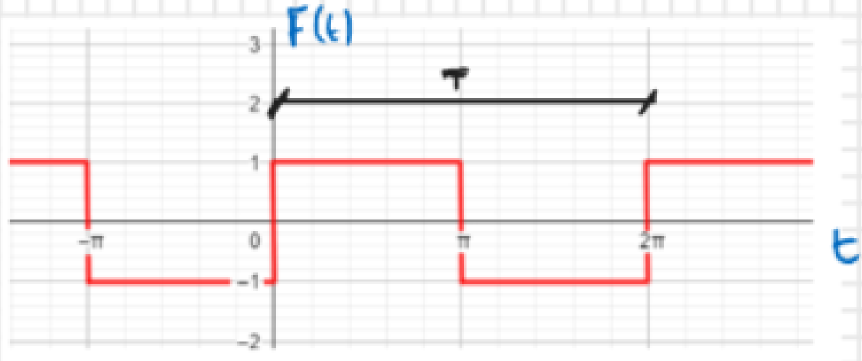
Oscillatore smorzato con forzante periodica

$F(t)$ periodica $\longrightarrow$ $F(a+T) = F(a) \quad \forall a$



$$F(t) = \underline{\underline{a_0}} + \sum_{n=1}^{\infty} \underline{\underline{a_n}} \cos(n\bar{\omega} t) + \underline{\underline{b_n}} \sin(n\bar{\omega} t)$$

$\bar{\omega}$ pulsazione fondamentale

$T = \frac{2\pi}{\bar{\omega}}$

$$a_n \cos \bar{\omega} t + b_n \sin \bar{\omega} t = C_n \sin(\bar{\omega} t + \psi_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2} \qquad \psi_n = \operatorname{arctg} \frac{a_n}{b_n}$$

a_0:

$$\int_0^T F(t)\,dt = \int_0^T a_0\,dt = a_0 T \longrightarrow a_0 = \frac{\int_0^T F(t)\,dt}{T}$$

$$\int_0^T \text{sen}\,t\,dt = 0 \qquad \int_0^T \cos t\,dt = 0$$

a_m e b_m

$$\int_0^T \sin^2(m\bar{\omega}t)\,dt = \frac{t}{2} - \frac{\sin 2m\bar{\omega}t}{4m\bar{\omega}}\Big|_0^T = \frac{T}{2} = \frac{\pi}{\bar{\omega}}$$

$$\int_0^T \cos^2(m\bar{\omega}t)\,dt = \frac{t}{2} + \frac{\sin 2m\bar{\omega}t}{4m\bar{\omega}}\Big|_0^T = \frac{T}{2} = \frac{\pi}{\bar{\omega}}$$

$$\int_0^T \text{sen}\,n\bar{\omega}t \cdot \text{sen}\,m\bar{\omega}t\,dt = 0 \qquad n \neq m$$

$$\int_0^T \cos n\bar{\omega}t \cdot \text{sen}\,m\bar{\omega}t\,dt = 0$$

$$\int_0^T \cos n\bar{\omega}t \cdot \cos m\bar{\omega}t\,dt = 0$$

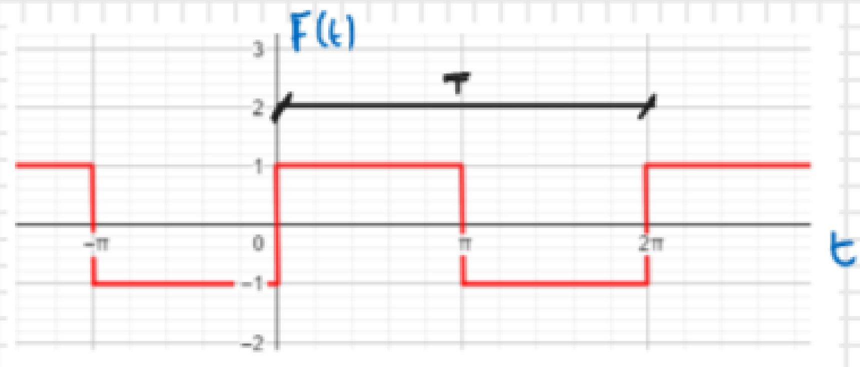
Si ha

$$\int_0^T F(t)\cos n\bar{\omega}t\,dt = a_n \int_0^T \cos^2 n\bar{\omega}t = a_n \frac{\pi}{\bar{\omega}}$$

$$\int_0^T F(t)\sin n\bar{\omega}t\,dt = b_n \int_0^T \text{sen}^2 n\bar{\omega}t = b_n \frac{\pi}{\bar{\omega}}$$

$$\left.\begin{aligned} a_0 &= \frac{1}{T}\int_0^T F(t)\,dt \\ a_n &= \frac{\bar{\omega}}{\pi}\int_0^T F(t)\cos n\bar{\omega}t\,dt \\ b_n &= \frac{\bar{\omega}}{\pi}\int_0^T F(t)\,\mathrm{sen}\, n\bar{\omega}\,dt \end{aligned}\right\}$$

Esempio



$F(t) = 1 \quad 0 < t < \pi$

$F(t) = -1 \quad \pi \leq t \leq 2\pi$

$T = 2\pi \quad \bar{\omega} = 1$

$a_0 = 0$

$$a_1 = \frac{1}{\pi}\cdot\left[\int_0^{\pi} 1\cdot\cos\bar{\omega}t\,dt + \int_{\pi}^{2\pi} -1\cos\bar{\omega}t\,dt\right] = 0$$

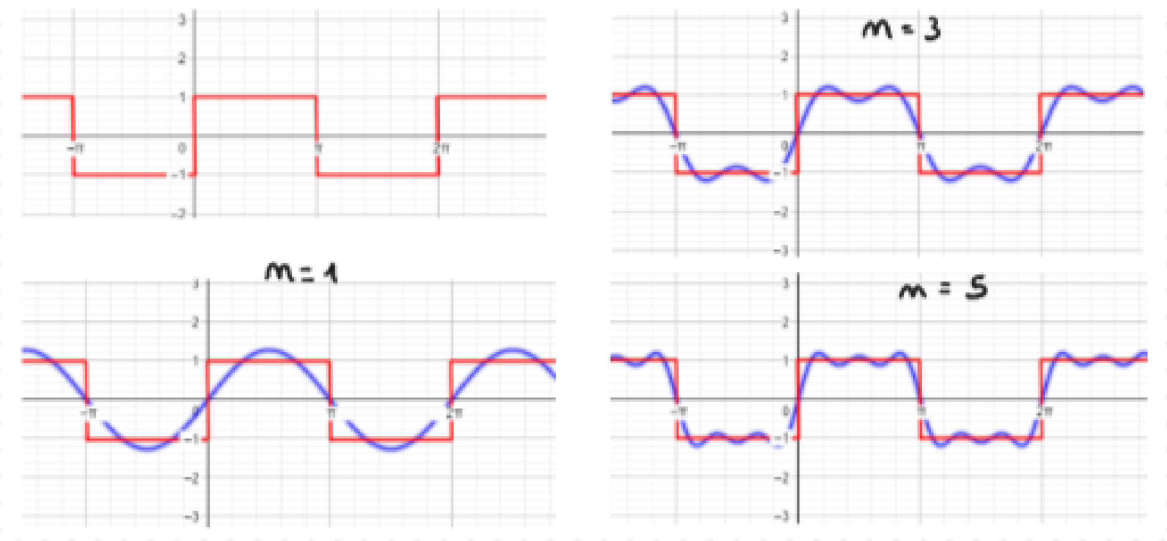
$$b_1 = \frac{1}{\pi}\cdot\left[\int_0^{\pi} 1\cdot\mathrm{sen}\,\bar{\omega}t\,dt + \int_{\pi}^{2\pi} -1\,\mathrm{sen}\,\bar{\omega}t\,dt = \frac{4}{\pi}\right.$$

$a_n = 0$

$$b_n = \begin{cases} \dfrac{4}{n\pi} & n \text{ è dispari} \\ 0 & n \text{ è pari} \end{cases}$$

$$F(t) = \underline{\underline{a_0}} + \sum_{m=1}^{\infty} \underline{\underline{a_m}} \cos(m\bar{\omega} t) + \underline{\underline{b_m}} \sin(m\bar{\omega} t)$$

$m=1 \qquad F(t) = \frac{4}{\pi} \sin \bar{\omega} t$



$m=3 \qquad F(t) = \frac{4}{\pi} \sin \bar{\omega} t + \frac{4}{3\pi} \sin 3\bar{\omega} t$

$m=5 \qquad F(t) = \frac{4}{\pi} \sin \bar{\omega} t + \frac{4}{3\pi} \sin 3\bar{\omega} t + \frac{4}{5\pi} \sin 5\bar{\omega} t$

- $m\ddot{x}(t) + b\,\dot{x}(t) + k\,x(t) = F(t) = a_0 + \sum_{m=1}^{\infty} C_m \operatorname{sen}(\bar{\omega} t + \psi_m)$

$$\ddot{x}(t) + 2\nu\omega\,\dot{x}(t) + \omega^2 x(t) = x_{st_0}\omega^2 + \sum_{m=1}^{\infty} x_{st_m}\omega^2 \operatorname{sen}(m\bar{\omega} t + \psi_m)$$

$$X(t) = \overset{\simeq 0}{x_{om}(t)} + x_p(t) \simeq x_{st_0} + \sum_{m=1}^{\infty} x_{st_m} N_m \operatorname{sen}(m\bar{\omega} t + \psi_m - \varphi_m)$$

(Regime)

$x_{st_0} = \frac{a_0}{K} \qquad x_{st_m} = \frac{C_m}{K} \qquad N_m = \frac{1}{\sqrt{\left(1-\left(\frac{m\bar{\omega}}{\omega}\right)^2\right)^2 + \left(2\nu \frac{m\bar{\omega}}{\omega}\right)^2}}$

$$tg\,\varphi_m = \frac{2\nu m\bar{\omega}/\omega}{1-\left(\frac{m\bar{\omega}}{\omega}\right)^2}$$