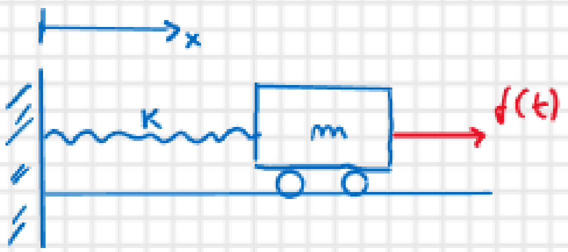
Titolo:

Oscillatore forzato

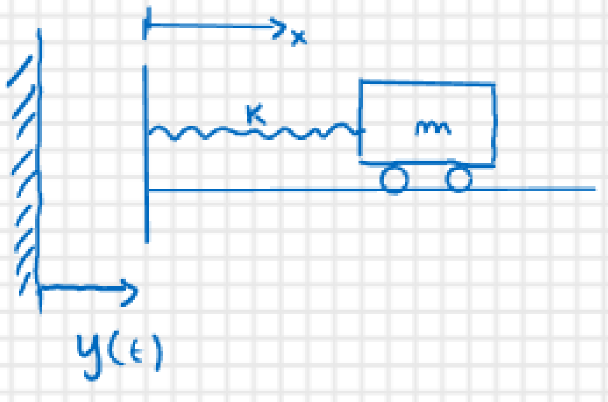


$f(t) = F \operatorname{sen} \bar{\omega} t$

ω pulsazione propria del sistema

- $m\ddot{x}(t) + kx(t) = F \operatorname{sen} \bar{\omega} t$ $\omega = \sqrt{\frac{k}{m}}$
- $\ddot{x}(t) + \omega^2 x(t) = \frac{F}{m} \operatorname{sen} \bar{\omega} t$
- $\ddot{x}(t) + \omega^2 x(t) = \frac{F}{k} \omega^2 \operatorname{sen} \bar{\omega} t = x_{st} \omega^2 \operatorname{sen} \bar{\omega} t$

$x_{st} = \frac{F}{k}$ spostamento dovuto alla forza applicata staticamente



$y(t)$ spostamento imposto

$m(\ddot{x} + \ddot{y}) + Kx = 0$

$m\ddot{x}(t) + Kx(t) = -m\ddot{y}(t)$
$= f(t)$

$x(t) = x_{om}(t) + x_p(t)$

$x_{om}(t)$ ↓ Oscillatore libero

$x_p(t)$ ↓ integrale particolare

Titolo:

$$x_p(t) = X \,\mathrm{sen}\,(\bar\omega t)$$

$$\dot x_p(t) = X \bar\omega \cos(\bar\omega t)$$

$$\ddot x_p(t) = -X \bar\omega^2 \sin(\bar\omega t)$$

Sostituire eq. moto

$$-X\bar\omega^2 \sin\cancel{(\bar\omega t)} + \omega^2 X \,\mathrm{sen}\cancel{(\bar\omega t)} = x_{st}\, \omega^2 \,\mathrm{sen}\cancel{\bar\omega t}$$

$$X = x_{st} \frac{\omega^2}{\omega^2 - \bar\omega^2} \longrightarrow X = x_{st} \frac{1}{1-\beta^2}$$

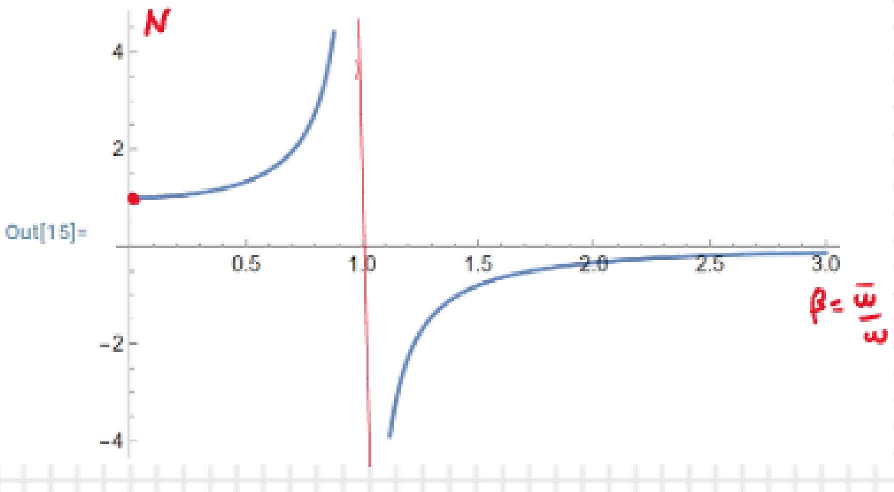
$\beta = \frac{\bar\omega}{\omega}$ Rapporto di frequenza

$$x_p(t) = X \,\mathrm{sen}\, \bar\omega t = x_{st} \frac{1}{1-\beta^2} \,\mathrm{sen}\, \bar\omega t$$

Fattore di amplificazione

$x_p(t) = x_{st}\, N \,\mathrm{sen}\, \bar\omega t$ $\quad N = \frac{1}{1-\beta^2}$

Se $\bar\omega > \omega$ $\quad \beta > 1$ $\quad N < 0$



- $\bar\omega = 0$
  $\beta = 0$ $\quad N = 1$
- $\bar\omega = \omega$
  $\beta = 1$ $\quad N \to \infty$
- $\beta \to \infty$
  $\bar\omega \gg \omega$
  $N \to 0$

Titolo:

• $x_p(t) = x_{st} \dfrac{1}{1-\beta^2} \operatorname{sen} \bar{\omega} t$

$N = \left| \dfrac{1}{1-\beta^2} \right|$ in valore assoluto

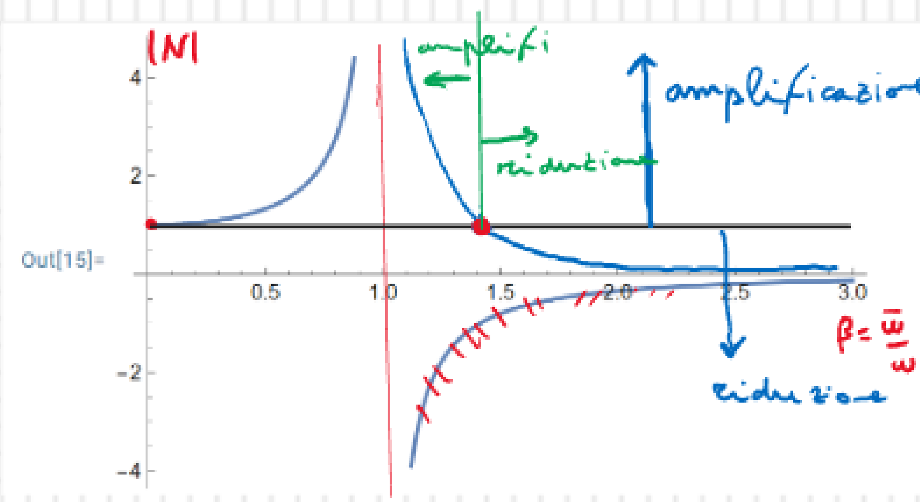
• $x_p(t) = x_{st} \, N \operatorname{sen}(\bar{\omega} t - \varphi)$

$\varphi = 0 \quad \beta < 1$

$\varphi = \pi \quad \beta > 1$

$\varphi = 0 \quad \beta < 1$ $x_p(t)$ e $f(t)$ fase

$\varphi = \pi \quad \beta > 1$ opposizione di fase



• $\bar{\omega} = \sqrt{2}\,\omega$

$|N| = 1$ $\beta = \sqrt{2}$

• $\beta < \sqrt{2}$

$N > 1$

• $\beta > \sqrt{2}$

$N < 1$

$x(t) = x_{om}(t) + x_p(t)$

$x(t) = A \operatorname{sen} \omega t + B \cos \omega t + x_{st} N \operatorname{sen} \bar{\omega} t$

A e B condizioni iniziali

Titolo:

Condizioni iniziali

1) $x(0) = x_0$

$$x(t) = A \operatorname{sen} \omega t + B \cos \omega t + x_{st} N \operatorname{sen} \bar{\omega} t$$

$$x(0) = B = x_0 \qquad B = x_0$$

2) $\dot{x}(0) = v_0$

$$\dot{x}(t) = A\omega \cos \omega t - B\omega \operatorname{sen} \omega t + x_{st} N \bar{\omega} \cos \bar{\omega} t$$

$$\dot{x}(0) = A\omega + x_{st} N \bar{\omega} = v_0 \qquad A = \frac{v_0}{\omega} - x_{st} N \beta$$

- $x(t) = \overline{x_0 \cos \omega t} + \overline{\frac{v_0}{\omega} \operatorname{sen} \omega t} + \underline{N x_{st} \big(}\overline{\operatorname{sen} \bar{\omega} t} - \beta \operatorname{sen} \omega t\big)$

Risposta del sistema

$$x_0, \; v_0, \; \omega, \; \bar{\omega}, \; x_{st}$$

$\bar{\omega} \simeq \omega$ battimenti

- Risonanza $\quad \bar{\omega} = \omega \quad N \to \infty \quad \beta = 1$

$$\cancel{x_p(t) = X \operatorname{sen} \bar{\omega} t} \qquad x_p(t) = X t \cos \bar{\omega} t$$

$$\dot{x}_p(t) = X \cos \bar{\omega} t - X t \bar{\omega} \sin \bar{\omega} t$$

$$\ddot{x}_p(t) = -X \bar{\omega} \sin \bar{\omega} t - X \bar{\omega} \sin \bar{\omega} t - X t \bar{\omega}^2 \cos \omega t$$

Titolo:

Sostituisco eq. moto

$$-2X\bar{\omega}\,\cancel{\sin\bar{\omega}t} - t\cancel{\bar{\omega}^2 X}\cos\bar{\omega}t + Xt\cancel{\omega^2}\cos\omega t = x_{st}\,\omega^2\,\cancel{\sin\bar{\omega}}t$$

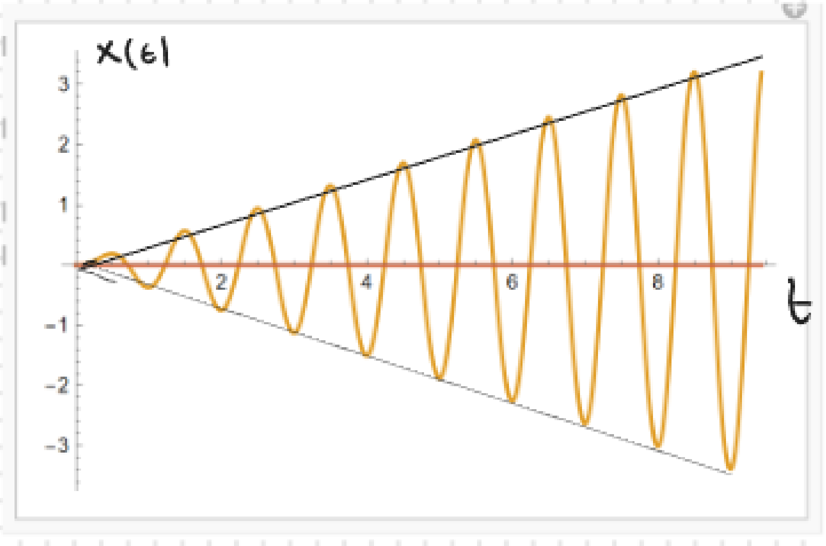
$$X = -\frac{x_s t}{2}\omega \qquad \omega = \bar{\omega}$$

$$x_p(t) = -\frac{x_{st}}{2}\omega\, t \cos\omega t$$

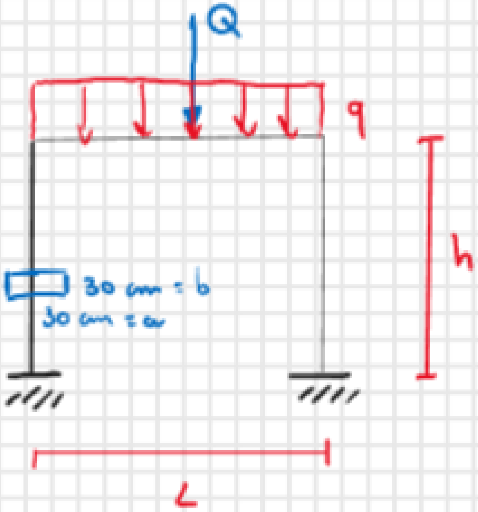
$$x(t) = A \,\text{sen}\, \omega t + B\cos\omega t - \frac{x_{st}}{2}\omega\, t \cos\omega t$$

termine espansivo

$$t \rightarrow \infty \qquad x(t) \rightarrow \infty$$



Risonanza



Pulsazione propria

$\omega = f(k, m)$

$h = 3m \quad l = 5\,m$

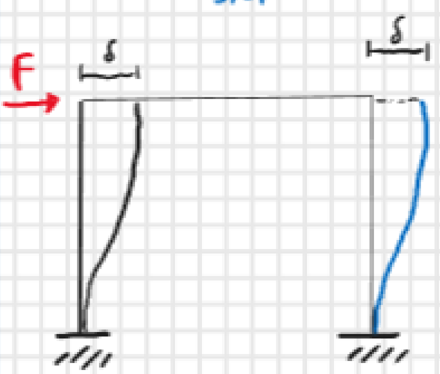
$q = 40\,kN\,m$

$E = 25.000\,N/mm^2$

$Q = q \cdot l = 200\,KN$

$m = Q/9{,}81 \simeq 20.000\,kg$

$I_x = \frac{b \cdot a^3}{12}$



$\delta = \frac{F\,h^3}{24\,E\,I}$ $\qquad F = K\,\delta$

$\frac{1}{K} \longrightarrow K = \frac{24\,E\,I_x}{h^3}$

$\omega = \sqrt{\frac{K}{m}} = 27.4\,\frac{rad}{s} \qquad T = \frac{2\pi}{\omega} = 0{,}23\,s$