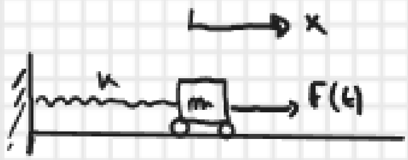
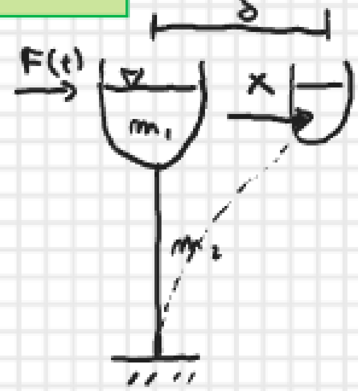
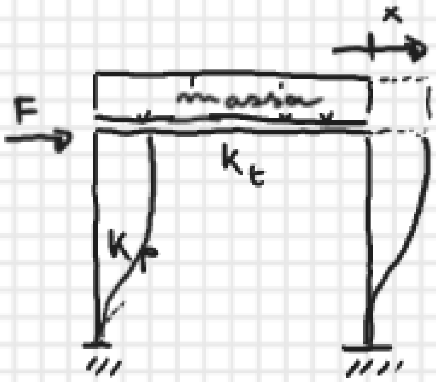
Titolo:

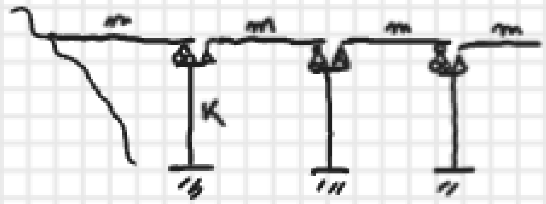


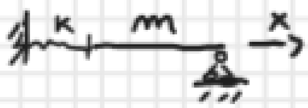


$$F = \overbrace{\frac{h^3}{3EJ}}^{k}\,\delta$$

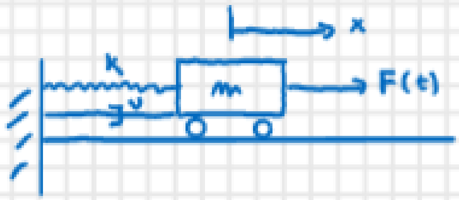


$$k_t^{Ass} \gg k_p^{fl}$$



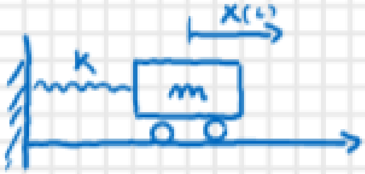


Sistemi



- Oscillazioni libere (k)
- smorzati k, ν
- Forzante esterna k, ν, F

• Oscillazioni libere



(k, m)

$F_{e(t)} = -k\,x(t)$

d'Alembert

1) $F = ma \longrightarrow -ma + F = 0$

$$-m\ddot{x}(t) - k\,x(t) = 0 \longrightarrow m\ddot{x}(t) + k\,x(t) = 0$$

Titolo:

2) Principio di conservazione energia

$$dT + dU = 0$$

$T = \frac{1}{2} m \dot{x}^2$ cinetica

$U = \frac{1}{2} k x^2$ elastica della molla

$$m \dot{x} \ddot{x}\, dt + k x \dot{x}\, dt = 0$$

$$(m \ddot{x}(t) + k x(t))\, \dot{x}(t)\, dt = 0 \qquad \dot{x}(t) \neq 0$$

$$m \ddot{x}(t) + k x(t) = 0$$

3) Eq. di Lagrange

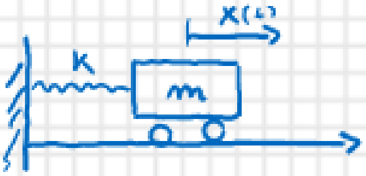
$$\frac{d}{dt} \frac{\partial T}{\partial \dot{q}} - \frac{\partial T}{\partial q} + \frac{\partial U}{\partial q} + \cancel{\frac{\partial D}{\partial \dot{q}}} = \cancel{Q}$$

D -> funzione di dissipazione

Q ->

$$\frac{d}{dt} m \dot{x}(t) + k x = 0 \quad m \ddot{x}(t) + k x(t) = 0$$

Titolo:



(k, m)

- $m\ddot{x}(t) + Kx(t) = 0$
- $\ddot{x}(t) + \frac{K}{m}x(t) = 0$
- $\ddot{x}(t) + \omega^2 x(t) = 0$ ✱

$$\omega = \sqrt{\frac{K}{m}}$$ pulsazione propria

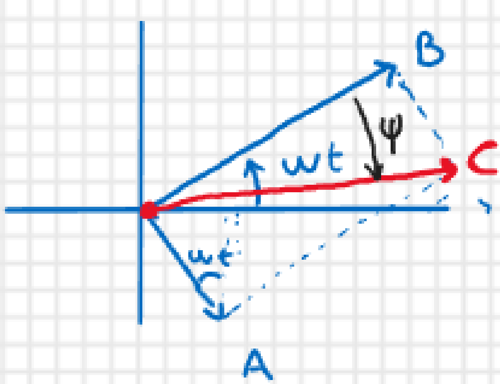
Soluzione

$$x(t) = A\sin\omega t + B\cos\omega t \quad (1)$$

$$v(t) = \dot{x}(t) = \omega A\cos\omega t - B\omega\sin\omega t$$

$$a(t) = \ddot{x}(t) = -\omega^2 A\sin\omega t - B\omega^2\cos\omega t$$

Verifica equazione del moto (sostituite in ✱)



$$C = \sqrt{A^2 + B^2} \qquad tg\psi = \frac{A}{B}$$

$$x(t) = C\cos(\omega t - \psi) \quad (2)$$

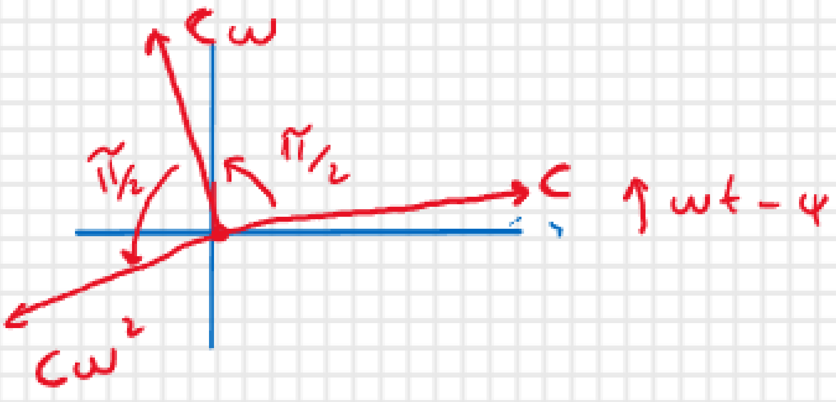
$C \longrightarrow x_{max}$ $\quad -1 < \cos < 1$

Titolo:

$$x(t) = C\cos(\omega t - \varphi)$$

$$\dot{x}(t) = -\omega C \sin(\omega t - \varphi) = \omega C\cos\left(\omega t - \varphi + \frac{\pi}{2}\right)$$

$$\ddot{x}(t) = -\omega^2 C\cos(\omega t - \varphi) = \omega^2 C\cos(\omega t - \varphi + \pi)$$



$$x(t) = C\cos(\omega t - \varphi) \qquad X_{max} = C \rightarrow \omega t - \varphi = 0$$

$$\dot{x}(t) = -\omega C\sin(\omega t - \varphi) \qquad v = 0 \quad \omega t - \varphi = 0$$

$$\ddot{x}(t) = -\omega^2 C\cos(\omega t - \varphi)$$

$$X_{max} \longleftrightarrow v = 0$$

- Soluzione $\quad Ce^{\lambda t}$ *

$$x(t) = Ce^{\lambda t} \qquad \dot{x} = C\lambda e^{\lambda t} \qquad \ddot{x} = C\lambda^2 e^{\lambda t}$$

$$mC\lambda^2 e^{\lambda t} + kCe^{\lambda t} = 0$$

$$(m\lambda^2 + k)Ce^{\lambda t} = 0 \longrightarrow m\lambda^2 + k = 0$$

Titolo:

$m\lambda^2 + k = 0 \qquad \lambda^2 = -\frac{K}{m} \qquad \lambda = \pm i w \qquad w = \sqrt{\frac{K}{m}}$

$$x(t) = c_1 e^{iwt} + C_2 e^{-iwt} \qquad (3)$$

$$e^{iwt} = \cos wt + i \operatorname{sen} wt$$
$$e^{-iwt} = \cos wt - i \operatorname{sen} wt$$

$$x(t) = C_1(\cos wt + i \operatorname{sen} wt) + C_2(\cos wt - i \operatorname{sen} wt)$$

$$x(t) = (C_1 + C_2)\cos wt + (iC_1 - iC_2)\operatorname{sen} wt$$

$$\Updownarrow$$

$$x(t) = A \sin wt + B \cos wt \qquad (1)$$

$$(iC_1 - iC_2) = A \qquad (C_1 + C_2) = B$$

$A = C\cos\varphi$

$B = C\sin\varphi$

- Ultimo caso

$$x(t) = C \sin(wt + \varphi) \qquad (4)$$

$$C = \sqrt{A^2 + B^2} \qquad tg\,\varphi = \frac{B}{A}$$

$$x(t) = C\cos\varphi \sin wt + C\sin\varphi \cos wt = C\sin(wt + \varphi)$$

Titolo:

- Condizioni iniziali

$$x(t) = A \sin \omega t + B \cos \omega t \quad (1)$$

$x(t=0) = x_0 \qquad x(0) = B = x_0$

$\dot{x}(t=0) = v_0 \qquad \dot{x}(t) = \omega A \cos \omega t - \omega B \sin \omega t$

$\dot{x}(0) = \omega A = v_0 \qquad A = v_0/\omega$

$B = x_0 \qquad A = v_0/\omega$

$$x(t) = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t$$

$$x(t) = C \cos(\omega t - \varphi)$$

$$\dot{x}(t) = -C\omega \sin(\omega t - \varphi)$$

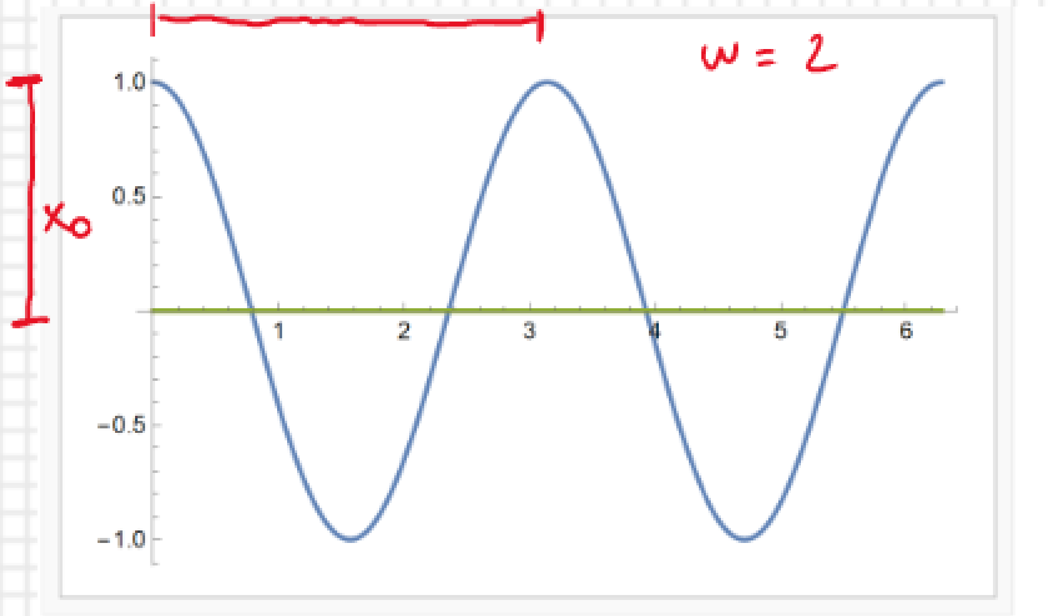
$$x(0) = C \cos \varphi = x_0$$

$$\dot{x}(0) = -C\omega \sin(-\varphi) = v_0 = C\omega \sin \varphi$$

$$\frac{\sin \varphi}{\cos \varphi} = \operatorname{tg} \varphi = \frac{v_0}{\omega x_0} \qquad C = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$$

Titolo:

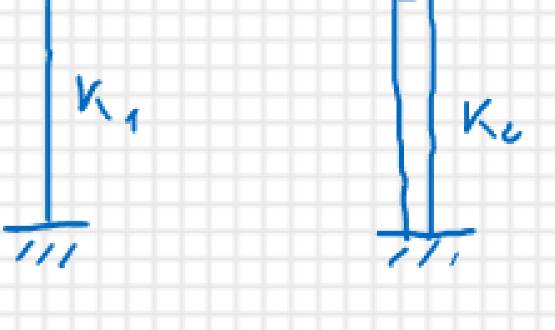
$$T = \frac{2\pi}{\omega} = \pi$$



$x_0 = 1 \qquad v_0 = 0 \qquad \omega = 2$

$$x(t) = \cancel{\frac{v_0}{\omega} \sin \omega t} + x_0 \cos \omega t$$

$$T = \frac{2\pi}{\omega} = \pi \qquad f = \frac{1}{T} = \frac{\omega}{2\pi}$$



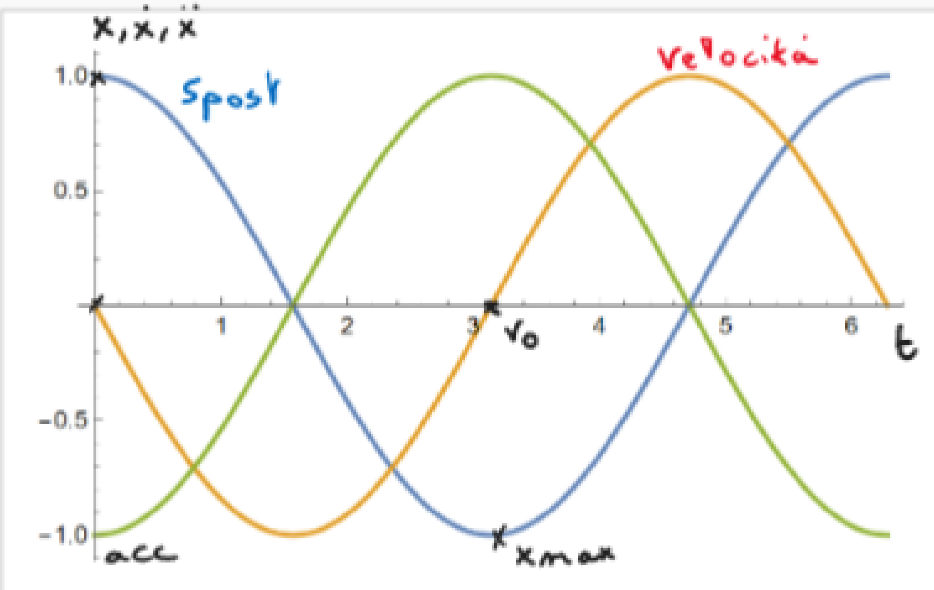
$m_1 = m_2$

$K_2 > K_1$

$\omega_2 > \omega_1 \qquad f_2 > f_1$

$T_2 < T_1$

Titolo:

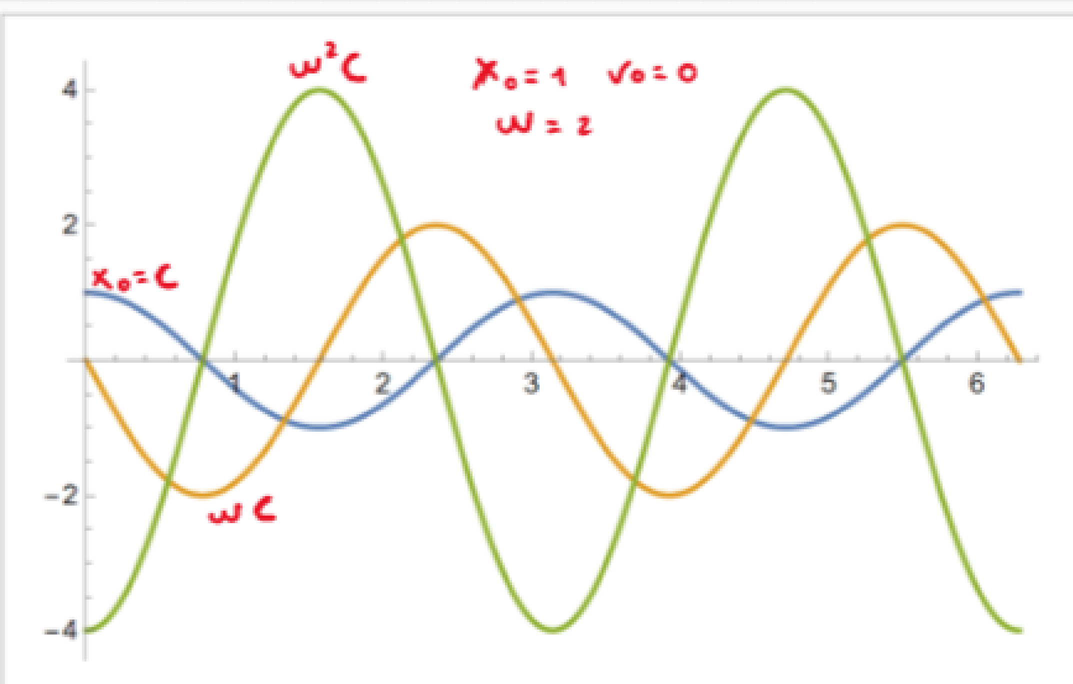


$x_{max} \leftrightarrow v_0$

$x_{max} = C$ $\quad C = x_0$ perché $v_0 = 0$

$\dot{x}_{max} = -\omega C$ $\quad C = \sqrt{A^2 + B^2}$

$\ddot{x}_{max} = -\omega^2 C$



Titolo:

Considerazioni energetiche

$x_0 \quad v_0$

$t = 0$

$T_0 = \frac{1}{2} m v_0^2$

$T_0 + U_0 = \frac{1}{2} m v_0^2 + \frac{1}{2} k x_0^2$

$U_0 = \frac{1}{2} k x_0^2$

$$x(t) = C \cos(\omega t - \varphi)$$

$\omega t - \varphi = 0$ istante nel quale $x(t) = x_{max}$

$x(t) = x_{max} = C \qquad \dot{x} = 0$

$T_1 = 0 \qquad U_1 = \frac{1}{2} k C^2$

$T_1 + U_1 = \frac{1}{2} k C^2 \longrightarrow \omega t - \varphi = 0$

$t = \varphi / \omega$

$C = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$

$$T_1 + U_1 = \frac{1}{2} k x_0^2 + \frac{1}{2} k \frac{v_0^2}{\omega^2} = \frac{1}{2} k x_0^2 + \frac{1}{2} m v_0^2 = T_0 + U_0$$