

Mathematics outdoor with

MathCityMap

Eugenia Taranto

eugenia.taranto@unikore.it



UNIVERSITÀ DEGLI STUDI DI ENNA "KORE"



Erasmus+

What is the talk about?

- Theoretical foundations on outdoor mathematical modeling
- Presentation of the MathCityMap system
- Methodology for conducting math trails with students
- Creating your own tasks: design criteria and response formats

Potential for outdoor learning

- Great potential is ascribed to learning outside of the classroom (DfES, 2006)
 - Creativity
 - Developing subject-related and interdisciplinary skills
 - Attitude towards learning
 - Motivation
- Long-lasting memories of learning outside (Dillon et al., 2006)

Potential for outdoor learning of mathematics

(Sauerborn & Brühne, 2009)

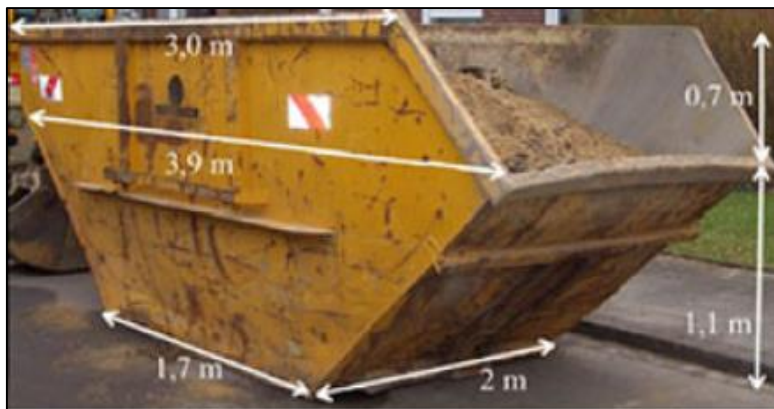
Potentials	Challenges
<ul style="list-style-type: none">• Primary experiences• Learning by active construction• Immediate application of theoretical concepts• Increased interest• Interdisciplinary Learning	<ul style="list-style-type: none">• Additional logistical, legal, and organisational effort• Curriculum and assessment• Abuse of freedom

Mathematical modelling

- Mathematical modelling is mostly done within the classroom
- The required data is usually provided in image or text form

Container task

The container is supposed to be filled up to the top.
How much sands fits inside the container?

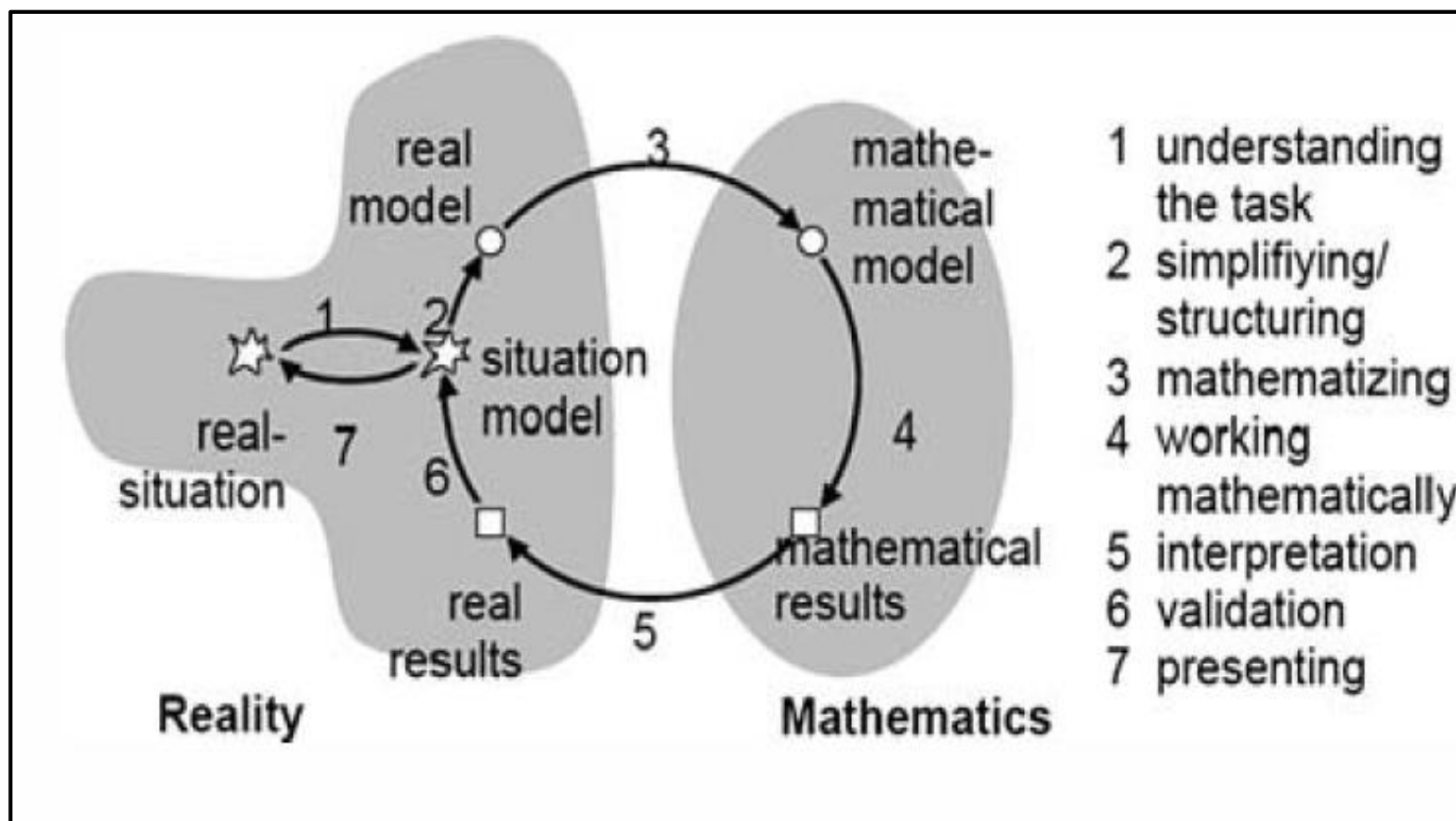


Tasks & Picture from:
Greefrath (2018)

Working on this task outdoors

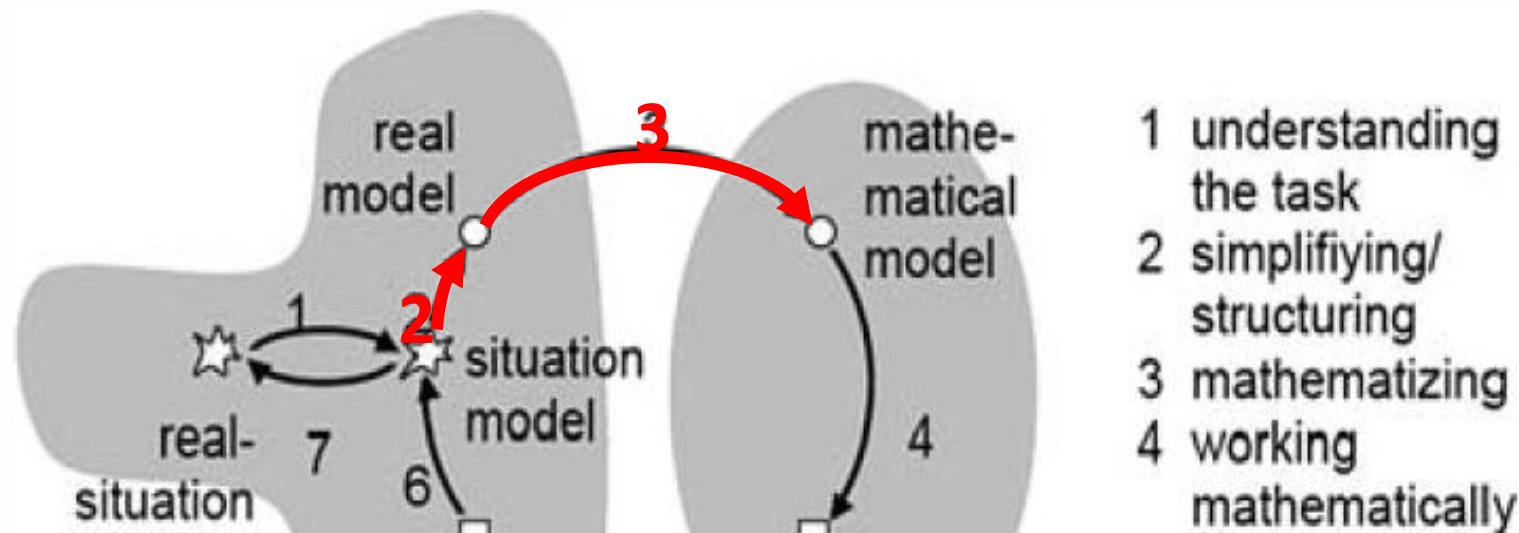


Modelling cycle according to Blum and Leiß, 2005



Modelling challenges (Schukajlow, 2006)

- Finding an adequate real model and a mathematical model for the given situation (steps 2 & 3)
- Deciding which data must be collected:
mathematization of the situation, definition of variables





Mathematical Modelling

Variety of possible models



Determine the weight of this stone at Camps Bay Beach, South Africa (in kg)
 1 cm^3 weighs approx. 2.6g.

Possible mathematical models:

- Sphere
- Cylinder
- Cuboid
- Prism
- Ellipsoid

Mathematical Modelling

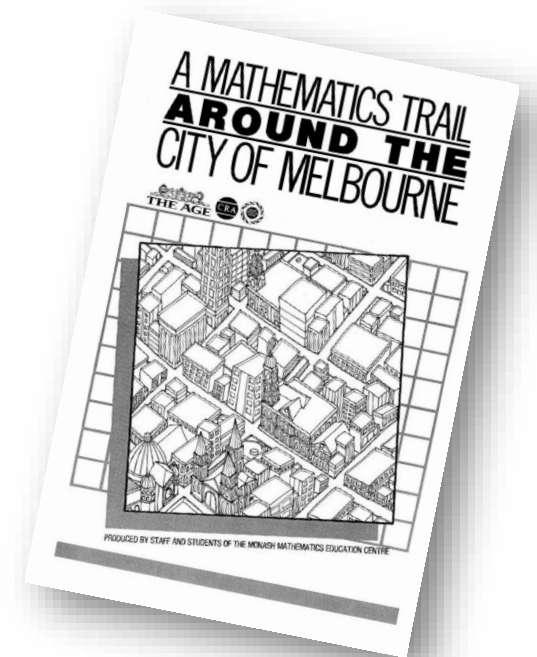
Variety of possible models

Mathematical model	Mathematical calculations (with measuring deviations of 2cm)	Result
Cuboid length: 112cm width: 78cm height: 58 cm	$(112m \pm 2cm) \cdot (78cm \pm 2cm) \cdot (58cm \pm 2cm)$ $= 507680cm^3 \pm 39500cm^3$	$1320kg \pm 103kg$
Cylinder	$536018cm^3 \pm 37008cm^3$	$1393kg \pm 96kg$
Ellipsoid	$199215cm^3 \pm 15369cm^3$	$518kg \pm 40kg$

Bad model!

Math trails

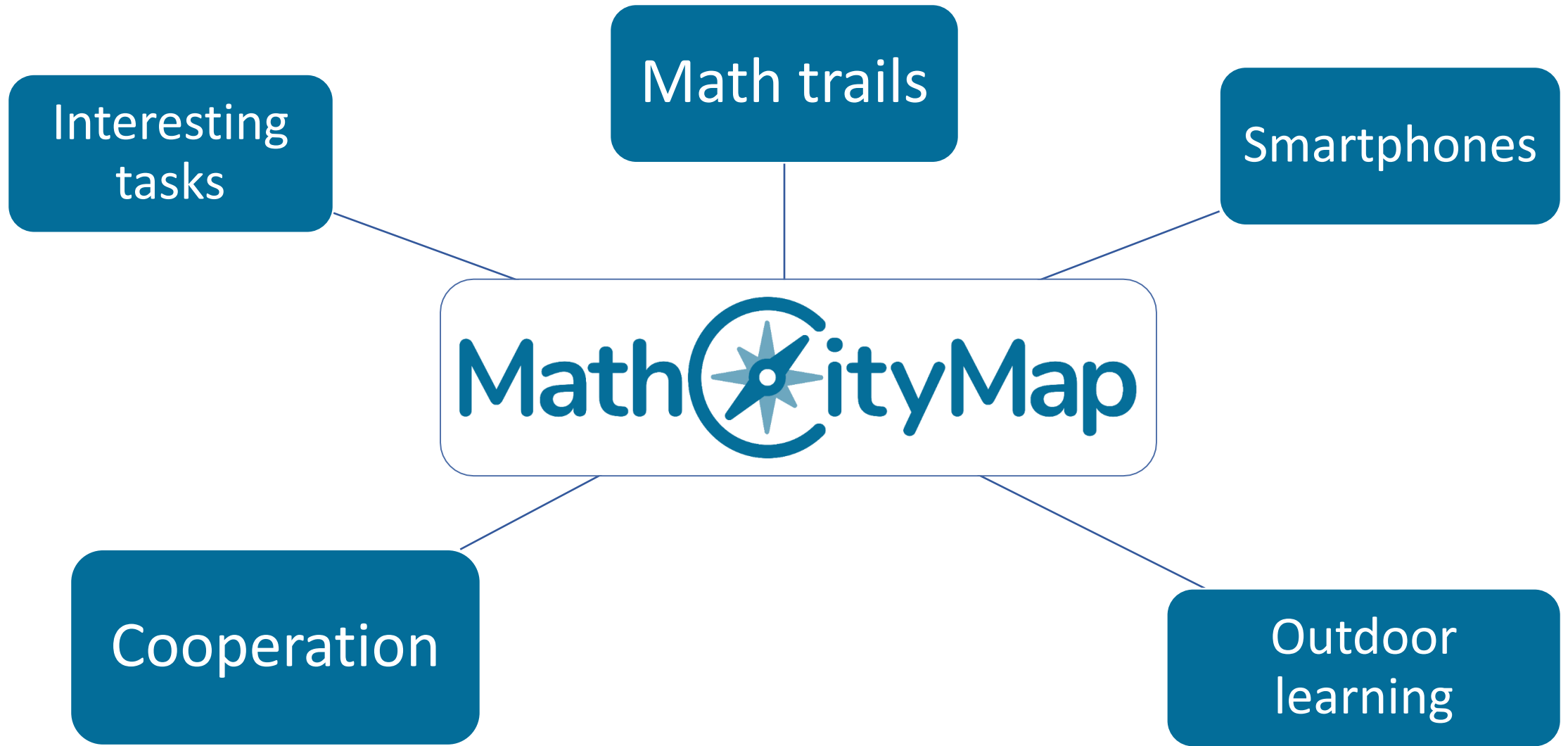
- A math trail is a mathematical walking trail
 - A walk through which mathematics can be experienced (Shoaf, Pollak & Schneider, 2004)
 - Discovering mathematics in interesting places and at interesting objects
- Can take place anywhere and is suitable for all age groups (Ludwig, Jesberg & Weiß, 2013)
- First math trail already over 30 years ago on Australia: A Mathematics Trail Around the City of Melbourne (Blane & Clarke, 1984)
- Helpful: A guide (real person or paper guide)



Math trails

Methodological characteristics:

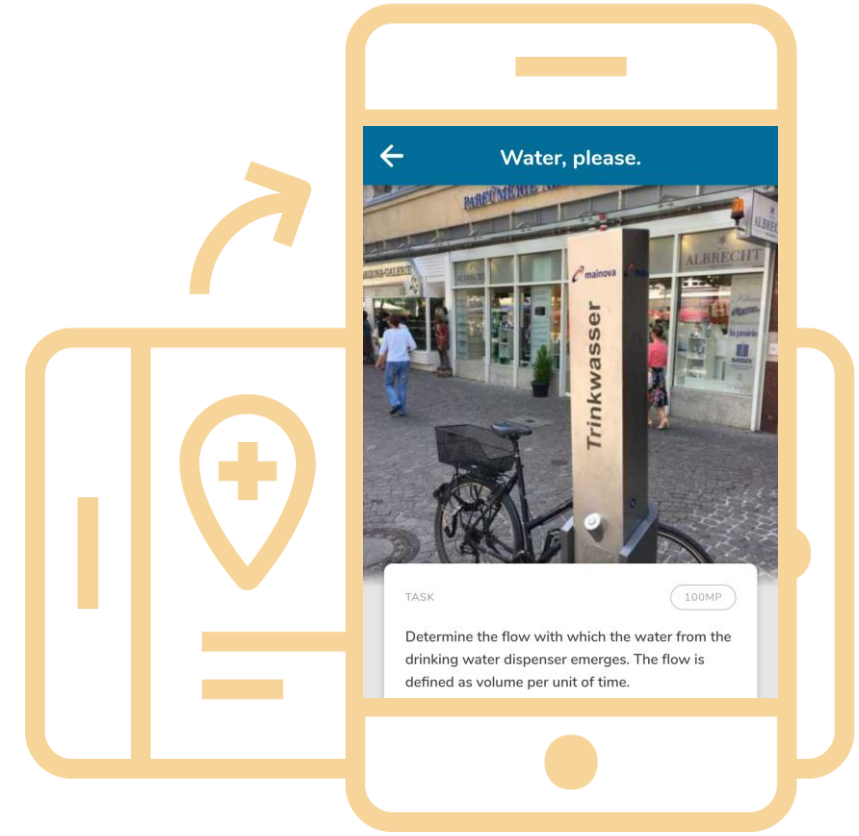
- Form of outdoor learning
- Working in small groups
- Realistic and authentic problems
- Interdisciplinary learning
- Importance of mathematics in everyday situations



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The MathCityMap system



MathCityMap idea: Web portal

- The tasks are marked on a map!

Web portal: Home page

MaSCE³ Web portal

Eugenia Taranto Level: 13
Role: admin

- Trails** Create and manage
- Tasks** Create and manage
- Groups** Create and manage
- Profile** Personal data, statistics
- Reviews** We ask for your opinion (62)
- Advanced** Advanced features

v.3.25

Web portal: Map view

MOM Web portal

English

Tasks

PUBLIC TASKS MY TASKS FOR ME FAVOURITES ALL TASKS

Order by Distance


- Sitzkreis
Messen Zählen Einheiten
5 5601.8 km 999318 [SHOW ON MAP](#)
- Infotafel
Messen Einheiten Fläche
5 5601.8 km 169324 [SHOW ON MAP](#)
- Grillhütte
Messen Fläche Einheiten
5 5601.8 km 829317 [SHOW ON MAP](#)

MathCityMap-Idea: Webportal





- The tasks are marked on a map
- **Tasks can be accessed in the portal**


Web portal: Calling up tasks

Grade level ↑ ↓





 50m to the North New


GPS line direction

  2  2.1 km  6817161 SHOW ON MAP



 Elevator in a glass box New


Geometry Volume Measure

  5  2.2 km  7917159 SHOW ON MAP





 Posters on the pillar New

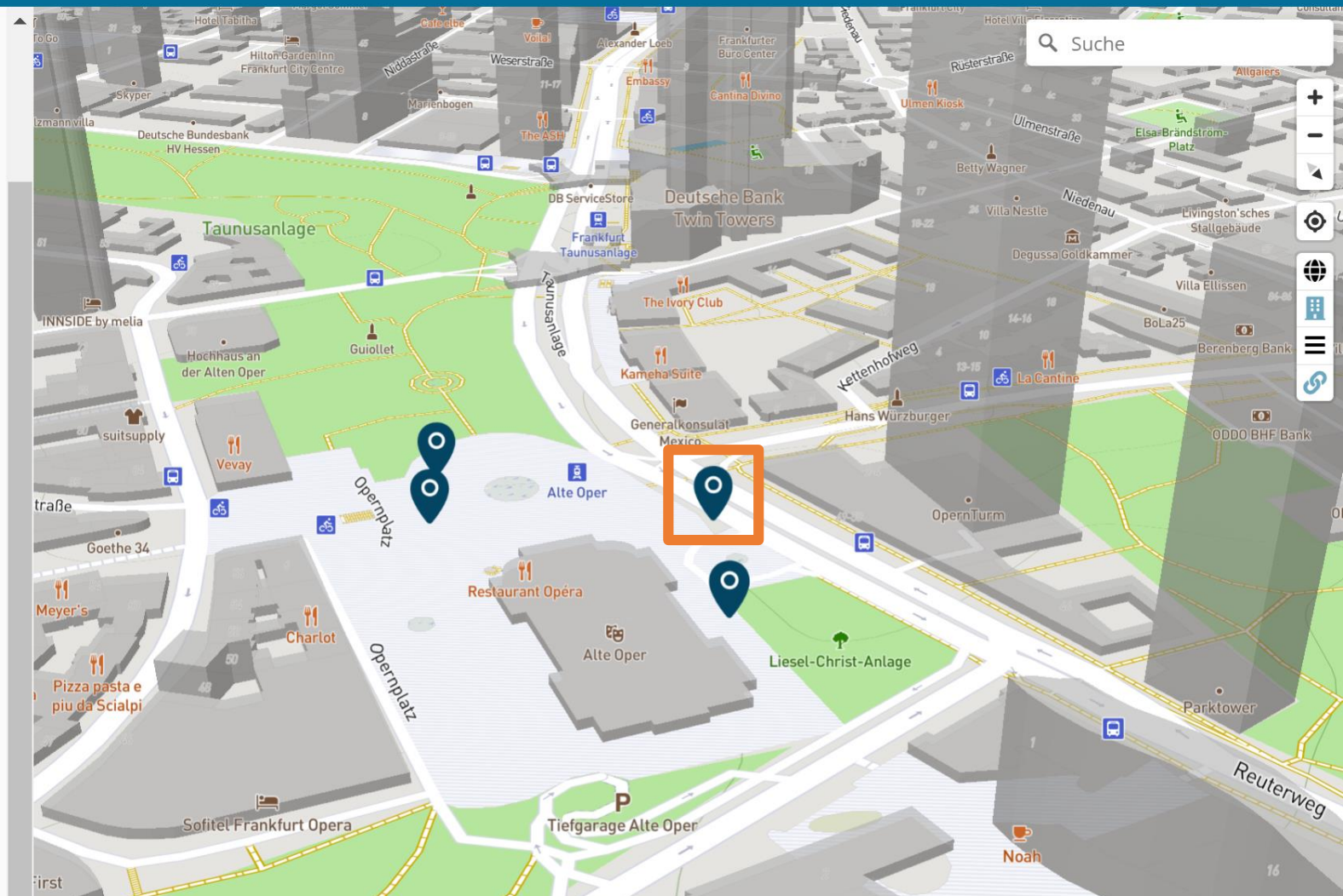
Number Poster DINA0

  6  2.2 km  3417162 SHOW ON MAP

 Hauptwache Escalator New

Speed Escalator

  8  1.8 km  4617160 SHOW ON MAP



Web portal: Calling up tasks

← Task: Elevator in a glass box



Elevator in a glass box

You can access the subway through an elevator in a glass box.
What is the volume of the glass box in m³?
(Note: Use the outer dimensions)

Geometry Volume Measure



MathCityMap idea: Web portal

- The tasks are marked on a map
- Tasks can be accessed or created in the portal
- **Several tasks form a Math trail**

Web portal: Home page

MaSCE³ Web portal

Eugenia Taranto Level: 13
Role: admin

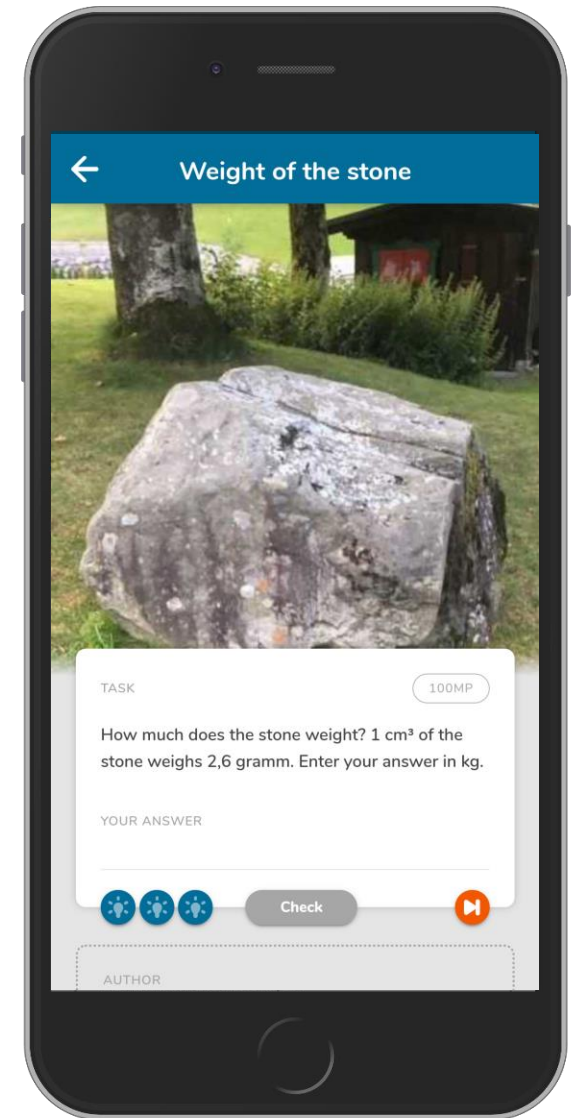
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v.3.25

mapbox © Mapbox © OpenStreetMap Improve this map

MathCityMap idea: Web portal

- The tasks are marked on a map
- Tasks can be accessed or created in the portal
- Several tasks form a math trail
- **The tasks are set in such a way that they can only be solved on site**
e.g. measurements must be collected



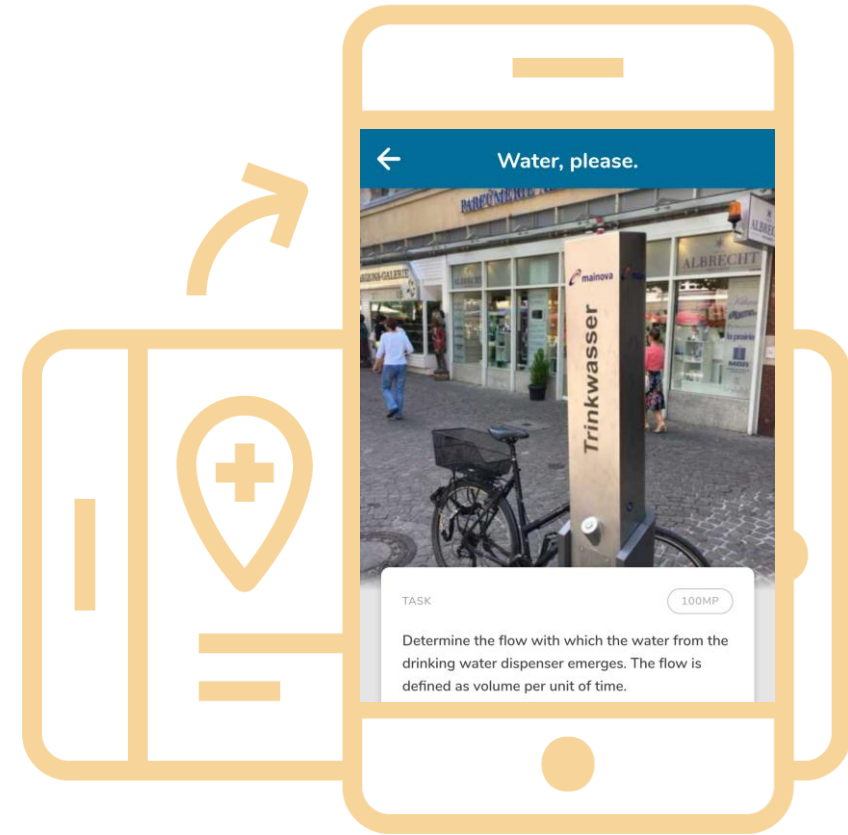
MathCityMap idea: Web portal

- The tasks are marked on a map
- Tasks can be accessed or created in the portal
- Several tasks form a math trail
- The tasks are set in such a way that they can only be solved on site
e.g. measurements must be collected
- **Everyone can participate and create their own tasks and math trails**

Create your own tasks & math trails

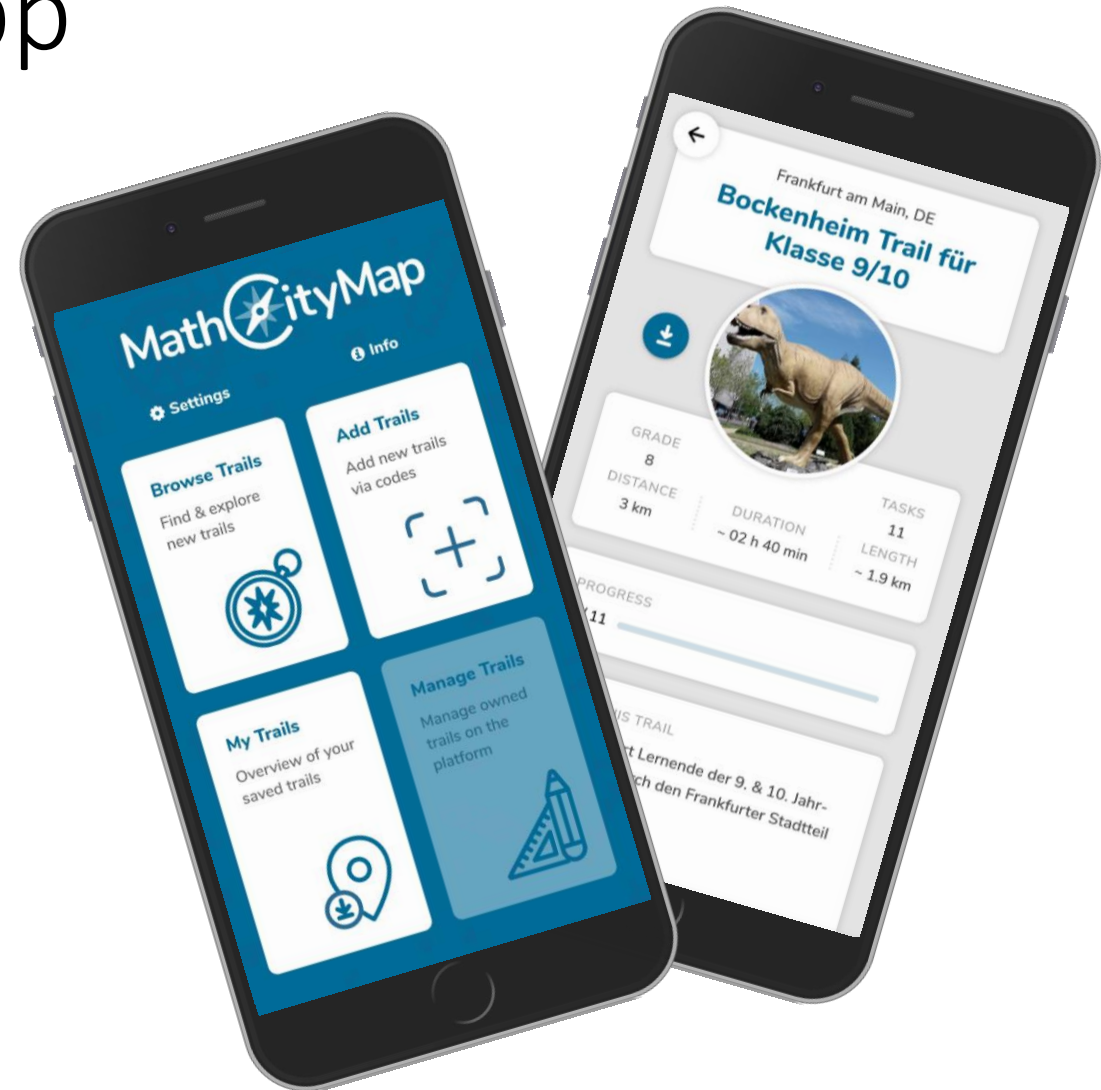
The screenshot shows the MOM Web portal interface. At the top, there is a navigation bar with the MOM logo, a search icon, the text 'English', and a user profile icon. Below this is a 'Tasks' section with a back arrow and a search bar. The 'Tasks' section is divided into 'PUBLIC TASKS', 'MY TASKS', 'FOR ME', and 'FAVOURITE'. Under 'PUBLIC TASKS', there is a filter for 'Distance' and a list of tasks. The first task is 'Treppenstufen' (Stairs) with the tag 'Kombinatorik' (Combinatorics), 8 tasks, 0.0 km distance, and 73181 views. The second task is 'Stairs' with tags 'combinatorics', 'systematic', and 'Fibonacci', 7 tasks, 0.0 km distance, and 186915 views. The third task is 'Institute of Mathematics and Computer Science Education' with tags 'Geometry' and 'slope'. On the right, a map of Frankfurt shows a math trail with green numbered markers (1-17) and blue location pins. A blue callout box with the text 'More about that later!' points to a specific location on the map. The map also shows various landmarks and streets.

The MathCityMap system

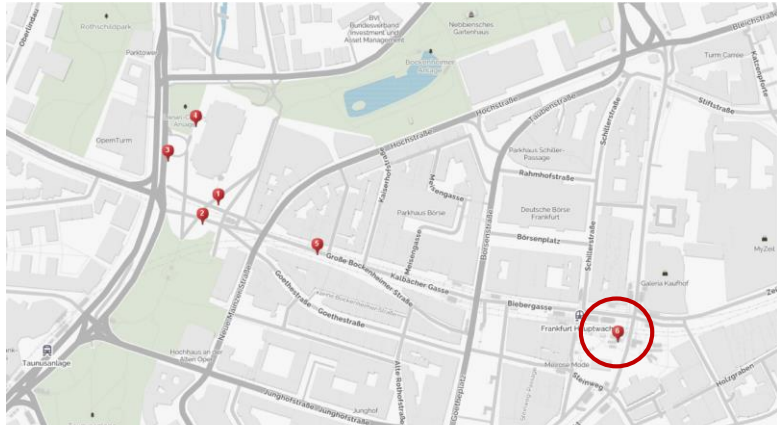


MathCityMap idea: App

- Trails are downloaded to the smartphone
- The MathCityMap app is free of charge and advertisements and it respects personal data protection (GDPR)
- The app is available for Android & iOS



Paper guide and/or smartphone app



6. Task: Hauptwache Escalator

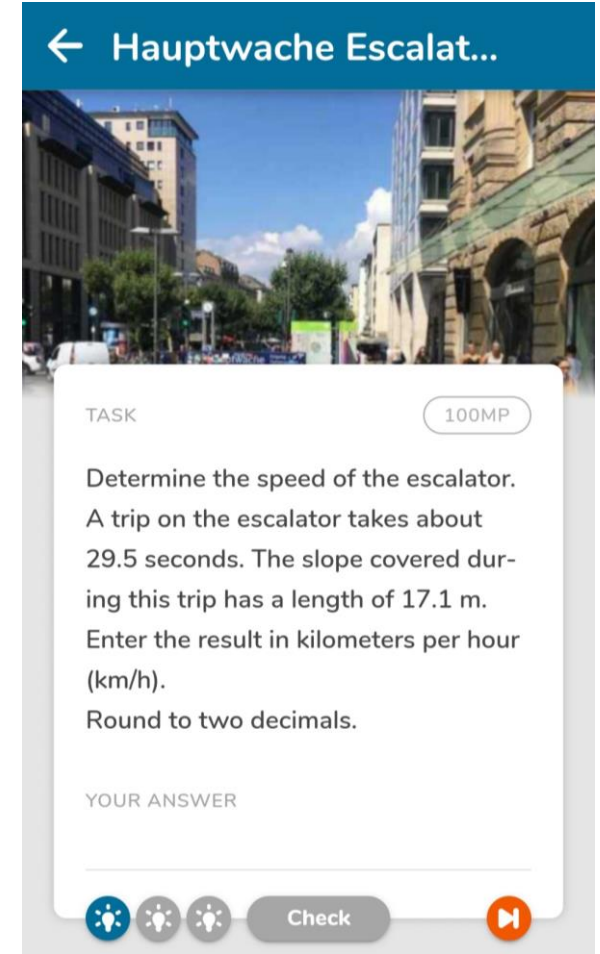
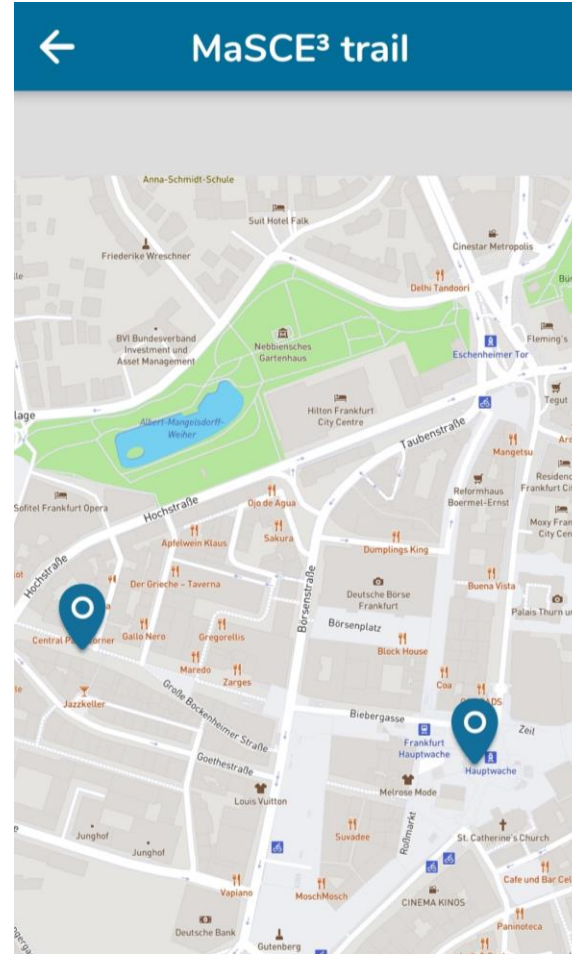


Calculation:

Determine the speed of the escalator. A trip on the escalator takes about 29.5 seconds. The slope covered during this trip has a length of 17.1 m. Enter the result in kilometers per hour (km/h). Round to two decimals.

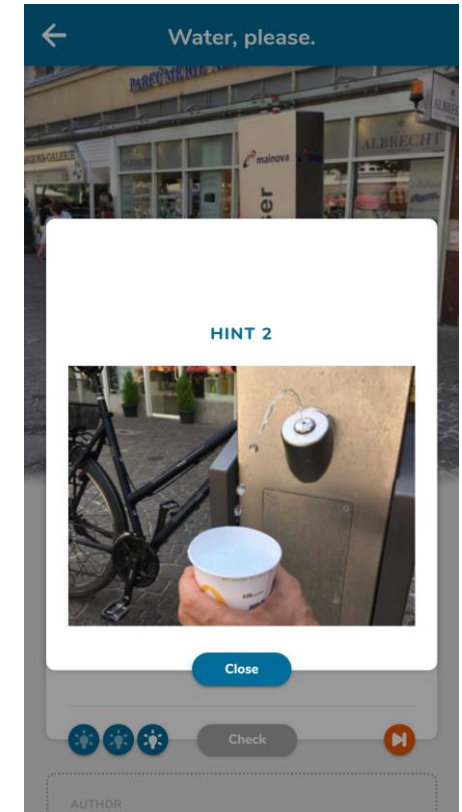
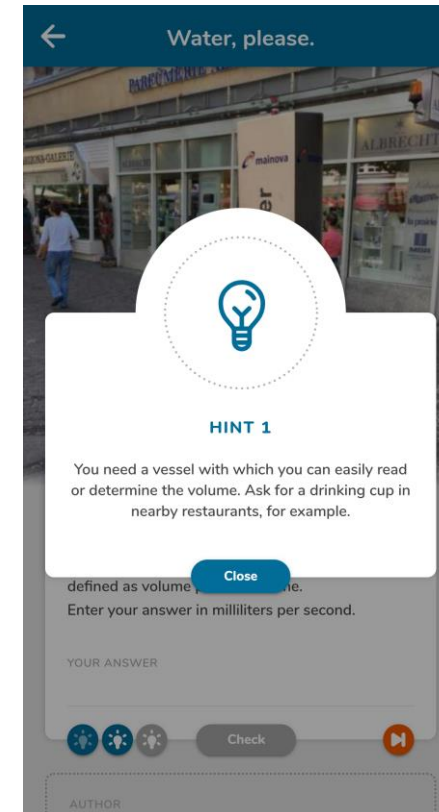
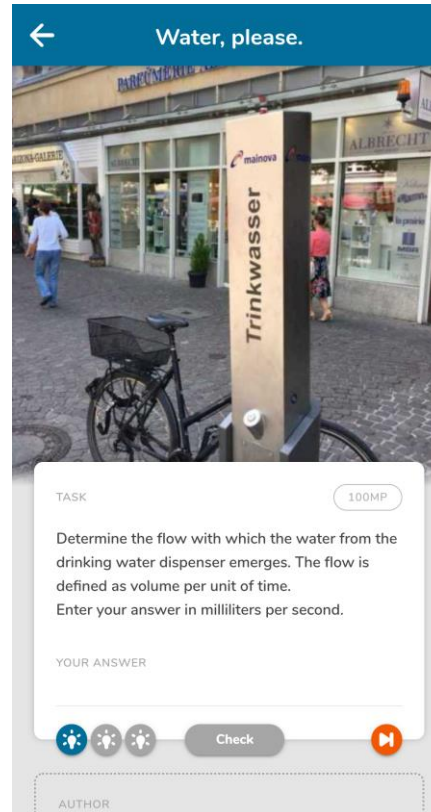
First guess:

The tasks are called up via the smartphone and/or a paper guide is used



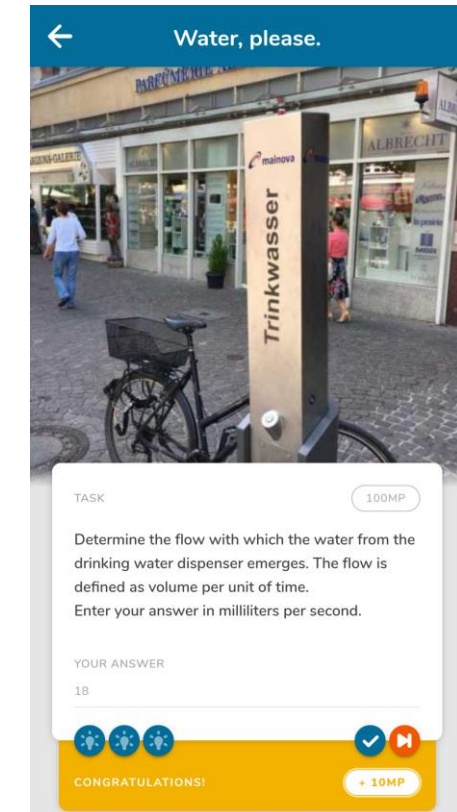
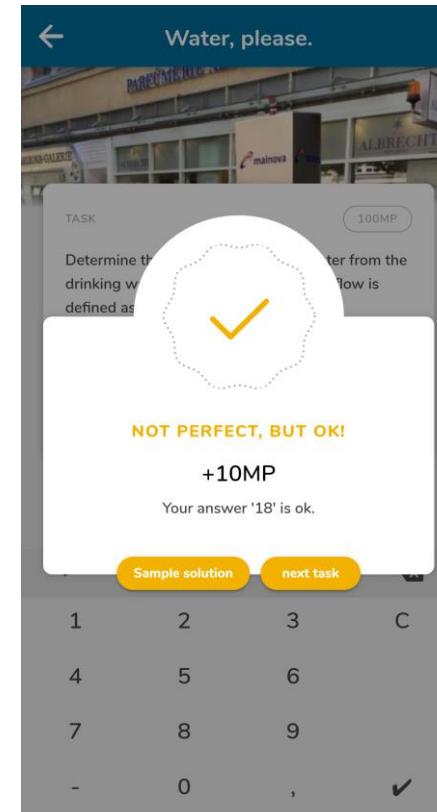
MathCityMap idea: App

- Trails are downloaded to the smartphone
- The tasks are called up via the smartphone and/or a paper guide is used
- **Help or hints can be called up via the app**



MathCityMap idea: App

- Trails are downloaded to the smartphone
- The tasks are called up via the smartphone and/or a paper guide is used
- Help or hints can be called up via the app
- **The solution is checked by the app**



What is the talk about?

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MathCityMap in class

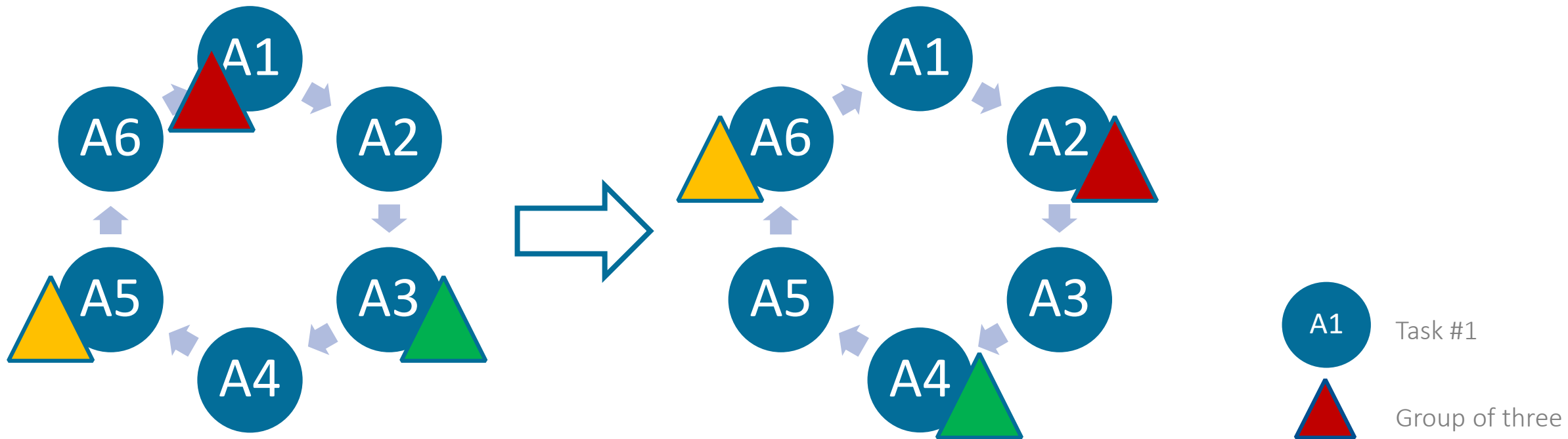
Preparation in the classroom

- Form groups of 3 members each
- Take one math trail set per group:
folding rule, measuring tape, trail guide, pen
- Each group needs only 1 active smartphone with the MCM app
- There are 3 roles – changing the rolls is desired:
 - Navigator operates the app, helps to find tasks
 - Measurement expert responsible for exact measurements and tools
 - Secretary records measured values and solution path

MathCityMap in class

Conduction of outdoor learning

Circle of stations: app guides groups to the next task

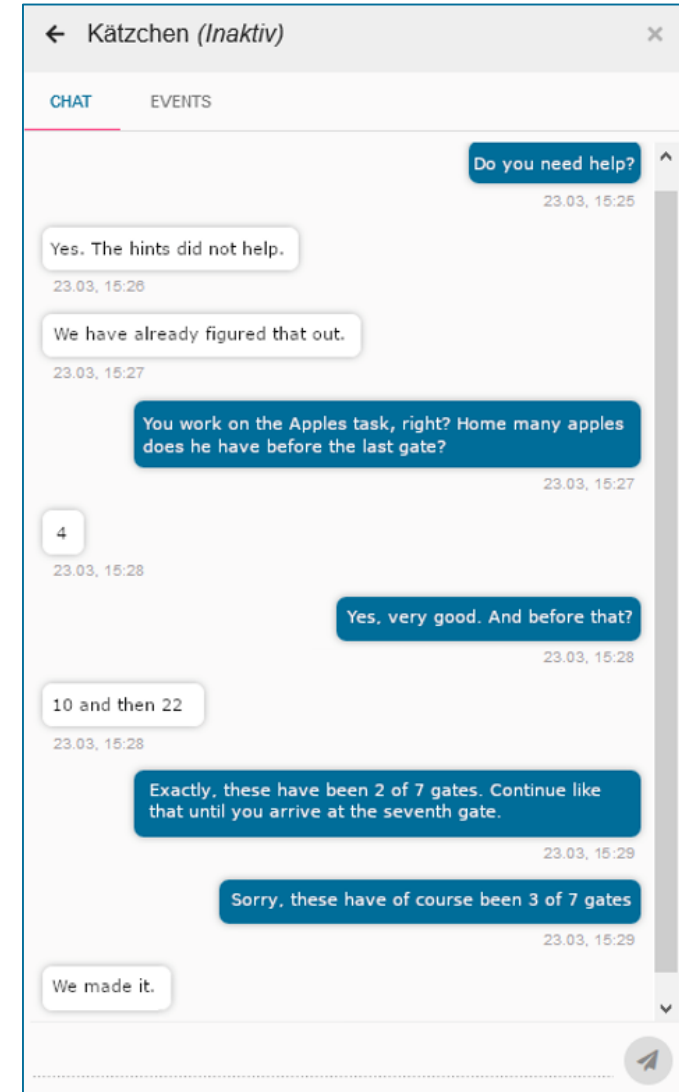




MathCityMap in class Digital classroom



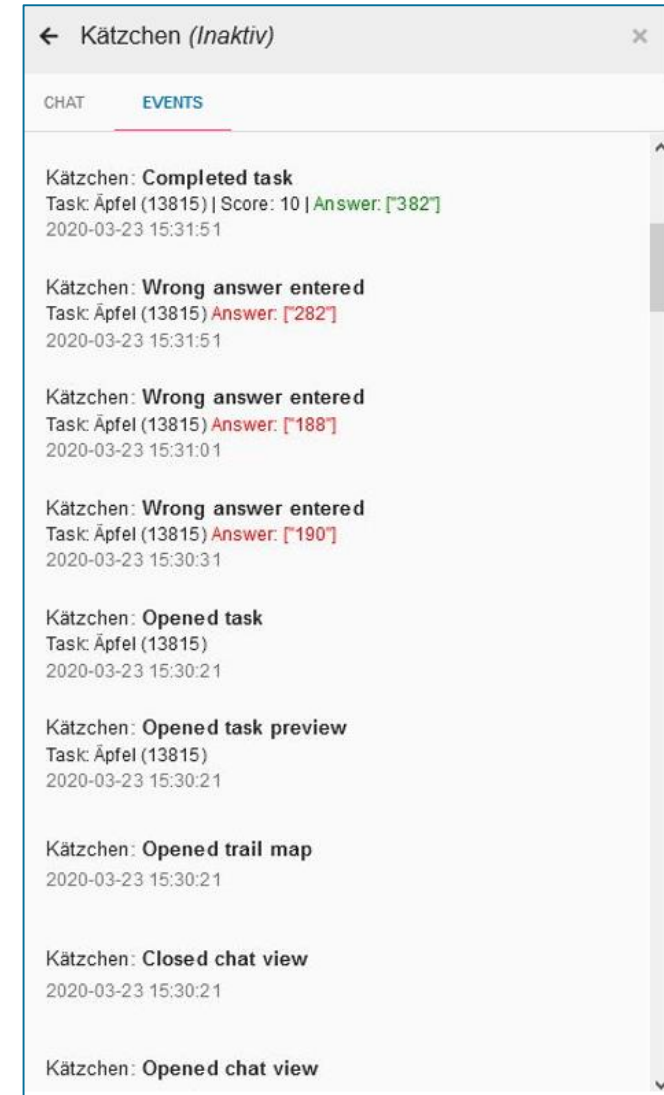
- Digital real-time representation of a class along a trail
- Three functions:
 - Organisational function
 - **Communication function: Chat between teacher & pupil groups**





MathCityMap in class Digital classroom

- Digital real-time representation of a class along a trail
- Three functions:
 - Organisational function
 - Communication function
 - **Evaluation function: Detection of potential errors through event view**



MathCityMap in class

Digital classroom – data protection

- The data is processed and stored in Germany (1&1 – Frankfurt).
- The collected data is not personalized
 - No registration for students necessary
 - Identification through random temporary keys (no clear names or e-mails necessary)
 - Encrypted data transmission (SSL)
- **MathCityMap complies with the provisions of the GDPR**

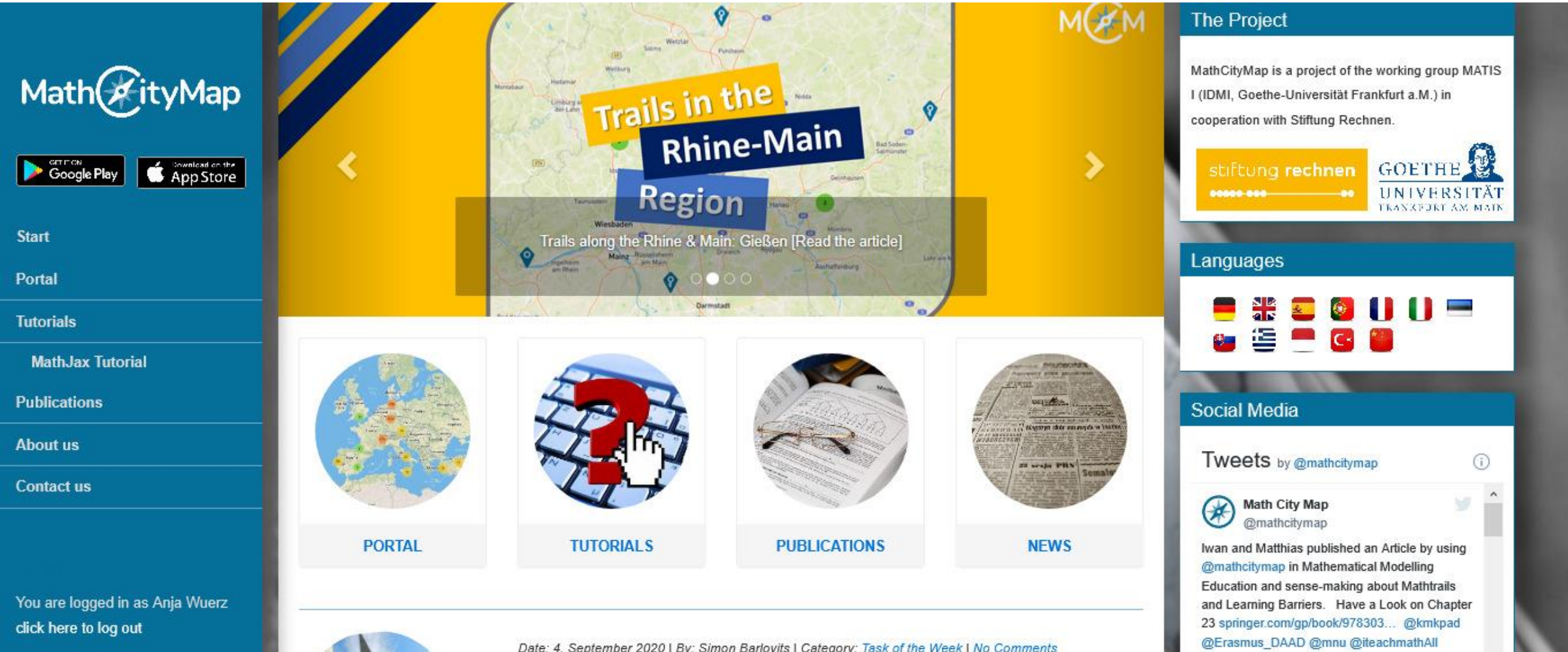
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Own tasks & Math trails

- MathCityMap is a participatory project:
Anyone can create a math trail in their own city
- Web portal: <https://mathcitymap.eu>

Own tasks & Math trails Web portal



The screenshot displays the MathCityMap website interface. On the left is a blue navigation sidebar with the MathCityMap logo and links for Start, Portal, Tutorials, MathJax Tutorial, Publications, About us, and Contact us. At the bottom of the sidebar, it shows the user is logged in as Anja Wuerz and provides a link to log out. The main content area features a large map of the Rhine-Main region with the title "Trails in the Rhine-Main Region" and a sub-header "Trails along the Rhine & Main: Gießen [Read the article]". Below the map are four circular icons representing the Portal, Tutorials, Publications, and News sections. On the right side, there are three sections: "The Project" which describes the project as part of the MATIS I working group at Goethe University Frankfurt a.M., in cooperation with Stiftung Rechnen; "Languages" which shows flags for German, English, Spanish, Portuguese, French, Italian, Dutch, Greek, and Chinese; and "Social Media" which displays a tweet from @mathcitymap about a publication by Iwan and Matthias.

The Project

MathCityMap is a project of the working group MATIS I (IDMI, Goethe-Universität Frankfurt a.M.) in cooperation with Stiftung Rechnen.

stiftung rechnen



Languages



Social Media

Tweets by @mathcitymap



Iwan and Matthias published an Article by using @mathcitymap in Mathematical Modelling Education and sense-making about Mathtrails and Learning Barriers. Have a Look on Chapter 23 springer.com/gp/book/978303... @kmpad @Erasmus_DAAD @mnu @iteachmathAll

Create your own tasks & Math trails

The screenshot displays the MOM Web portal interface. On the left, a sidebar contains a 'Tasks' section with a search icon, a list of filters (PUBLIC TASKS, MY TASKS, FOR ME, FAVOURITES, ALL TASKS), and an 'Order by Distance' dropdown menu. Below this, three task cards are visible:

- Treppenstufen**: Category 'Kombinatorik', 8 tasks, 0.0 km, 73181 views. Includes a 'SHOW ON MAP' link.
- Stairs**: Categories 'combinatorics', 'systematic', 'Fibonacci', 7 tasks, 0.0 km, 186915 views. Includes a 'SHOW ON MAP' link.
- Institute of Mathematics and Computer Science Education**: Categories 'Geometry', 'slope'.

The main area on the right shows a 3D map of Frankfurt, Germany, with several green circular markers numbered 2 through 11. A search bar at the top right contains the text 'Suche'. The bottom of the map shows the 'mapbox' logo and copyright information: '© Mapbox © OpenStreetMap Improve this map'.

Own tasks & Math trails

- MathCityMap is a participatory project:
Anyone can create a math trail in their own city
- Web portal: <https://mathcitymap.eu>
- **Registration is required to create tasks**
- **To create a math trail, several tasks are combined into one such trail**

Own tasks & Math trails

- Own content (tasks & trails) are private by default
- They can be shared by **code** or in a **working group**
- Publication of own contents possible on request
 - Tasks go through a review process
 - Tasks are checked for the following **quality criteria**

Criteria for a MCM task I

- **Clarity:**

For each task, a **picture** must be created that allows the clear identification of the situation or the object the task is about.

- **Presence:**

The task can only be solved **on site**, i.e. the task data must be collected on site. This also means that the picture or the task description must not be sufficient to successfully solve the task.

- **Activity:**

The person who solves the task must be **active** and do something (e.g. measuring or counting).

Criteria for a MCM task II

- **Multiple solutions:**
The task should be solvable in **different ways**.
- **Reality:**
The task should be **application-oriented**, realistic and not too contrived.
- **Graduated hints:**
At least two **hints** should be added to each task.
- **School mathematics and „tags“:**
The task should have a clear relation to school mathematics: Use the prepared tags or add new **terms**. The task should also be assigned to a **class level**.

Criteria for a MCM task III

- **Solution formats:**

The solution of the task should be expressed through one of the **response formats** that the sistem provides.

- **Tools:**

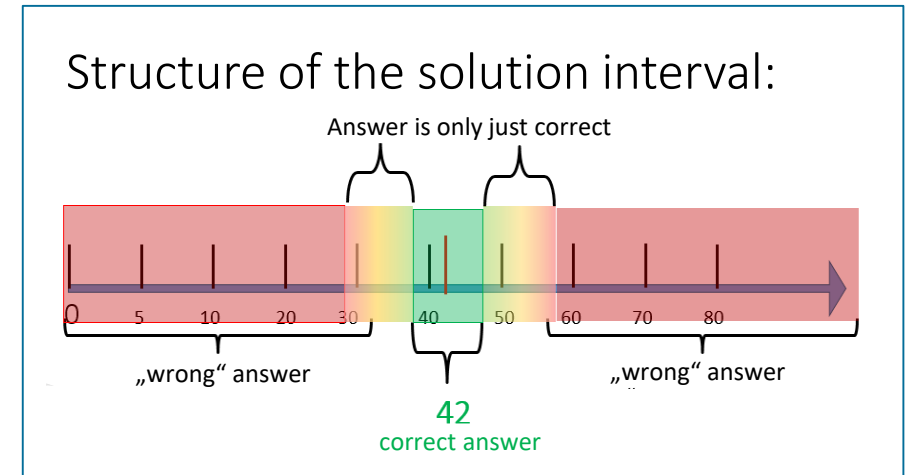
No special tools should be required to solve the task.

- **Sample solution:**

One should offer a **detailed solution and hints** (only visable in the portal) for teachers.

Response formats

- Exact solutions
- Multiple choice
- Intervals



Which response format fits best to my task idea?

Response formats: Exact solution

- Usage scenarios:
 - Combinatorial questions with an exact answer
 - Counting tasks where the number can be determined very accurately
- Example:
 - How many windows are visible on the south side of the building?
Answer: 40
- **Author** defines in the web portal: Number
- **User** enters in app: Number

Response formats: Interval

- Most common task type
- BUT: Badly calculated intervals can cause frustration!
- Usage scenarios:
 - All tasks that require measuring in the process.
- Example:
 - Determine the area of the (rectangular) schoolyard in m².
Solution: $A = a \cdot b = 100\text{m} \cdot 50\text{m} = 5000 \text{ m}^2$
 - BUT: Other measured values are conceivable (e.g. $a = 99,5 \text{ m}$; $b = 49,8 \text{ m}$)

The interval should contain all results that arise from acceptable deviations in the measurement!

Response formats

Interval: calculate interval limits

The „measured value“ method:

- A deviation is defined for each measured value which is still accepted (e.g. **3%-deviation for green** & **7%-deviation for orange interval**):

Acceptable measurement error	Own measurement	Acceptable measurement error
$a_1 = 97\text{m}$	$a = 100\text{m}$	$a_2 = 103\text{m}$
$b_1 = 48,5\text{m}$	$b = 50\text{m}$	$b_2 = 51,5\text{m}$
$A = a_1 \cdot b_1 = 4704,5\text{m}^2$	$A = a \cdot b = 5000\text{m}^2$	$A = a_2 \cdot b_2 = 5304,5\text{m}^2$

- Same procedure for the orange interval
- This leads to the following interval limits:



Response formats: Interval

- Usage scenarios:
 - All tasks that require measuring in the process
- Example:
 - Determine the area of the (rectangular) schoolyard in m².
Solution: $A = a \cdot b = 100\text{m} \cdot 50\text{m} = 5000 \text{ m}^2$

4325

4704

5305

5725

- **Author** defines in the web portal: 4 numbers (limits for green & orange interval)
- **User** enters in app: Number

Response formats: Multiple choice

- Usage scenarios:
 - Tasks that have several correct answers
 - Tasks that have terms as answers (equations, technical terms)
 - Tasks that present equivalent answers (e.g. $3 \cdot 2$, 6)
- Example:
 - What geometric shapes and bodies can you recognize in the Leaning Tower of Pisa?
 - a) squares
 - b) triangles
 - c) cylinders
 - d) circles
- **Author** defines in the web portal: 4 answers of which 1-4 may be correct
- **User** enters in app: Checks all correct answers

Response format: some news

<https://mathcitymap.eu/it/the-mathcitymap-task-formats/>

- Vector (interval)
- Vector (exact value)
- Set
- Fraction
- Information station
- Fill in the blanks
- Support tasks

For more information

Contact in Italy:
Eugenia Taranto

eugenia.taranto@unikore.it



www.mathcitymap.eu



www.masce.eu

MathCityMap

Co-funded by the
Erasmus+ Programme
of the European Union



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Same math trails as example

MathCityMap

MathCityMap - Liceo Matematico - Sample solution

Code: 1413844

Emanuele Amico, Salvatore Andrea Frazzetta, Simona Costa, Luana Nicosia, Claudia Alfano, Francesco Pulvirenti, Antonino Casto, Veronica Sambataro, Eugenia Taranto, Maria Flavia Mammana



MathCityMap

Matematica in giro ad UniSa - Sample solution

Code: 6813326

Eugenia Taranto



MathCityMap

Matematica in centro - Torino - Sample solution

Code: 1810212

Eugenia Taranto, Giulia Ferrari



MathCityMap

Matematica al Parco di San Giovanni - Trieste - Esempio di soluzione

Code: 1922725

Eugenia Taranto



Now is your turn!

Divide into groups to create 8 tasks
(this is the minimum number to generate
a trail)

With your group, find 2 interesting
objects outside (up to 18:10).

Create your tasks in the MCM system
(up to 18:30) and share it in the
"Trieste" group (code **145186**).

