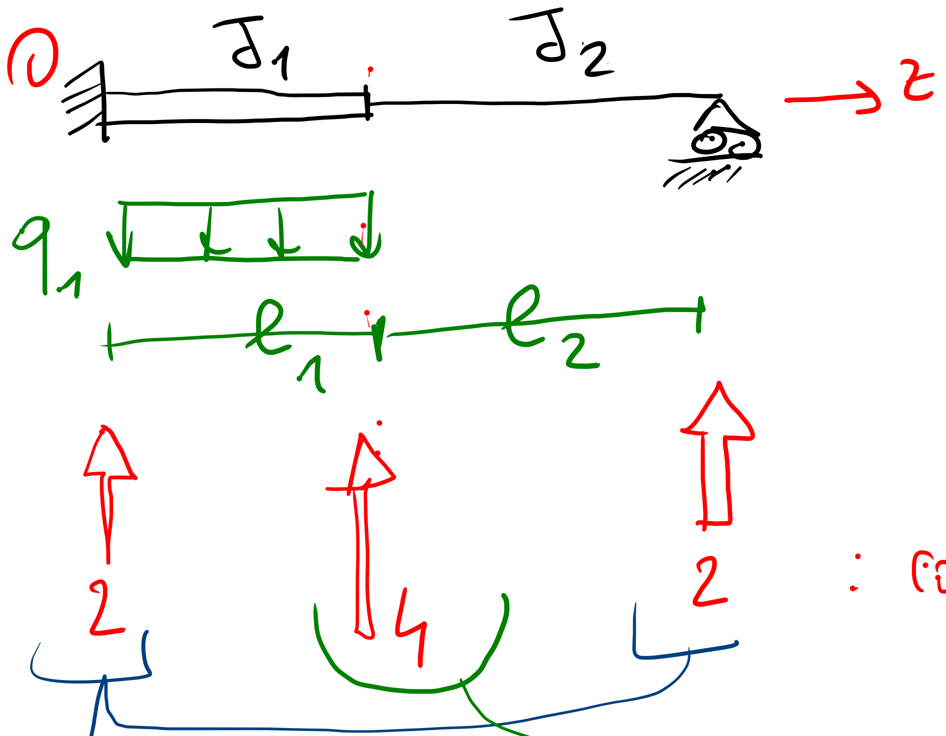
LINEA EL. IV ORDINE: CONDIZIONI DI RACCORDO PER PIÙ CAMPI DI INTEGRAZIONE

2/10/24

2 CAMPI DI INTEGRAZIONE

① $z \in [0, \ell_1]$

② $z \in]\ell_1, \ell_1+\ell_2]$



: CONDIZ. DA IMPORRE

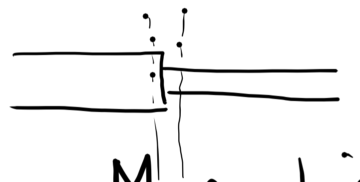
① $EJ_1 v_1'''' = q_1 \rightsquigarrow C_1, C_2, C_3, C_4$

② $EJ_2 v_2'''' = 0 \rightsquigarrow C_5, C_6, C_7, C_8$

8 COSTANTI INCOGNITE

$$\begin{cases} v_1(0) = 0 \\ v_1'(0) = 0 \\ v_2(\ell_1+\ell_2) = 0 \\ v_2''(\ell_1+\ell_2) = 0 \end{cases}$$

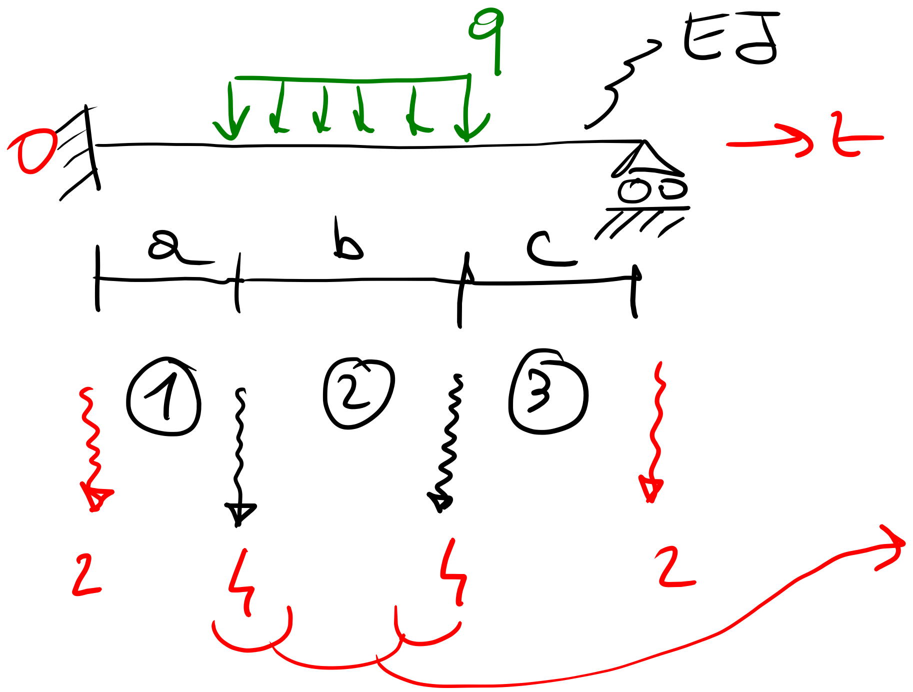
$$\begin{cases} v_1(\ell_1) = v_2(\ell_1) \\ v_1'(\ell_1) = v_2'(\ell_1) \\ -EJ_1 v_1''(\ell_1) = -EJ_2 v_2''(\ell_1) \\ -EJ_1 v_1'''(\ell_1) = -EJ_2 v_2'''(\ell_1) \end{cases}$$



M continuo

T continui

ALTRI CASI: 1



CAMPI

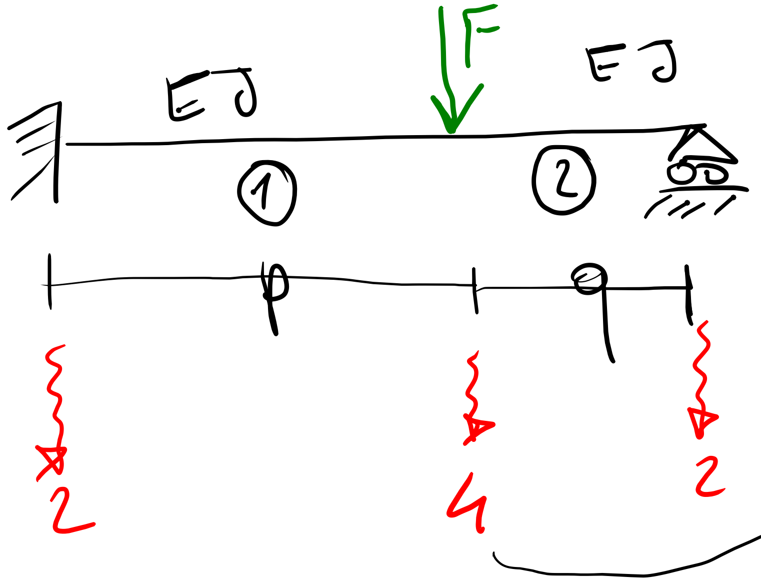
$z_1 \in [0, a]$ $\qquad EJ\, v_1^{IV}(z) = 0$

$z_2 \in \,]a, a+b]$ $\qquad EJ\, v_2^{IV}(z) = q$

$z_3 \in \,]a+b, a+b+c]$ $\qquad EJ\, v_3^{IV}(z) = 0$

CONTINUITÀ DI v, v', M, T

2

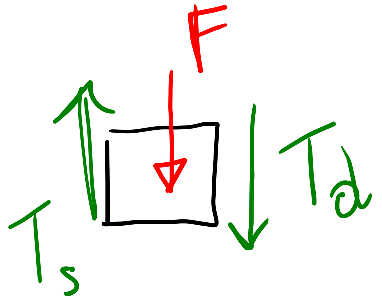


$z_1 \in [p, q]$ $\qquad EJ\, v_1^{IV}(z) = 0$

$z_2 \in \,]q, p+q]$ $\qquad EJ\, v_2^{IV}(z) = 0$

$$\begin{cases} v_1(p) = v_2(p) \\ v_1'(p) = v_2'(p) \\ v_1''(p) = v_2''(p) \\ v_1'''(p) - v_2'''(p) = -\dfrac{F}{EJ} \end{cases}$$

$T = -EJ\, v'''$



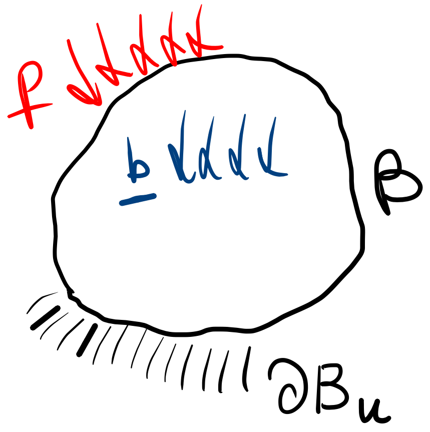
$T_d + F - T_s = 0$

$$-EJ\, v_2'''(p) + F = -EJ\, v_1'''(p)$$

PRINCIPIO DI STAZIONARIETA' DELL'EN. POTENZIALE TOTALE

NELLA MECCANICA DEI SIST. DISCRETI QUESTO PRINCIPIO PERMETTE DI STUDIARE LE CONFIG. DI EQUILIBRIO

$$\underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla \underline{u} + \nabla \underline{u}^T\right) \; ; \; \underset{\sim}{\sigma} = \mathbb{C}\, \underset{\sim}{\varepsilon}$$



VINCOLI DI NATURA CINEMATICA

$\underline{u}(\underline{x})$: CAMPO DI SPOST. NELLA CONFIG. EQUILIBRATA

$\underline{x} \in B$

$$\Pi(\underline{u}) = \underbrace{\frac{1}{2}\int_B \underset{\sim}{\sigma} \cdot \underset{\sim}{\varepsilon}\, dV}_{\Phi\text{ : EN ELASTICA}} - \underbrace{\left\{ \int_B \underline{b} \cdot \underline{u}\, dV + \int_{\partial B} \underline{p} \cdot \underline{u}\, dA \right\}}_{\text{POTENZ. CARICHI.}}$$

E.P.T.

APPLICO UNA "VARIAZIONE" $\delta \underline{u}$ (ANALOGO ALLO SPOST. VIRTUALE)

$$\Pi(\underline{u}) \to \Pi(\underline{u} + \delta \underline{u})$$

$$\Pi(\underline{u}+\delta\underline{u}) = \frac{1}{2}\int_B \mathbb{C}(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})\cdot(\underset{\sim}{\varepsilon}+\delta\underset{\sim}{\varepsilon})\,dV - \int_B \underline{b}\cdot(\underline{u}+\delta\underline{u})\,dV - \int_{\partial B} \underline{p}\cdot(\underline{u}+\delta\underline{u})\,dA$$

$$= \frac{1}{2}\int_B \cancel{\mathbb{C}\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\varepsilon}} + 2\,\mathbb{C}\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon} + \mathbb{C}\,\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\,dV - \cancel{\int_B \underline{b}\cdot\underline{u}\,dV} - \int_B \underline{b}\cdot\delta\underline{u}\,dV - \cancel{\int_{\partial B}\underline{p}\cdot\underline{u}\,dA} - \int_{\partial B}\underline{p}\cdot\delta\underline{u}\,dA$$

$$\cancel{\Pi(\underline{u})} + \delta\Pi =$$

$$\Rightarrow \delta\Pi = \underbrace{\int_B \mathbb{C}\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon} - \int_B \underline{b}\cdot\delta\underline{u}\,dV - \int_{\partial B}\underline{p}\cdot\delta\underline{u}\,dA}_{\delta\Pi^{(1)}:\ \text{VARIAZIONE PRIMA}} + \underbrace{\frac{1}{2}\int_B \mathbb{C}\,\delta\underset{\sim}{\varepsilon}\cdot\delta\underset{\sim}{\varepsilon}\,dV}_{\delta\Pi^{(2)}\ \text{VARIAZ. SECONDA}}$$

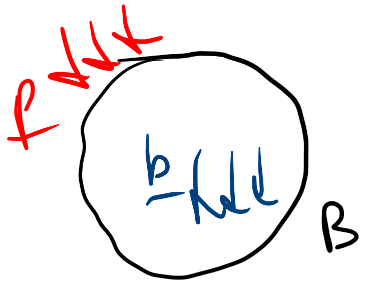
VISTO CHE $\underline{u}(\underline{x})$ DEFINISCE UNA CONFIG. DI EQUILIBRIO, PER IL T.L.V.,

$$\delta\Pi^{(1)} = 0 \quad \forall\,\delta\underline{u} \quad \Rightarrow$$

LA FUNZIONE $\Pi(\underline{u})$ è STAZIONARIA IN $\underline{u}$

RICORD. IL T.L.V.

SIST. A STAT. AMMISS.



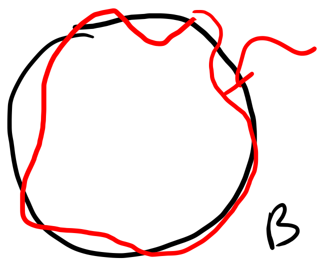
SOLUZ. REALE

$\Uparrow$

$\underset{\sim}{\sigma}$ IN EQUIL. CON $\underline{p}$ e $\underline{b}$

$$\begin{cases} \operatorname{div} \underset{\sim}{\sigma} + \underline{b} = \underline{0} \\ \underset{\sim}{\sigma}\, \underline{n} = \underline{p} \end{cases}$$

SIST B CINEMATIC. AMMISS.



$\delta \underline{u} \rightarrow \delta \underset{\sim}{\varepsilon} = \frac{1}{2}\left(\nabla \delta \underline{u} + \nabla \delta \underline{u}^T\right)$

$$L_{ve} = L_{vi}$$

$$\int \underset{\sim}{\sigma} \cdot \delta \underset{\sim}{\varepsilon}\, dV = \int \underline{p} \cdot \delta \underline{u}\, dA + \int \underline{b} \cdot \delta \underline{u}\, dV$$

HO UTILIZZATO

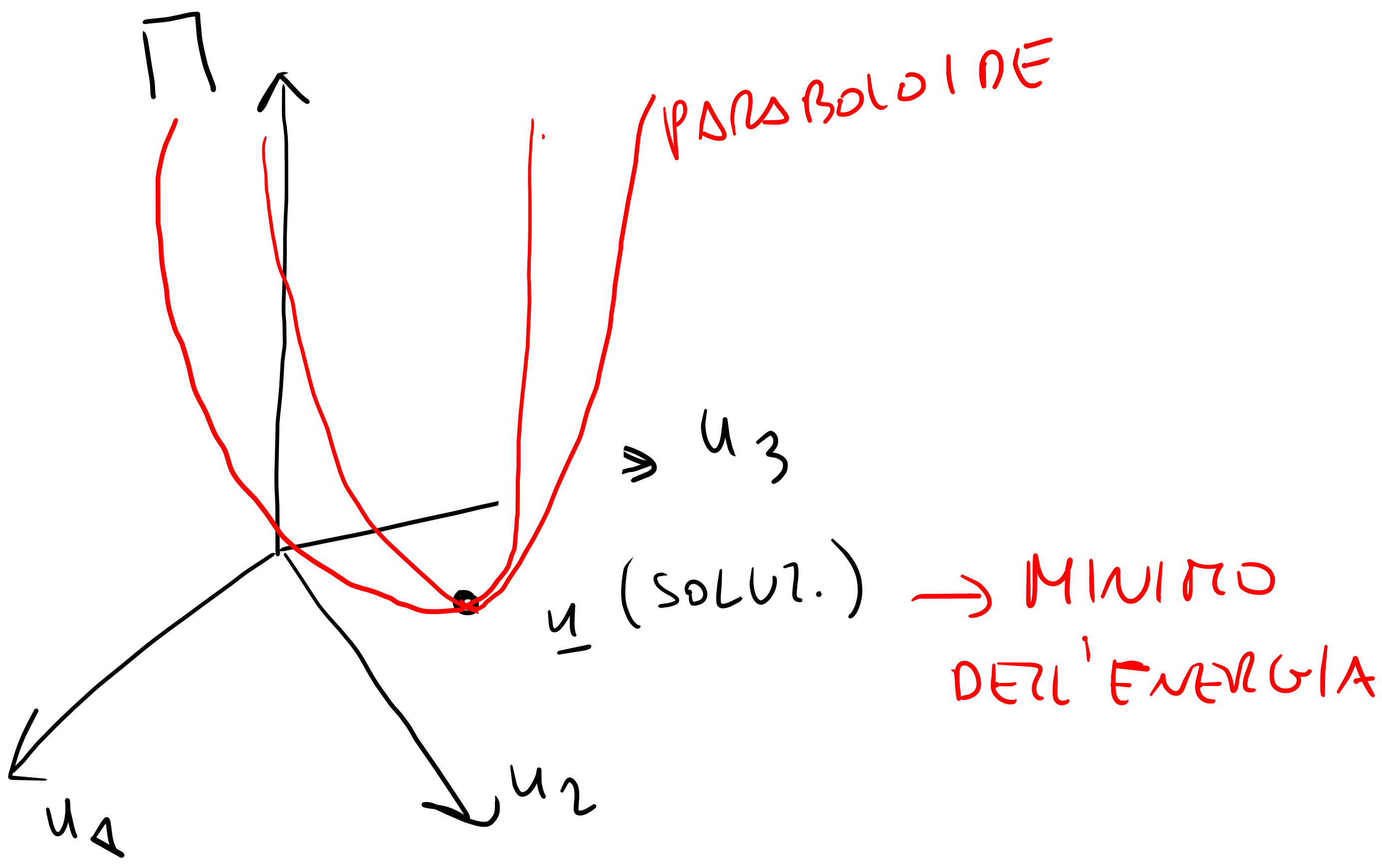
$L_{ve} - L_{vi} = 0$

ALL'EQUILIBRIO:

$$\delta \Pi = \delta \Pi^{(2)} \Rightarrow \delta \Pi = \frac{1}{2} \int \mathbb{C}\, \delta \underset{\sim}{\varepsilon} \cdot \delta \underset{\sim}{\varepsilon} > 0 \quad (\forall \delta \underset{\sim}{\varepsilon})$$

$\mathbb{C}$: TENSORE DEFINITO POSITIVO

$\rightsquigarrow$ LA CONFIG. DI EQUILIBRIO $\underline{u}(\underline{x})$ È UN MINIMO DELL'E.P.T.



PRINCIPIO DI STAZ. DELL' E.P.T.

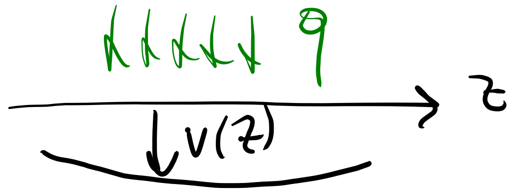
TRA TUTTE LE CONFIGURAZIONI $\underline{u}(\underline{x})$ CONGRUENTI, LA SOLUZ. EQUILIBRATA RENDE STAZIONARIA L'E.P.T.

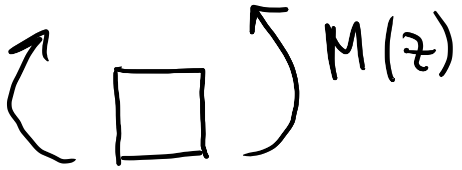
$\underline{u}$: CONGRUENTE $\Rightarrow \delta\Pi^{(1)} = 0 \Rightarrow \underline{u}(\underline{x})$ SOLUZ.

RICORDIAMOCI CHE $\exists$ L'EN. POT. TOTALE COMPLEMENTARE

$$\Pi^c(\underset{\sim}{\sigma}) = \underbrace{\frac{1}{2}\int_B \underset{\sim}{\sigma} \cdot \mathbb{C}^{-1} \underset{\sim}{\sigma}\, dV}_{\Phi^c:\ \text{EN. ELASTICA COMPL.}} - \int_B \underline{b} \cdot \underline{u}\, dV - \int_{\partial B} \underline{p} \cdot \underline{u}\, dA$$

E.P.T. $\Pi(\underline{u})$ DI STR. INFLESSE (EULERO – BERNOULLI)
DA VINCI





$$d\varphi = \kappa\, dz$$

CURVATURA $= -v''(z)$

$$d\Phi = \frac{1}{2} M\, d\varphi = \frac{1}{2} M \kappa\, dz = -\frac{1}{2} M v''(z)\, dz \rightsquigarrow \Phi = -\frac{1}{2}\int_{STR} M v''(z)\, dz$$

EN ELASTICA
NEL CONCIO
ELEMENTARE
(MOM. FLETTENTE

$$M = -EJ v''(z) \longrightarrow \Phi = \frac{1}{2}\int_{STR} EJ \left(v''(z)\right)^2 dz$$

$$\Pi(v) = \underbrace{\frac{1}{2}\int_{STR} EJ\, v''^2(z)\, dz}_{\text{EN ELASTICA}} - \underbrace{\int_{STR} q\, v(z)\, dz}_{\text{POT CARICHI ESTERNI}}$$

E.P.T. TRAVI SNELLE
INFLESSE