometry" OR "Mathematical". Then the inclusion criteria were applied. First, the articles went through a manual screening (title and abstract). Then, the articles were screened further to evaluate their relevance in answering the research questions. Out of the 1570 articles found, only 19 articles met the criteria that suit the needs of the inclusion criteria. Finally, we read the articles thoroughly to extract the relevant data which will provide support in answering our research questions.

3.2 Data extraction

We extracted the following relevant information from the included articles:

- Year of publication and the researchers' country of origin.
- Types of AR development tools to develop mathematics applications.
- Main contributions (AR implementation, development and effectiveness).

- Benefits of AR in math learning.
- Problems of AR in math learning.
- Research approaches to study AR in math learning.
- Methods to evaluate the effectiveness of AR as a math learning tool.
- The main topics in mathematics that implement AR for learning.

Fig. 2 shows the trends of publication included in this review since 2015. A significant increase in articles number can be seen from 2018 to 2019.

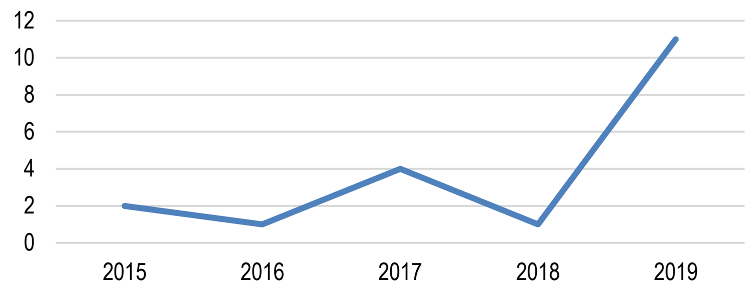

Fig. 2. Number of articles by year

Table 3 shows the list of articles categorized by the researchers' country of origin.

Table 3. List of articles by country

Country	Ref.	f	%
Mexico	[18][19][20]	3	16
Spain	[13][14][15]	3	16
China	[25][26][27]	3	16
Turkey	[9][10]	2	11
Taiwan	[11][12]	2	11
Indonesia	[21][22]	2	11
Malaysia	[16]	1	5
Greece	[17]	1	5
Cyprus	[23]	1	5
USA	[24]	1	5

Table 3 shows the articles that were published by ten countries. Mexico, Spain and China produced the highest number of articles (n=3). Meanwhile, Turkey, Taiwan and Indonesia published two articles. The remaining four countries only published one article each. Fig. 3 illustrates the patterns and trends in studies related to AR for learning mathematics by colours (2015–2019 periods) under the eight themes: Type of AR markers (marker-based, marker-less); development tools (Blippar, ENTiTi Creator, HDAugmentedReality, Metaio SDK, HP Reveal, Unity 3D & Vuforia SDK); contribution types (implementation, development, effectiveness); benefits of AR for math (increased confidence, enhanced visualization, interactive learning); math learning problems (visualization, understanding); research approaches (TAM, ARCS, ANCOVA, quantitative); testing methods (questionnaire, pre- & post-test, SUS, interviews); math subtopics (geometry, algebra, Euclidean vectors, probability, statistic, others).

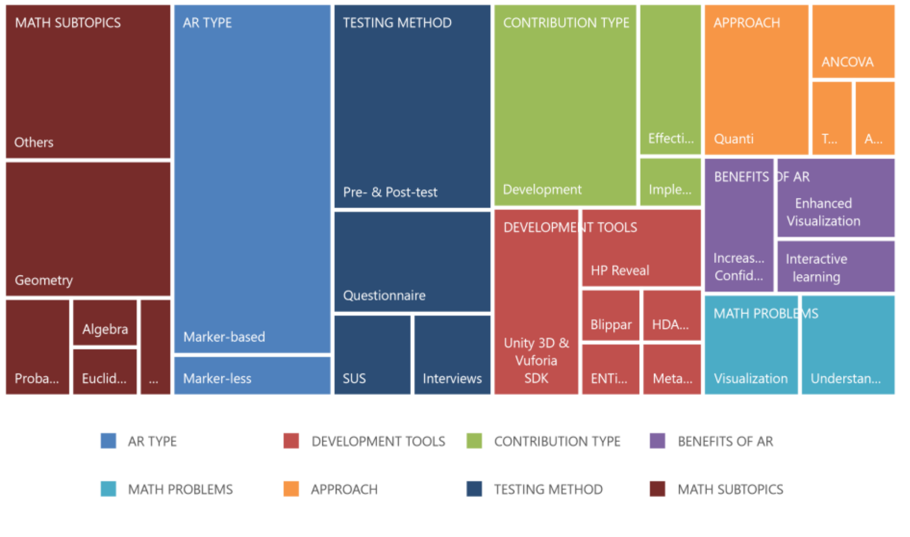

Fig. 3. Distribution of analyzed themes

3.3 AR types

RQ1: What are the most popular types of AR development tools used for learning mathematics?

AR can be divided into two types which are marker-based, and marker-less (location-based). These two main types of AR provide different specific purposes and use a different approach to develop it. Table 4 shows the articles that fall into two main categories of AR.

Table 4. Articles by AR types

AR Type	Ref.	f	%
Marker-based	[10][11][12][14][15][16][17][19][20][21][22][23][24][25][26][27][28]	17	89
Marker-less (Location-based)	[13][18]	2	11

Marker-based AR: Majority of the articles (89%) deployed marker-based AR. The first study used it to enhance 3D geometric thinking skills to understand math teachers' perceptions of the AR tutoring system [9]. In another study by Hanafi et al. [16], the impact of mobile AR application in the understanding of mathematics among primary school students were analyzed. Demitriadou, Stavroulia, and Lanitis [23] evaluated the potential of AR technology for teaching geometric lessons to primary school children. Improvement in the teaching and learning process of mathematics can be done through the development of spatial visualization and problem skills by tackling real-world problems [20]. Martin-Gonzalez, Chi-Poot, and Uc-Cetina [19] described the development and evaluation of AR for teaching Euclidean vectors. A study by Lin, Chen, and Chang [11] assessed the effectiveness of learning by using an AR-assisted learning system that also used marker-based AR. Meanwhile, Chen [12] also used marker-based AR to study the effect of mobile AR on learning performance, motivation, and math anxiety. Cai et al. [25] developed an app (consists of three parts: Seven, Super Spaces, and Magic Coins) to study the impact of student conceptions in learning statistics. A study by Gecu-Parmaksiz and Delialioglu [10] used marker-based AR to improve the understanding of geometric shapes among preschool students. Another study by de Ravé [15] developed DiedricAR, a marker-based mobile AR designed for ubiquitous geometry learning. Meanwhile, Andrea et al. [22] conducted a study regarding elementary school geometry textbook using marker-based AR.

Marker-based AR is also used by studies that are related to disabilities and special needs. For instance, a study by Kazanidis and Pellas [17] developed marker-based AR to provide a framework to implement AR as a teaching tool for secondary students with disabilities. AR was also used as the instructional tool to teach mathematics to secondary students with disabilities [24]. Similarly, the study by Cascales-Martínez et al. [14] showed that AR determines the feasibility of using a multi-touch tabletop system for mathematics learning in primary education with students with special needs (SEN).

Markerless AR (Location-based): In regard to markerless AR, only two articles were included and analyzed. The first study was AR in the context of mobile learning to enhance the comprehension of Science, Technology, Engineering and Mathematics (STEM) [13]. The second study is the usage of the immersion methods to enhance collaborative creativity using AR to support creative math learning [18].

3.4 AR development tools

RQ2: What are the popular types of AR tools to develop mathematics applications?

Only 11 out of 19 articles mentioned the AR development tools that were used to develop a math learning mobile application. Most of the researchers used Unity 3D with Vuforia SDK while most of the articles (37%) used Unity 3D with Vuforia SDK as the development tools (Table 5).

Table 5. Articles based on AR development tools

AR Development Tools	f	%	Ref.
Unity 3D with Vuforia SDK	5	48	[15][20][21][26][28]
HP Reveal	2	18	[12][24]
Blippar	1	9	[17]
ENTiTi Creator	1	9	[23]
HDAugmentedReality	1	9	[13]
Metaio SDK	1	9	[27]

The first study that used Unity 3D with Vuforia SDK was by Hernández-Ordoñez et al. [28] who developed an app to enable the enhancement for students' 3D geometric skills. Article by Medina-Herrera [20] aimed to improve the teaching and learning process of mathematics through the development of spatial visualization and problem-solving skills. Next, a study was developed for AR media applications for geometry [21].

Another study was by de Ravé [15] that helped to assist the understanding of descriptive geometry topics among children. The last study that used Unity 3D with Vuforia SDK is an article on intelligence AR tutoring system for mathematics teaching and learning [26].

The second most-used AR development tool is HP Reveal. This development tool was mentioned in three studies. Two research indicated that HP Reveal might be used to enhance mathematics teaching and learning. Kellems, Cacciatore, and Osborne [24] developed and evaluated an AR tool using HP Reveal to help educators to apply modern teaching techniques for math concepts. The last study that used HP Reveal was by Chen [12] who investigated how mobile AR affects learning, motivation and math anxiety. The other remaining four development tools, which are Blippar, ENTiTi Creator, HDAugmentedReality, and Metaio SDK were mentioned in each one of the remaining three articles.

3.5 Implementation, development and effectiveness of AR

RQ3: What did research say about the implementation, development and effectiveness of AR for learning mathematics?

Three main discussion points were identified as the focus of the articles: (i) implementation of AR in math education; (ii) development of math apps, and (iii) the effectiveness of AR for learning math. However, only one article by Lin, Chen, and Chang [11] discussed the implementation of AR in math. They evaluated the effectiveness of learning solid geometry among 76 students from Tainan City, Taiwan.

The second category of AR main findings is in the development of apps. Nine articles discussed the development of math apps. Nagata, García-Bermejo Giner, and Abad [13] developed a mobile pedestrian navigation-AR platform. On the other hand, De Ravé [15] developed a descriptive geometry app. Another research by Kazanidis and Pellas [17] developed an application for teaching and learning mathematics. Meanwhile, Martin-Gonzalez, Chi-Poot, and Uc-Cetina [19] developed a geometry-based application. Buchori et al. [21] developed an app called AR Media for Geometry. Andrea et al. [22] developed an AR tool as a technique for teaching mathematics concepts. Kellems, Cacciatore, and Osborne [24] developed the AR system of Euclidean Vectors. The study by Cai et al. [25] developed statistics and probability lessons using. Lastly, Hsieh and Chen [26] developed an intelligent AR tutoring system (IARTS) for mathematics.

The third category of AR main findings was studying its effectiveness (n = 3). Chen [12] examined whether mobile AR affects learning motivation and math anxiety among students. Another article by Cascales-Martínez et al. [14] assessed the effectiveness of AR learning tools in making the learning process of the European currency system simpler. Meanwhile, Hanafi et al. [16] studied the effect of mobile AR application in understanding mathematics subjects among primary school students in Malaysia. Table 6 describes the implementation, development and study on the effectiveness of AR for learning mathematics.

Table 6. Implementation, development and effectiveness of AR for learning mathematics

Issues	Description	Ref.
Implementation	Implementation of AR technology in teaching activities.	[11]
Development	Development of an AR application.	[13]
	Development of an application for learning descriptive Geometry.	[15]
	Development and assessment of math application.	[17]
	Development of geometry-based apps.	[19]
	Development of AR media for geometry.	[21]
	Development of AR tools as a modern technique for teaching mathematics.	[22]
	Development of an AR system for teaching Euclidean Vectors.	[24]
	Development of design statistics and probability lessons using AR.	[25]
	Development of an interactive intelligence AR tutoring system.	[26]
Effectiveness	Study on the effectiveness of AR for learning and motivation.	[12]
	Study on the effectiveness of AR for learning the European monetary.	[14]
	Study on the effectiveness of mobile AR on math understanding.	[16]

3.6 Benefits of AR in math learning

RQ4: What are the significant benefits of AR in math learning?

Three major benefits can be obtained from AR in math learning: (i) increased confidence and understanding; (ii) enhanced visualization; (iii) interactive learning. Three studies ([10][11][22]) found the benefits of AR in increasing students' confidence and understanding of graphical math content that eases the learning of shape and geometry. They found an increased knowledge of geometric shapes after using AR.

The implementation of AR in math education, which managed to enhance visualization was reported in three studies. The study by Medina-Herrera [20] has shown that the application of AR in math was able to strengthen math theory visualization using 3D animation. The 3D visualization showed the imaginary movement of objects and eased in the identification of geometry objects from different angles. Buchori et al. [21] stated that AR had enhanced visualization of geometry, while a study by Chen [12] found that AR provides interesting visual experiences for learners.

Meanwhile, AR has made mathematics learning more interactive. Two articles discussed this benefit. A study by Kazanidis and Pellas [17] stated that AR provides an interactive experience, thus leading to the enhancement in students' achievement. Another study by Demitriadou, Stavroulia, and Lanitis [23] has shown that AR can make learning more interactive. Table 7 shows the major benefits of AR in mathematics education.

Table 7. The major benefits of AR for math education

Benefits	Explanation	Ref.
Increased confidence and understanding	Increased understanding of geometric shapes.	[10]
	Increased students understanding using graphical content.	[11]
	Increased students understanding of learning 3D geometry and shape.	[22]
Enhanced visualization	Provided interesting visual experiences.	[12]
	Provided 3D visualization and imaginary movements of the object.	[20]
	Enhanced visualization of geometrical shape.	[21]
Interactive learning	Provided interactive learning experience and better performance.	[17]
	Improved students' interactivity and interest in mathematics.	[23]

3.7 Problems in math learning

RQ5: What are major problems in math learning?

Two major problems in math learning have been identified from this review. The first problem among students is that they cannot clearly visualize mathematics shapes and objects. A study by de Ravé [15] indicated that students have difficulties in visualizing and categorizing geometric shapes. Whereas Sanabria and Arámburo-Lizárraga [18] emphasized that students have difficulties in visualizing 3D geometrical shapes. According to Medina-Herrera [20], students faced problems in the visualization of the calculation process. Similarly, Cai et al. [27] reported difficulties in visualizing abstract applications of probability.

The second challenge among the students is in understanding mathematics concepts. A research by Martin-Gonzalez, Chi-Poot, and Uc-Cetina [19] stated that students are facing the problems in understanding 3D vector properties. Meanwhile, a study by Buchori et al. [21] indicated that students are having difficulties in understanding geometry subjects while their educators are having problems in finding suitable teaching aids. Demitriadou, Stavroulia, and Lanitis [23] argued that students have difficulties in understanding the difference between 2D and 3D geometric shapes. Table 8 shows the major problems found in math learning in the context of AR.

Table 8. Major problems in math learning

Problem	Explanation	Ref.
Visualization	Difficulty in visualizing and categorizing geometric shapes.	[15]
	Difficulty in visualizing and interacting with 3D images.	[18]
	Difficulty in visualizing information.	[20]
	Difficulty in visualizing abstract applications of probability in mathematics.	[27]
Understanding	Difficulty in understanding 3D vector and 2D visualization.	[19]
	Difficulty in understanding geometric shapes.	[21]
	Difficulty to understand the 2D and 3D geometric shapes.	[23]

3.8 Research approaches

RQ6: What are the research approaches used to study AR in math learning?

Among the articles included in this study, only eight stated the research approaches they used in their study. The two research approaches are the Technology Acceptance Model (TAM) and Quantitative and Quasi-Experimental. The research by Ibili, Resnyansky and Billinghurst [9] used TAM to determine the acceptance level among teachers in using Augmented Reality Geometry Tutorial System (ARGTS). They found that Perceived Ease of Use (PEU) has a direct effect on the Perceived Usefulness (PU) of their system. Two studies ([25][27]) used a one-way analysis of covariance (ANCOVA) using pre- and post-test.

Another study by Hsieh and Chen [26] used the Instructional Material Motivational Survey (IMMS) based on Attention Relevance Confidence Satisfaction (ARCS) motivation model. Meanwhile, the Quantitative and Quasi-Experimental approach was adopted by five articles. An article by Gecu-Parmaksiz and Delialioglu [10] tested AR-based geometric shapes app among preschool children. Another study by Lin, Chen, and Chang [11] assessed the effectiveness of learning solid geometry using an AR-assisted learning system, while Cascales-Martínez et al. [14] deployed a quasi-experimental design with pre-and post-test without a control group.

However, the study by Hanafi et al. [16] used a quasi-experimental procedure using the pre-test and post-test with control group among 78 primary school students. The last study by Andrea et al. [22] also used a quantitative approach with Non-equivalent Control Group Design. Table 9 summarizes the research approaches used in 9 out of 19 articles.

Table 9. Research approaches used

Research approach	Ref.
Technology Acceptance Model (TAM)	[9]
Attention Relevance Confidence Satisfaction (ARCS) motivation model	[26]
Analysis of Covariance (ANCOVA)	[25][27]
Quantitative and Quasi-Experimental	[10][11][14][16][22]

3.9 AR effectiveness testing methods

RQ7: What are the methods used to evaluate the effectiveness of AR math tools?

Four methods were used to evaluate the effectiveness of the AR math tools. The most popular method used was pre-test and post-test (n=7). The studies that used this method consist of a different targeted group. A study by Lin, Chen, and Chang [11] focused on secondary school students as the target group which involved 76 students. Medina-Herrera [20] targeted 993 undergraduate students. The second most popular method was the questionnaire method (n=4, 50%). This method was used by de Ravé [15] involving 20 university students. Research by Kazanidis and Pellas [17] also used the questionnaire method through a focus group among 78 undergraduate students. On the other hand, Andrea et al. [22] targeted primary school students for their study. Meanwhile, Hsieh and Chen [26] conducted a study which was participated by 137 junior high school students.

Another two studies used pre-test and post-test in their experiment but the focused group was primary school students. Cascales-Martínez et al. [14] conducted a study involving 22 students. In contrast, a study by Chen [12] involved 137 students for Abbreviated Math Anxiety Scale (AMAS) pre-test and 82 students for pre-test and post-test for Algebra and Geometry. The study by Gecu-Parmaksiz and Delialioglu [10] also focused on primary school students. Another study by Cai et al. [25] focused on 101 junior high school students. Two other studies by Hanafi et al. [16] and Demitriadou, Stavroulia, and Lanitis [23] also used pre-test and post-test methods.

The third method adapted by researchers was the System Usability Scale (SUS). Only one article used this method [19]. The fourth testing method is the interview. Lin, Chen, and Chang [11] conducted a study involving 76 secondary students, while the study by Cai et al. [27] involved 68 students from junior high school. Nine studies included less than 100 student respondents, while four studies involved more than 100 students. The majority are from primary schools (n=7), followed by junior high school (n=3). Table 10 shows the testing methods used to test AR math apps.

Table 10. Testing methods

Method	Students	Level	Ref.	f	%
Pre-test and Post-test	72	Primary school	[10]	7	50
	137	Primary school	[12]		
	22	Primary school	[14]		
	78	Primary School	[16]		
	993	Undergraduates	[20]		
	30	Primary School	[23]		
	101	Junior high school	[25]		
Questionnaire	20	University students	[15]	4	29
	78	Undergraduates	[17]		
	NA	Primary school	[22]		
	137	Junior high school	[26]		
System Usability Scale (SUS)	18	Undergraduates	[19]	1	7
Interviews	76	Secondary school	[11]	2	14
	68	Junior high school	[27]		

3.10 Topics of math learning using AR

RQ8: What are the most popular mathematics topics chosen for AR apps?

Five popular topics were chosen in the included articles: Geometry, Algebra, Euclidean Vectors, Statistic, and Probability. Six articles focused on geometry ([9][10][11][15][22][23]). An article by Chen [12] studied Algebra topics (multiplication of fractions, prime factors, and greatest common divisor) while the app developed by Martin-Gonzalez, Chi-Poot, and Uc-Cetina [19] focused on Euclidean Vectors (addition, subtraction and cross product). Cai et al. [27] focused on probability,whereas the study by Cai et al. [25] focused on the statistics topic. The remaining seven articles, however, did not mention any specific topics. Table 11 shows the list of mathematics subtopics chosen by the reviewed articles.

Table 11. Mathematics subtopics

Topic	References	f	%
Geometry	[9][10][11][15][22][23]	6	33
Algebra	[12]	1	6
Euclidean Vectors	[19]	1	6
Probability	[27]	1	6
Statistic	[25]	1	6
Others	[13][14][16][17][20][21][24][26]	8	44

4 Discussions and Limitations

4.1 Discussions

This review helps to understand a systematic and comprehensive analysis over the last five years of research in AR for learning mathematics. At the same time, it provides an updated analysis that reveals educational and technological necessities for deeper studies to be conducted in the future. Limited by the scope of this review (see data extraction sub-section), eight themes were discovered: AR type, development tools, contribution type, benefits of AR for math, math learning problems, research approaches, testing methods, and math subtopics. Clearly, AR significantly helps to ease the process of learning mathematics [29].

AR-based math learning could help to visualize the content better as marker-based AR is more popular compared to the markerless AR apps developed mostly using Unity 3D with Vuforia SDK. This is because Unity 3D and Vuforia SDK are better in terms of usability to support the development process. This review also found that AR usage in math teaching and learning provides students with fun and interactive learning processes, increased understanding, and enhanced visualization. Algebra is the most popular subtopic being rendered as AR apps. This review gives us a clearer overview of the often-used methods and research methods deployed by the articles included in this study.

Nevertheless, we need to conduct more research to answer several questions on why AR technology is popular among educators? Why are several AR development tools are more preferred than others? Besides the positive impacts, researchers may also systematically review the negative side effects of AR adaptation in learning mathematics as well as their learning experience [30] such as through game-based learning [31].

4.2 Limitations

This study is limited by the search terms used. Only the articles published in Scopus were analyzed. It is possible to find research articles on the educational uses of AR in other databases, such as Web of Science, Google Scholar, or Dimensions. Second, only journal articles published from 2015 until 2019 were included. Our review results were very much determined and limited by our review method. Thus, our results may differ from other reviews if any of these factors are altered: research questions, conceptual approach, selection criteria (inclusion or exclusion), and review process (identification, screening, eligibility).

A more rigorous and systematic review method may involve citation counting, reference analysis, and article evaluation by experts. The article's impact analysis based on established indexing databases may also be considered. Our systematic literature review may also be extended into meta-analysis and bibliometric analysis.

5 Conclusion and Future Works

Overall, the main contribution of this systematic literature review is that AR for learning mathematics has a promising trend. We have also enumerated a summary of challenges related to our research questions. Thus, the eight themes of research focus are AR apps types, AR apps development tools, research contribution categories, benefits of AR for math, learning problems, testing method, and math subtopics.

Furthermore, this study opens a fresh insight into the trends of AR implementation for mathematics education. This systematic review fills the gap in understanding AR research and development patterns and trends in learning mathematics. Moreover, this synthesized review on learning of mathematics through AR could benefit researchers and educators, thereby suggesting more avenues for future research.

Therefore, the following research questions are presented to guide future, and wider researches on a systematic review of AR usage in mathematics research:

- What are the user interaction and experience (UI/UX) issues about AR usage among students?
- What are the current industrial revolution 4.0 advancements that could be linked with AR and mathematics education?
- What are the factors that contribute towards students' satisfaction in using AR in mathematics education?

6 Acknowledgement

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