theNumberline

Learning to Think Mathematically with the Number Line

A Resource for Teachers, A Tool for Young Children

Jeff Frykholm, Ph.D.

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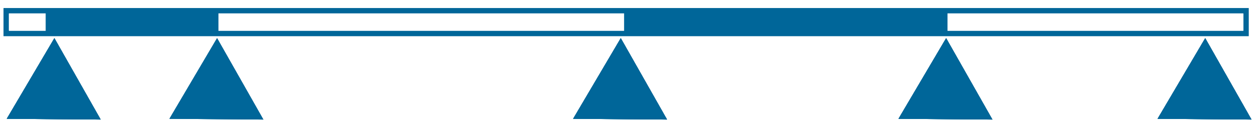

Overview: This book prepares teachers with the theoretical basis, practical knowledge, and expertise to use the number line as a vigorous model for mathematical learning in grades K–5. While the number line is a common artifact in elementary school classroom, it is seldom used to its potential. *Learning to Think Mathematically with the Number Line* helps teachers present lesson activities that foster students' confidence, fluency, and facility with numbers. Working effectively with the number line model, students can develop powerful intuitive strategies for single- and multiple-digit addition and subtraction.

About the Author

Dr. Jeffrey Frykholm is an Associate Professor of Education at the University of Colorado at Boulder. As a former public school mathematics teacher, Dr. Frykholm has spent the last 20 years of his career teaching young children, working with beginning teachers in preservice teacher preparation courses, providing professional development support for practicing teachers, and working to improve mathematics education policy and practices across the globe (in the U.S., Africa, South America, Central America, and the Caribbean).

Dr. Frykholm has authored over 30 articles in various math and science education journals for both practicing teachers, and educational researchers. He has been a part of research teams that have won in excess of six million dollars in grant funding to support research in mathematics education. He also has extensive experience in curriculum development, serving on the NCTM Navigations series writing team, and having authored two highly regarded curriculum programs: An integrated math and science, K-4 program entitled ***Earth Systems Connections*** (funded by NASA in 2005), and an innovative middle grades program entitled, ***Inside Math*** (Cambium Learning, 2009). This book, ***Learning to Think Mathematically with the Number Line***, is part of his latest series of textbooks for teachers. Other books in this series include: ***Learning to Think Mathematically with the Rekenrek; Learning to Think Mathematically with the Ratio Table;*** and ***Learning to Think Mathematically with the Double Number Line.***

Dr. Frykholm was a recipient of the highly prestigious National Academy of Education Spencer Foundation Fellowship, as well as a Fulbright Fellowship in Santiago, Chile to teach and research in mathematics education.

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Learning to think Mathematically: An Introduction

The *Learning to Think Mathematically* Series

One driving goal for elementary level mathematics education is to help children develop a rich understanding of numbers – their meanings, their relationships to one another, and how we operate with them. In recent years, there has been growing interest in ***mathematical models*** as a means to help children develop such number sense. These models – e.g., the number line, the rekenrek, the ratio table, etc. – are instrumental in helping children develop structures – or *ways of seeing* – mathematical concepts.

This textbook series has been designed to introduce some of these models to teachers – perhaps for the first time, perhaps as a refresher – and to help teachers develop the expertise to implement these models effectively with children. The approaches shared in these books are unique; they are also easily connected to more traditional strategies for teaching and developing number sense. Toward that end, we hope they will be helpful resources for your teaching. In short, these books are designed with the hope that they will support teachers' content knowledge and pedagogical expertise toward the goal of providing a meaningful and powerful mathematics education for all children.

How to Use this Book

This book contains numerous lesson plans, each of which can be modified for use with students in various grade levels. So in that sense, this is not a book "for second grade," for example. Lessons have been divided into several clusters; sometimes these groupings of lessons have to do with common mathematical themes, and other times they might be grouped together because they use similar pedagogical strategies. In any event, each lesson plan contains detailed notes for teachers regarding the objectives for the lesson, some background information about the concepts being emphasized, and specific, step-by-step instructions for how to implement the lessons.

Of course, the lesson plan itself is only ink on paper. We hope that teachers will apply their own expertise and craft knowledge to these lessons to make them relevant, appropriate (and better!) in the context of their own classrooms, and for their own students. In many cases, a lesson may be extended to a higher grade level, or perhaps modified for use with students who may need additional support. Ideas toward those pedagogical adaptations are provided in the lesson plans.

The final section of this book contains an appendix that includes activity sheets for some (not all) of the lesson activities. Typically, there are several distinct activity sheets per lesson – the same content, but problems designed to reflect a different age group. Again, these activity sheets have been designed as templates – they contain ideas and can certainly be used "as is." However, teachers may choose to build upon, change, or enhance these activity sheets.

The Number Line: An Overview

One of the most overlooked tools of the elementary and middle school classroom is the number line. Typically displayed above the chalkboard right above the alphabet, the number line is often visible to children, though rarely used as effectively as it might be. When utilized in the elementary classroom, the number line has often used to help young children memorize and practice counting with ordinal numbers. Less often, perhaps, the number line is used like a ruler to illustrate the benchmark fractions like ½ or ¼. Beyond an illustration for these foundational representations of whole numbers and some fractions, however, the number line is underutilized as a mathematical model that could be instrumental in fostering number sense and operational proficiency among students.

Recently, however, there has been a growing body of research to suggest the importance of the number line as a tool for helping children develop greater flexibility in mental arithmetic as they actively construct mathematical meaning, number sense, and understandings of number relationships. Much of this emphasis has come as a result of rather alarming performance of young learners on arithmetic problems common to the upper elementary grades. For example, a study about a decade ago of elementary children in the Netherlands – a country with a rich mathematics education tradition – revealed that only about half of all students tested were able to solve the problem 64-28 correctly, and even fewer students were able to demonstrate flexibility in using arithmetic strategies. These results, and other research like them, prompted mathematics educators to question existing, traditional models used to promote basic number sense and computational fluency.

Surprising to some, these research findings suggested that perhaps the manipulatives and mathematical models typically used for teaching arithmetic relationships and operations may not be as helpful as once thought. Base-10 blocks, for example, were found to provide excellent conceptual understanding, but weak procedural representation of number operations. The hundreds chart was viewed as an improvement on arithmetic blocks, but it too was limited in that it was an overly complicated model for many struggling students to use effectively. On the other hand, the number line is an easy model to understand and has great advantages in helping students understand the relative magnitude and position of numbers, as well as to visualize operations. As a result, Dutch mathematicians in the 90's were among the first in the world to return to the "empty number line," giving this time-tested model a new identity as perhaps the most important construct within the realm of number and operation. Since that time, mathematics educators across the world have similarly turned to this excellent model with great results.

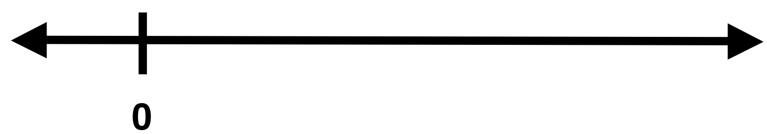
The intent of this book is to share some of the teaching strategies that have emerged in recent years that take advantage of the number line in productive and powerful ways.

The Big Ideas

As noted above, the number line stands in contrast to other manipulatives and mathematical models used within the number realm. Some of the reasons for developing the number line as a foundational tool are illustrated below as key ideas for this textbook.

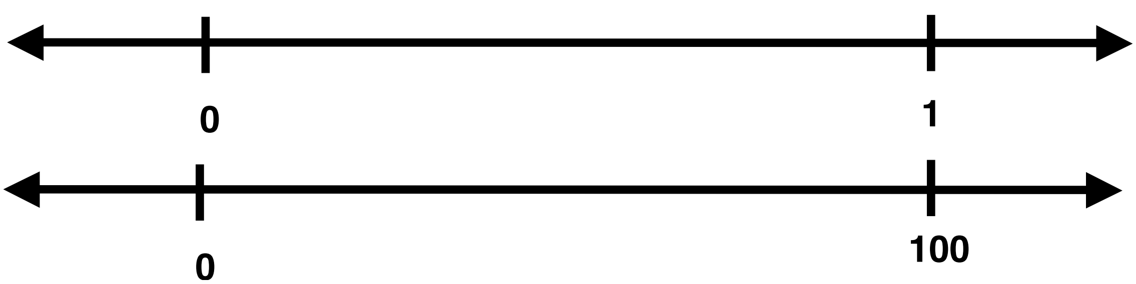
<u>Key Concept #1: The linear character of the number line</u>. The number line is well suited to support informal thinking strategies of students because of its inherent linearity. In contrast to blocks or counters with a "set-representation" orientation, young children naturally recognize marks on a number line as visual representations of the mental images that most people have when they learn to count and develop understanding of number relationships. It is important to note the difference between an "open number line" (shown below) and a ruler with its predetermined markings and scale.

An open number line:



The open line allows students to partition, or subdivide, the space as they see fit, and as they may need, given the problem context at hand. In other words, the number line above could be a starting point for any variety of number representations, two of which are shown below: the distance from zero to 1, or the distance from zero to 100. Once a second point on a number line has been identified, the number line moves from being an open number line, to a closed number line. In addition, the open number line allows for flexibility in extending counting strategies from counting by ones, for example, to counting by tens or hundreds all on the same sized open number lines.

Closed number lines:

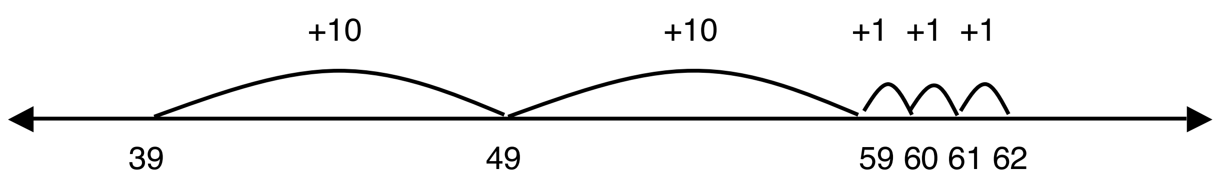


<u>Key Concept #2: Promoting creative solution strategies and intuitive reasoning</u>. A prevalent view in math education reforms is that students should be given freedom to develop their own solution strategies. But to be clear, this perspective does not mean that it is simply a matter of allowing students to solve a problem however they choose. Rather, the models being promoted by the teacher should themselves refine and push the student toward more elegant, sophisticated, and reliable strategies and procedures. This process of formalizing mathematics by having students recognize, discuss, and

internalize their thinking is a key principle in math education reforms, and is one that can be viewed clearly through the use of a model like the number line – a tool that can be used both to model mathematical contexts, but also to *represent methods, thinking progressions, and solution strategies* as well. As opposed to blocks or number tables that are typically cut off or grouped at ten, the open number line suggests continuity and linearity -- a representation of the number system that is ongoing, natural, and intuitive to students. Because of this transparency and intuitive match with existing cognitive structures, the number line is well suited to model subtraction problems, for example, that otherwise would require regrouping strategies common to block and algorithmic procedures.

Key Concept #3: Cognitive engagement. Finally, research studies have shown that students using the empty number line tend to be more cognitively active than when they are using other models, such as blocks, which tend to rely on visualization of stationary groups of objects. The number line, in contrast, allows students to engage more consistently in the problem as they jump along the number line in ways that resonate with their intuitions. While they are jumping on the number line, they are able to better keep track of the steps they are taking, leading to a decrease in the memory load otherwise necessary to solve the problem. For example, imagine a student who is trying to solve the problem: 39 + 23. Under the traditional addition algorithm, or with base-10 manipulatives as well, the "regrouping" strategy, or the "carry the one" algorithm is significantly different cognitively than thinking of this addition problem as a series of jumps. Specifically, the student might represent the problem as: 1) Start at 39; 2) Jump 10 to get to 49; 3) Jump 10 more to get to 59; 4) jump three to get to 62. These steps are highlighted on the number line as the student traces her thinking with a pencil.

39 + 23 = 62



Like any mathematical tool, the more teachers are aware of both the benefits and constraints of the model, the more likely they are to use it effectively with students. Throughout this book, the previous big ideas – though theoretical in nature – are drawn upon repeatedly as students view and subsequently manipulate various open number lines that are used to represent numerous mathematical contexts and operations.

Teaching Ideas

A large portion of this book is devoted to helping students develop a rich sense of numbers and their relationships to one another. The number line is centrally related to this task. As noted above, perhaps the most important teaching point to convey regarding the number line is the notion that, unlike a ruler, it is open and flexible. Given this starting point, students will quickly recognize that they need to create their own actions on the number lines to give the model meaning. Throughout the book, activities include opportunities for students to partition a number line as they see fit. The important thing for students to recognize is that one point alone on a number line does not tell us much about the **scale or magnitude** of numbers being considered. In the number line below, we know very little about this mathematical context other than the fact that it identifies the number 0 on a line.

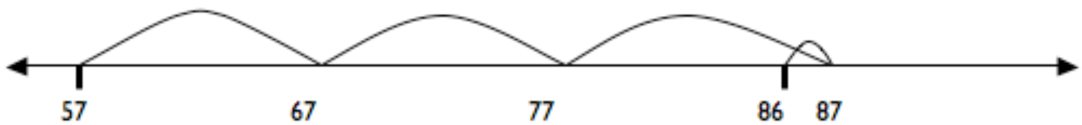
Yet, by putting a second mark on the line, suddenly each number line below takes on its own significant meaning, and to work with each of these respective lines would require a different kind of mathematical thinking.

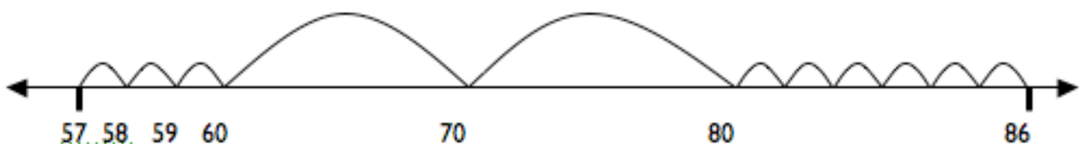
In the first number line above, students will likely begin thinking immediately in terms of tens and twenties — perhaps 50 — as they imagine how they might partition a line from 0 to 100. They will use doubling and halving strategies, among others, as they mark the number line. In the second line, fractional distances between zero and one are likely to come to mind. Once again, students may be using halving strategies if they are finding a number like 1/2 or 1/4. Finding thirds or fifths requires a different type of thinking, of course, which may be beyond K-3 learners. The point here is to recognize the notably different outcomes that might be pursued with these two, simple number lines that each shared a common beginning above (i.e., an open number line with zero identified). Throughout the book, activities will take advantage of this principle.

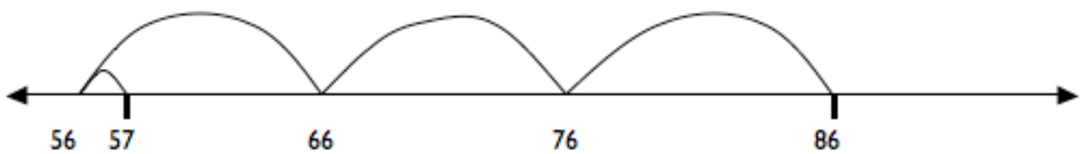
In subsequent sections of the book, the number line is developed as a reliable tool to help students add and subtract. Developed in the book is the idea of a “skip jump” – progression along a number line that is done in specific increments. In this way, the number line becomes a helpful model to mirror how students add and subtract mentally. Students become quite adept at skip jumping by 10’s or 100’s, for example, and eventually begin to make mental adjustments to the number sentences at hand in order to take advantage of more sophisticated (or for them, easier) intervals for skip jumping. Consider the following problem, for example:

> <u>The Problem</u>: Kerri was trying to set her record for juggling a soccer ball. On her first attempt, she juggled the ball a total of 57 times before it hit the ground. On her second attempt, she only got a total of 29 juggles. Combining both her first and second attempts, how many times did she juggle the ball in total?

Using a number line flexibly, students may choose to solve this problem in any number of ways, each of which anchors on fundamental understandings of number. The first solution, for example, shows a student who counts on from 57, first by 10's. After skip-jumping forward by 30 (three jumps of 10), the student realizes that she needs to compensate by hopping back by one to arrive at the correct answer → 57 + ***29***.







What are the thinking strategies of the other two students? Take a moment to consider how the number line can be used to mirror the actual thoughts contributing to the solution strategies of these two children… In the second, add 3 to get to an even "decade" number of 60. Now it becomes rather trivial to add 26 to 60. In the third example… take one away (57-1=56), and now we add 30 (instead of 29) to 56 to arrive at a total of 86.

Summary

I have often been asked about the reason for focusing on the number line itself in a book like this rather than on, for example, number concepts, addition, or subtraction. The number line models the natural ways in which we think about all number relationships and number operations. The premise underlying this book-- and the series as a whole-- is that a curriculum best serves students when it provides powerful mathematical tools and understandings that can be used in numerous mathematical contexts and with different types of numbers. Specifically, the tools that students learn to employ in this book can be used with larger whole numbers, integers, fractions, and decimals as well. In short, the number line is an extremely powerful model. The intent is that, perhaps without fully recognizing, students will gain rich intuitions about numbers and operations in this book that will serve them well in years to come.

Activity Set 1: The Life-Sized Number Line

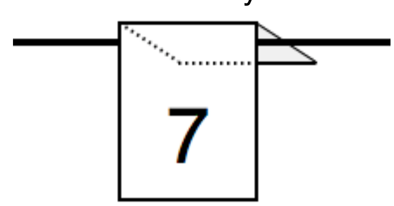
We begin this book with ten lesson plans and related activities that illustrate a powerful **teaching methodology** that we hope you will adopt throughout your use of the book. While many of the activities in this book are designed to be done by individual students or small groups at their desks (often with pencil and paper), it can also be extremely effective to model number line concepts with a "life-sized" number line. I have had the good fortune of observing master teachers use such a number line with great success -- a rope across the front of the room, clothespins, and large number cards that are used to develop conceptual understanding while at the same time fostering student engagement and participation.

The intent of the following 10 lesson plans in **Activity Set 1** is to provide some examples of how a large-scale number line can be used. **These activities are not necessarily designed to follow one another sequentially**. Rather, they are included at the beginning of this book as example lesson plans that illustrate the power of the demonstration number line. It is my hope that teachers will adopt this methodology -- the life sized number line -- when appropriate throughout the remainder of the activities in this book.

While many of these lessons are designed for use in K-3 classrooms, **upper elementary teachers (grades 3-5) may find the ideas and methodology of the life-sized number line to be helpful for their students.** I would encourage all teachers to look at the lessons, and make a determination as to how the numbers or problem contexts could be altered so as to be relevant, meaningful, and appropriate for their own classroom and students. The power and impact of the visualization that students do while engaging the life-sized number line is significant.

The basic idea for these lessons is to extend a rope or clothesline across a large, open space in the classroom. Using large number cards, the teacher can solicit the help of students to place numbers on the line as required in a given lesson progression. Teachers may also ask students to stand in various locations as representations of numbers on the number line. It is easy to imagine the ways in which young children might become engaged as participants in this activity, placing their cards, if not themselves, appropriately on a large-scale number line.

You can **make number cards** in several ways. Clothespins and index cards work well. Alternatively, and perhaps easier for students, you may use the backs of recycled paper, cut into 4 or 5 inch rectangles. Fold the top end of the paper, making a small crease that can then be used to hang the number card on the number line. (See picture.) The advantage of this method is that the cards are easily slid along the number line. Of course, number cards can easily be reused for other lessons.



Lesson 1: Zero -10 with the Life Sized Number Line

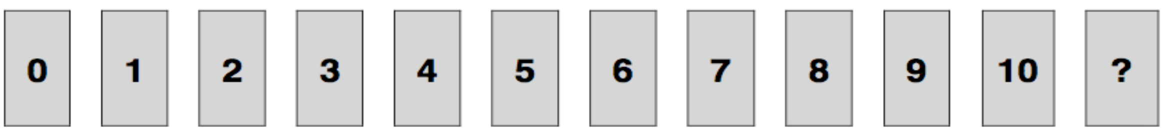
Lesson Objectives

- Students will locate (place) numbers between zero and ten on a fixed number line. (For more experienced students, extend the range of the numbers being used.)
- Students will give the number name for specific locations on a number line.

Activity Background and Introduction

- This lesson uses a demonstration number line to develop number sense between zero and ten. It is important for students to be able to both place numbers on a fixed number line (e.g., **"Can you show me where the number 8 belongs?"**), as well as to be able to identify the value of a given location on a number line (e.g., **"Can you tell me what number should go in the box?**). These key ideas are developed in this lesson.

Lesson Progression



- Begin by stringing rope (the number line) across the front of the room. Prepare the number cards shown above (large size) in advance of the class. You may wish to choose other larger numbers for more advanced students.
- Begin by asking students: **"Where should I place zero on the number line?"** It is important for them to realize that zero can go anywhere. Based on student input, pin the zero card on the number line, preferably leaving considerable space to the right of zero for other numbers.
- Next ask students: **"Where should I place 10?"** Take advantage of the open number line to foster discussion about where ten could go (anywhere, because we have not yet determined a scale for the number line). After sufficient discussion and input from students, pin the number ten on the number line, preferably leaving plenty of space between zero and ten.
- Next, students should place various cards on the number line. You might want to distribute the number cards to various students so that they can participate in hanging the cards on the number line.
- Begin with the number 5 card. Ask: **"If I know zero, and I know 10, do I know where to place 5?"** Listen for students to note that 5 would be halfway between zero and ten. Place the 5 card on the line based on student comments. Note: If students decide to place a number in the wrong spot initially, that is okay. You might choose to continue without correcting. Eventually students will run into a logical contradiction, or a card that does not feel intuitively correct given its proximity to two or more other cards. Look for these moments to solidify students' number sense.

- Proceed with other cards, encouraging students to justify how they know where a card might be placed. Listen for comments such as, for example, “4 is half of 8, so it should be placed halfway between zero and 8.”... “6 is one more than 5, so put it one to the right of 5.”... etc. These comments will give you an indication of the developing number sense of your students.
- Next, clear all the number off the line except for the numbers zero and ten. Now, place two empty box cards (with the question marks) on the line. Place one at the midpoint between zero and 10 (i.e., 5), and another card roughly where one would expect to see the number two. Ask students: **“Can you tell me what numbers would go in each box?”** Vary the location of the empty boxes. Also, you may include other numbers as anchor points to help students determine what goes in the empty box. For example, place zero, ten, and 8. Next, place an empty box between 8 and 10. **“What number goes in the box?”**

Lesson Adaptation

For younger children, or for those who are struggling with numbers to ten, begin with numbers between zero and 5. For older or more advanced children, extend the number line to 20, 50, 100, etc. You might also use number ranges that do not include zero – for example, use endpoints of 15 and 75. Adapt the number cards and student tasks appropriately to reflect these different number ranges.

Lesson 2: Next to, Far Away

Lesson Objectives

- Students will use the number line to examine relationships between groups of three numbers (e.g., greater/less than, relative distances from one another, etc.)

Activity Background and Introduction

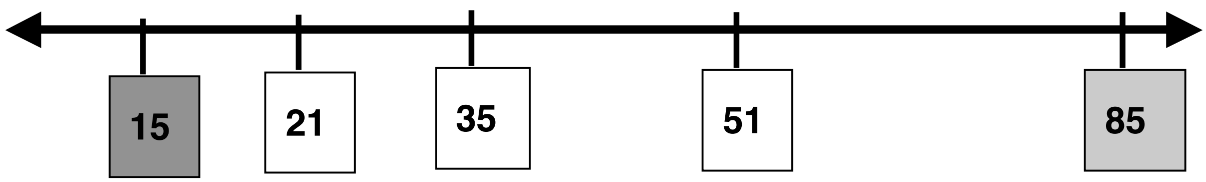
- This lesson allows students to explore the relative positions of numbers. Students will be encouraged to think about the distances separating three numbers, and how the number line can help them determine the differences between those numbers.
- As students work through this lesson, they will have opportunities, though perhaps implicit, to think about number combinations, addition and subtraction facts, etc.
- This lesson can be done with the life-sized number line, and/or with pencil and paper.

Lesson Progression

- Begin with a life sized number line strung across the front of a room. Label zero, or another appropriate starting value. Depending on the age and ability of your students, choose an appropriate endpoint, e.g., 10, 20, 40, or perhaps 100.
- Next, ask students to draw three cards from a hat (or roll combinations of dice, or otherwise randomly select three numbers between the endpoints of your number line).
- Students are then asked to place their number cards on the number line as accurately as possible. Listen to their justifications and rationale for placing the cards where they do. For example… Given endpoints of zero and 20, students could place 4, 7, and 16 in the following manner:

- Next, begin a series of questions that will motivate children to think about the distances separating these 5 numbers, their relationships to each other (e.g., greater than, less than), etc. There are many questions you could ask about these five numbers. For example:
    - **"Which two numbers are closest together? How close are they?"**
    - **"How far is the 7 from the 16?"**
    - **"What would you have to add to the 4 in order to get to 16?"**
    - **"What number card would be exactly halfway between 4 and 16?"**
    - **"Which is greater -- the distance between 4 and 7, or between 16 and 20?"**
    - **"What number is less than 16, but greater than 4?"**
- After you have exhausted the questions relevant to your first set of three numbers, begin again with a new set of three numbers. You will find that different numbers will lead to different questions and concepts.

- For example (for use with older students), select endpoints of 15 and 85. Give students the following cards: 21, 35, 51
- Questions might include:
    - **Which two of the white cards are closest to each other?**
    - **How close are they?**
    - **Which is a greater distance: from 21 to 85? Or from 51 to 15?**
    - **Is 51 closer to 35, or is it closer to 85?**
    - **What number is exactly halfway between 35 and 85?**
    - **Is 35 closer to 21 or 51? How much closer to each number?**
    - There are many other questions to be asked…



Lesson 3: Finding the Middle (Halving)

Lesson Objectives

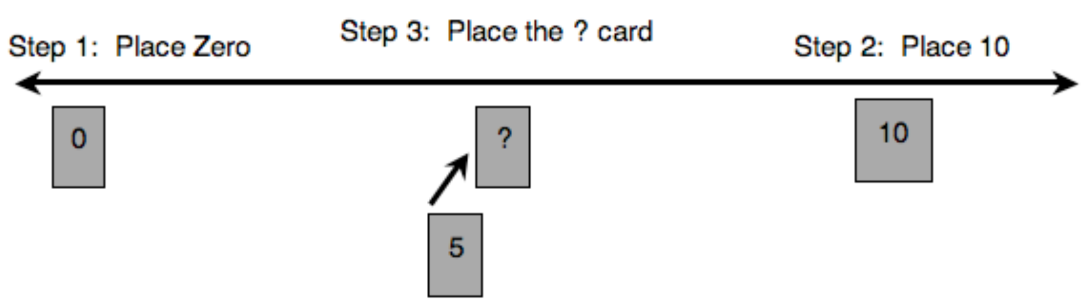
- Students will use the number line to find the middle (half) between two numbers.
- Students will begin to see the number line as a tool that can model mathematical thinking like, for example, “halving” a given quantity.

Activity Background and Introduction

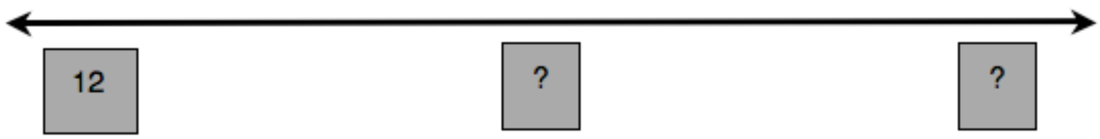
- For many reasons, it is important for students to be able to determine “half” of a given quantity. In this lesson students are given plenty of opportunities to examine the half/double relationships using visualization strategies developed through the number line. Even young children -- pre-kindergarten -- have an intuitive understanding of what “half” means. This lesson confirms and develops those intuitions by utilizing visualization strategies inherent in the number line.

Lesson Progression

- You will need a life sized number line, and various number cards, including zero, 3, 5, 6, 10, and 12. (Others are appropriate as well, including larger numbers for more advanced students.) You should also include an empty box number card.
- **Step 1**: Begin by placing zero on the number line.
- **Step 2:** Next, ask students **where to place the number 10**. Given their input, place 10, leaving ample distance between zero and ten.
- **Step 3**: Next, ask for a student volunteer to place an empty number card at the point of the rope that looks to be exactly halfway between the zero and the 10 number cards.
- Ask students what number would go in the empty box halfway between zero and 10.
- Replace the empty box with the number 5.
- While leaving the existing cards on the number line (zero, 5, 10), repeat the activity with a new set of numbers. Ask the students **where they should put the number 12 on the line.** Then, ask them to place the number 6 on the number line in the correct place. It is important to listen to their justifications.
- Ask the to describe their thinking: **How do you know where to place the number 6?**
- Some students may note that 6 is right next to 5. Others will use the halving strategy, working off the number 12. In either case, make sure that students discuss both methods for determining the middle of the number line.



- Repeat this activity again with different numbers. (You may use larger or smaller numbers depending on the level of students.)
- To vary the activity, begin by placing a number on the number line (e.g., 12). Then, to the right, place two empty boxes, equally spaced. Ask students what two numbers might go in the boxes. Of course, their answers will vary. Be sure to listen to their reasoning to understand in what ways they may be using halving strategies to determine the middle value.



Lesson 4: Doubles

Lesson Objectives

- This lesson is designed to help children understand, visualize, and use "doubles" e.g., 2 and 4 … 4 and 8 … 5 and 10.

Activity Background and Introduction

- A firm sense of doubles – and the use of "doubling" as a mental computational strategy – are important to the development of early number sense. When children can visualize doubles, they can use those visualizations in various ways as they work informally with numbers. For example, many children use "doubles +1" or "doubles -1" strategies to compute number facts such as 6 + 7
- Doubles and doubling strategies should be connected explicitly to the notion of "halving" as described in **Lesson 2** in this book.

Lesson Progression

- You may wish to use contexts to motivate conceptual understanding of doubles. For example:

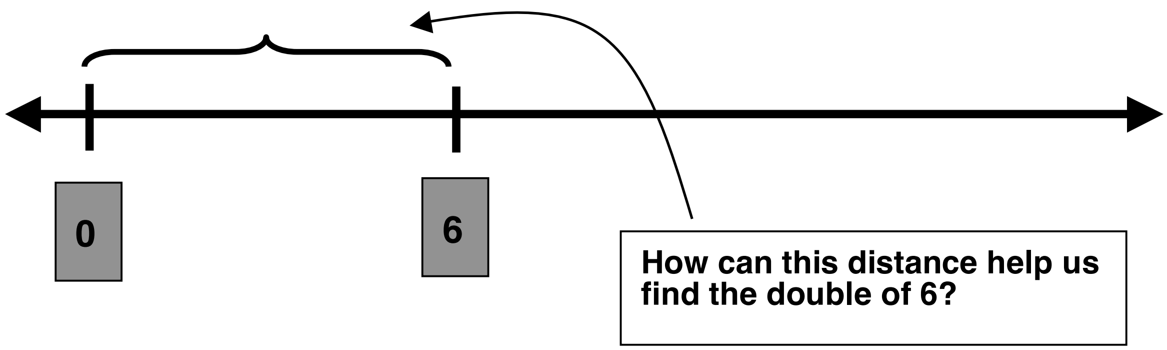
  "There are four children sitting at each of your tables. Without looking down, how many shoes would you find under your table?"

- You might build on this context by constructing a table that extends the context. Have children help you complete the table. It is not necessary to go in sequence (from 1-10, for example, on the top row). Rather, vary the order of the children so that they are encouraged to think in terms of doubling and halving.

Number of Children at your table	1	2	3	6	5	10			
Number of Shoes under the table	2	4	6						

- As you are working through the table, you might model the pairs of numbers on the demonstration number line. It is important that children can solve the problem both by making *visual estimates* of where the "doubles" will fall on the number line, and also by *counting equal intervals* on the number line. It is important to begin with zero labeled on the number line so that the students can determine a reasonable visual scale for doubling.
- Begin by placing the 2 on the number line. Ask students: **"If 4 is the double of 2, where should we place the 4 card?"** After the cards have been placed, take a piece of string and measure from zero to 2. Ask: **"How can we use this string to know if we put the 4 card in exactly the right place?"** Listen for students to articulate that the distance between zero and 2 must be the same as the distance between 2 and 4.

- Continue with another example. **"Suppose there were 6 children at the table. How many shoes would there be under the table? Let's use the number line to help…"**
- Begin by asking the children where to place the number 6 on the line. To highlight a visual strategy for determining the double of 6, you might take a string and measure the distance from zero to 6. Ask the children how that string could be used determine the double of 6.
- Continue similarly with other doubles pairs.



Lesson 5: What's in the box?

Lesson Objectives

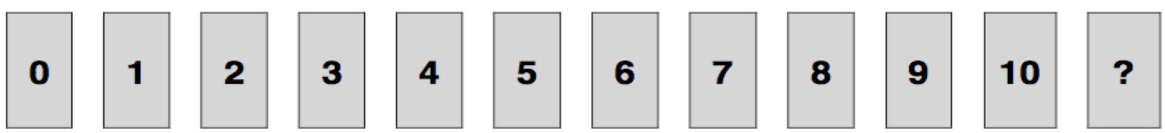
- Students will use both intuitive and informal strategies to identify missing values on a number line.
- Students will build on their sense of numbers, number relationships, and visualizations of the number line to find missing values.

Activity Background and Introduction

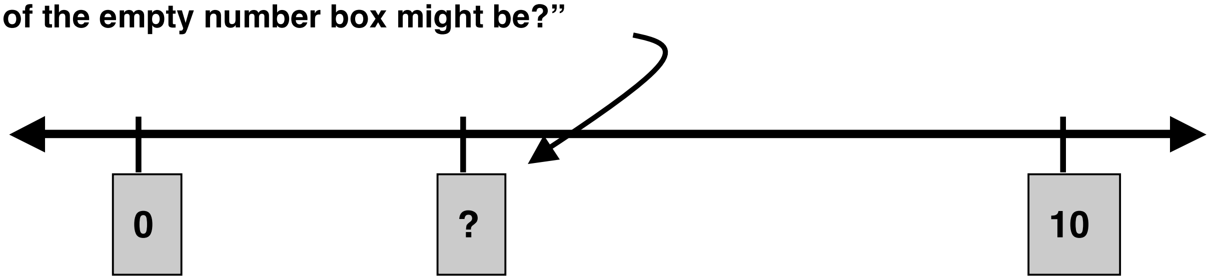
- Throughout this activity, teachers will be able to discern the thinking strategies and number sense of students as they exhibit their reasoning about numbers and their relationships to each other.
- Students will use strategies that can be formalized by teachers (e.g., doubling or halving) in order to determine missing number values.

Lesson Progression

- Begin with the large-scale number line strung across the front of the room. Prepare various number cards appropriate for your students, including several number cards that are blank (or have a question mark inside). Teachers of younger children might wish to focus on numbers between zero and 10. More experienced children can be asked to focus on larger numbers (e.g., up to 100), and also a larger number range (e.g., numbers between 35 and 95).

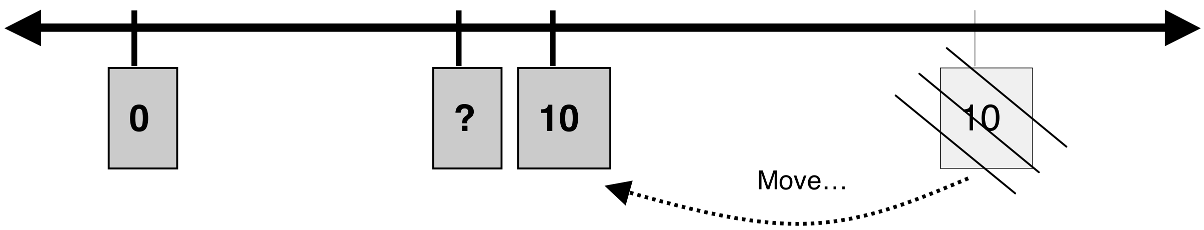


- Place zero (or some other starting value) on the number line. Next, hold up an empty number card. Ask: **"Where should I place this empty number box on the number line?"** Encourage students to think about the fact that the empty number card can go anywhere on the line as the number line is open, and the card could represent any number on the line. Place the card on the number line.
- Next, take a second card with a number on it (e.g., 10). Place it on the number line <u>to the right</u> of the empty card. Ask: **"Now, do we have some idea of what the value of the empty number box might be?"**



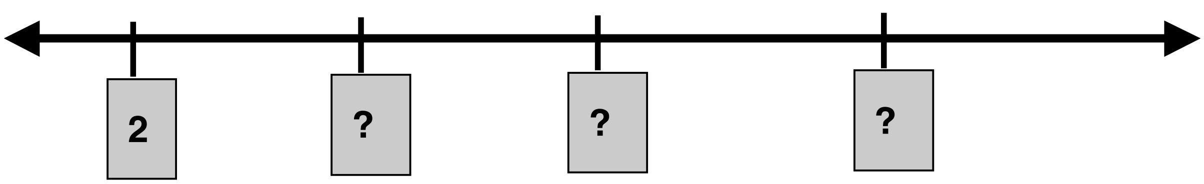
- Listen to the answers and justifications of the children. They may be using visual strategies (e.g., "It is not quite halfway to ten."), or other informal methods for determining the appropriate value in the box. Come to some group consensus as to what should go in the empty box. Pin that number to the bottom of the empty card.

- Next, move the 10-card closer to the empty box. Ask: **"Ok… now I have moved the 10 card much closer to the empty box. Do we have to change the number we selected for our empty box? What is a better number to put in there?"**

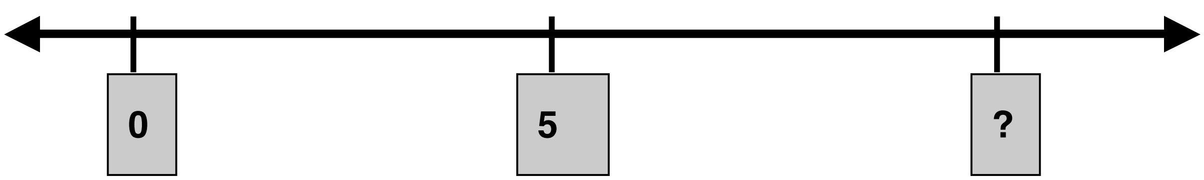


- Listen to the explanations of the children as they discuss what new number should go in the box. This technique of moving one or more of the number cards is extremely effective in promoting mathematical thinking and understanding.
- You may continue this lesson with many other similar variations. A few examples of differing levels of difficulty are provided below. Continue to probe students understanding by allowing for classroom discussion. By changing the initial card (left side endpoint), you can facilitate different kinds of mathematical thinking.

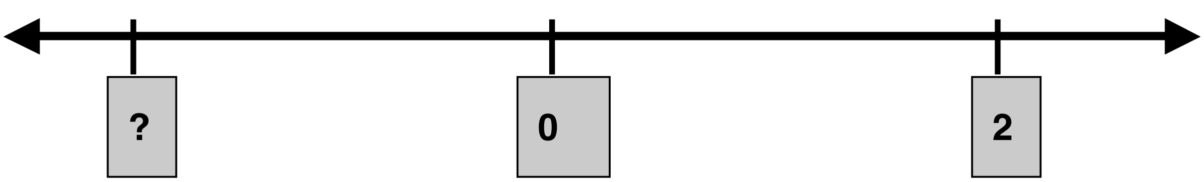
Ask: **"If I put a 10 in the middle unknown box, what are the other two values?"**
"What if I put 100 in the middle box. What are the other two values?"



Ask: "**What if I changed the 5 to a 10? Would the empty box have to change?"**



Ask: "**What goes in the empty box?"**



Lesson 6: What number am I?

Lesson Objectives

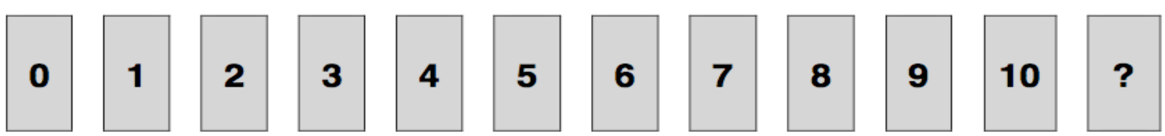
- Students will be given number riddles that they will model as necessary, and solve, using the life sized number line.
- Students will use both intuitive and informal strategies to identify missing values on a number line.
- Students will build on their sense of numbers, number relationships, and visualizations of the number line to solve the number riddles and identify the missing number.

Activity Background and Introduction

- Throughout this activity, teachers will be able to discern the thinking strategies and number sense of students as they exhibit their reasoning about numbers and their relationships to each other.
- Students begin by solving riddle statements presented by the teacher, using the number line when necessary. With experience, students can create riddles for their peers.
- Students will use strategies that can be formalized with the help of teachers (e.g., doubling or halving) in order to create visual solutions to the number riddles.
- Teachers can vary the riddles to focus on various number ranges depending on the level and experience of the students.

Lesson Progression

- Begin with the large-scale number line strung across the front of the room. Prepare various number cards appropriate for your students, including several number cards that are blank (or have a question mark inside). Teachers of younger children might wish to focus on numbers between zero and ten. More experienced children can be asked to focus on larger numbers (e.g., up to 100), and also a larger number range (e.g., numbers between 35 and 95). For students who are ready, perhaps a focus on integers (e.g., -10 to 10) might be appropriate.



- Begin by modeling the first number riddle with students on the large-scale number line. As students are ready and able, they can take control of modeling their own solution strategies on the number line.
- It is important to note that some students may have mental strategies that they can use to solve the number riddles. It might be a bit of a struggle with these children, but try to have them create a visual model of their thinking and strategies on the number line. Though they may initially resist (because they are using some other efficient mental strategy), it is important for them to be able to see the connections between their intuitive strategies and the number line as a computational tool. Also, other students will benefit from seeing the solution strategies modeled on the large scale number line.

- <u>Example Riddles for Early Learners, number between zero and 10</u> (Adapt as necessary depending on the level of your students):
  - "I am two more than five, and three less than 10. What number am I?"
  - "You can find me when you double 5, and then add 2. What number am I?"
  - "If you double 2, and then add 1, you'll find me. What number am I?"
  - "I am halfway between 10 and 2. What number am I?"
  - "Count *backwards* from 10 until you get to 6. How many steps did you take?"
  - "I am twice as far away from zero as the number 2. What number am I?"

- <u>Example riddles for more advanced learners, using positive and negative integers</u>
  - "Start at -3. Now add 8. What number am I?"
  - "I am a number between -10 and 10. I am the same distance away from zero as the number 5. What number am I?"
  - "I am halfway between -8 and zero. What number am I?"
  - "You can find me when you take 7 away from 5. What number am I?"
  - "I am halfway between -6 and 8. What number am I?
  - "If you added four to me, you would be at zero. What number am I?"

- <u>Example riddles for more advanced learners, numbers between 10 and 100</u>
  - "If you doubled 23 you would find me. What number am I?"
  - "I am a number between 1 and 100. Half of me is 25. What number am I?"
  - "Start at 13. Count by 10, five times. What number am I?"
  - "I am a number between 10 and 100. If you took half of me, and then doubled that amount, where would you end up?"
  - "Start at 20. If you added <u>half of me</u> to 20, you would now be at 60. What number am I?
  - "I am halfway between 85 and 45. What number am I?"

- As an alternative activity, students may be asked to develop the riddle for a given number. For example, each student is assigned a number between a given range (e.g., 1-10). The students must create a riddle for their respective numbers, and then the riddles may be shared and solved with a partner.

Lesson 7: Ordering numbers on a number line

Lesson Objectives

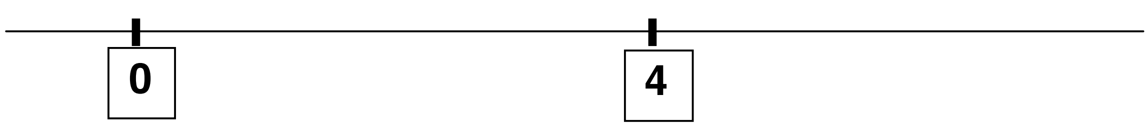
- Given a list of numbers in a given range, students will work together to place the respective number cards, in order and properly spaced, on the life sized number line.

Activity Background and Introduction

- This activity reinforces number relationships and, specifically, number order. Students may be given numbers in consecutive sequence (e.g., 5, 6, 7, 8, …), or numbers across a wider range (e.g., 12, 23, 45, 67…) for this activity. Increasing the range of numbers given increases the difficulty of the activity.
- The activity is made more challenging when students are given numbers out of numerical order. In addition, by giving students endpoints for the number line they must use (that are not elements of the list of numbers to be arranged), this activity can become more challenging. For example, give students the following set of numbers: {43, 19, 33, 54, 67}. Next, place two number cards on the number line: 10 and 85. Students then must place their given number cards on the line with respect to the endpoints (that determine the scale for the number line) that have been fixed on the number line.

Lesson Progression

- Determine the range of numbers that will provide the context for this activity with your students. Depending on the students with whom you work, you might consider a range between zero and 10 (young children), 10 to 100 (3rd or 4th grade children), or even a range of numbers that includes negative numbers (e.g., -10 to 10) for more advanced children who are ready work with negative numbers.
- Present children with a set of numbers within the given range of focus. Children can make number cards (as described previously) for each number in the set.
- Next, ask children to arrange their number cards on the number line. You may ask students to do this individually (one at a time), or as a whole group. There are advantages to both.
- If you choose to have students place numbers one at a time, be aware that the first cards placed on the number line may have a big bearing on how easily the remaining cards may be placed. For example, imagine the following set of numbers:
    - {4, 8, 12, 16, 20}
    - If a student places the 4 on the number line as shown below, it will be impossible to place the remaining cards to scale on this number line.



- Think strategically about the number sets you provide your students. You may want to focus on multiples (e.g., multiples of 4 as shown in the example above), numbers separated by a given increment (e.g., 14, 24, 34, 44), prime numbers, even numbers, odd numbers, etc. Different sets of numbers will elicit different kinds of mathematical thinking. In other words, use this activity to stress not only number order and sequence, but also other mathematical concepts that are appropriate for your students.

- Do not always ask the student with the smallest (or largest) number to go first. This will be a natural tendency of the children, but it is often a more cognitively complex task to start with a number in the middle of the range.

Lesson 8: Greater Than? Less than? The same?

Lesson Objectives

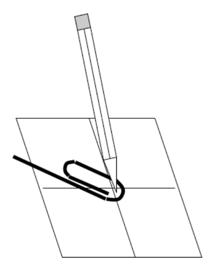
- Given a set of numbers, students will work in small groups (or individually) with the number line to explore "greater than," "less than," and "equal to" relationships.

Activity Background and Introduction

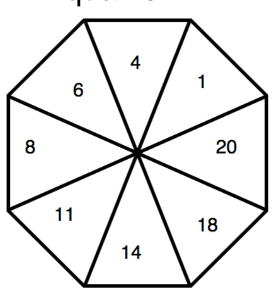
- This activity is designed as a game for students to play in groups. Depending on the size of the class, split students into small teams of 3 to 4 students. (Students may also play the game on their own, competing with other individuals or groups.)
- It is best to have no more than four groups playing against each other at a time in order to keep the game moving and to keep students motivated.
- This game also takes advantage of the notion of "chance" as students use spinners as part of the game. Probability contexts are great opportunities to foster number sense as illustrated in this game.
- You will need number cards for each group of students in a given range (zero to 20, for example).

Lesson Progression

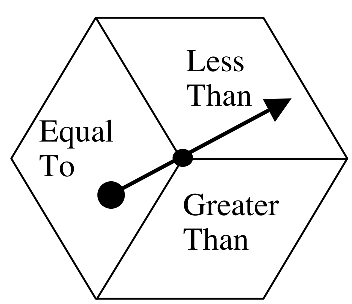
- **Prepare number cards** within a given number range. As always, choose a suitable range of numbers given the ability level of your students. You may choose to make a number card for each number in the range (e.g., numbers between zero and 20: 0, 1, 2, 3, …) or you may choose a larger range and select a subset of cards within that range (e.g., numbers between zero and 200: 0, 15, 30, 40, 45, 72, …). Create at least 20 cards in the range.



- **Prepare a spinner template** to reflect the number range selected. If you do not have a spinner, it is easy to make one with a paperclip and a pencil. Extent one end of the paperclip, and use a pencil or other pointed object as the anchor of the spinner. When using a paperclip spinner it is easy to change the template to reflect the number range of interest.
- When you prepare the spinner template, you do not need to use every number in the range. For example, if the number range of interest is zero to 20, you might use a spinner template that has the following numbers: 1, 4, 6, 8, 11, 14 18, 20. **Create number cards** for each number that appears on the spinner.
- Next, **prepare a second spinner template** with three options: "Greater than" … "Less than" … "Equal to."



Spinner #1



Spinner #2

- Next, divide students into pairs or teams of 3-4 students each. (Or students may play individually). Shuffle and **distribute the number cards** equally among the groups playing against each other. Optimally, each group should have at least 5 cards.
- Groups competing against each other will use one life-sized number line.
- Play begins with Team 1. A member of Team 1 spins the first spinner to reveal a number somewhere in the range (e.g., spins a 14 on the spinner above). The teacher (or a student) places the 14 number card on the number line at the appropriate place.
- Next, another member of the group spins the second spinner to receive either "greater than," "less than," or "equal to" (e.g., spins "Less Than").
- The group members then must look at their own stack of cards. If possible, they select (and then place on the number line) one of their cards that is "Less Than 14". If they do not have a card that is less than 14, they must pass to the next group.
- Play continues with the Team 2. They spin the number spinner. Assume they spin a new number – e.g., 6. The teacher (or a student) places 6 on the number line in the appropriate place. Then, a second child spins the second spinner (e.g., "Greater Than"). If possible, students in Team 2 select a card from their stack that is "greater than 6" and place it on the number line. If they do not have a number card greater than 6, they must pass to the next team.
- Play continues in this fashion, each team taking turns spinning the two spinners. **The team to place all their cards on the number line first is the winning team.**
- It is important to leave all the cards on the number line, and for the students (and perhaps teachers) to make sure that the cards are placed appropriately on the number line. In some cases (when an "equal to" spin is obtained), two cards may be placed in the same location.

Lesson 9: Zero – 100 … Zero – 10 … Zero - 1

Lesson Objectives

- Students will examine the similarities between three number lines: 0 – 100, 0 – 10, and 0 – 1.
- Students will explore the structure of our base-10 system – how our number line can be fruitfully divided into demarcations of 10, and how those tenths can be used to make sense of numbers and their relationships with one another.

Activity Background and Introduction

- Throughout this activity, students will be asked to find the similarities between three different number lines as we change the endpoint from 100 to 10, and then from 10 to 1.
- The focus of the activity is to focus on divisions of 10 on a given number line. First students explore multiples of 10 – from zero to 100. Then, the teacher changes the ending card from 100 to 10, and students must adjust the number cards in between.
- This method is repeated again, now focusing on ten equal divisions of the number line between zero and one. Obviously, this step introduces students to fractions (or decimals). The key idea is to show that the structure of the number line remains the same though the numbers used differ. Students will see that they can use one of these number lines to make sense of a similar one.

Lesson Progression

- **Prepare three sets** of cards: {0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100}
  {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
  {1/10, 2/10. 3/10, 4/10, 5/10, 6/10, 7/10, 8/10, 9/10, 1}
- Using different colors for each set of number cards will be helpful.
- Begin by soliciting the help of the children in **placing the first set of cards** on the life-sized number line. Start with zero and 100 at the respective ends of the line, and then ask students how they know where to place the remaining cards. For example…
  - Ask: "*Which of the remaining cards would be the easiest to place?"*
  - Listen for students to use informal reasoning (e.g., "50 is in the middle.").
- After all the cards in the first set have been placed on the number line appropriately and with student input, **change the endpoint of the number line from 100 to 10**. Do this by putting the 10 card directly on top of the 100 card.
- Ask: *"Ok… I just changed the endpoint from 100 to 10. Now… all the other cards on the number line are out of place. How can we fix this by using the second set of cards?"*
- Use this opportunity to help students make the connection between the decades that exist between zero and 100 (10, 20, 30, etc.), and whole numbers between zero and 10 (1, 2, 3, etc.). As students place the new set of number cards on the number line, have them place them directly on top of the previous correlated number (e.g., 3 is placed on top of 30). Continue to solicit informal reasoning from the students that reinforces the links between the two number lines (e.g., "Since 40 is halfway to 80, then I know 4 is halfway to 8.").

- Repeat the same series of steps one more time, beginning by placing the 1 directly on top of the 10 (and the 100) cards. Continue to build on students' informal knowledge and intuitions. For example, it is likely that the students will understand that the number line continues to be broken into ten equal parts. Use this information to motivate the idea of "tenths" -- that the number line between zero and 1 can be broken into ten equal parts just like the number line between zero and 100. Help students refer to these demarcations as tenths… 1 tenth… 2 tenths… 5 tenths… etc.
- Solicit students' input to help place the third set of cards on the number line. Draw on their knowledge of the first two sets of numbers, and listen to their informal reasoning (e.g., "Since 50 was halfway to 100, and since 5 was halfway to 10, then 5 tenths must be halfway from zero to 1.").
- Depending on the level of your students, this activity can be either shortened (do not do the third set with the fractions), or deepened by looking more closely at the set of fractions (tenths) in the third set. Regardless, the point of this lesson is to help students recognize the base-10 structure of our number system.

Lesson 10: Benchmark Fractions and Decimals

Lesson Objectives

- Students will use the number line to explore basic benchmark fractions ($\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$) and their relationships to one another.
- Students will compare the relative sizes of fractions (e.g., $\frac{1}{3}$ is larger than $\frac{1}{4}$).
- Students will compare fractions with the unit whole (e.g., there are four $\frac{1}{4}$'s in 1 whole).

Activity Background and Introduction

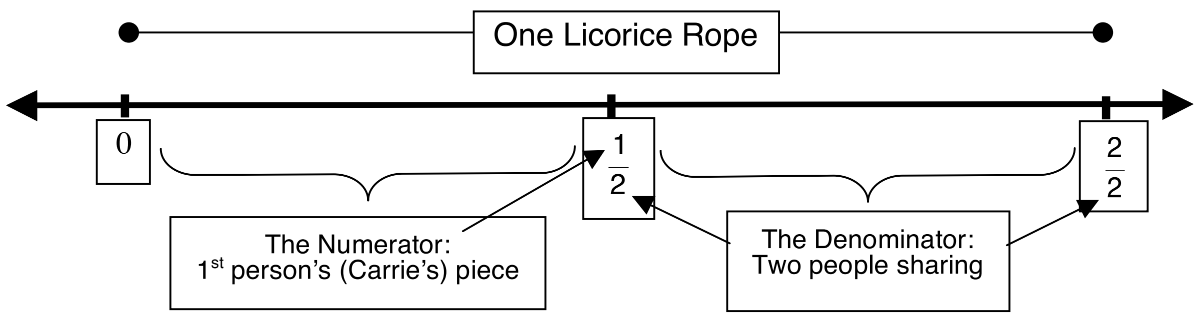
- Throughout this activity, students will focus on the number line between zero and one. If students have had opportunity to engage in the previous lesson in this book, they should have a basic understanding of the way a number line can be partitioned – for example, that other numbers (fractions and decimals) can be found between two given numbers. This is an essential understanding for young learners, and lies at the foundation of this lesson.
- It is important to make it clear that the structure of the number line, as it functions for whole numbers, behaves the same way for numbers between zero and 1. Work with students to help them apply the tools and understandings they learned in the context of whole numbers (e.g., doubling, halving, etc.) to those numbers (fractions or decimals) between zero and one.
- Although not an explicit focus of this lesson, it is important to help students know that the fractions that can be found between zero and one also exist between other whole numbers. For example, if possible, make the connection between $\frac{1}{4}$ and, say, $2\frac{1}{4}$.

Lesson Progression

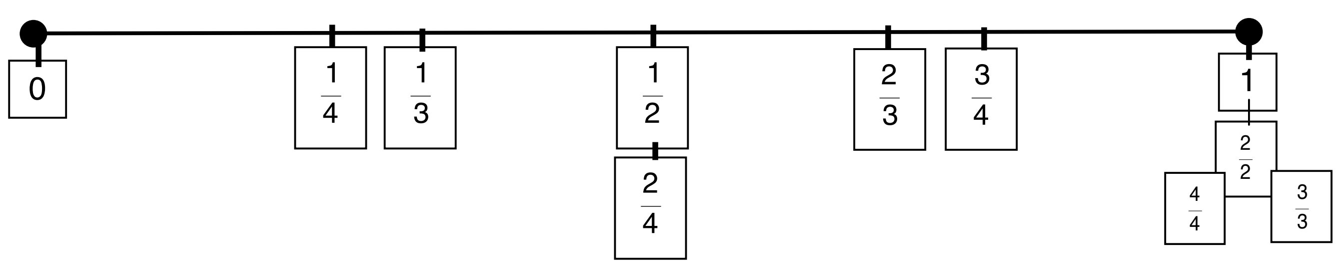
- Begin by preparing the following sets of number cards (preferably in different colors):
  - $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}\}$
  - $\{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}\}$
  - $\{0, \frac{1}{2}, \frac{2}{2}\}$
- Place two cards on the life-sized number line – zero and 1. Leave plenty of space between the two cards.
- The notion of a "fair share" is a very helpful tool in generating understanding about fractional pieces. Toward that end, the following context might be helpful with your students as you begin this lesson. If possible, bring in several long licorice ropes as visual models and helpful motivators.

  "Imagine that you have a long rope of licorice. The zero on our number line represents the beginning of the licorice rope, and the 1 represents the end. Now… suppose you were going to share that licorice rope with one friend. What would be a fair share for each of you?"

- Next, have a student place the ½ number card in the appropriate place. Be sure to emphasize the meaning of the action with respect to the fraction: "***We have split the licorice (as represented by the number line) into two equal pieces, to be shared by Carrie and Josh. Our fraction helps us understand: The bottom number (denominator) tells us, in this case, how many people want to share the licorice. The top (numerator) number shows us how many pieces we have given out so far.***" Next, place the second card, $\frac{2}{2}$, on the line, pinning it to the existing card (the number one). Ask students to similarly articulate the meaning of the second card: "The licorice was split fairly between two people, and this marks the end of the second piece of licorice."



- Continue the lesson with thirds. Ask: ***"We know how much Carrie and Josh got. They each got ½ of the licorice. Now, we have another piece of licorice. This time three people want to share it equally. How would we divide that licorice evenly between Ale, Carlos, and Paul?"*** Distribute the set of thirds fraction number cards. Before asking students to place the thirds on the number line, ask: "*Who will get the most licorice – Carrie (the first group), or Ale (the second group?"*
- Students may now place the next three cards on the number line: $\{\frac{1}{3}, \frac{2}{3}, \frac{3}{3}\}$.
- Continue the lesson in this fashion, next splitting the number line into fourths. If you use a large number line, students will be able to see clearly the difference between the thirds, quarters, and halves. By the end of the activity, the completed number line (see below) will be a great context to generate meaningful discussion. Some of these important discussion questions might include:
  - Which group (halves, thirds, fourths) got the largest piece of licorice? (halves)
  - Which group split the licorice rope into the most pieces? (quarters)
  - Which is larger, $\frac{1}{4}$ or $\frac{1}{3}$? (one-third)
  - How many quarters do you need to make a whole licorice rope? (four)
  - How many thirds do you need to make a whole? (three)
  - What if we had a group of five friends who wanted to split the licorice evenly?



Activity Set 2: Activities for Pencil and Paper

The following two lessons are designed for students to work either individually, or in small groups. They build on the main concepts developed with the large-scale number line described in Activity Set 1. They focus primarily on locating points on a number line (Lesson 11), and subsequently marking the number line with multiples, or "skip jumps" (Lesson 12).

For each lesson activity that follows, teaching objectives and notes are provided for teachers. To accompany the teacher lesson plan, you will find a black and white, reproducible activity sheets for each lesson in Appendix 1 (back of the book). It is the intent that these activity sheets will be photocopied, and distributed to students. For some lessons that are appropriate across various grade levels, multiple activity sheets follow one another, each designed (as noted) for an appropriate grade level (L1 = roughly designated as K-2; L3 = roughly designated as 2-4; L5 = roughly designated as 4-6). Or, teachers may be given specific recommendations to alter the lesson in order to make it relevant for different grade levels. Of course, these grade level indications are only rough guidelines. Teachers may choose to use the activity sheets flexibly across the grade levels. Teachers may also choose to alter the activity sheets prior to photocopying in order to make them more or less challenging. This might be particularly necessary for children in Kindergarten given the wide range of informal understandings 4 and 5-year-old children bring to the Kindergarten classroom.

Lesson Content

The first objective of the lessons in this activity set is for students to use the number line to develop a richer sense of numbers and their relationships to one another. This includes opportunities for students to place numbers on a number line, and to determine the numeric value of a location on a number line based on other clues given in the context.

The second objective of the lessons in this set is to introduce the idea of "skip counting" with the number line – essentially an introduction to the mathematical concept of ***multiples***. The problems in this group of lessons prepare students for operations with the number line.

Lesson 11: Graphing Points on a Number Line

Lesson Objectives

- Students will label numbers on a blank number line.
- Students will identify the numeric value of a designated location on a number line.

Activity Background and Introduction

- It is important for students to develop an intuitive understanding of the scale of a number line. Given the fixed number lines as a starting context, students must recognize the relative value of given numbers – both those that are already on the line, and those that they need to place.
- Watch for students to demonstrate the ability to reasonably partition a number line based on a particular scale. In some cases students are asked to plot the same number values, but on two different lines. This is an important step in helping students determine the scale of the number line, and in helping them recognize that the distance between two given numbers may change depending on the scale of the number line in use.

Lesson Progression

- Depending on the level of your students, you may wish to introduce the lesson by working a similar problem on the board. Draw a number line with zero and 10 as endpoints (or some other appropriate interval). Ask students to locate a point or two on the number line.
- If there are no questions about the intent of the activity, distribute the activity sheet to students and monitor their progress.
- It is likely that you will be able to illustrate the solutions to these problems within the same class period.

Materials Needed

Three lesson activity sheets that accompany this lesson may be found in the Appendix of this book. Select the activity sheet that is best suited for the ability level of your students. For this activity, use the following lesson sheets:

- L1: Lesson 11 Activity Sheet (K-2)*
- L3: Lesson 11 Activity Sheet (2-4)
- L5: Lesson 11 Activity Sheet (4-6)

*NOTE: Some Kindergarten children may struggle with the range of numbers on this activity sheet, while other young children will be just fine. Be prepared to adapt for some students.

Lesson 12: Skip Counting and Multiples

Lesson Objectives

- Students will explore multiples (e.g., counting by 2's, 3's, 10's, etc.) by labeling numbers on a blank number line using a "skip counting" technique.
- Students will use clues on the number line to help them determine equal intervals (i.e., multiples) for a given problem context.

Activity Background and Introduction

- Multiples are a key building block for multiplication and division (e.g., "repeated addition"). They are also helpful in various problems related to number patterns (e.g., counting by two's or three's, anchoring on 5's and 10's, etc.) In this lesson, students will think about multiples by using the notion of "skip counting" – progressing up or down a number line by using a consistent interval.
- In order to be successful with intervals, students must be comfortable with the idea of scale – that the distance between zero and one is the same as the distance between one and two. The power of the number line becomes apparent in this context insomuch as it provides visual confirmation (i.e., space between numbers) that students will eventually transfer to cognitive understanding of scale, intervals, and multiples.

Lesson Progression

- All three activity sheets begin with the same concept – marking a number line by a given multiple. How you introduce that idea to your students may vary depending on the grade level, but it is important to begin this lesson by visually highlighting that the physical gap between multiples is always the same.
- Introduce the lesson by working a problem similar to number one on the board – dividing the number line (or circling numbers that are already placed on the line) in some sort of pattern: e.g., multiples of two, multiples of 5, etc.
- As you introduce the lesson, use the notion of a "skip jump" with students – i.e., "jumping" along the number line by a given interval. This interval might be thought of in numeric terms (each jump is 3 units), or in visual terms (each jump is a certain length that corresponds to various units). This idea is important for understanding multiplication and division as repeated addition/subtraction.
- Distribute activity sheets. It is likely that you will be able to illustrate the solutions to these problems within the same class period.

Materials Needed

The three lesson activity sheets that accompany this lesson may be found in the Appendix of this book. Select the activity sheet that is best suited for the ability level of your students.

Lesson 13: Hit the Target

Lesson Objectives

- The objective of this engaging activity is to encourage students to work with given multiples as they aim for a "target" number on the number line.

Activity Background and Introduction

- This activity can easily be adapted by the teacher to reinforce various multiples (e.g., multiples of 3, 5, etc.). The range of numbers a teacher may wish students to engage may also be dictated easily by changing the target number at hand.
- As noted in the directions on the activity sheet, students are given a starting number, a target number, and several options of multiples that they can use to "skip jump" along the number line. For example, I may need to start at zero, use jumps of 1, 2, or 5, and aim for a destination target number of 23. Jumps may be made in either direction on the line (addition or subtraction). What is the fewest number of jumps it would take me to get to that number?
- With some forethought, it is easy for teachers to make challenging problems for students of various levels. Also, by allowing students to jump forward and backwards on the line, students may realize with time that it actually requires less jumps to arrive at a particular destination if you jump ***past*** the mark, and then back track to the target.

Lesson Progression

- Begin by distributing the activity sheet to students (select the sheet that is most appropriate for your students from those provided in the Appendix).
- Show students the example problem, working it together on the board. Students may then progress either individually or in pairs to try to solve the problems in as few jumps as possible.
- Extra number lines are provided so that students may think about multiple strategies for getting to the same target point.
- Students enjoy making their own restrictions (target value and available multiples) and sharing their problems with a peer.

Materials Needed

The three lesson activity sheets that accompany this lesson may be found in the Appendix of this book. Select the activity sheet that is best suited for the ability level of your students. For this activity, use the following lesson sheets:

- L1: Lesson 13 Activity Sheet
- L3: Lesson 13 Activity Sheet
- L5: Lesson 13 Activity Sheet

Activity Set 3: Operations with the Number Line

In this final group of activities, students will use the number line to develop confidence and fluency with addition and subtraction. Specifically, the lessons are designed to foster students' ability to:

- Visualize addition and subtraction using the number line as a model.
- Develop understanding and facility with adding on along the number line.
- Apply number-line strategies in various addition and subtraction problems.
- Compare different solution strategies for addition and subtraction.
- Develop efficiency in calculating addition and subtraction problems (2- and 3-digit) mentally.

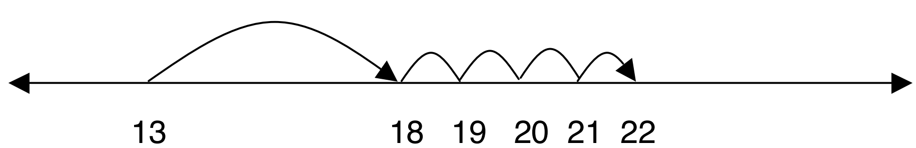
These lesson plans and accompanying activity sheets (included in the Appendix) build on the technique of skip counting that was introduced previously. This technique helps students navigate along the number line by taking advantage of familiar number relationships, such as adding on by 5's or 10's. Throughout the problems, children should be encouraged to estimate as often as possible – estimating before they start a problem is especially important. As students develop proficiency with the number line, they will naturally become better at estimating reasonable answers.

It is also important to reinforce the idea that multiple strategies may be used to complete the same operation (i.e., there are multiple solution strategies for each problem context). Some students may be comfortable using increments of 10, while others need to use increments of 5. Some students may be comfortable overshooting an intended target, and then backtracking to the desired result. Others may not be able to use such a strategy initially. For example, when given the following problem to solve, consider the differences in mental strategies two students might use: 13 + 9

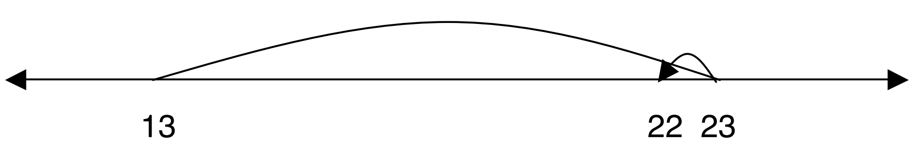
> <u>Student 1</u>: "13 + 9… I will start by adding 5: 13 +5 = 18… I need to add 4 more… 19, 20, 21, 22. 22 is my final answer."
>
> <u>Student 2</u>: "13 + 9… that is close to 13 + 10. 13 + 10 = 23. Now I need to subtract one because I only need to add 9, not 10."

These strategies can (and should) be modeled and compared on a number line:

Student 1:



Student 2:



The lessons that follow are designed to foster this sort of mathematical thinking as students learn to use the number line as a powerful model for addition and subtraction.

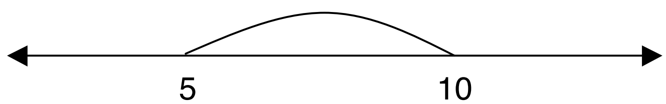
Lesson 14: How far?

Lesson Objectives

- Students will use number lines and skip counting techniques to determine the difference between two numbers.
- Students will explore how subtraction and addition can both be used to determine the distance between two numbers that are represented on a number line.

Activity Background and Introduction

- As suggested in the objectives above, it is extremely important that students understand the relationship between addition and subtraction – one undoes the other. For example, when asked to find the difference between 8 and 15, students could find the answer by building up (e.g., 8 and 2 is 10, ten and 5 more is 15), or they could find the answer by subtracting (15 – 5 is 10, 10 - 2 is 8, so the difference between 15 and 8 is 7).
- The number line is instrumental in helping students see this connection between addition and subtraction. For example, on the diagram below, the visual nature of the number line clearly indicates a jump of 5, but it is impossible to tell in which direction the jump was made – from 5 to 10, or from 10 to 5.



- A key concept that must be in place if students are to use the number line as a mental strategy for computation is the notion of ***subitizing***… the ability to see "inside" a number. Seven, for example, might be thought of as 5 and 2 more. When students can subitize, they are able to use meaningful skip jumps when adding or subtracting.

Lesson Progression

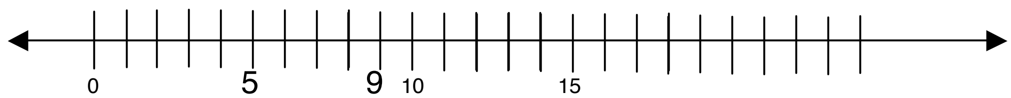
- The problems on the activity sheets (found in the appendix) for this lesson ask students to find the difference between two numbers – e.g., how far is it between 15 and 20? You may choose to use a context to motivate this idea – a context that does not necessarily imply either subtraction or addition. For example:

  John lives on the corner of 15th street and Broadway. His friend, Michael, lives on the corner of 21st street and Broadway. How many blocks does John have to walk in order to get to Michael's house?

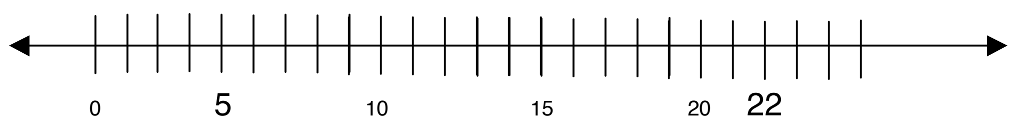
- Depending on the level of your students, begin by highlighting (on a number line drawn on the board) the various skip jumps that might be helpful in finding the distance between points. For example, begin with skip jumps of 5. Pick a number on the number line (a multiple of five will be helpful for younger students). For example, start on 10. Ask students where a skip jump of 5 would leave them. See if students realize that a skip jump of five might be done in either direction – ending at 15, or ending at 5. Do the same with skips of 1, 2, 10, and perhaps other multiples of 10 up to 100.

- Next, ask students to see certain "skip amounts" inside a given number. For example, see if students can see a skip of 5 within the number 7. See if students can see a skip of 10 within the number 12. Model these with the number line, starting (at first) with zero. Model the notion of skipping a total of seven, in two jumps: 5, and 2.
- After this introduction (which is a review of earlier lessons), students should be ready to answer the next question – what is the difference (distance) between two numbers? Do several examples that are relevant for the level of your students. For example, with younger children, you might do the following:

  - *Here are two points on the number line: 5 and 9. How far apart are they?*



  - While students might count by ones for this problem (which is fine), gradually progress so that they learn to skip count by 5's and 10's.
  - For example: *How far apart are 5 and 22?* Model the solution by drawing skip jumps (in this case, using either 5's or 10's) in the same way you will expect students to do so on their activity sheets.



- Look for students to use appropriate skip jumps (in this case, jumps of 5, 10, and 1) as they solve the problems on the activity sheet. Teachers may easily vary the problems that appear **on the three activity sheets (L1, L3 and L5)** to match the levels of their students.

Lesson 15: Adding by anchoring on 5 and 10

Lesson Objectives

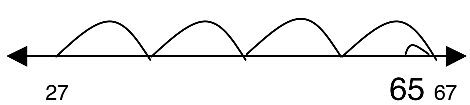
- Students will use skip jumps of 5 and 10 to visually represent solution strategies for addition and subtraction on the number line.

Activity Background and Introduction

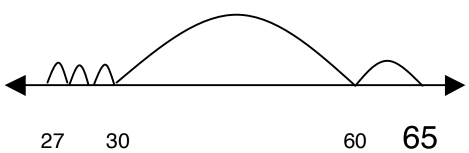
- Activities similar to those highlighted in this lesson are important insofar as they help students develop visual solution strategies for addition and subtraction. If we hope that students will be able to develop confidence in solving such problems mentally, we must give them practice developing visual models (that can be used mentally with success) such as those fostered by the number line. Procedures that require steps such as "carrying the one," for example, are extremely difficult for students (even adults) to juggle mentally. On the other hand, given the natural tendency and aptitude that most children have for visualizing, mental models such as the number line can be used by students to solve addition and subtraction problems both accurately and efficiently.
- One of the most positive features of number line solutions is that students may choose their own solution strategies – steps that make sense to them and their interpretation of the problem. That is not true with standard algorithms. Consider the following problem: 27 + 38 = ___. Using the standard algorithm, students are forced to follow steps that might not be intuitively obvious: 1) rewrite the problem vertically; 2) add 8 and 7; 3) put a 5 underneath the 8; 4) carry the one; 5) add 2 and 3; 6) add the 1 that was carried. All students using the traditional method must follow that procedure closely.

$$\begin{array}{r} 27 \\ \underline{+38} \end{array}$$

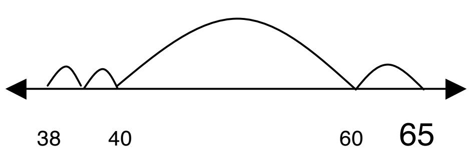
- In contrast, students may use a variety of visual strategies to solve the same problem – working in skip jumps and number partitions that resonate with their intuitions. See below a sample of strategies students might use:



Bounce by 10's, 4 times (40). Subtract 2.



Bounce up 3 to 30, bounce 30 to 60, add 5 more.



Start at 38, bounce 2 to 40, bounce 20, add 5.

- In these three example (and there could be plenty of others), students exhibit strategies that resonate with their thinking, and illustrate their strengths (e.g., skip counting by 10, skip counting by 5, decomposing seven into parts of 5 and 2, overshooting the answer and then backtracking, etc.).

- If students are able to skip jump by 5 and 10 comfortably, and if they are able to add on (or subtract) by 1's and 2's, **there is no number they cannot represent on the number line!** Imagine… 38. 38 can be found by four jumps of 10, and then subtract 2. Imagine… 47. 47 can be found by 4 jumps of 10, one jump of 5, add on two. Once students realize the power they have to represent any number by simply counting in 5's and 10's, they begin to work with both confidence and accuracy. This is a very powerful bi-product of this lesson.

Lesson Progression

- Depending on the level of your students, work several examples similar to those found on the activity sheet (**three levels of activities for this lesson may be found in the Appendix**).
- Emphasize the use of jumping by 5's and 10's.
- Emphasize that in addition, one can start with either number, and then add the second. Bring to the attention of students that starting with one number (instead of the other) might be an easier problem.

Lesson 16: Fill in the Box

Lesson Objectives

- Students use the number line to visualize solution strategies to open ended number sentences involving addition and subtraction.

Activity Background and Introduction

- One of the problems in U.S. math education is that we do not vary the presentation of addition and subtraction problems. Children get many opportunities to solve problems such as: 3 + 4 = __?__. However, rarely are they asked to solve the same problem, but in a different form: 3 + __?__ = 7.
- This lesson helps students develop a relational view of the equal sign (as in the second example), rather than developing the misconception that the equal sign only means, “Here comes the answer…”
- Students may use the number line flexibly to solve the **problems on the activity sheets for this lesson** (found in the Appendix).

Lesson Progression

- Depending on the level of your students, model several problems. Be sure to model problems where the position of the missing is varied.
- The activity sheets for this lesson represent only a few of the many problems that could be generated to address the objectives of this lesson.

Lesson 17: Addition and Subtraction Story Problems

Lesson Objectives

- Students will apply the skills they have developed throughout the previous lessons to story problem contexts involving addition and subtraction.

Activity Background and Introduction

- Very rarely do addition and subtraction problems come to us (children and adults) in the form of flash cards! Rather, we often have to find the mathematics – to say nothing of the mathematical solution – in the context of authentic problems. This lesson provides just a beginning – a handful of problem contexts that require students to add and/or subtract.
- The hope of this textbook is to prepare students to solve these sorts of imaginable contexts using intuitive tools and strategies highlighted throughout the lessons.

Lesson Progression

- Depending on the level of your students, work several problems similar to those **on the activity sheets corresponding to this lesson** that may be found in the Appendix.
- You may wish to use the life-sized number line to model the problems. Seek opportunities to reinforce the strategies students have practiced throughout this book.

Appendix 1: Lesson Activity Sheets

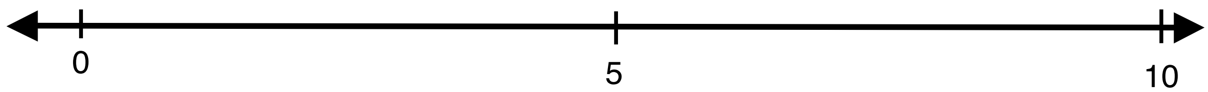
The following activity sheets are to be used in conjunction with various lesson plans as outlined in the previous pages. Note the following ***rough*** guidelines for the various levels of the activity sheets. Teachers may need to further adapt these activity sheets to better match the ability of their students.

Level 1 (L1): Suitable for most kindergarten through second grade students.
Level 2 (L2): Suitable for most second through fourth grade students.
Level 3 (L3): Suitable for most fourth through sixth grade students.

Lesson 11: L1 Activity Sheet **Name**: _______________

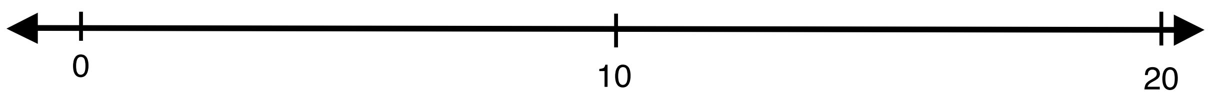
1. Where do these numbers belong on the number line below?

a) 4 b) 1 c) 9 d) 3 e) 7



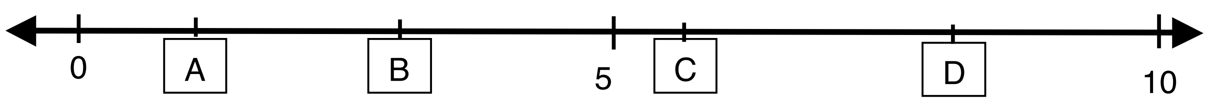
2. Where do these numbers belong on the number line below?

a) 5 b) 15 c) 11 d) 19 e) 1



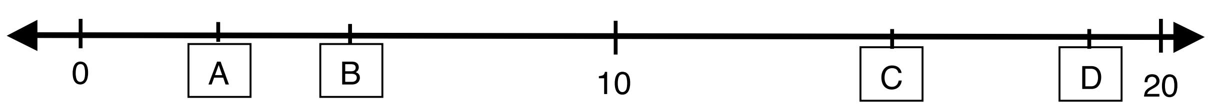
3. What is the number value of each letter shown on the number line below?

A = _____ B = _____ C = _____ D = _____



4. What is the number value of each letter shown on the number line below?

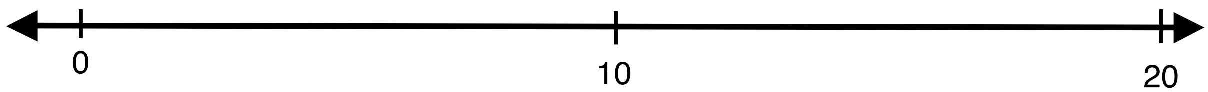
A = _____ B = _____ C = _____ D = _____ E = _____ F = _____

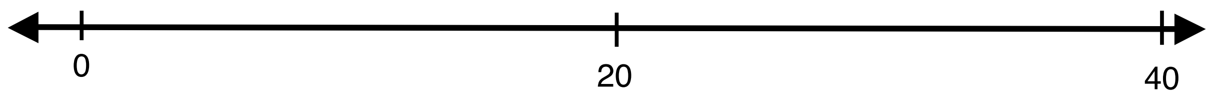


Lesson 11: L2 Activity Sheet **Name: ________________**

1. Graph these numbers on the TWO number lines below.

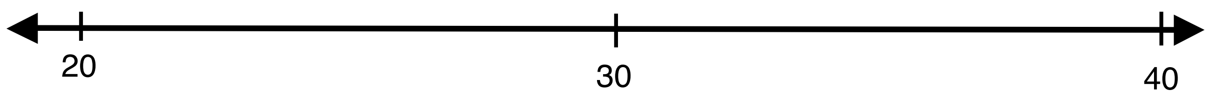
a) 5 b) 15 c) 11 d) 3 e) 18 f) 9

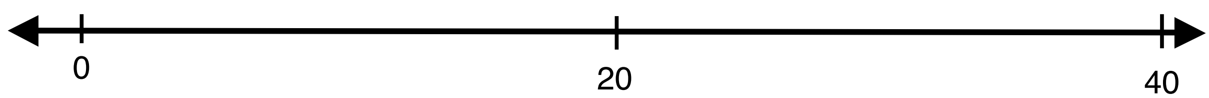




2. Graph these numbers on the TWO number lines below.

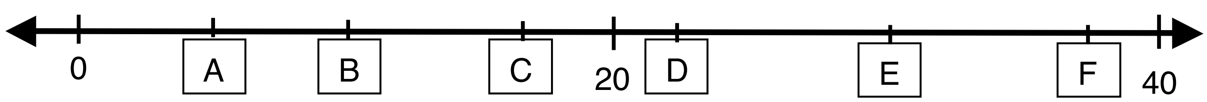
a) 25 b) 28 c) 35 d) 22 e) 31 f) 38





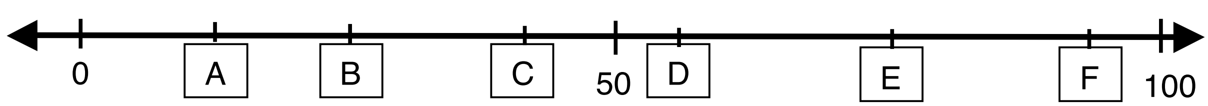
3. What is the number value of each letter shown on the number line below?

A = _____ B = _____ C = _____ D = _____ E = _____ F = _____



4. What is the number value of each letter shown on the number line below?

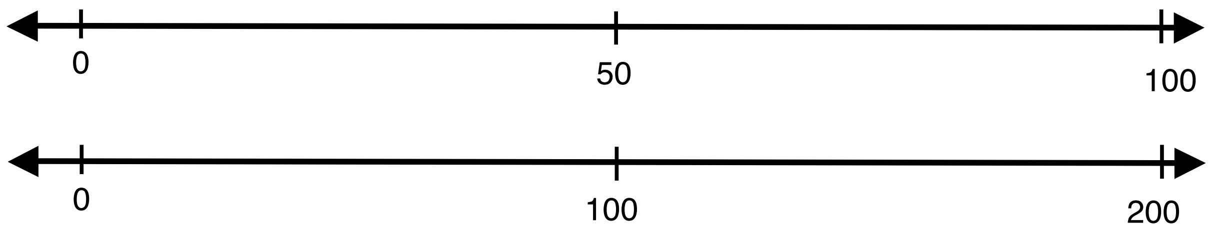
A = _____ B = _____ C = _____ D = _____ E = _____ F = _____



Lesson 11: L3 Activity Sheet **Name:** ________________

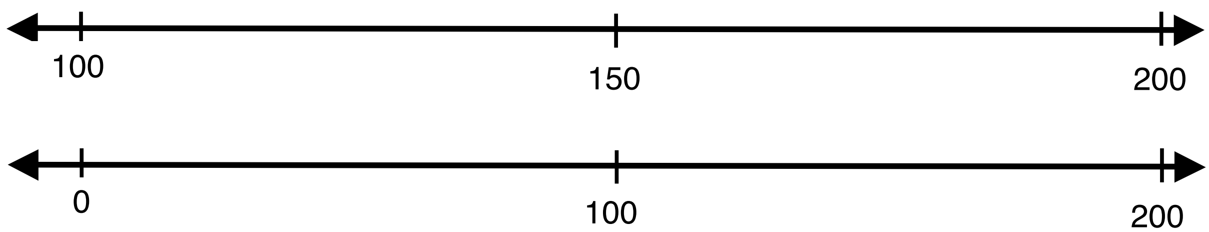
1. Graph these numbers on the TWO number lines below.

a) 25 b) 15 c) 60 d) 30 e) 80 f) 95



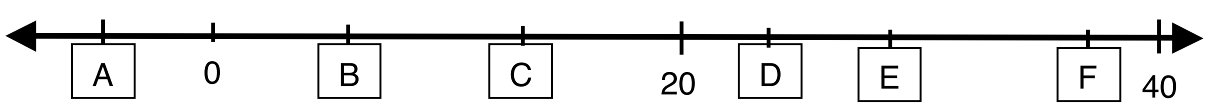
2. Graph these numbers on the TWO number lines below.

a) 125 b) 140 c) 175 d) 110 e) 195 f) 165



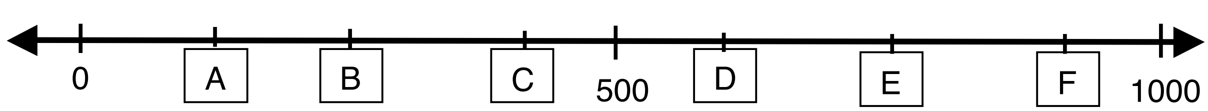
3. What is the number value of each letter shown on the number line below?

A = _____ B = _____ C = _____ D = _____ E = _____ F = _____



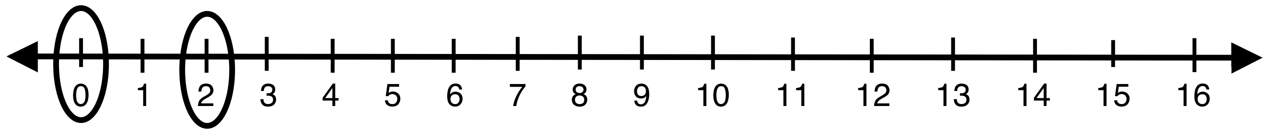
4. What is the number value of each letter shown on the number line below?

A = _____ B = _____ C = _____ D = _____ E = _____ F = _____

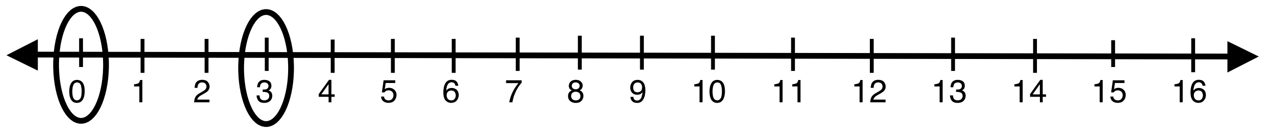


Lesson 12: L1 Activity Sheet **Name**: ________________

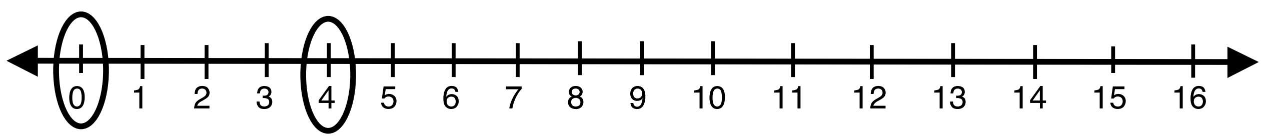
1. Can you count by 2's? Starting at zero, put a circle around every second number.



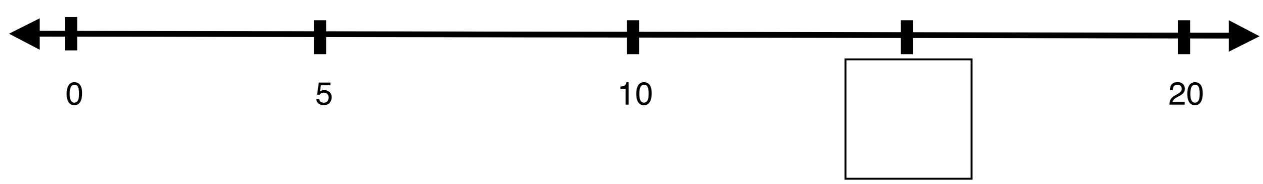
2. Can you count by 3's? Starting at zero, put a circle around every third number.



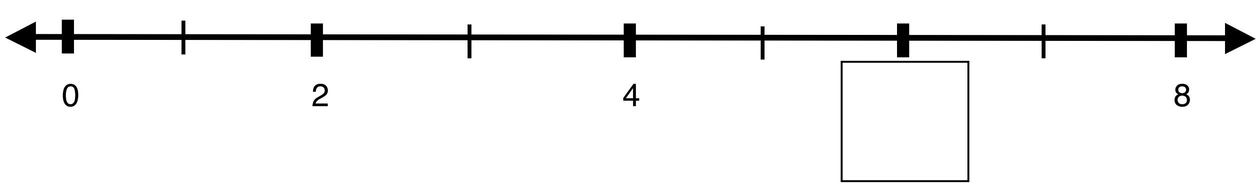
3. Can you count by 4's? Starting at zero, put a circle around every fourth number.



4. What number goes in the box?

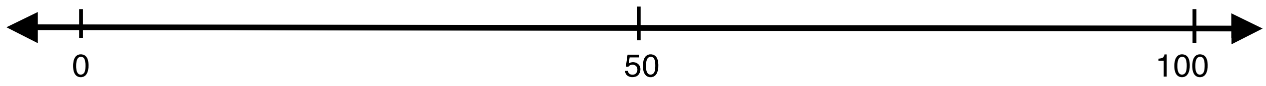


5. What number goes in the box?

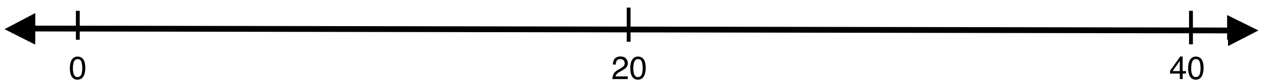


Lesson 12: L2 Activity Sheet **Name**: _______________

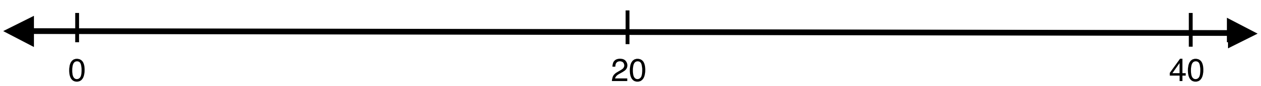
1. Skip count by 10, up to 100. How many 10's are in 100? _____



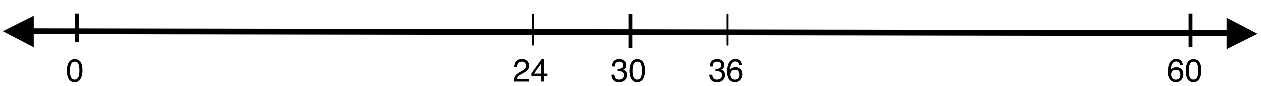
2. Skip count by 5 up to 40 (e.g., 5, 10, …). How many 5's are in 40? ____



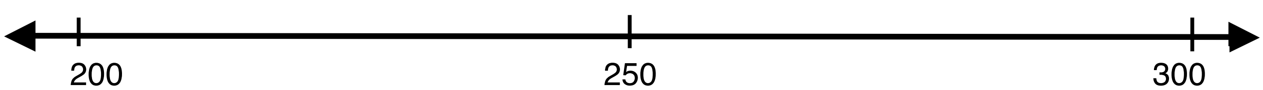
3. Skip count by 4 up to 40 (e.g., 4, 8, 12…). How many 4's are in 40? ____ How many 4's are in 20? ____



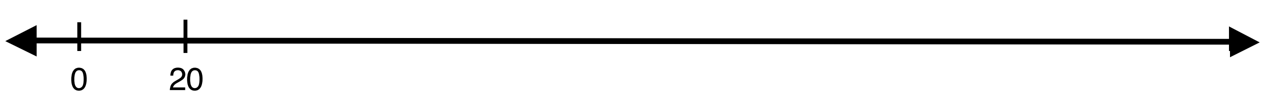
4. Skip count by 6 up to 60 (e.g., 6,12, 18…). How many 6's are in 60? ____ How many 6's are in 30? ____



5. Skip count by 10, starting at 200, and finishing at 300. How many 10's are there between 200 and 300? _____

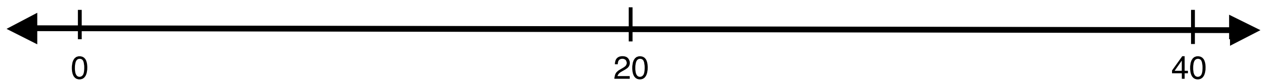


6. Find where the number 200 would be on this line.

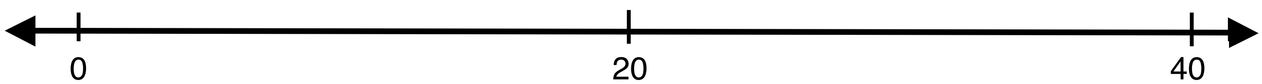


Lesson 12: L3 Activity Sheet **Name**: ________________

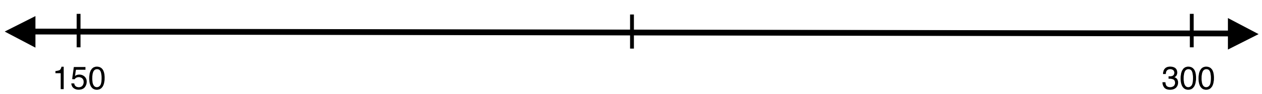
1. Skip count by 5 up to 40 (e.g., 5, 10, …). How many 5's are in 40? ____



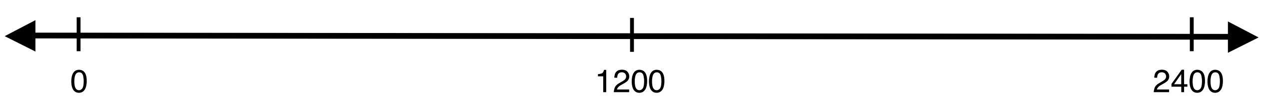
2. Skip count by 8 up to 40 (e.g., 8, 16…). How many 8's are in 40? ____



3. Graph this number line below in multiples of 10, beginning at 150. How many 10's are there between 150 and 300? _____



4. Graph this number line below in multiples of 300. How many 300's are in 2400? ____
How many 300's are in 1200? ____ How many 150's are in 1200?? ___

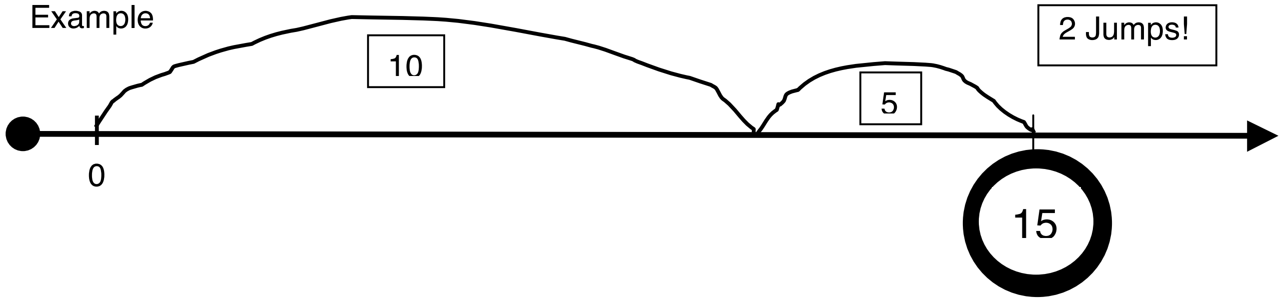


5. Draw a number line. **Start with zero**. Make the next **number 300**. Then, extend the number line **to 3000.**

Lesson 13: L1 Activity Sheet **Name**: ________________

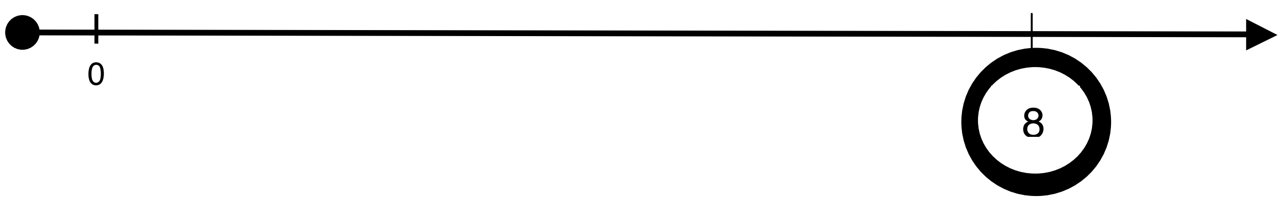
Hit the Target! Use skip **jumps of 1, 5, or 10** to hit the target.

Example

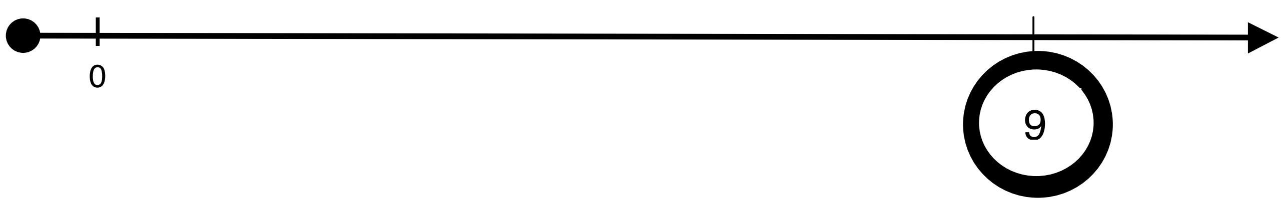


1. Go from zero to 6 in as few jumps as possible.

2. Go from zero to 8 in as few jumps as possible.



3. Go from zero to 9 in as few jumps as possible.

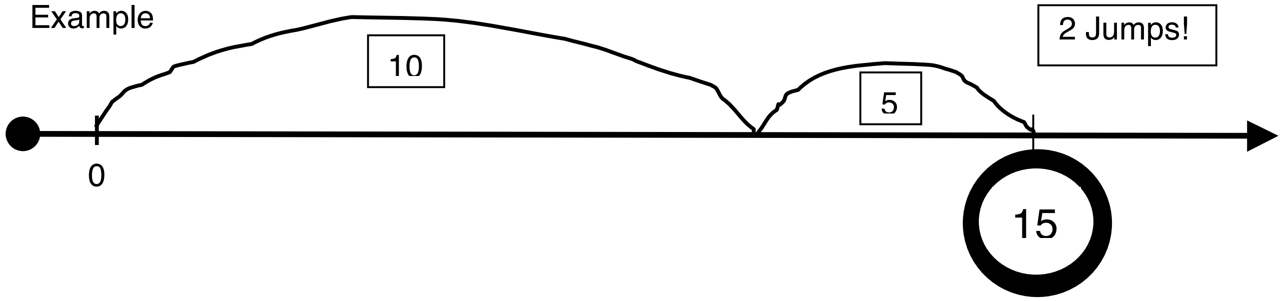


4. Go from zero to 12 in as few jumps as possible.

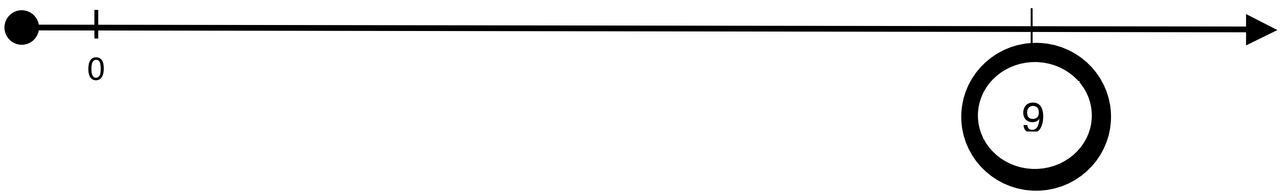
<u>Lesson 13: L2 Activity Sheet</u> **Name**: _______________

<u>Hit the Target!</u> Use **skip jumps of 1, 5, or 10** to hit the target.

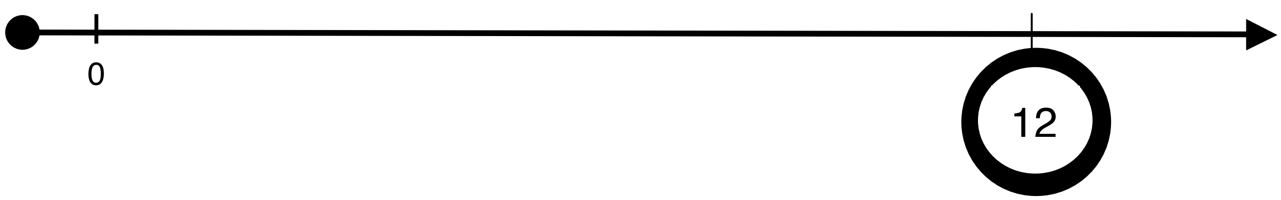
Example



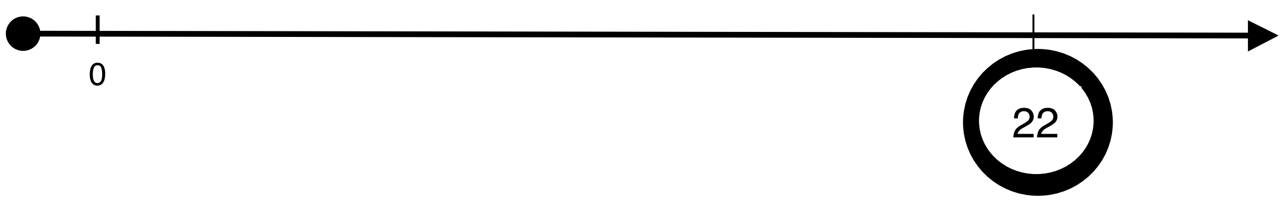
1. Go from zero to 9 in as few jumps as possible.



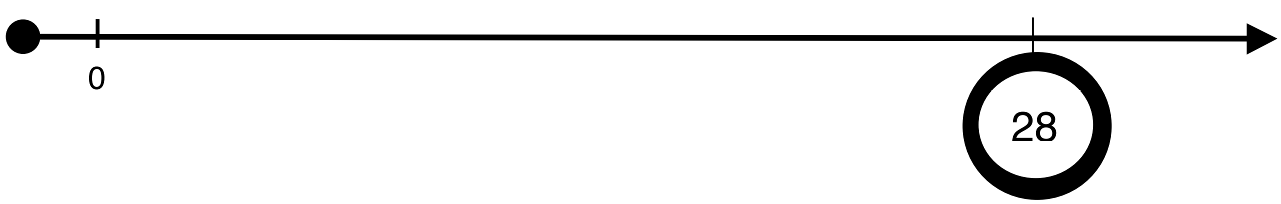
2. Go from zero to 12 in as few jumps as possible.



3. Go from zero to 22 in as few jumps as possible.



4. Go from zero to 28 in as few jumps as possible.

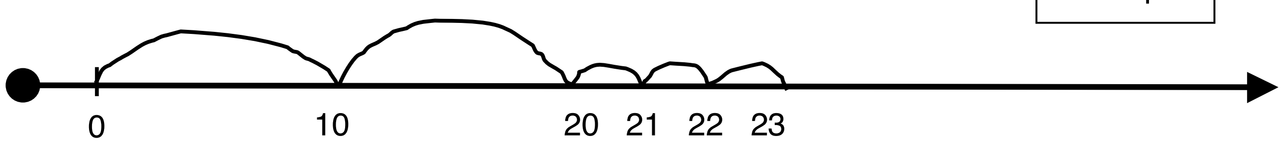


Lesson 13: L3 Activity Sheet **Name**: ________________

Hit the Target! Use skip **jumps of 1, 10, or 100** to hit the target.

Example: Go from 0 to 23 with as few jumps as possible.

5 Jumps!

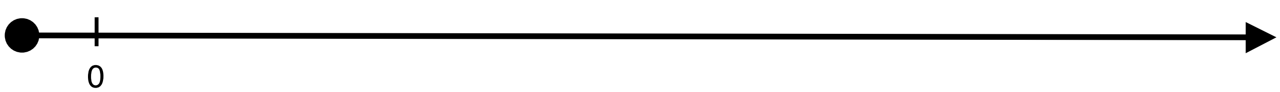


1. Go from zero to 53 in as few jumps as possible.

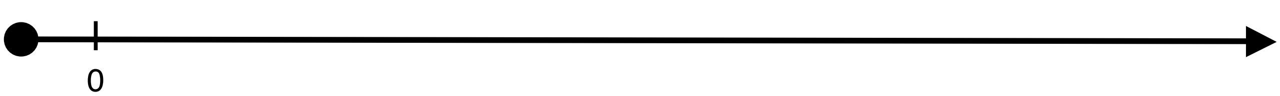
2. Go from 35 to 77 in as few jumps as possible.

3. Go from 108 to 240 in as few jumps as possible.

4. Go from 46 to 153 in as few jumps as possible.

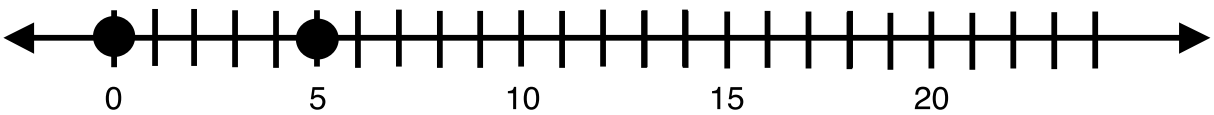


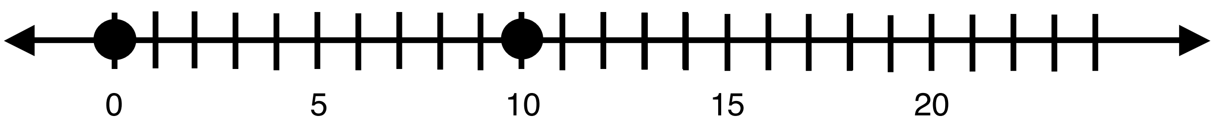
5. Go from 5 to 93 in as few jumps as possible.

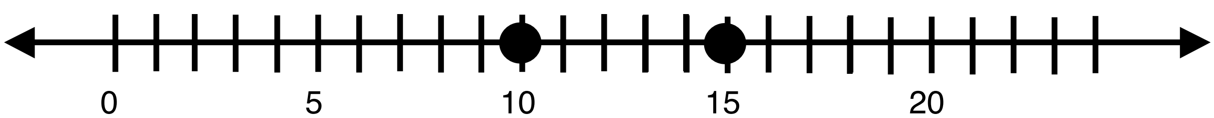


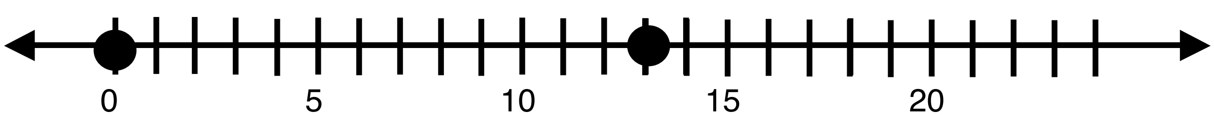
Lesson 14: L1 Activity Sheet **Name**: _______________

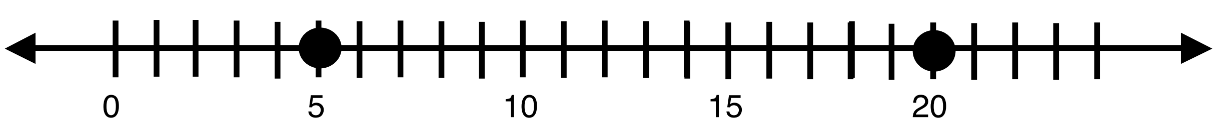
How far is it between the dots shown on the number lines? Use Skip Jumps to help find the answer.

1.


2.


3.


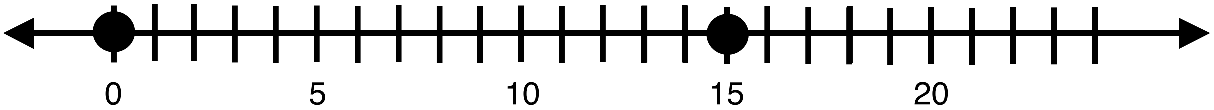
4.


5.


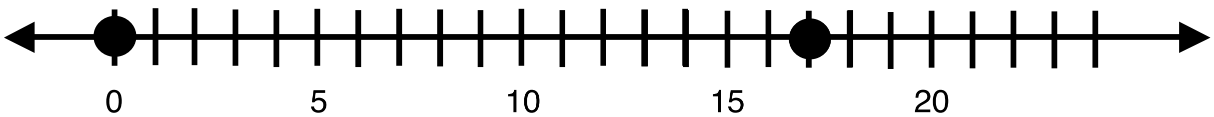
Lesson 14: L2 Activity Sheet **Name**: _______________

How far is it between the dots shown on the number lines? Use Skip Jumps to help find the answer.

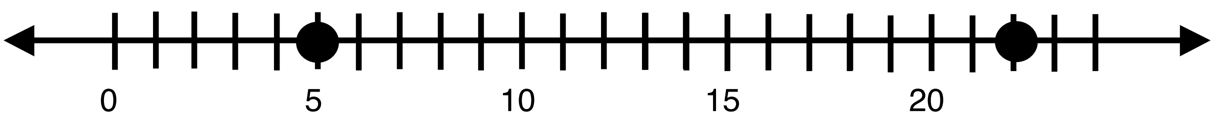
1.



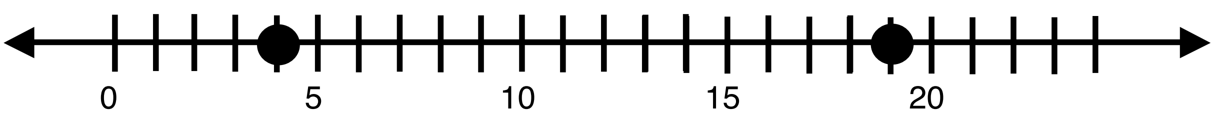
2.



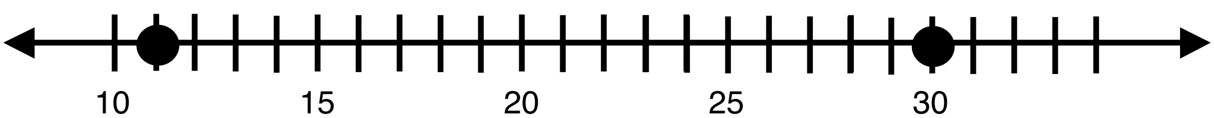
3.



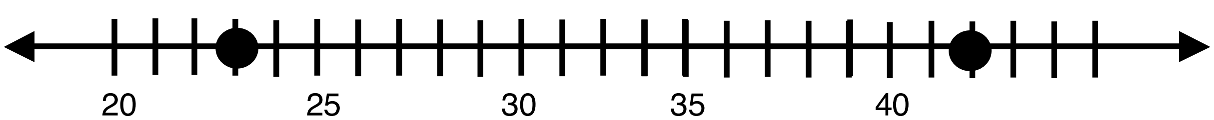
4.



5.



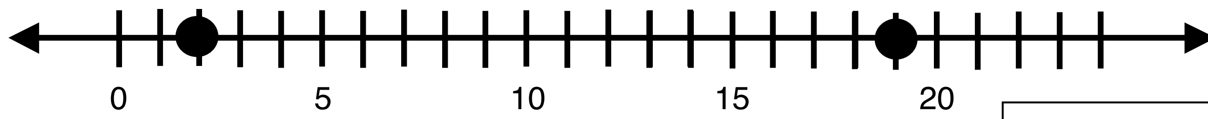
6.



Lesson 14: L3 Activity Sheet **Name**: ________________

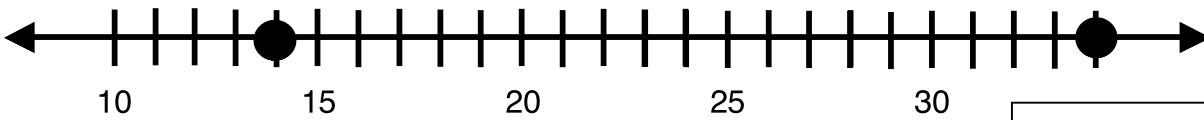
How far is it between the dots shown on the number lines? Use Skip Jumps to help find the answer.

1.



Answer:

2.



Answer:

3. How far is it from 16 to 76? _____ Show this by graphing points and using skip-count arrows on the number line below.

Using what you just did to find the distance between 16 and 76...

a) How far is it from 16 to 75? _____

b) How far is it from 16 to 80? _____

c) How far is it from 15 to 77? _____

4. How far is it from 138 to 267? _____ Show this by graphing points and using skip-count arrows on the number line below.

Using what you just did to find the distance between 138 and 267...

a) How far is it from 140 to 270? _____

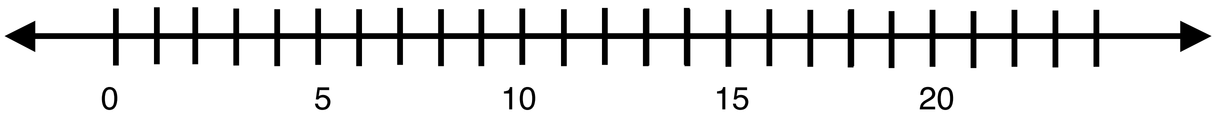
b) How far is it from 138 to 268? _____

c) How far is it from 135 to 265? _____

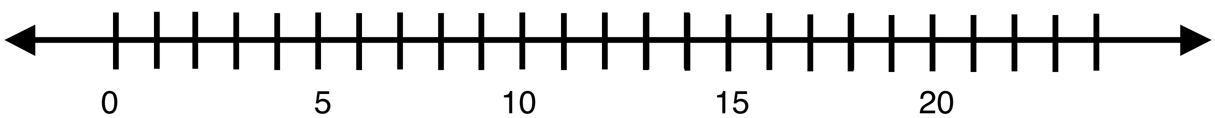
Lesson 15: L1 Activity Sheet **Name**: ________________

Use the number line to add the two numbers.

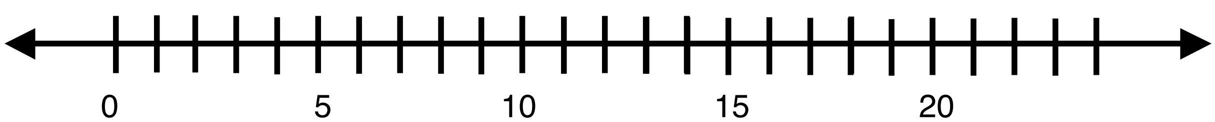
1) 5 + 5 = ______



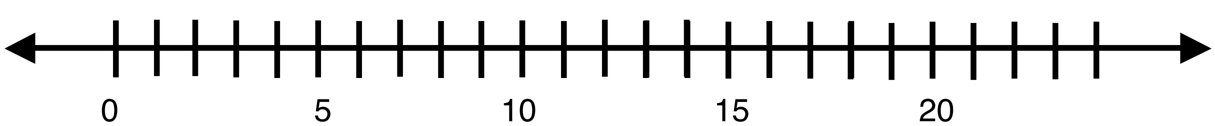
2) 5 + 7 = ______



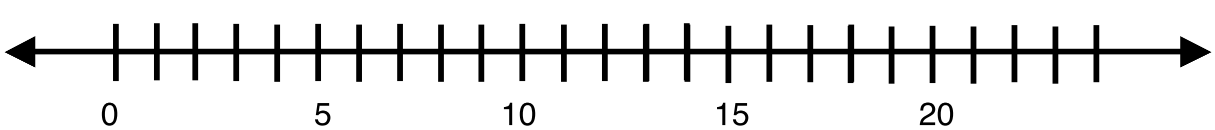
3) 3 + 10 = ______



4) 8 + 7 = ______



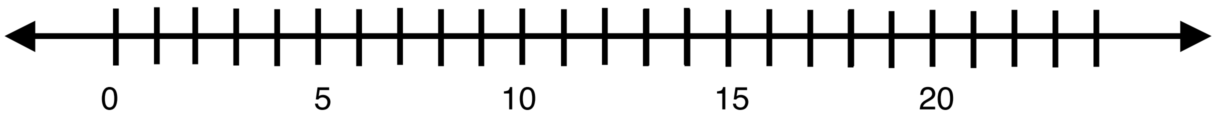
5) 5 + 12 = ______



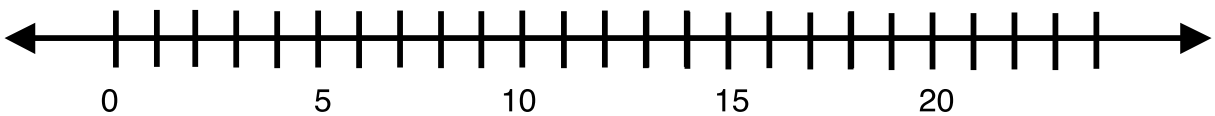
<u>Lesson 15: L2 Activity Sheet</u> **Name**: ________________

Use the number line to add the two numbers.

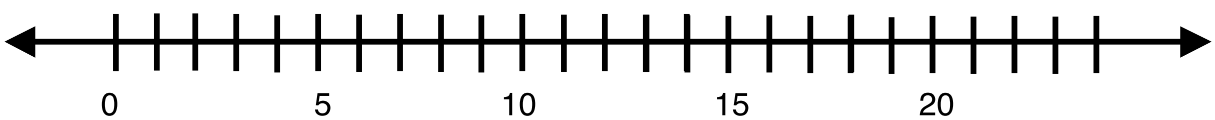
1) 5 + 10 = ______



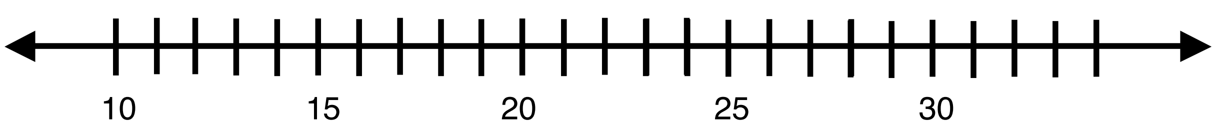
2) 5 + 12 = ______



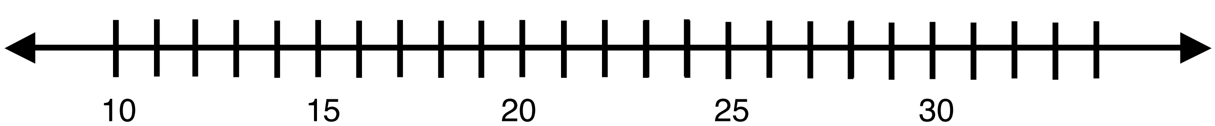
3) 13 + 11 = ______



4) 12 + 19 = ______

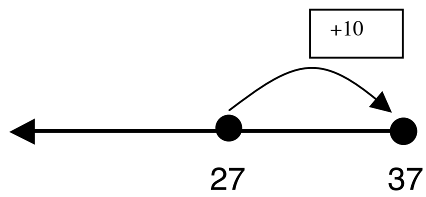


5) 21 + 13 = ______



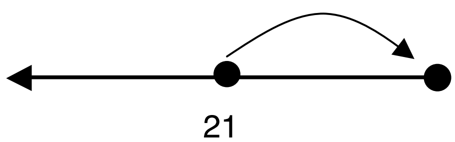
Lesson 15: L3 Activity Sheet **Name**: ________________

1) Use skip counting to extent the following number line to 67. For each "skip" that you use, extend the line, add a point, and label the number.

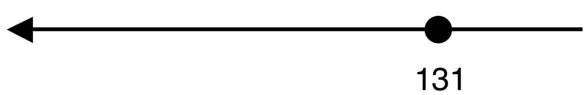


2) Starting with the number 21, add 62 by skip counting. Record each skip jump on the number line, extending it as you go. Complete the number sentence:

21 + 62 = ____



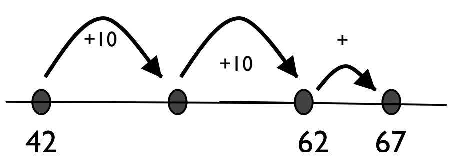
3) What is the sum if you start with 131, and add 40?

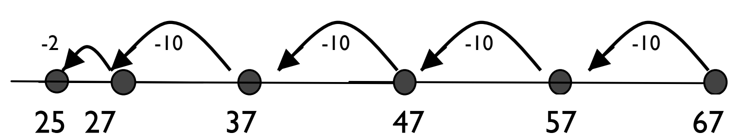


4) What is the sum if you start with 131, and add 42?

5) What is the sum if you start with 131, and add 38?

6) In Denver, Colorado, the average high temperature in October is 67 degrees. At night, the average low temperature is 42 degrees. How much warmer is the average daytime temperature? _____ Both of the number lines below show this problem.



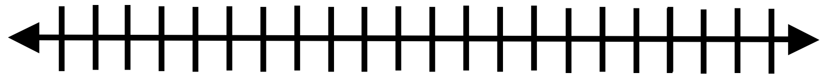


Explain how the two number lines are different:

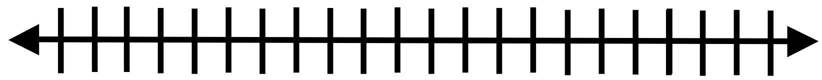
Lesson 16: L1 Activity Sheet Name: ________________

Put the correct number in the box. You may use a number line to help you.

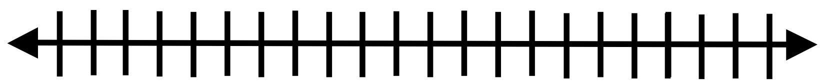
1) $3 + 4 = \square$



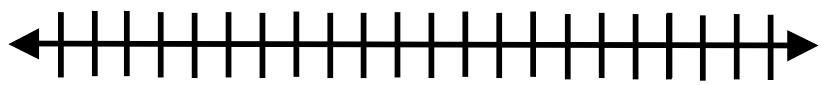
2) $4 + 6 = \square$



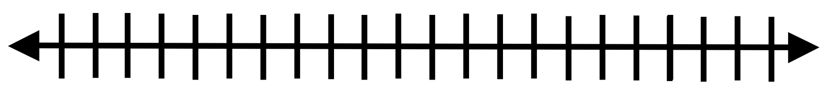
3) $8 - 3 = \square$



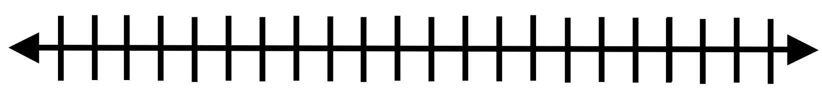
4) $4 + \square = 6$



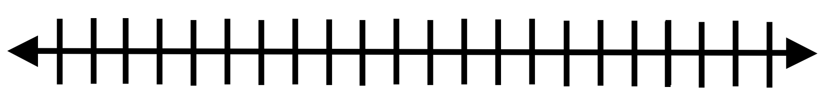
5) $\square + 4 = 10$



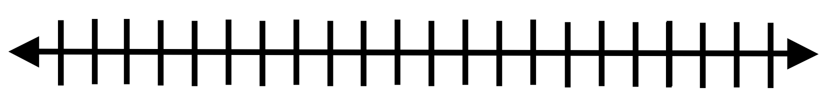
6) $4 + 7 = \square$



7) $4 + \square = 11$



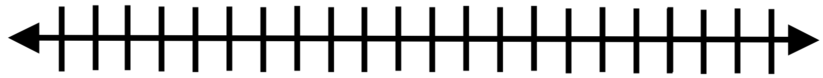
8) $\square - 5 = 10$



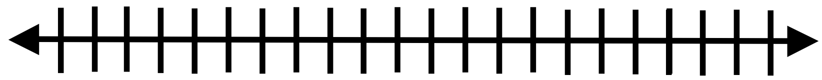
Lesson 16: L2 Activity Sheet Name: ________________

Put the correct number in the box. You may use a number line to help you.

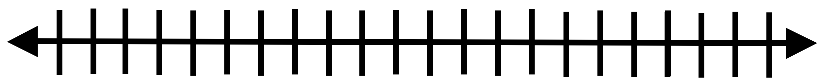
1) $13 + 4 = \boxed{}$



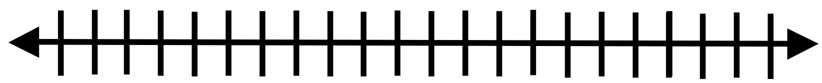
2) $14 + 6 = \boxed{}$



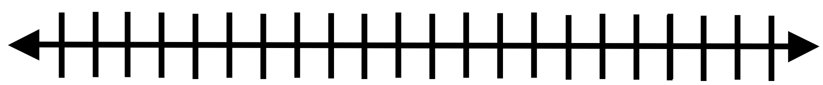
3) $13 - 8 = \boxed{}$



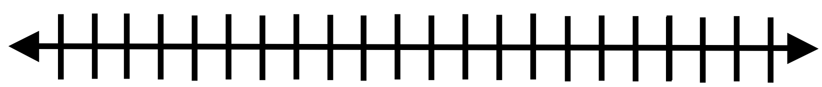
4) $14 + \boxed{} = 26$



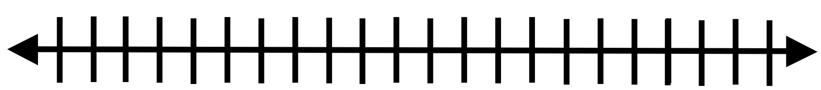
5) $\boxed{} + 8 = 40$



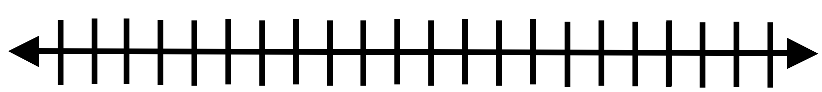
6) $24 + 7 = \boxed{}$



7) $24 + \boxed{} = 41$



8) $\boxed{} - 15 = 63$



Lesson 16: L3 Activity Sheet **Name**: ________________

1. Use the number line and skip counting to fill in the box, and make the following number sentences true.

a) $48 + 93 = \boxed{}$

Explain your thinking:

b) $84 - 57 = \boxed{}$

Explain your thinking:

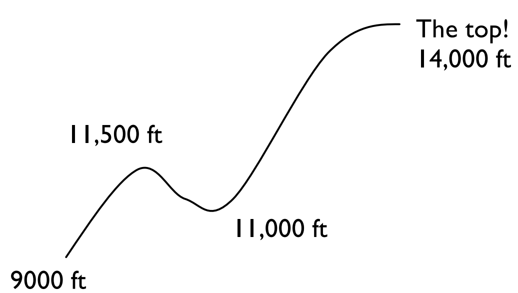
c) $78 + \boxed{} = 236$

Explain your thinking:

d) $112 - \boxed{} = 39$

Explain your thinking:

2. Santiago went hiking to the top of a mountain in Colorado. He started at an elevation of 9000 feet above sea level. He climbed to the top of a hill that was at 11,500 feet of elevation. The trail then went down to the bottom of a valley at 11,000 feet of elevation. The trail then went up steeply again for the last 4 miles to the top, at 14,000 feet high! How many **total feet of elevation** did Santiago climb during the hike up? Use a number line and skip jumps to help find your answer.



Explain:

Lesson 17: L1 Activity Sheet **Name**: _______________

1. Katie has 5 dolls. Her friend Emma has 4 dolls. How many do they have together?

2. Katie has 5 dolls. She gave some of her dolls to her friend Olivia. Now she has 3 dolls. How many did she give to Emma?

3. Jason plays baseball. He has 5 baseballs in his house. Kyle gave him some more baseballs. Now Jason has 11 baseballs. How many baseballs did Kyle give him?

4. There are 5 puppies for sale. There are 13 kittens for sale. How many animals are for sale?

5. Recess lasts for 10 minutes. The teacher gave her class an extra 5 minutes for recess. How many minutes of recess did the students get?

Lesson 17: L2 Activity Sheet **Name**: _______________

1. The soccer team scored 4 goals in the first game, 3 goals in the second game, and 6 goals in the third game. How many goals did they score in all three games?

2. Carrie scored some of the goals in the first game. She scored 2 goals in the second game. In both games, Carrie scored 5 goals. How many did she score in the first game?

3. There are 15 puppies playing together at the park. Alejandra and Josie were at the park, and they counted 8 of the dogs swimming in the lake. How many dogs were not in the lake?

4. There are 24 colored pencils in the container. There are 8 students in the class that want to use the pencils. How many pencils does each student get (if they split them evenly)?

5. Jennie’s family was on vacation. Lunch cost $22. The dinner was more expensive than the lunch. Together, her parents spent $54 on both dinner and lunch. How much did the dinner cost?

Lesson 17: L3 Activity Sheet **Name**: _______________

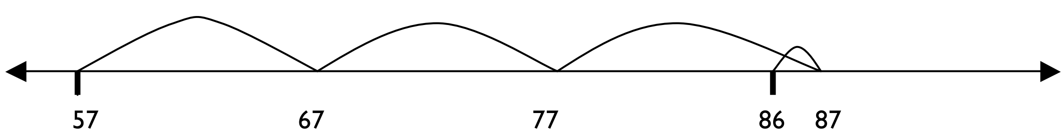
1. Jon is reading a book with 246 pages. He has already read 117 pages. How many pages does he need to read to finish the book?

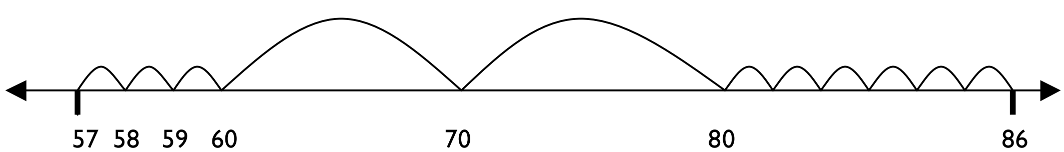
2. Stephanie had $396 in her bank account. She spent $99 on a new bicycle. How much does she have left?

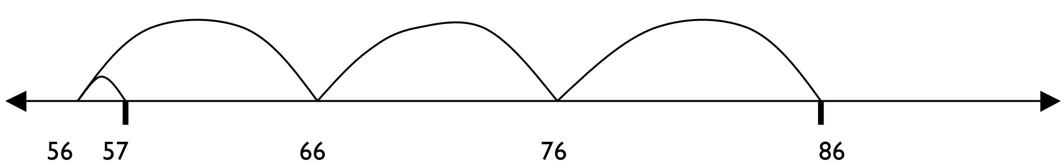
3. Peter wanted to buy a new skateboard that cost $125. He had saved $46 in his bank. He earned another $80 mowing lawns in his neighborhood. Does he have enough to buy the skateboard? How do you know?

4. The following 3 number lines were drawn by students as they tried to figure out the problem below. Study each number line, and be prepared to explain the process used for each solution strategy. Each strategy starts skipping from the number 57.

The Problem: Kerri was collecting empty water bottles to recycle. On Saturday, she found 57 bottles at the park. On Sunday, she found 29 bottles in the parking lot of a school. Combining both days, how many bottles did she find in all?

Solution #1


Solution #2


Solution #3


How are these strategies different from each other? Which of these methods would you use if you had to do this problem mentally? Explain.

The primary goal for elementary level mathematics education is to help children develop a rich understanding of numbers – their meanings, their relationships to one another, and how we operate with them. In recent years, there has been a resurgence of interest in the number line as a powerful mathematical model to be used toward those ends. This book has been created to illustrate a number of innovative ways in which the number line can help young students develop confidence in their intuitive strategies and mathematical insights, as well as efficient and fruitful strategies for addition and subtraction.

The book highlights ways in which teachers can use a "life sized" number line to foster meaningful conversations about significant mathematics. The lesson activities provide students with opportunities to:

- Explore both open and closed number lines;
- Locate and order numbers on a number line;
- Use the number line to understand fractions and decimals;
- "Skip count" by anchoring on fives and tens;
- Add and subtract numbers using visual strategies;
- Become adept at using various strategies and mathematical relationships such as doubling, halving, repeated addition, etc.
- Develop informal, mental strategies for addition and subtraction.

"Marcia lives at the corner of Broadway and 12th street. Her school is on the corner of Broadway and 34th street. How many blocks does Marcia need to walk to get to school?"

Jeffrey Frykholm, Ph.D.

An award winning author, Dr. Jeffrey Frykholm is a former classroom teacher who now focuses on helping teachers develop pedagogical expertise and content knowledge to enhance mathematics teaching and learning. In his Learning to Think Mathematically series of textbooks for teachers, he shares his unique approach to mathematics teaching and learning by highlighting ways in which teachers can use mathematical models (e.g., the rekenrek, the ratio table, the number line, etc.) as fundamental tools in their classroom instruction. These books are designed with the hope that they will support teachers' content knowledge and pedagogical expertise toward the goal of providing a meaningful and powerful mathematics education for all children.