MADS – 1ª PARTE (PROF. GEI)

MADS, 26/9/24

$\varphi(z) = -v'(z)$

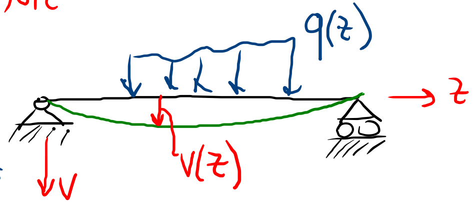
LINEA ELASTICA IV ORDINE:

• LINEA ELASTICA DEL II ORDINE (TRAVI SNELLE)

MOM. FLETTENTE NOTO

$$\begin{cases} v''(z) = -\dfrac{M(z)}{EJ} \\ \text{2 COND. LIMITI} \end{cases}$$

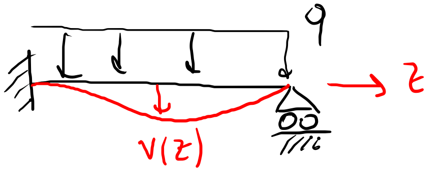
CURVATURA ELASTICA



STR. STAT. DET.

PER STRUTTURE STATIC. DETERMINATE (ISOSTATICHE)

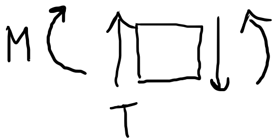
DOMANDA: SIAMO IN GRADO DI RISOLVERE QUESTA STRUTTURA CON LA "LINEA ELASTICA DEL II ORD."?

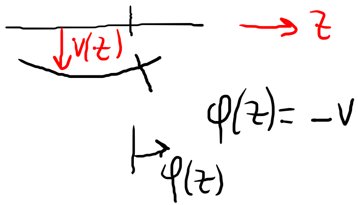


NO: È UNA STR. IPERSTATICA PERCHÈ NON CONOSCO IL $M(z)$

OSS: PER ORA SAPPIAMO RISOLVERE LA STR. COL METODO DELLE FORZE

LINEA EL. IV ORDINE





$\varphi(z) = -v'(z)$

- EQUILIBRIO $\quad \dfrac{dM}{dz} = T \; (*) \quad ; \quad \dfrac{dT}{dz} = -q$

$M, T, q, v, \varphi, \kappa, EJ$ sono funzioni di z

- CONGRUENZA (INTERNA) $\quad \kappa = \dfrac{d\varphi}{dz} \quad ; \quad \dfrac{dv}{dz} = -\varphi$

CURVATURA

- LEGAME COSTITUTIVO $\quad \kappa = \dfrac{M}{EJ}$

$$\frac{M}{EJ} = \kappa = -\frac{d^2 v}{dz^2} \leadsto M = -EJv'' \leadsto M' = -(EJv'')' \overset{(*)}{\leadsto} T = -(EJv'')' \leadsto$$

$$\leadsto T' = -(EJv'')'' \leadsto q = (EJv'')'' \Rightarrow \boxed{(EJ(z)\, v''(z))'' = q(z)}$$

L. EL. IV ORDINE
TRAVI SNELLE

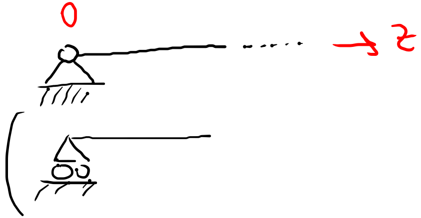
- PER L'INTEGR. SERVONO 4 C. LIMITI
- TERMINI NOTI: $EJ(z)$, $q(z)$

SEMPLICAZIONE QUANDO $EJ(z) = \text{COST.}$

$$\boxed{EJ\, v''''(z) = q(z)}$$

$$T = -EJ\, v'''(z)$$

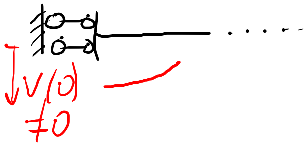
CONDIZ. AI LIMITI



$$v(0) = 0$$

$$M(0) = \underbrace{-EJ\, v''(0) = 0}$$

CERNIERA
o CARRELLO



$$\varphi(0) = \underbrace{0 = -v'(0)}$$

$$T(0) = \underbrace{-EJ\, v'''(0) = 0}$$

DOPPIO PENDOLO
PATTINO

$$v(0) = 0$$

$$\varphi(0) = \underbrace{-v'(0) = 0}$$

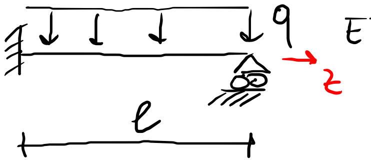
INCASTRO

$$M(0) = \overbrace{-EJ\, v''(0) = 0}$$

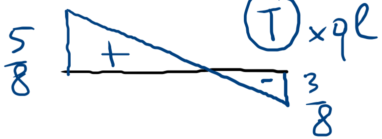
$$T(0) = \underbrace{-EJ\, v'''(0) = 0}$$

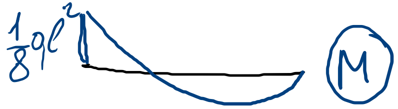
ESTREMO
LIBERO

ES



EJ : cost





$$\begin{cases} EJ\, v''''(z) = q \\ v(0) = 0 \\ v'(0) = 0 \\ v(l) = 0 \\ v''(l) = 0 \end{cases} \Rightarrow \begin{cases} C_3 = 0 \\ C_4 = 0 \\ C_1 = -\frac{5}{8}\frac{ql}{EJ} \\ C_2 = \frac{1}{8}\frac{ql^2}{EJ} \end{cases}$$

$$v'''(z) = \frac{q}{EJ} z + C_1$$

$$v''(z) = \frac{q}{EJ}\frac{z^2}{2} + C_1 z + C_2$$

$$v'(z) = \frac{q}{EJ}\frac{z^3}{6} + C_1\frac{z^2}{2} + C_2 z + C_3$$

$$v(z) = \frac{q}{EJ}\frac{z^4}{24} + C_1\frac{z^3}{6} + C_2\frac{z^2}{2} + C_3 z + C_4$$

$$M(z) = -\left(q\frac{z^2}{2} - \frac{5}{8}qlz + \frac{1}{8}ql^2\right)$$

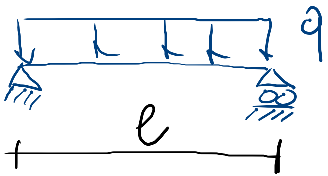
$$M(0) = -\frac{1}{8}ql^2 \qquad \text{OK}$$

$$T(z) = -\left(qz - \frac{5}{8}ql\right)$$

$$T(0) = +\frac{5}{8}ql \qquad \text{OK}$$

$$T(l) = -\left(ql - \frac{5}{8}ql\right) = -\frac{3}{8}ql \qquad \text{OK}$$

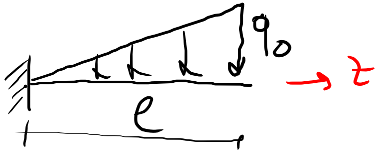
ES



$$\begin{cases} EI\, v''''(z) = q \\ v(0) = 0 \qquad v(l) = 0 \\ v''(0) = 0 \qquad v''(l) = 0 \end{cases}$$

VERIFICA CHE $v(z)$ SIA LA STESSA DEL PROBL $v'' = -\frac{M(z)}{EI}$

ES

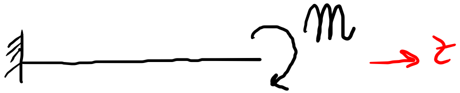


$$q(z) = \frac{q_0}{l} z$$

$$\boxed{EI, EJ \text{ cost.}}$$

$$\begin{cases} EJ\, v''''(z) = q(z) \\ v(0) = 0 \qquad v''(l) = 0 \\ v'(0) = 0 \qquad v'''(l) = 0 \end{cases}$$

ES



$$EJ\, v''''(z) = 0$$

$$v(0) = 0 \qquad v'''(l) = 0$$

$$v'(0) = 0 \qquad EJ\, v''(l) = m$$

$$M(z) = -EJ\, v''(z)$$

$$M(l) = -EJ\, v''(l)$$

osservo che

$$M(l) = -m$$

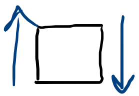
ES

$$\begin{cases} EJ\,v''''(z)=0 \\ v(0)=0 \qquad v''(\ell)=0 \\ v'(0)=0 \qquad -EJ\,v'''(\ell)=F \end{cases}$$

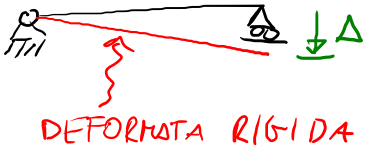
$$T(\ell) = -EJ\,v'''(\ell)$$

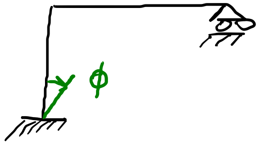
osservo che

$$T(\ell) = +F$$

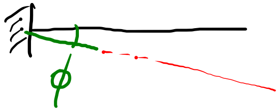


CEDIMENTI ANELASTICI





(CEDIM. ANGOLARE)



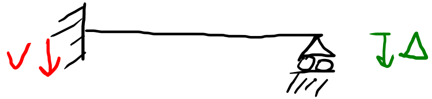
STR IPERST.

$$T(\Delta,\phi),\ M(\Delta,\phi) \neq 0$$

STR ISOSTATICA

$$T(\Delta,\phi),\ M(\Delta,\phi) = 0$$

ES

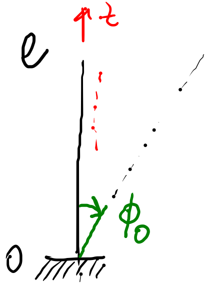


$$EJ\,v''''(z) = 0$$

$$v(0)=0 \qquad v(\ell)=+\Delta$$

$$v'(0)=0 \qquad v''(\ell)=0$$

ES



$$EJ\, v''''(z) = 0$$

$v''(\ell)=0$ $\quad v(0)=0$

$v'''(\ell)=0$ $\quad v'(0)=+\phi_0$

$\varphi(z) = -v'(z)$

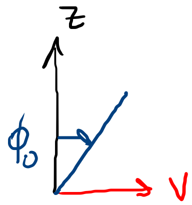
$\varphi(z)>0$

Risolvo e ottengo $v(z)=\alpha z$

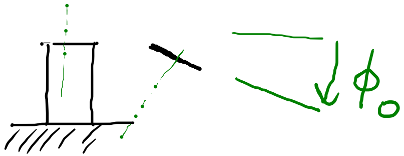
(termini in $z^3, z^2 \Rightarrow$ sono annullati dalle costanti)

$T(z), M(z) \equiv 0$; $v'(z)=\text{cost.}$

(anche il termine "C_4" si annulla)



$v'(0)=\phi_0$

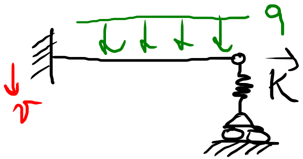


$\varphi(0)<0$

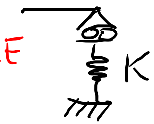
$\varphi(0) = -\phi_0 = -v'(0)$

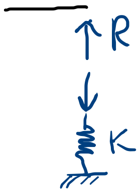
$v'(0)=+\phi_0$

CEDIMENTI ELASTICI



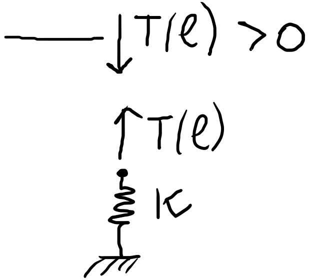
MOLLA LINEARE





$R \neq 0$

$v \neq 0 \quad \left(|v| = \frac{R}{K} \right)$



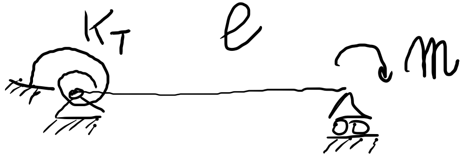
$$T(\ell) = -K\, v(\ell)$$

$$-EJ\, v'''(\ell) = -K\, v(\ell)$$

$$\begin{cases} EJ\, v''''(z) = q \\ v(0) = 0 \\ v'(0) = 0 \\ v''(\ell) = 0 \\ EJ\, v'''(\ell) = K\, v(\ell) \end{cases}$$

C. LIMITI MOLLA LINEARE

Molle ROTAZIONALE (1 v. IPERST.)



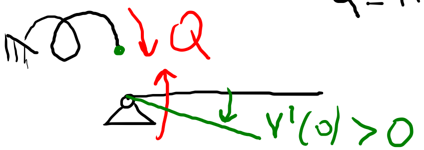
INCASTRO CEDEVOLE

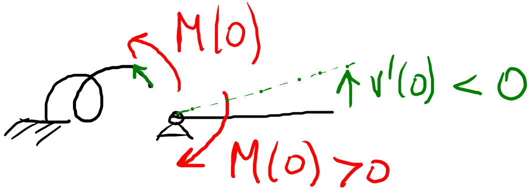
MOM. ROTAZ.

$$Q = K_T \, v'(0)$$

$$[K_T] = [FL]$$

Nm





$$M(0) = -K_T \, v'(0)$$

$$-EJ \, v''(0) = -K_T \, v'(0)$$

$$\begin{cases} EJ \, v''''(z) = 0 \\ v(\ell) = 0 \\ -EJ \, v''(\ell) = -\mathcal{M} \\ EJ v''(0) = K_T \, v'(0) \\ v(0) = 0 \end{cases}$$

C. LIMITI MOLLA ROTAZ.

NELLE CONDIZIONI AI LIMITI PER VINCOLI CEDEVOLI ELASTICAM. OCCORRE PORRE ATTENZIONE ALLE CONVENZ. ADOTTATE.